

# A FORTRAN 90 numerical library

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# Contents

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<b>Contents</b>	<b>i</b>
<b>List of Tables</b>	<b>xiii</b>
<b>Listings</b>	<b>xiv</b>
<b>Generalities</b>	<b>xvii</b>
Installing . . . . .	xviii
<b>1 MODULE NumTypes</b>	<b>1</b>
1.1 Description . . . . .	1
1.2 Examples . . . . .	1
<b>2 MODULE Constants</b>	<b>3</b>
2.1 Name conventions . . . . .	3
2.2 $\pi$ -related constants . . . . .	3
2.2.1 Real . . . . .	3
2.2.2 Complex . . . . .	3
2.3 Square roots and log related constants . . . . .	3
2.4 Other mathematical constants . . . . .	4
<b>3 MODULE Error</b>	<b>5</b>
3.1 Defined variables . . . . .	5
3.1.1 <code>stderr</code> . . . . .	5
3.2 Subroutine <code>perror([routine], msg)</code> . . . . .	5
3.2.1 Description . . . . .	5
3.2.2 Arguments . . . . .	5
3.2.3 Examples . . . . .	6
3.3 Subroutine <code>abort([routine], msg)</code> . . . . .	6
3.3.1 Description . . . . .	6
3.3.2 Arguments . . . . .	6
3.3.3 Examples . . . . .	6
<b>4 MODULE Integration</b>	<b>9</b>
4.1 Function <code>Trapezio(a, b, Func, [Tol])</code> . . . . .	9
4.1.1 Description . . . . .	9
4.1.2 Arguments . . . . .	9

4.1.3	Output	10
4.1.4	Examples	10
4.2	Function <code>Simpson(a, b, Func, [Tol])</code>	11
4.2.1	Description	11
4.2.2	Arguments	11
4.2.3	Output	11
4.2.4	Examples	11
4.3	Function <code>TrapezioAb(a, b, Func, [Tol])</code>	12
4.3.1	Description	12
4.3.2	Arguments	12
4.3.3	Output	13
4.3.4	Examples	13
4.4	Function <code>SimpsonAb(a, b, Func, [Tol])</code>	14
4.4.1	Description	14
4.4.2	Arguments	14
4.4.3	Output	14
4.4.4	Examples	14
4.5	Function <code>SimpsonInfUp(a, Func, [Tol])</code>	15
4.5.1	Description	15
4.5.2	Arguments	15
4.5.3	Output	16
4.5.4	examples	16
4.6	Function <code>SimpsonInfDw(a, Func, [Tol])</code>	17
4.6.1	Description	17
4.6.2	Arguments	17
4.6.3	Output	17
4.6.4	examples	17
4.7	Function <code>SimpsonSingUp(a, b, Func, [Tol], gamma)</code>	18
4.7.1	Description	18
4.7.2	Arguments	18
4.7.3	Output	19
4.7.4	Examples	19
4.8	Function <code>SimpsonSingDw(a, b, Func, [Tol], gamma)</code>	20
4.8.1	Description	20
4.8.2	Arguments	20
4.8.3	Output	20
4.8.4	Examples	20
4.9	Function <code>Euler(Init, Xo, Xfin, Feuler, [Tol])</code>	21
4.9.1	Description	21
4.9.2	Arguments	22
4.9.3	Output	22
4.9.4	Examples	22
4.10	Function <code>Rgnkta(Init, Xo, Xfin, Feuler, [Tol])</code>	24
4.10.1	Description	24
4.10.2	Arguments	24
4.10.3	Output	25
4.10.4	Examples	25

4.11	Function IntQuadrilateral(P1,P2,P3,P4,Fval)	26
4.11.1	Description	26
4.11.2	Arguments	26
4.11.3	Output	26
4.11.4	Examples	26
<b>5</b>	<b>MODULE Optimization</b>	<b>29</b>
5.1	Subroutine Bracket(X1, X2, X3, Func)	29
5.1.1	Description	29
5.1.2	Arguments	29
5.1.3	Example	30
5.2	Subroutine LineSrch(X, Func[, Tol])	31
5.2.1	Description	31
5.2.2	Arguments	31
5.2.3	Example	32
5.3	Subroutine ConjGrad(X, F, Fd[, Tol])	33
5.3.1	Description	33
5.3.2	Arguments	33
5.3.3	Example	34
5.4	Function Step(X, FStep[, Tol])	35
5.4.1	Description	35
5.4.2	Arguments	35
5.4.3	Output	36
5.4.4	Example	36
5.5	Function MaxPosition(FVal, IpX, IpY)	37
5.5.1	Description	37
5.5.2	Arguments	37
5.5.3	Output	37
5.5.4	Example	37
<b>6</b>	<b>MODULE MinuitAPI</b>	<b>39</b>
6.1	Subroutine Minimize(Func, X, Fval, [Bound], [Release], [logfile])	39
6.1.1	Description	39
6.1.2	Arguments	39
6.1.3	Example	40
6.2	Subroutine Migrad(Func, X, Fval, [Bound], [Release], [logfile])	41
6.2.1	Description	41
6.2.2	Arguments	41
6.2.3	Example	42
6.3	Subroutine Misimplex(Func, X, Fval, [Bound], [Release], [logfile])	43
6.3.1	Description	43
6.3.2	Arguments	43
6.3.3	Example	44
6.4	Subroutine Miseek(Func, X, Fval, [Bound], [Release], [logfile])	45
6.4.1	Description	45
6.4.2	Arguments	45
6.4.3	Example	46

6.5	Subroutine <code>Miscan(Func, X, Fval, [Bound], [Release], [logfile])</code> . .	47
6.5.1	Description . . . . .	47
6.5.2	Arguments . . . . .	47
6.5.3	Example . . . . .	48
<b>7</b>	<b>MODULE Linear</b>	<b>51</b>
7.1	Subroutine <code>Pivoting(M, I piv, Idet)</code> . . . . .	51
7.1.1	Description . . . . .	51
7.1.2	Arguments . . . . .	51
7.1.3	Examples . . . . .	51
7.2	Subroutine <code>LU(M, I piv, Idet)</code> . . . . .	52
7.2.1	Description . . . . .	52
7.2.2	Arguments . . . . .	52
7.2.3	Examples . . . . .	53
7.3	Subroutine <code>LUsolve(M, b)</code> . . . . .	54
7.3.1	Description . . . . .	54
7.3.2	Arguments . . . . .	54
7.3.3	Examples . . . . .	54
7.4	Function <code>Det(M)</code> . . . . .	55
7.4.1	Description . . . . .	55
7.4.2	Arguments . . . . .	55
7.4.3	Output . . . . .	55
7.4.4	Examples . . . . .	55
<b>8</b>	<b>MODULE NonNum</b>	<b>57</b>
8.1	Subroutine <code>Swap(X, Ind1, Ind2)</code> . . . . .	57
8.1.1	Description . . . . .	57
8.1.2	Arguments . . . . .	57
8.1.3	Examples . . . . .	57
8.2	Subroutine <code>Insrt(X[, Ipt])</code> . . . . .	58
8.2.1	Description . . . . .	58
8.2.2	Arguments . . . . .	58
8.2.3	Examples . . . . .	58
8.3	Subroutine <code>Qsort(X[, Ipt])</code> . . . . .	59
8.3.1	Description . . . . .	59
8.3.2	Arguments . . . . .	59
8.3.3	Examples . . . . .	59
8.4	Function <code>Locate(X, X<sub>0</sub>[, Iin])</code> . . . . .	60
8.4.1	Description . . . . .	60
8.4.2	Arguments . . . . .	60
8.4.3	Output . . . . .	60
8.4.4	Examples . . . . .	60
<b>9</b>	<b>MODULE SpecialFunc</b>	<b>63</b>
9.1	Function <code>GammaLn(X)</code> . . . . .	63
9.1.1	Description . . . . .	63
9.1.2	Arguments . . . . .	63

9.1.3	Output	63
9.1.4	Examples	63
9.2	Function <code>erf(X)</code>	64
9.2.1	Description	64
9.2.2	Arguments	64
9.2.3	Output	64
9.2.4	Examples	64
9.3	Function <code>erfc(X)</code>	64
9.3.1	Description	64
9.3.2	Arguments	65
9.3.3	Output	65
9.3.4	Examples	65
9.4	Function <code>inverf(X)</code>	65
9.4.1	Description	65
9.4.2	Arguments	65
9.4.3	Output	65
9.4.4	Examples	65
9.5	Function <code>Theta(i, z, tau[, Prec])</code>	66
9.5.1	Description	66
9.5.2	Arguments	66
9.5.3	Output	66
9.5.4	Examples	67
9.6	Function <code>ThetaChar(a, b, z, tau[, Prec])</code>	67
9.6.1	Description	67
9.6.2	Arguments	67
9.6.3	Output	67
9.6.4	Examples	68
9.7	Function <code>Hermite(n,x[, Dval])</code>	68
9.7.1	Description	68
9.7.2	Arguments	68
9.7.3	Output	69
9.7.4	Examples	69
9.8	Function <code>HermiteFunc(n, x[, Dval])</code>	69
9.8.1	Description	69
9.8.2	Arguments	69
9.8.3	Output	69
9.8.4	Examples	70
9.9	Function <code>Basis(X1, X2, n, s, q, itau[, Prec])</code>	70
9.9.1	Description	70
9.9.2	Arguments	70
9.9.3	Output	70
9.9.4	Examples	71
9.10	Function <code>Factorial(N)</code>	71
9.10.1	Description	71
9.10.2	Arguments	71
9.10.3	Output	71
9.10.4	Examples	71

9.11	Function <code>Legendre(l, m, X)</code> . . . . .	72
9.11.1	Description . . . . .	72
9.11.2	Arguments . . . . .	72
9.11.3	Output . . . . .	72
9.11.4	Examples . . . . .	72
9.12	Function <code>SphericalHarmonics(l, m, th,ph)</code> . . . . .	73
9.12.1	Description . . . . .	73
9.12.2	Arguments . . . . .	73
9.12.3	Output . . . . .	73
9.12.4	Examples . . . . .	73
<b>10</b>	<b>MODULE Statistics</b> . . . . .	<b>75</b>
10.1	Function <code>Mean(X)</code> . . . . .	75
10.1.1	Description . . . . .	75
10.1.2	Arguments . . . . .	75
10.1.3	Output . . . . .	75
10.1.4	Examples . . . . .	75
10.2	Function <code>Median(X)</code> . . . . .	76
10.2.1	Description . . . . .	76
10.2.2	Arguments . . . . .	76
10.2.3	Output . . . . .	76
10.2.4	Examples . . . . .	76
10.3	Function <code>WMedian(X, w)</code> . . . . .	76
10.3.1	Description . . . . .	76
10.3.2	Arguments . . . . .	76
10.3.3	Output . . . . .	77
10.3.4	Examples . . . . .	77
10.4	Function <code>WPercentile(X, w, p)</code> . . . . .	77
10.4.1	Description . . . . .	77
10.4.2	Arguments . . . . .	77
10.4.3	Output . . . . .	77
10.4.4	Examples . . . . .	77
10.5	Subroutine <code>WConfInt(X, w, Xmin, Xmax)</code> . . . . .	78
10.5.1	Description . . . . .	78
10.5.2	Arguments . . . . .	78
10.5.3	Examples . . . . .	78
10.6	Function <code>Var(X)</code> . . . . .	79
10.6.1	Description . . . . .	79
10.6.2	Arguments . . . . .	79
10.6.3	Output . . . . .	79
10.6.4	Examples . . . . .	79
10.7	Function <code>Stddev(X)</code> . . . . .	79
10.7.1	Description . . . . .	79
10.7.2	Arguments . . . . .	80
10.7.3	Output . . . . .	80
10.7.4	Examples . . . . .	80
10.8	Function <code>Moment(X, k)</code> . . . . .	80



10.8.1	Description	80
10.8.2	Arguments	80
10.8.3	Output	80
10.8.4	Examples	81
10.9	Subroutine Histogram(Val, Ndiv, Ntics, Vmin, Vmax, h)	81
10.9.1	Description	81
10.9.2	Arguments	81
10.9.3	Examples	81
10.10	Subroutine LinearReg(X, Y, Yerr, [Func], Coef, Cerr, ChisqrV)	82
10.10.1	Description	82
10.10.2	Arguments	82
10.10.3	Examples	83
10.11	Subroutine NonLinearReg(X, Y, Yerr, Func, Coef, Cerr, ChisqrV)	85
10.11.1	Description	85
10.11.2	Arguments	85
10.11.3	Examples	86
10.12	Subroutine SetLuxLevel(Ilevel)	88
10.12.1	Description	88
10.12.2	Arguments	88
10.12.3	Examples	89
10.13	Subroutine PutLuxSeed([ISeed(25)])	89
10.13.1	Description	89
10.13.2	Arguments	89
10.13.3	Examples	89
10.14	Subroutine GetLuxSeed(ISeed(25))	90
10.14.1	Description	90
10.14.2	Arguments	90
10.14.3	Examples	90
10.15	Subroutine Normal(X, [Rm], [Rsig])	90
10.15.1	Description	90
10.15.2	Arguments	91
10.15.3	Examples	91
10.16	Subroutine Levy(X, $\alpha$ , [c], [ $\beta$ ], [ $\mu$ ])	91
10.16.1	Description	91
10.16.2	Arguments	91
10.16.3	Examples	92
10.17	Subroutine FishTipp(X, Rm, Rb)	92
10.17.1	Description	92
10.17.2	Arguments	92
10.17.3	Examples	93
10.18	Subroutine Laplace(X, Rm, Rb)	93
10.18.1	Description	93
10.18.2	Arguments	93
10.18.3	Examples	93
10.19	Subroutine/Function Irand([Irnd], N, M)	94
10.19.1	Description	94
10.19.2	Arguments	94

10.19.3 Examples . . . . .	94
10.20 Subroutine <code>Permutation</code> (Idx) . . . . .	95
10.20.1 Description . . . . .	95
10.20.2 Arguments . . . . .	95
10.21 Subroutine <code>BootStrap</code> (Ibt) . . . . .	95
10.21.1 Description . . . . .	95
10.21.2 Arguments . . . . .	95
10.21.3 Examples . . . . .	95
10.22 Subroutine <code>SaveBstrp</code> (Ibt, Filename) . . . . .	96
10.22.1 Description . . . . .	96
10.22.2 Arguments . . . . .	96
10.22.3 Examples . . . . .	96
10.23 Subroutine <code>ReadBstrp</code> (Ibt, Filename) . . . . .	97
10.23.1 Description . . . . .	97
10.23.2 Arguments . . . . .	97
10.23.3 Examples . . . . .	97
10.24 Subroutine <code>EstBstrp</code> (Data, Ibt, Func, Val, Err[, Rest]) . . . . .	97
10.24.1 Description . . . . .	97
10.24.2 Arguments . . . . .	98
10.24.3 Examples . . . . .	98
10.25 Subroutine <code>BstrpConfInt</code> (Data, Ibt, alpha, Func, dmin, dpls)) . . . . .	99
10.25.1 Description . . . . .	99
10.25.2 Arguments . . . . .	99
10.25.3 Examples . . . . .	100
10.26 Function <code>Prop_Error</code> (X, Dx, Func[, N]) . . . . .	101
10.26.1 Description . . . . .	101
10.26.2 Arguments . . . . .	101
10.26.3 Output . . . . .	102
10.26.4 Examples . . . . .	102
10.27 Subroutine <code>MCIntegration</code> (X1, X2, Func, Val, Err[, Tol]) . . . . .	103
10.27.1 Description . . . . .	103
10.27.2 Arguments . . . . .	103
10.27.3 Examples . . . . .	103
<b>11 MODULE Polynomial</b> . . . . .	<b>105</b>
11.1 Type <code>Pol</code> . . . . .	105
11.1.1 Description . . . . .	105
11.1.2 Components . . . . .	105
11.1.3 Examples . . . . .	105
11.2 Type <code>CmplxPol</code> . . . . .	106
11.2.1 Description . . . . .	106
11.2.2 Components . . . . .	106
11.2.3 Examples . . . . .	106
11.3 Assignment . . . . .	106
11.3.1 Description . . . . .	106
11.3.2 Examples . . . . .	106
11.4 Operator <code>+</code> . . . . .	107

11.4.1	Description	107
11.4.2	Examples	107
11.5	Operator -	108
11.5.1	Description	108
11.5.2	Examples	108
11.6	Operator *	109
11.6.1	Description	109
11.6.2	Examples	109
11.7	Subroutine <code>Init(P, Dgr)</code>	110
11.7.1	Description	110
11.7.2	Arguments	110
11.7.3	Examples	110
11.8	Function <code>Degree(P)</code>	110
11.8.1	Description	110
11.8.2	Arguments	111
11.8.3	Output	111
11.8.4	Examples	111
11.9	Function <code>Value(P, X)</code>	112
11.9.1	Description	112
11.9.2	Arguments	112
11.9.3	Output	112
11.9.4	Examples	112
11.10	Function <code>Deriv(P)</code>	113
11.10.1	Description	113
11.10.2	Arguments	113
11.10.3	Output	113
11.10.4	Examples	113
11.11	Function <code>Integra(P[, Cte])</code>	114
11.11.1	Description	114
11.11.2	Arguments	114
11.11.3	Output	114
11.11.4	Examples	114
11.12	Function <code>InterpolValue(X, Y, Xo)</code>	115
11.12.1	Description	115
11.12.2	Arguments	115
11.12.3	Output	115
11.12.4	Examples	115
11.13	Function <code>Interpol(X, Y)</code>	116
11.13.1	Arguments	116
11.13.2	Output	116
11.13.3	Examples	116
11.14	Subroutine <code>Spline(X, Y, Ypp0, YppN, Pols)</code>	117
11.14.1	Description	117
11.14.2	Arguments	117
11.14.3	Examples	117

12.1 Subroutine <code>RootPol(a, b, [c, d], z1, z2, [z3, z4])</code> . . . . .	119
12.1.1 Description . . . . .	119
12.1.2 Arguments . . . . .	119
12.1.3 Examples . . . . .	119
12.2 Function <code>Newton(Xo, Fnew, [Tol])</code> . . . . .	120
12.2.1 Description . . . . .	120
12.2.2 Arguments . . . . .	120
12.2.3 Output . . . . .	121
12.2.4 Examples . . . . .	121
12.3 Function <code>Bisec(a, b, Fbis, [Tol])</code> . . . . .	122
12.3.1 Description . . . . .	122
12.3.2 Arguments . . . . .	122
12.3.3 Output . . . . .	123
12.3.4 Examples . . . . .	123
<b>13 MODULE Fourier</b> . . . . .	<b>125</b>
13.1 Type <code>Fourier_Serie</code> . . . . .	125
13.1.1 Description . . . . .	125
13.1.2 Components . . . . .	125
13.1.3 Examples . . . . .	125
13.2 Type <code>Fourier_Serie_2D</code> . . . . .	126
13.2.1 Description . . . . .	126
13.2.2 Components . . . . .	126
13.2.3 Examples . . . . .	126
13.3 Assignment . . . . .	126
13.3.1 Description . . . . .	126
13.3.2 Examples . . . . .	126
13.4 Operator <code>+</code> . . . . .	127
13.4.1 Description . . . . .	127
13.4.2 Examples . . . . .	127
13.5 Operator <code>-</code> . . . . .	128
13.5.1 Description . . . . .	128
13.5.2 Examples . . . . .	128
13.6 Operator <code>*</code> . . . . .	128
13.6.1 Description . . . . .	128
13.6.2 Examples . . . . .	128
13.7 Operator <code>**</code> . . . . .	129
13.7.1 Description . . . . .	129
13.7.2 Examples . . . . .	129
13.8 Subroutine <code>Init_Serie(FS, Ns)</code> . . . . .	130
13.8.1 Description . . . . .	130
13.8.2 Arguments . . . . .	130
13.8.3 Examples . . . . .	130
13.9 Function <code>Eval_Serie(FS, X, [Y], Tx, [Ty])</code> . . . . .	131
13.9.1 Description . . . . .	131
13.9.2 Arguments . . . . .	131
13.9.3 Output . . . . .	131

13.9.4 Examples . . . . .	131
13.10 Function <code>Unit(FS, Ns)</code> . . . . .	132
13.10.1 Description . . . . .	132
13.10.2 Arguments . . . . .	132
13.10.3 Examples . . . . .	132
13.11 Function <code>DFT(Data, Is)</code> . . . . .	133
13.11.1 Description . . . . .	133
13.11.2 Arguments . . . . .	133
13.11.3 Output . . . . .	133
13.11.4 Examples . . . . .	133
13.12 Function <code>Conjg(FS)</code> . . . . .	134
13.12.1 Description . . . . .	134
13.12.2 Arguments . . . . .	134
13.12.3 Output . . . . .	134
13.12.4 Examples . . . . .	134
13.13 Subroutine <code>Save_Serie(FS, File)</code> . . . . .	135
13.13.1 Description . . . . .	135
13.13.2 Arguments . . . . .	135
13.13.3 Examples . . . . .	135
13.14 Subroutine <code>Read_Serie(FS, File)</code> . . . . .	135
13.14.1 Description . . . . .	135
13.14.2 Arguments . . . . .	136
13.14.3 Examples . . . . .	136
<b>14 MODULE Time</b>	<b>137</b>
14.1 Type <code>tm</code> . . . . .	137
14.1.1 Description . . . . .	137
14.1.2 Components . . . . .	137
14.1.3 Example . . . . .	137
14.2 Function <code>gettime()</code> . . . . .	138
14.2.1 Description . . . . .	138
14.2.2 Arguments . . . . .	138
14.2.3 Output . . . . .	138
14.2.4 Example . . . . .	138
14.3 Function <code>isleap(Nyr)</code> . . . . .	139
14.3.1 Description . . . . .	139
14.3.2 Arguments . . . . .	139
14.3.3 Output . . . . .	139
14.3.4 Example . . . . .	139
14.4 Function <code>asctime(t)</code> . . . . .	139
14.4.1 Description . . . . .	139
14.4.2 Arguments . . . . .	140
14.4.3 Output . . . . .	140
14.4.4 Example . . . . .	140
14.5 Function <code>Day_of_Week(Day, Month, Year)</code> . . . . .	140
14.5.1 Description . . . . .	140
14.5.2 Arguments . . . . .	140

---

14.5.3	Output . . . . .	140
14.5.4	Example . . . . .	141
<b>GNU Free Documentation License</b>		<b>143</b>
1.	APPLICABILITY AND DEFINITIONS . . . . .	143
2.	VERBATIM COPYING . . . . .	145
3.	COPYING IN QUANTITY . . . . .	145
4.	MODIFICATIONS . . . . .	145
5.	COMBINING DOCUMENTS . . . . .	147
6.	COLLECTIONS OF DOCUMENTS . . . . .	147
7.	AGGREGATION WITH INDEPENDENT WORKS . . . . .	148
8.	TRANSLATION . . . . .	148
9.	TERMINATION . . . . .	148
10.	FUTURE REVISIONS OF THIS LICENSE . . . . .	148
	ADDENDUM: How to use this License for your documents . . . . .	149
<b>Bibliography</b>		<b>151</b>
<b>Index</b>		<b>153</b>

---

## List of Tables

---

2.1	$\pi$ -related real constants defined in the <code>MODULE constants</code> . . . . .	3
2.2	$\pi$ -related complex constants defined in the <code>MODULE constants</code> . . . . .	4
2.3	Square roots and log related constants defined in the <code>MODULE constants</code> . . . . .	4
2.4	Other mathematical constants defined in the <code>MODULE constants</code> . . . . .	4

---

# Listings

---

1.1	Definition of data types. . . . .	1
3.1	Standard error unit. . . . .	5
3.2	Print error message. . . . .	6
3.3	Print error message and stop a program. . . . .	6
4.1	Example of integration of a function using <b>Trpecio</b> . . . . .	10
4.2	Exmaple of integration of a function using <b>Simpson</b> . . . . .	11
4.3	Integrating a function using the open trapezoid rule. . . . .	13
4.4	Exmaple of integration using the open Simpson rule. . . . .	14
4.5	Integration of a function between 0 and $\infty$ . . . . .	16
4.6	Integrating a function between $-\infty$ and 0. . . . .	17
4.7	Integrating functions with singularities in the upper limit. . . . .	19
4.8	Integrating functions with singularities in the lower limit. . . . .	21
4.9	Integrating differential equations with Euler. . . . .	23
4.10	Integrating differential equations with the Runge-Kutta method . . . . .	25
4.11	Computing the area of a quadrilateral. . . . .	26
5.1	Bracketing a minimum. . . . .	30
5.2	Minimising a function. . . . .	32
5.3	Minimising a function. . . . .	34
5.4	Minimising a function. . . . .	36
5.5	Example of the usage of the routine <b>MaxPosition</b> . . . . .	38
6.1	Using minuit library to minimize a function. . . . .	40
6.2	Using minuit library to minimize a function. . . . .	42
6.3	Using minuit library to minimize a function. . . . .	44
6.4	Using minuit library to minimize a function. . . . .	46
6.5	Using minuit library to minimize a function. . . . .	48
7.1	Pivoting data of a matrix label . . . . .	51
7.2	Making the LU decomposition. . . . .	53
7.3	Solving systems of linear equations. . . . .	54
7.4	Computing the determinant of a matrix. . . . .	55
8.1	Sorting data. . . . .	57
8.2	Sorting data. . . . .	58
8.3	Sorting data. . . . .	59
8.4	Searching data position in an ordered list. . . . .	60
9.1	Computing the Gamma Function. . . . .	63
9.2	Computing the Error Function. . . . .	64
9.3	Computing the complementary Error Function. . . . .	65



9.4	Computing the Inverse Error Function. . . . .	66
9.5	Computing the Jacobi Theta functions. . . . .	67
9.6	Computing the Jacobi Theta functions with characteristics. . . . .	68
9.7	Computing the first 31 Hermite numbers. . . . .	69
9.8	Compute the Hermite functions. . . . .	70
9.9	Computing the bassi of a special Hilbert space (details in [1]). . . . .	71
9.10	Computing the factorial. . . . .	72
9.11	Computing some legendre polynomials. . . . .	72
9.12	Computing some spherical harmonics. . . . .	73
10.1	Computing the Mean of a vector of numbers. . . . .	75
10.2	Computing the Median of a vector of numbers. . . . .	76
10.3	Computing the Weighted Median of a vector of numbers. . . . .	77
10.4	Computing the Weighted Median in two ways. . . . .	77
10.5	Computing the 1 sigma confidence interval in two ways. . . . .	78
10.6	Computing the Variance of a set of numbers. . . . .	79
10.7	Computing the standard deviation. . . . .	80
10.8	Computing the $k^{\text{th}}$ moment of a data set. . . . .	81
10.9	Making Histograms. . . . .	81
10.10	Doing linear regressions. . . . .	83
10.11	Doing non linear regressions. . . . .	87
10.12	Setting the Luxury level of the pseudo random number generator. . . . .	89
10.13	Using a previously saved point in the generating process. . . . .	89
10.14	Saving a point in the generating process. . . . .	90
10.15	Obtaining numbers with a normal distribution. . . . .	91
10.16	Obtaining numbers with a Levy skew stable distribution. . . . .	92
10.17	Obtaining numbers with a Fisher-Tippet distribution. . . . .	93
10.18	Obtaining numbers with a Laplace distribution. . . . .	93
10.19	Obtaining integer random numbers. . . . .	94
10.20	Obtaining a permutation. . . . .	95
10.21	Resampling some data. . . . .	95
10.22	Reading the resampling info. . . . .	96
10.23	Saving the resampling info. . . . .	97
10.24	Estimating the average. . . . .	98
10.25	Giving a confidence interval. . . . .	100
10.26	Propagating errors in a function. . . . .	102
10.27	Integrating a two dimensional function. . . . .	103
11.1	Defining a polynomial. . . . .	105
11.2	Defining a polynomial. . . . .	106
11.3	Assigning polynomials. . . . .	106
11.4	Adding polynomials. . . . .	107
11.5	Subtracting polynomials. . . . .	108
11.6	Computing the product of two polynomials. . . . .	109
11.7	Initialising a polynomial data type. . . . .	110
11.8	Returns the degree of a polynomial. . . . .	111
11.9	Computes the values of a polynomial at some points. . . . .	112
11.10	Computing the derivative of a polynomial. . . . .	113
11.11	Computing the integral of a polynomial. . . . .	114

11.12	Compute values of the Interpolation polynomial. . . . .	116
11.13	Computes the interpolation polynomial. . . . .	116
11.14	Computes the cubic spline interpolation polynomial. . . . .	117
12.1	Computing roots of polynomials. . . . .	119
12.2	Computing roots of non-linear functions with the Newton method. . . . .	121
12.3	Computing roots with the bisection method. . . . .	123
13.1	Defining a Fourier serie. . . . .	125
13.2	Defining a two-dimensional Fourier serie. . . . .	126
13.3	Assigning Fourier series. . . . .	126
13.4	Adding Fourier series. . . . .	127
13.5	Subtracting Fourier series. . . . .	128
13.6	Computing the convolution of Fourier series. . . . .	128
13.7	"Exponentiating" Fourier series. . . . .	129
13.8	Initialising a Fourier series. . . . .	130
13.9	Evaluating a Fourier series at a point. . . . .	131
13.10	Obtaining a constant Fourier series. . . . .	132
13.11	Computing the Discrete Fourier Transform. . . . .	133
13.12	Computing the Conjugate Fourier Series. . . . .	134
13.13	Saving a Fourier Serie in a file. . . . .	135
13.14	Reading a Fourier serie from a file. . . . .	136
14.1	Defining a Time data type. . . . .	137
14.2	Obtaining the current date and time. . . . .	138
14.3	Are we in a leap year?. . . . .	139
14.4	Printing current date/time. . . . .	140
14.5	Day of week of the first of January 1900. . . . .	141

---

# Generalities

---

This is the documentation of a total of fourteen **FORTRAN 2003** modules with different utilities. This code is well documented, and can be useful for several people, although the idea is *not* to produce fast, high performance code, but to have nice data structures and **INTERFACE** definitions so that complex problems can be solved fast, writing only a couple of lines of code.

The code of all these modules is *free software*, this means that you can redistribute and/or modify all the code under the terms of the GNU General Public License<sup>1</sup> as published by the Free Software Foundation; either version 2 of the License, or (at your option) any later version. Note that the code is distributed in the hope that it will be useful, but **without any warranty; without even the implied warranty of merchantability or fitness for a particular purpose**. See the GNU General Public License for more details.

The code has been written using standard **FORTRAN 2003**, this means that it should run on any machine and with any compiler that conforms the standard. In particular the code of all these modules has been compiled using **GNU gfortran**, **INTEL ifort**.

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Traditionally the source of the code has been hosted in <http://www.sourceforge.net>. Due to the access blocking of some countries the code has been moved to <http://github.com>.

To obtain a copy of the source code and documentation, follow one of the following two options

- If you have **git** installed in your system, simply type

```
user@pc $ git clone git://github.com/ramos/afnl.git
```

- In any other case, go to <http://github.com/ramos/afnl/downloads> and get the latest version of the library.

## Minuit minimisation routines

The library includes an API to minuit minimisation routines. If you want to use this routines, you should have the fortran version of minuit available in your system. There is the option of building the **afnl** library without minuit support (see next section).

---

<sup>1</sup><http://www.gnu.org/copyleft/gpl.html>

## Installation

To install this library in a Unix/Linux environment, simply edit the `Makefile` file, and set the `F90` and `F900PT` variables to whatever your compiler and your favourite optimisation flags are. After running `make` you should obtain a file called `libafnl.a`, and probably (that depends on the particular compiler) some `.mod` files. Copy the `libafnl.a` library and the `.mod` files to any place you like, and compile and link your program to that files. With GNU `gfortran` this is done using the flags `-I<path> -L<path> -lf90`, where `<path>` has to be substituted by the path you have chosen.

In case that you do not have a version of the fortran minuit library in your system, use `make lib-no-minuit` to build a version of `afnl` without the minuit API.

In other environments, you should ask the local guru/administrator about how to generate a library. In particular in a Windows environment the best option is to repartition you hard drive, eliminate Windows and install any Unix like free operating system, like Linux or FreeBSD.

```

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```

# One

---

## MODULE NumTypes

---

This is the documentation of the `MODULE NumTypes`, that contains the definition of Single Precision, and Double Precision data. All the other numerical modules use this data type definitions.

### 1.1 Description

The `MODULE NumTypes` provides the definition of the Single Precision and Double Precision real and complex data in a portable way. When we want to define a single precision real we *will* do it with a statement like `Real (kind=DP)`, instead of `Real (kind=4)`. What we mean with DP is defined in this module. The different data types are:

**SP:** Single precision real.

**DP:** Double precision real.

**SPC:** Single precision complex.

**DPC:** Double precision complex.

To make all the code as portable as possible, all the data definitions should make use of this module.

### 1.2 Examples

Here we will define `A` as a single precision real, `D` as a double precision real, `Ac` as a single precision complex, and `Dc` as a double precision complex.

Listing 1.1: Definition of data types.

```
1  Program Types_of_Data
2  USE NumTypes
3
4
5  Real (kind=SP) :: A
6  Real (kind=DP) :: D
```

```
7      Complex (kind=SPC) :: Ac
      Complex (kind=DPC) :: Dc
9
      Write(*,*) Kind(A), Kind(Aa)
11
13     End Program Types_of_Data
```

# Two

---

## MODULE Constants

---

This is the documentation of the `MODULE Constants`, that contains the definition of the most used mathematical constants. This module uses numerical types defined in the `MODULE NumTypes`.

### 2.1 Name conventions

All the real simple precision constants ends with `_SP`, the real double precision constants with `_DP`, the complex simple precision with `_SPC` and the complex double precision with `_DPC`.

If a there exist a real or complex constant of simple precision defined, then it exist other with the same name (except for the suffix) of double precision and viceversa.

### 2.2 $\pi$ -related constants

#### 2.2.1 Real

The complex  $\pi$ -related defined in this module and its values can be seen in the table (2.1)

SP Name	DP Name	Value
PI_SP	PI_DP	$\pi$
TWOPI_SP	TWOPI_DP	$2\pi$
HALFPI_SP	HALFPI_DP	$\frac{\pi}{2}$

Table 2.1:  $\pi$ -related real constants defined in the `MODULE constants`.

#### 2.2.2 Complex

The complex  $\pi$ -related defined in this module and its values can be seen in the table (2.2)

### 2.3 Square roots and log related constants

We have only real constants defined here. We can see a list of names-vlues in the table (2.3)

SPC Name	DPC Name	Value
UNITIMAG_SPC	UNITIMAG_DPC	$\iota$
PI_IMAG_SPC	PI_IMAG_DPC	$\pi\iota$
TWOPI_IMAG_SPC	TWOPI_IMAG_DPC	$2\pi\iota$
HALFPI_IMAG_SPC	HALFPI_IMAG_SDC	$\frac{\pi}{2}\iota$

Table 2.2:  $\pi$ -related complex constants defined in the MODULE constants.

SP Name	DP Name	Value
SR2_SP	SR2_DP	$\sqrt{2}$
SR3_SP	SR3_DP	$\sqrt{3}$
SRe_SP	SRe_DP	$\sqrt{e}$
SRpi_SP	SRpi_DP	$\sqrt{\pi}$
LG102_SP	LG102_DP	$\log_{10} 2$
LG103_SP	LG103_DP	$\log_{10} 3$
LG10e_SP	LG10e_DP	$\log_{10} e$
LG10pi_SP	LG10pi_DP	$\log_{10} \pi$
LGe2_SP	LGe2_DP	$\log_e 2$
LGe3_SP	LGe3_DP	$\log_e 3$
LGe10_SP	LGe10_DP	$\log_e 10$

Table 2.3: Square roots and log related constants defined in the MODULE constants.

## 2.4 Other mathematical constants

In this section we have only the Euler  $\gamma$  constant. We can see the name-value pair in the table (2.4)

SP Name	DP Name	Value
GEULER_SP	GEULER_DP	$\gamma(= 0.5772\dots)$

Table 2.4: Other mathematical constants defined in the MODULE constants.



# Three

---

## MODULE Error

---

This is the documentation of the `MODULE Error`, a set of `FORTRAN 90` routines that allow to write errors.

### 3.1 Defined variables

#### 3.1.1 `stderr`

##### Description

This variable has the unit number of standard error.

##### Examples

Listing 3.1: Standard error unit.

```
1 Program Test
  USE Error
3
  Write(stderr,*) 'This is printed in standard error.'
5
  Stop
7 End Program Test
```

### 3.2 Subroutine `perror([routine], msg)`

#### 3.2.1 Description

Prints the error message `msg` in standard error. If the optional argument `routine` is given, it is used as the routine where the program has crashed.

#### 3.2.2 Arguments

**routine:** Character string with arbitrary length. It should be the routine or program name where the error has occurred. It is an optional argument.

**msg:** Character string with arbitrary length. It should be the message that you want to print.

### 3.2.3 Examples

Listing 3.2: Print error message.

```

1 Program Test
  USE Error
3
  Integer :: N1, N2
5
  Write(*,*) 'Two integer numbers: '
7  Read(*,*)N1,N2
9
  If (N2 == 0) Then
    CALL Perror( 'Test', 'Division by cero. See the product: ')
11  Write(*,*)N1*N2
  Else
13  Write(*,*)N1/N2
  End If
15
  Stop
17 End Program Test

```

## 3.3 Subroutine abort([routine], msg)

### 3.3.1 Description

Prints the error message **msg** in standard error, and stops the program. If the optional argument **routine** is given, it is used as the routine where the program has crashed.

### 3.3.2 Arguments

**routine:** Character string with arbitrary length. It should be the routine or program name where the error has occurred. It is an optional argument.

**msg:** Character string with arbitrary length. It should be the message that you want to print.

### 3.3.3 Examples

Listing 3.3: Print error message and stop a program.

```

1 Program Test
  USE Error
3
  Integer :: N1, N2
5
  Write(*,*) 'Two integer numbers: '
7  Read(*,*)N1,N2
9
  If (N2 == 0) Then

```

```
11      CALL abort( 'Test ', 'Division by zero ')  
      Else  
13      Write(*,*)N1/N2  
      End If  
  
15      Stop  
End Program Test
```



# Four

---

## MODULE Integration

---

This is the documentation of the `MODULE Integration`, a set of `FORTRAN 90` routines that performs numerical integration and solves the initial value problem for a specified system of first-order ordinary differential equations. This module make use of the `MODULE NumTypes`, so please read the documentation of this module *before* reading this.

### 4.1 Function Trapecio(a, b, Func, [Tol])

#### 4.1.1 Description

Calculates the integral of the function `Func` between `a` and `b` with precision `Tol` (optional) using the trapezoid rule.

#### 4.1.2 Arguments

**a, b:** Real single or double precision. The limits of the integral.

**Func:** The function to be integrated. It must be a function of only one argument of the same type as the function itself. If it is an external function an interface block like the following should be declared:

```
Interface
  Function Fint(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
  End Function Fint
End Interface
```

**Tol:** Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter, and the default is `Tol = 0.01`.

### 4.1.3 Output

If the arguments are real of single (double) precision, the result will also be a real of single (double) precision. The value of the integral.

### 4.1.4 Examples

Listing 4.1: Example of integration of a function using Trapecio.

```

Program Test
2  USE NumTypes
  USE Integration
4
  Real (kind=DP) :: Tol
6
  Interface
8    Function Fint(X)
      USE NumTypes
10
      Real (kind=DP), Intent (in) :: X
12      Real (kind=DP) :: Fint
      End Function Fint
14 End Interface

16 Tol = 1.0E-6_DP
  Write(*,*) 'Integral of x**2 between 0 and 1:'
18  Write(*,*) Trapecio(0.0_DP, 1.0_DP, Fint, Tol)

20 Stop
End Program Test

22
23 ! *****
24 ! *
Function Fint(X)
26 ! *
27 ! *****
28
  USE NumTypes
30
  Real (kind=DP), Intent (in) :: X
32  Real (kind=DP) :: Fint
34
  Fint = X**2
36
  Return
End Function Fint

```

## 4.2 Function Simpson(a, b, Func, [Tol])

### 4.2.1 Description

Calculates the integral of the function `Func` between `a` and `b` with precision `Tol` (optional) using the Simpson's rule.

In general this routine is better than `Trapeccio`.

### 4.2.2 Arguments

**a, b:** Real single or double precision. The limits of the integral.

**Func:** The function to be integrated. It must be a function of only one argument of the same type as the function itself. If it is an external function an interface block like the following should be declared:

```
Interface
  Function Fint(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
  End Function Fint
End Interface
```

**Tol:** Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter, and the default is `Tol = 0.01`.

### 4.2.3 Output

If the arguments are reals of single (double) precision, the result will also be a real of single (double) precision. The value of the integral.

### 4.2.4 Examples

Listing 4.2: Exmaple of integration of a function using Simpson.

```
1 Program Test
  USE NumTypes
3  USE Integration

5  Real (kind=DP) :: Tol

7  Interface
    Function Fint(X)
9    USE NumTypes

    Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
13 End Function Fint
```

```

15      End Interface
16      Tol = 1.0E-6_DP
17      Write(*,*) 'Integral of x**2 between 0 and 1: '
18      Write(*,*) Simpson(0.0_DP, 1.0_DP, Fint, Tol)
19
20      Stop
21 End Program Test
22
23 ! *****
24 ! *
25 Function Fint(X)
26 ! *
27 ! *****
28
29 USE NumTypes
30
31 Real (kind=DP), Intent (in) :: X
32 Real (kind=DP) :: Fint
33
34 Fint = X**2
35
36 Return
37 End Function Fint

```

### 4.3 Function TrapecioAb(a, b, Func, [Tol])

#### 4.3.1 Description

Calculates the integral of the function `Func` between `a` and `b` with precision `Tol` (optional) using the open trapezoid rule.

#### 4.3.2 Arguments

**a, b:** Single (SP) or double (DP) precision. They are the limits of the integral.

**Func:** The function to be integrated. It must be a function of only one argument of the same type as the function itself. If it is an external function an interface block like the following should be declared:

```

Interface
  Function Fint(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
  End Function Fint
End Interface

```



**Tol:** Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result.  
It is an optional parameter, and the default is  $Tol = 0.01$ .

### 4.3.3 Output

If the arguments are single (double) precision, the result will also be of single (double) precision.  
The value of the integral.

### 4.3.4 Examples

Listing 4.3: Integrating a function using the open trapezoid rule.

```

1  Program Test
   USE NumTypes
3   USE Integration

5   Real (kind=DP) :: Tol

7   Interface
   Function Fint(X)
9       USE NumTypes

11      Real (kind=DP), Intent (in) :: X
      Real (kind=DP) :: Fint
13  End Function Fint
End Interface

15
Tol = 1.0E-6_DP
17 Write(*,*) 'Integral of x**2 between 0 and 1:'
Write(*,*) TrapecioAb(0.0_DP, 1.0_DP, Fint, Tol)
19
   Stop
21 End Program Test

23 ! *****
24 ! *
25 Function Fint(X)
26 ! *
27 ! *****

29 USE NumTypes

31 Real (kind=DP), Intent (in) :: X
Real (kind=DP) :: Fint
33
Fint = X**2
35
   Return
37 End Function Fint

```

## 4.4 Function SimpsonAb(a, b, Func, [Tol])

### 4.4.1 Description

Calculates the integral of the function `Func` between `a` and `b` with precision `Tol` (optional) using the open Simpson's rule.

In general better than `TrapezioAb`

### 4.4.2 Arguments

**a, b:** Single (SP) or double (DP) precision. They are the limits of the integral.

**Func:** The function to be integrated. It must be a function of only one argument of the same type as the function itself. If it is an external function an interface block like the following should be declared:

```
Interface
  Function Fint(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
  End Function Fint
End Interface
```

**Tol:** Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter, and the default is `Tol = 0.01`.

### 4.4.3 Output

If the arguments are single (double) precision, the result will also be of single (double) precision. The value of the integral.

### 4.4.4 Examples

Listing 4.4: Exmaple of integration using the open Simpson rule.

```
1 Program Test
  USE NumTypes
3  USE Integration

5  Real (kind=DP) :: Tol

7  Interface
    Function Fint(X)
9    USE NumTypes

11     Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
13  End Function Fint
```

```

15      End Interface
17      Tol = 1.0E-6_DP
18      Write(*,*) 'Integral of x**2 between 0 and 1: '
19      Write(*,*) SimpsonAb(0.0_DP, 1.0_DP, Fint, Tol)
21      Stop
22  End Program Test
23
24  ! *****
25  ! *
26  ! *
27  ! *****
28
29  USE NumTypes
30
31  Real (kind=DP), Intent (in) :: X
32  Real (kind=DP) :: Fint
33
34  Fint = X**2
35
36  Return
37 End Function Fint

```

## 4.5 Function SimpsonInfUp(a, Func, [Tol])

### 4.5.1 Description

Calculates the integral of the function **Func** between **a** and  $\infty$  with precision **Tol** (optional) using the Simpson rule and a change of variables.

### 4.5.2 Arguments

**a**: Single (SP) or double (DP) precision. They are the limits of the integral.

**Func**: The function to be integrated. It must be a function of only one argument of the same type as the function itself. If it is an external function an interface block like the following should be declared:

```

Interface
  Function Fint(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
  End Function Fint
End Interface

```

This routine does not check if the integral exist, so the function must obviously decay fast for large  $x$  to obtain a finite value.

**Tol:** Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter, and the default is  $\text{Tol} = 0.01$ .

### 4.5.3 Output

If the arguments are single (double) precision, the result will also be of single (double) precision. The value of the integral.

### 4.5.4 examples

Listing 4.5: Integration of a function between 0 and  $\infty$ .

```

1  Program Test
   USE NumTypes
3   USE Integration

5   Real (kind=DP) :: Tol

7   Interface
   Function Fint(X)
9       USE NumTypes

11      Real (kind=DP), Intent (in) :: X
       Real (kind=DP) :: Fint
13  End Function Fint
End Interface

15  Tol = 1.0E-6_DP
17  Write(*,*) 'Integral of e**(-x**2) between 0 and infinity:'
19  Write(*,*) SimpsonInfUp(0.0_DP, Fint, Tol)

21  Stop
End Program Test

23  ! *****
25  ! *
Function Fint(X)
27  ! *****

29  USE NumTypes

31  Real (kind=DP), Intent (in) :: X
   Real (kind=DP) :: Fint

33  Fint = exp(-X**2)

35  Return
37  End Function Fint

```

## 4.6 Function SimpsonInfDw(a, Func, [Tol])

### 4.6.1 Description

Calculates the integral of the function `Func` between  $-\infty$  and `a` with precision `Tol` (optional) using the Simpson rule and a change of variables.

### 4.6.2 Arguments

**a:** Single (SP) or double (DP) precision. They are the limits of the integral.

**Func:** The function to be integrated. It must be a function of only one argument of the same type as the function itself. If it is an external function an interface block like the following should be declared:

```
Interface
  Function Fint(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
  End Function Fint
End Interface
```

This routine does not check if the integral exist, so the function must obviously decay fast for large  $-x$  to obtain a finite value.

**Tol:** Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter, and the default is `Tol = 0.01`.

### 4.6.3 Output

If the arguments are single (double) precision, the result will also be of single (double) precision. The value of the integral.

### 4.6.4 examples

Listing 4.6: Integrating a function between  $-\infty$  and 0.

```
1 Program Test
  USE NumTypes
3  USE Integration

5  Real (kind=DP) :: Tol

7  Interface
  Function Fint(X)
9    USE NumTypes

11   Real (kind=DP), Intent (in) :: X
```

```

13      Real (kind=DP) :: Fint
      End Function Fint
      End Interface

15
      Tol = 1.0E-6_DP
17      Write(*,*) 'Integral of e**(-x**2) between -infinity and 0:'
      Write(*,*) SimpsonInfDw(0.0_DP, Fint, Tol)

19
      Stop
21 End Program Test

23 ! *****
24 ! *
25 Function Fint(X)
26 ! *
27 ! *****

29 USE NumTypes

31 Real (kind=DP), Intent (in) :: X
32 Real (kind=DP) :: Fint

33
34 Fint = exp(-X**2)
35
36 Return
37 End Function Fint

```

## 4.7 Function SimpsonSingUp(a, b, Func, [Tol], gamma)

### 4.7.1 Description

Calculates the integral of the function **Func** between **a** and **b** with precision **Tol** (optional) using the Simpson's rule. The function may have an integrable singularity of the type:

$$f(x+b) \approx \frac{c}{(x-b)^\gamma} + \dots$$

with  $0 < \gamma < 1$ .

### 4.7.2 Arguments

**a, b:** Single (SP) or double (DP) precision. They are the limits of the integral.

**Func:** The function to be integrated. It must be a function of only one argument of the same type as the function itself. If it is an external function an interface block like the following should be declared:

```

Interface
  Function Fint(X)
    USE NumTypes

```

```

        Real (kind=DP), Intent (in) :: X
        Real (kind=DP) :: Fint
    End Function Fint
End Interface

```

**Tol:** Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter, and the default is  $Tol = 0.01$ .

**gamma:** The “degree of divergence” of the function in  $x \approx b$ .

### 4.7.3 Output

If the arguments are single (double) precision, the result will also be of single (double) precision. The value of the integral.

### 4.7.4 Examples

Listing 4.7: Integrating functions with singularities in the upper limit.

```

1  Program Test
    USE NumTypes
3   USE Integration

5   Real (kind=DP) :: Tol

7   Interface
    Function Fint(X)
9       USE NumTypes

11      Real (kind=DP), Intent (in) :: X
        Real (kind=DP) :: Fint
13      End Function Fint
    End Interface

15
    Tol = 1.0E-6_DP
17    Write(*,*) 'Integral of 1/sqrt(-x) between -1 and 0: '
    Write(*,*) SimpsonSingUp(-1.0_DP, 0.0_DP, Fint, Tol, 0.5_DP)

19
    Stop
21 End Program Test

23 ! *****
24 ! *
25 Function Fint(X)
26 ! *
27 ! *****

29 USE NumTypes

31 Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
33

```

```

35   Fint = Sqrt(-X)
37   Return
End Function Fint

```

## 4.8 Function SimpsonSingDw(a, b, Func, [Tol], gamma)

### 4.8.1 Description

Calculates the integral of the function **Func** between **a** and **b** with precision **Tol** (optional) using the Simpson's rule. The function may have an integrable singularity of the type:

$$f(x+a) \approx \frac{c}{(x-a)^\gamma} + \dots$$

with  $0 < \gamma < 1$ .

### 4.8.2 Arguments

**a, b:** Single (SP) or double (DP) precision. They are the limits of the integral.

**Func:** The function to be integrated. It must be a function of only one argument of the same type as the function itself. If it is an external function an interface block like the following should be declared:

```

Interface
  Function Fint(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
  End Function Fint
End Interface

```

**Tol:** Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter, and the default is **Tol** = 0.01.

**gamma:** The “degree of divergence” of the function in  $x \approx a$ .

### 4.8.3 Output

If the arguments are single (double) precision, the result will also be of single (double) precision. The value of the integral.

### 4.8.4 Examples



Listing 4.8: Integrating functions with singularities in the lower limit.

```

1 Program Test
  USE NumTypes
3  USE Integration

5  Real (kind=DP) :: Tol

7  Interface
    Function Fint(X)
9      USE NumTypes

11     Real (kind=DP), Intent (in) :: X
    Real (kind=DP) :: Fint
13 End Function Fint
End Interface

15 Tol = 1.0E-6_DP
17 Write(*,*) 'Integral of 1/sqrt(x) between 0 and 1: '
19 Write(*,*) SimpsonSingDw(0.0_DP, 1.0_DP, Fint, Tol, 0.5_DP)

21 Stop
End Program Test

23 ! *****
24 ! *
25 Function Fint(X)
26 ! *
27 ! *****

29 USE NumTypes

31 Real (kind=DP), Intent (in) :: X
32 Real (kind=DP) :: Fint
33
34 Fint = Sqrt(X)
35
36 Return
37 End Function Fint

```

## 4.9 Function Euler(Init, Xo, Xfin, Feuler, [Tol])

### 4.9.1 Description

Integrate the first order set of ODE defined by the function **Feuler**, with initial conditions given by the vector **Init** in **Xo**, until **Xfin**, with a precision given by **Tol** (optional).

A set of first order ODE's is given by the first derivatives of the variables involved:

$$\frac{y_i(x)}{x} = f_i(y_j, x)$$

and the initial conditions:

$$y_i(x_0)$$

After the integration we get:

$$y_i(x_{\text{fin}})$$

So to define a set of first order ODE's we need the value of the derivative of the variable  $i$  in the point  $x$  (this is done by **Feuler**), a vector of initial conditions (**Init**) and the point where this initial conditions are defined (**Xo**), and finally the point where we want the solution (**Xfin**)

### 4.9.2 Arguments

**Init(:)**: Single (SP) or double (DP) precision vector of one dimension with the initial conditions.

**Xo**: Single (SP) or double (DP) precision. The point where the initial conditions are defined.

**Xfin**: Single (SP) or double (DP) precision. The point where we want the value of the functions.

**Feuler**: The function that defines the set of first order ODE's. If it is an external function an interface block like the following should be declared:

```
Interface
  Function Feuler(X, Y) Result (Func)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X, Y(:)
    Real (kind=DP) :: Func(Size(Y))
  End Function Feuler
End Interface
```

The function must return a vector with the values of the first derivatives of the functions  $y_i(x)$  in the point  $X$ .

**Tol**: Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter.

### 4.9.3 Output

Real single or double precision (same as input) one dimensional array. The array contains the values of the functions  $y_i$  in the point **Xfin**.

### 4.9.4 Examples

This example below will integrate the set of first order ODE's defined by the equations:

$$\frac{y_1(x)}{x} = y_2(x); \quad \frac{y_2(x)}{x} = -y_1(x)$$

whose solution is:

$$y_1(x) = A \cos(x) + B \sin(x)$$

With the initial conditions  $y_1(0) = 0$ ;  $y_2(0) = 1$ , the solution is:

$$y_1(x) = \sin(x); \quad y_2(x) = \cos(x)$$

so if we plot  $y_1(1)$  and  $y_2(1)$  we will obtain the values  $\sin(1)$  and  $y_2(1)$ . In the following example, we will compare the result of integrating the differential equations with the exact values.

Listing 4.9: Integrating differential equations with Euler.

```

1 Program Test
  USE NumTypes
3  USE Integration

5  Real (kind=DP) :: Tol, In(2)

7  Interface
  Function Feuler(X, Y) Result (Func)
9    USE NumTypes

11     Real (kind=DP), Intent (in) :: X, Y(:)
    Real (kind=DP) :: Func(Size(Y))
13  End Function Feuler
End Interface

15

17  Tol = 1.0E-2_DP
  In(1) = 0.0_DP
19  In(2) = 1.0_DP
  Write(*,*) 'Values of sin(1) and cos(1): '
21  Write(*,*) Euler(In, 0.0_DP, 1.0_DP, Feuler, Tol)
  Write(*,*) Sin(1.0_DP), Cos(1.0_DP)
23

  Stop
25 End Program Test

27 ! *****
! *
29 Function FEuler(X, Y) Result (Func)
! *
31 ! *****

33     Real (kind=8), Intent (in) :: X, Y(:)
    Real (kind=8) :: Func(Size(Y))

35     Func(1) = Y(2)
37     Func(2) = -Y(1)

39     Return
End Function FEuler

```

## 4.10 Function Rgnkta(Init, Xo, Xfin, Feuler, [Tol])

### 4.10.1 Description

Integrate the first order set of ODE defined by the function **Feuler**, with initial conditions given by the vector **Init** in **Xo**, until **Xfin**, with a precision given by **Tol** (optional). This method uses a Runge-Kutta algorithm and is much more exact than the previous **Euler** function.

A set of first order ODE's is given by the first derivatives of the variables involved:

$$\frac{y_i(x)}{x} = f_i(y_j, x)$$

and the initial conditions:

$$y_i(x_0)$$

After the integration we get:

$$y_i(x_{\text{fin}})$$

So to define a set of first order ODE's we need the value of the derivative of the variable  $i$  in te point  $x$  (this is done by **Feuler**), a vector of initial conditions (**Init**) and the point where this initial conditions are defined (**Xo**), and finally the point where we want the solution (**Xfin**)

### 4.10.2 Arguments

**Init(:)**: Single (SP) or double (DP) precision vector of one dimension with the initial conditions.

**Xo**: Single (SP) or double (DP) precision. The point where the initial conditions are defined.

**Xfin**: Single (SP) or double (DP) precision. The point where we want the value of the functions.

**Feuler**: The function that defines the set of first order ODE's. If it is an external function an interface block like the following should be declared:

```
Interface
  Function Feuler(X, Y) Result (Func)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X, Y(:)
    Real (kind=DP) :: Func(Size(Y))
  End Function Feuler
End Interface
```

The function is the same as in the previos function.

**Tol**: Single (SP) or double (DP) precision. An estimation of the desired accuracy of the result. It is an optional parameter.

### 4.10.3 Output

Real single or double precision (same as input) one dimensional array. The array contains the values of the functions  $y_i$  in the point **Xfin**.

### 4.10.4 Examples

This example below will integrate the set of first order ODE's defined by the equations:

$$\frac{y_1(x)}{x} = y_2(x); \quad \frac{y_2(x)}{x} = -y_1(x)$$

whose solution is:

$$y_1(x) = A \cos(x) + B \sin(x)$$

With the initial conditions  $y_1(0) = 0; y_2(0) = 1$ , we have:

$$y_1(x) = \sin(x); \quad y_2(x) = \cos(x)$$

so if we plot  $y_1(1)$  and  $y_2(1)$  we will obtain the values  $\sin(1)$  and  $y_2(1)$ . In the following example, we will compare the values obtained with **Euler**, with **Rgnkta** and the exact ones.

Listing 4.10: Integrating differential equations with the Runge-Kutta method

```

Program Test
2  USE NumTypes
  USE Integration
4
  Real (kind=DP) :: Tol, In(2)
6
  Interface
8    Function Feuler(X, Y) Result (Func)
      USE NumTypes
10
      Real (kind=DP), Intent (in) :: X, Y(:)
12      Real (kind=DP) :: Func(Size(Y))
    End Function Feuler
14 End Interface

16
  Tol = 1.0E-3_DP
18  In(1) = 0.0_DP
  In(2) = 1.0_DP
20  Write(*,*) 'Values of sin(1) and cos(1): '
  Write(*,*) ' Euler: '
22  Write(*,*) Euler(In, 0.0_DP, 1.0_DP, Feuler, Tol)
  Write(*,*) ' Runge-Kutta: '
24  Write(*,*) Rgnkta(In, 0.0_DP, 1.0_DP, Feuler, Tol)
  Write(*,*) ' Exact: '
26  Write(*,*) Sin(1.0_DP), Cos(1.0_DP)

28  Stop
End Program Test
30

```

```

32  ! *****
33  ! *
34  ! *
35  ! *****
36
37      Real (kind=8), Intent (in) :: X, Y(:)
38      Real (kind=8) :: Func(Size(Y))
39
40      Func(1) = Y(2)
41      Func(2) = -Y(1)
42
43      Return
44  End Function FEuler

```

## 4.11 Function IntQuadrilateral(P1,P2,P3,P4,Fval)

### 4.11.1 Description

Given four 2D points (P1(2), P2(2), P3(2), P4(2)), and the values of a function in this points, this routine computes the bilinear interpolation polynomial, and returns the value of this polynomial in the quadrilateral.

### 4.11.2 Arguments

**P1(:), P2(:), P3(:), P4(:):** Real single or double precision two component vectors. Four points that delimitate a quadrilateral. The corners of the quadrilateral must be given in (counter-)clock wise order.

**Fval:** Real singler or double precision one dimensional array of four elements. The value of a functio in the four corners of the quadrilateral.

### 4.11.3 Output

Real single or double precision (same as input). An aproximation of the value of the integral of the function over the quadrilateral.

### 4.11.4 Examples

Listing 4.11: Computing the area of a quadrilateral.

```

2  Program TQ
3
4  USE NumTypes
5  USE Integration
6  USE SpaceGeometry
7
8  Real (kind=DP) :: P1(3), P2(3), P3(3), P4(3), F(4), Tp(3)
9
10 P1(:) = (/ -2.0_DP, -1.4.0_DP, 0.0_DP /)

```

```
10  P2(:) = (/1.2_DP, 0.0_DP, 0.0_DP/)
11  P3(:) = (/2.0_DP, 3.0_DP, 0.0_DP/)
12  P4(:) = (/0.0_DP, 2.5_DP, 0.0_DP/)

14  F(:) = (/1.0_DP, 1.0_DP, 1.0_DP, 1.0_DP/)

16
17  Write(*,*) IntQuadrilateral(P1(1:2),P2(1:2),P3(1:2),P4(1:2),F)
18  Tp(:) = Cross_Product(P1,P2)
19  Tp(:) = Tp + Cross_Product(P2,P3)
20  Tp(:) = Tp + Cross_Product(P3,P4)
21  Tp(:) = Tp + Cross_Product(P4,P1)
22  Write(*,*) Dot_Product(/0.0_DP,0.0_DP,0.5_DP/),Tp

24
25  Stop
26 End Program TQ
```





# Five

---

## MODULE Optimization

---

This is the documentation of the MODULE `Optimization`, a set of routines to Optimise (maximise or minimise) functions of one or several variables. Lot of work is needed to improve this module (simplex, quasi-Newton methods, etc...).

### 5.1 Subroutine Bracket(X1, X2, X3, Func)

#### 5.1.1 Description

The routine `Bracket(X1, X2, X3, FStep)` “brackets” a minimum of the function `Func`. That is to say, after calling, ‘X1, X2 and X3 obey

$$X1 < X2 < X3 \quad (5.1)$$

and

$$\text{Func}(X1) > \text{Func}(X2) \quad \text{and} \quad \text{Func}(X3) > \text{Func}(X2) \quad (5.2)$$

This assures that `Func` has a minimum in the interval  $(X1, X3)$ . The bracketing of the minimum obey the golden rule, that is to say

$$(X2 - X1) = \Phi(X3 - X2) \quad (5.3)$$

where  $\Phi$  is the golden number

$$\Phi = \frac{1 + \sqrt{5}}{2} \quad (5.4)$$

#### 5.1.2 Arguments

**X1,X2,X3:** Real single or double precision. At output, they bracket a minimum of `Func`. If you have a guess of the minimum of `Func`, introduce it in `X2`. of the minimum.

**Func:** The function whose minimum we want to bracket. An interface like the following should be declared

```
Interface
  Function Func(Xo)
```

```

        USE NumTypes

        Real (kind=DP), Intent (in) :: Xo
        Real (kind=DP) :: Func
    End Function Func
End Interface

```

### 5.1.3 Example

Listing 5.1: Bracketing a minimum.

```

2  Program TestMin
3
4  USE NumTypes
5  USE Optimization
6
7  Real (kind=DP) :: X1, X2, X3
8
9  Interface
10     Function FstepM(Xo)
11         USE NumTypes
12
13         Real (kind=DP), Intent (in) :: Xo
14         Real (kind=DP) :: FstepM
15     End Function FstepM
16 End Interface
17
18 ! Initial guess of the position of the minimum
19 X2 = 2.0_DP
20
21 CALL Bracket(X1,X2,X3, FstepM)
22 Write(*,*)
23 Write(*,*) 'Minimum bracketed: '
24
25 Write(*, '(1A,1ES33.25) ') 'X1: ', X1
26 Write(*, '(1A,1ES33.25) ') 'X2: ', X2
27 Write(*, '(1A,1ES33.25) ') 'X3: ', X3
28
29 Stop
30 End Program TestMin
31
32 ! *****
33 ! *
34 Function FstepM(Xo)
35 ! *
36 ! *****
37
38 USE NumTypes
39
40 Real (kind=DP), Intent (in) :: Xo
41 Real (kind=DP) :: FstepM

```

```

42
44     FstepM = Sin(Xo)
46
48     Return
End Function FstepM

```

## 5.2 Subroutine LineSrch(X, Func[, Tol])

### 5.2.1 Description

The function `LineSrch(X, FStep, Tol)` returns the position of the minimum of the Function `Fstep` with an optional precision `Tol`. This routine does not need the values of the derivative(s) of the function to work.

### 5.2.2 Arguments

**X[(:)]:** Real single or double precision. An initial guess of the position of the minimum when input. At output, the position of the minimum.

**Func:** The function that we want to minimise. It can be a function of one or several variables. In the case of one variable functions an interface like the following should be declared

```

Interface
  Function Func(Xo)
    USE NumTypes

    Real (kind=DP), Intent (in) :: Xo
    Real (kind=DP) :: Func
  End Function Func
End Interface

```

In the case of a function of several variables, the interface block should be like the following

```

Interface
  Function Func(Xo)
    USE NumTypes

    Real (kind=DP), Intent (in) :: Xo(:)
    Real (kind=DP) :: Func
  End Function Func
End Interface

```

**Tol:** Real single or double precision. As estimation of the precision of the result. The default value is  $10^{-3}$ .

### 5.2.3 Example

Listing 5.2: Minimising a function.

```

Program TestMin
2
  USE NumTypes
4  USE Optimization

  Integer, Parameter :: Ndim = 4
  Real (kind=DP) :: XoM(Ndim)

8
  Interface
    Function FstepM(Xo)
10      USE NumTypes

12      Real (kind=DP), Intent (in) :: Xo(:)
14      Real (kind=DP) :: FstepM
    End Function FstepM
16 End Interface

18
  ! Initial guess of the position of the minimum
20 XoM(1) = 1.373_DP
21 XoM(2) = 1.373_DP
22 XoM(3) = 1.373_DP
23 XoM(4) = 1.373_DP
24

26 Write(*,*) 'Initial Position: '
  Do I = 1, Ndim
28     Write(*, '(1A,1I4,1A,1ES33.25)') 'Variable ', I, " : ", XoM(I)
  End Do

30
  CALL LineSrch(XoM, FstepM, 1.0E-7_DP)
32 Write(*,*)
  Write(*,*) 'Position of the minimum: '
34 Do I = 1, Ndim
    Write(*, '(1A,1I4,1A,1ES33.25)') 'Variable ', I, " : ", XoM(I)
36 End Do

38 Stop
End Program TestMin

40
! *****
42 ! *
Function FstepM(Xo)
44 ! *
! *****

46
  USE NumTypes

48
  Real (kind=DP), Intent (in) :: Xo(:)

```

```

50  Real (kind=DP) :: FstepM
52
53  FstepM = (Xo(1)-1.0_DP)**2 + &
54          & (Xo(2)-2.0_DP)**2 + &
55          & (Xo(3)+3.0_DP)**4 + &
56          & (Xo(4)-4.0_DP)**8
57
58  Return
59 End Function FstepM

```

## 5.3 Subroutine ConjGrad(X, F, Fd[, Tol])

### 5.3.1 Description

The function `ConjGrad(X, F, Fd[, Tol])` returns the position of the minimum of the Function `F` with an optional precision `Tol`. This routine uses the conjugate gradient method, and should be much faster than `LineSrch`, but you must be able to compute the derivatives of `F`

### 5.3.2 Arguments

**X(:):** Real single or double precision. An initial guess of the position of the minimum when input. At output, the position of the minimum.

**F:** The function that we want to minimise. An interface block like the following should be defined.

```

Interface
  Function F(Xo)
    USE NumTypes

    Real (kind=DP), Intent (in) :: Xo(:)
    Real (kind=DP) :: F
  End Function F
End Interface

```

**Fd:** The gradient of the function that we want to minimise. An interface block like the following should be defined.

```

Interface
  Subroutine Fd(X, D)
    USE NumTypes
    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP), Intent (out) :: D(Size(X))
  End Subroutine Fd
End Interface

```

where the vector  $D$  returns the value of the derivatives.

**Tol:** Real single or double precision. As estimation of the precision of the result. The default value is  $10^{-3}$ .

### 5.3.3 Example

Listing 5.3: Minimising a function.

```

1 Program TestMin
2
3 Program OOOO
4
5     USE NumTypes
6     USE Optimization
7
8     Real (kind=DP) Xx(2)
9
10    Interface
11        Function F(X)
12            USE NumTypes
13            Real (kind=DP), Intent (in) :: X(:)
14            Real (kind=DP) :: F
15        End Function F
16    End Interface
17
18    Interface
19        Subroutine Fd(X, D)
20            USE NumTypes
21            Real (kind=DP), Intent (in) :: X(:)
22            Real (kind=DP), Intent (out) :: D(Size(X))
23        End Subroutine Fd
24    End Interface
25
26
27    Xx = 112.0_DP
28
29    CALL ConjGrad(Xx, F, Fd, 1.E-10_DP)
30    Write(*,*) 'Minimo: ', Xx
31
32    Stop
33 End Program Test
34
35
36 Function F(X)
37
38     USE NumTypes
39
40     Real (kind=DP), Intent (in) :: X(:)
41     Real (kind=DP) :: F
42
43
44     F = (X(1)-2.23_DP)**2 + (X(2)+1.23_DP)**2 + X(1)*X(2) + Sin(X(1)*X(2))

```

```

46      Return
48 End Function F

50 Subroutine Fd(X, D)
52   USE NumTypes

54   Real (kind=DP), Intent (in) :: X(:)
54   Real (kind=DP), Intent (out) :: D(Size(X))

56   D(1) = (X(1)-2.23_DP)*2 + X(2) + Cos(X(1)*X(2))*X(2)
58   D(2) = (X(2)+1.23_DP)*2 + X(1) + Cos(X(1)*X(2))*X(1)

60   Return
End Subroutine Fd

```

## 5.4 Function Step(X, FStep[, Tol])

### 5.4.1 Description

The function `Step(X, FStep, Tol)` returns the position of the minimum of the Function `Fstep` with an optional precision `Tol`. Unless you know very well what you are doing, you should use `LineSrch` or `ConjGrad` to minimize functions.

### 5.4.2 Arguments

**X:** Real single or double precision. An initial guess of the position of the minimum.

**Fstep:** The function that we want to minimise. It can be a function of one or several variables. In the case of one variable functions an interface like the following should be declared

```

Interface
  Function Fstep(Xo)
    USE NumTypes

    Real (kind=DP), Intent (in) :: Xo
    Real (kind=DP) :: Fstep
  End Function Fstep
End Interface

```

In the case of a function of several variables, the interface block should be like the following

```

Interface
  Function Fstep(Xo)
    USE NumTypes

```

```

        Real (kind=DP), Intent (in) :: Xo(:)
        Real (kind=DP) :: Fstep
    End Function Fstep
End Interface

```

Tol: Real single or double precision. As estimation of the precision of the result. The default value is  $10^{-3}$ .

### 5.4.3 Output

Real Single or double precision (same as the output). The position of a minimum of Fstep.

### 5.4.4 Example

Listing 5.4: Minimising a function.

```

1  Program TestMin
3
4      USE NumTypes
5      USE Optimization
6
7      Integer, Parameter :: Ndim = 4
8      Real (kind=DP) :: XoM(Ndim), Xmin(Ndim)
9
10     Interface
11         Function FstepM(Xo)
12             USE NumTypes
13
14             Real (kind=DP), Intent (in) :: Xo(:)
15             Real (kind=DP) :: FstepM
16         End Function FstepM
17     End Interface
18
19     ! Initial guess of the position of the minimum
20     XoM(1) = 1.373_DP
21     XoM(2) = 1.373_DP
22     XoM(3) = 1.373_DP
23     XoM(4) = 1.373_DP
24
25
26     Write(*,*) 'Initial Position: '
27     Do I = 1, Ndim
28         Write(*, '(1A,1I4,1A,1ES33.25)') 'Variable ', I, " : ", XoM(I)
29     End Do
30
31     Xmin = Step(XoM, FstepM, 1.0E-7_DP)
32     Write(*,*)
33     Write(*,*) 'Position of the minimum: '
34     Do I = 1, Ndim
35         Write(*, '(1A,1I4,1A,1ES33.25)') 'Variable ', I, " : ", Xmin(I)
36     End Do

```



```

37      Stop
39 End Program TestMin

41 ! *****
41 ! *
43 Function FstepM(Xo)
44 ! *
45 ! *****

47 USE NumTypes

49 Real (kind=DP), Intent (in) :: Xo(:)
49 Real (kind=DP) :: FstepM

51

53 FstepM = (Xo(1)-1.0_DP)**2 + &
           & (Xo(2)-2.0_DP)**2 + &
55           & (Xo(3)+3.0_DP)**4 + &
           & (Xo(4)-4.0_DP)**8

57

59 Return
End Function FstepM

```

## 5.5 Function MaxPosition(FVal, IpX, IpY)

### 5.5.1 Description

Given a two dimensional array of values `FVal(:, :)`, the function `MaxPosition(FVal, IpX, IpY)` returns the number of local maxima of `FVal(:, :)` and its positions in the one dimensional arrays `IpX(:)` and `IpY(:)`.

### 5.5.2 Arguments

`Fval(:, :)`: Real single or double precision. The values of a function in a two dimensional grid of points.

`IpX(:)`: Integer. A one dimensional array that contains the value of  $X$  for the positions of the maxima.

`IpY(:)`: Integer. A one dimensional array that contains the value of  $Y$  for the positions of the maxima.

### 5.5.3 Output

Integer. The number of local maxima of the input. `FVal(:, :)`.

### 5.5.4 Example

Listing 5.5: Example of the usage of the routine MaxPosition.

```

2  Program MaxLoc
4      USE NumTypes
4      USE Constants
4      USE Optimization
6      USE Error

8      IMPLICIT NONE

10     Integer :: I, J, IsX, IsY, Nmax
10     Character (len=200) :: Filename
12     Real (kind=DP) :: DnullX, DnullY
12     Real (kind=DP), Allocatable :: F(:, :)
14     Integer :: IpX(10), IpY(10)

16     Write(stderr, *) "SizeX, SizeY, Filename"
16     Read(*, *) IsX, IsY, Filename
18
18     Allocate(F(IsX, IsY))
20     Open (Unit=666, File=Trim(Filename), Action="READ")
20     Do I = 1, IsX
22         Do J = 1, IsY
22             Read(666, *) DnullX, DnullY, F(I, J)
24             Write(stderr, *) DnullX, DnullY, F(I, J)
24         End Do
26     End Do
26     Close(666)

28     Nmax = MaxPosition(F, IpX, IpY)
30     Write(*, *) "# Number of maxima: ", Nmax
30     Write(*, *) "# Positions of the maxima: "
32     Do I = 1, Nmax
32         Write(*, *) IpX(I), IpY(I)
34     End Do

36     Stop
End Program MaxLoc

```

# Six

---

## MODULE MinuitAPI

---

This is the documentation of the MODULE `MinuitAPI`, a set of routines to Optimise (maximise or minimise) functions of one or several variables. This module is a simple API to the CERN minuit library<sup>1</sup>.

### 6.1 Subroutine `Minimize(Func, X, Fval, [Bound], [Release], [logfile])`

#### 6.1.1 Description

The routine `Minimize`, minimises the function of several variables `Func`. As an output you get the position of the minima in the vector `X(:)`, and the value of the function in the minima in the variable `Fval`. It uses a series of minimization calls to different minuit strategies: `MINIMIZE` → `SEEK` → `MIGRAD`. This is a safe and robust minimizer for almost everything.

Several optional parameters can be used to put boundaries in the values of the parameters, or to specify a release order for the parameters.

#### 6.1.2 Arguments

**Func:** The function we want to minimise. An interface like the following should be declared

```
Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func
  End Function Func
End Interface
```

**X(:):** Real double precision one dimensional array. As input an estimate of the position of the minima. As output the position of the minima.

---

<sup>1</sup><http://lcgapp.cern.ch/project/cls/work-packages/mathlibs/minuit/home.html>

**Fval:** Real double precision. Output. The value of the function at the minima.

**Bound(:):** Real double precision one dimensional array. Optional. Parameter limits. The minimum value for parameter  $n$  is given by **Bound(2n-1)**, and the maximum value is given by **Bound(2n)**. If both limits are 0.0D0 the parameter has no limits.

**Release(0,:):** Integer two dimensional array. Optional. The integer two dimensional array can be used to specify a release order of parameters. The first dimension of **Release(0,:)** contains the steps in which you want to release the parameters. The vector **Release(0,:)** contains the number of parameters released in each step. **Release(1,:)** contains the parameters released in the first step. **Release(2,:)** contains the parameters released in the second step, etc...

For example, the array (an \* means that the value of this element of the array is irrelevant).

$$R_{ij} = \begin{pmatrix} 2 & 2 & 3 & * \\ 1 & 2 & * & * \\ 3 & 4 & * & * \\ 5 & 6 & 7 & * \end{pmatrix} \quad (6.1)$$

Means that parameters 1,2 are free in the first step of the fit. Parameters (1,2,3,4) are free in the second step, and finally parameters (1,2,3,4,5,6,7) are released in the last step of the fit.

**logfile:** Character (len=\*). Optional. A file name where MINUIT will write some information about the minimisation process.

### 6.1.3 Example

Listing 6.1: Using minuit library to minimize a function.

```

1  Program TestAPI
3
4      USE MinuitAPI
5      USE Statistics
6      USE Constants
7
8      Integer, Parameter :: N = 2
9      Real (kind=8) :: X(N), Y(N), Ye(N), C(2), Ch
10
11     Interface
12         Function Func(X)
13             Real (kind=8), Intent (in) :: X(:)
14             Real (kind=8) :: Func
15         End Function Func
16     End Interface
17
18     X(:) = -23.0D0
19     CALL Minimize(Func, X, Ch)
20     Write(*,*)Ch
21     Write(*,*)X

```

```

23   Write(*,*) Tan(X(1)), Cos(X(2))
25   Stop
27 End Program TestAPI

27 Function Func(X)
29   Real (kind=8), Intent (in) :: X(:)
31   Real (kind=8) :: Func
33   Func = (X(1)-tan(X(1)))**2 + (X(2) - Cos(X(2)))**2
35   Return
End Function Func

```

## 6.2 Subroutine Migrad(Func, X, Fval, [Bound], [Release], [logfile])

### 6.2.1 Description

The routine `Migrad`, minimises the function of several variables `Func` using Minuit `MIGRAD` minimizer. As an output you get the position of the minima in the vector `X(:)`, and the value of the function in the minima in the variable `Fval`.

From the minuit documentation:

This is the best minimizer for nearly all functions. It is a variable-metric method with inexact line search, a stable metric updating scheme, and checks for positive-definiteness.

Several optional parameters can be used to put boundaries in the values of the parameters, or to specify a release order for the parameters.

### 6.2.2 Arguments

**Func:** The function we want to minimise. An interface like the following should be declared

```

Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func
  End Function Func
End Interface

```

**X(:):** Real double precision one dimensional array. As input an estimate of the position of the minima. As output the position of the minima.

**Fval:** Real double precision. Output. The value of the function at the minima.

**Bound(:):** Real double precision one dimensional array. Optional. Parameter limits. The minimum value for parameter  $n$  is given by **Bound(2n-1)**, and the maximum value is given by **Bound(2n)**. If both limits are 0.0D0 the parameter has no limits.

**Release(0,:):** Integer two dimensional array. Optional. The integer two dimensional array can be used to specify a release order of parameters. The first dimension of **Release(0,:)** contains the steps in which you want to release the parameters. The vector **Release(0,:)** contains the number of parameters released in each step. **Release(1,:)** contains the parameters released in the first step. **Release(2,:)** contains the parameters released in the second step, etc...

For example, the array (an \* means that the value of this element of the array is irrelevant).

$$R_{ij} = \begin{pmatrix} 2 & 2 & 3 & * \\ 1 & 2 & * & * \\ 3 & 4 & * & * \\ 5 & 6 & 7 & * \end{pmatrix} \quad (6.2)$$

Means that parameters 1, 2 are free in the first step of the fit. Parameters (1, 2, 3, 4) are free in the second step, and finally parameters (1, 2, 3, 4, 5, 6, 7) are released in the last step of the fit.

**logfile:** Character (len=\*). Optional. A file name where MINUIT will write some information about the minimisation process.

### 6.2.3 Example

Listing 6.2: Using minuit library to minimize a function.

```

1  Program TestAPI
3
4      USE MinuitAPI
5      USE Statistics
6      USE Constants
7
8      Integer, Parameter :: N = 2
9      Real (kind=8) :: X(N), Y(N), Ye(N), C(2), Ch
10
11     Interface
12         Function Func(X)
13             Real (kind=8), Intent (in) :: X(:)
14             Real (kind=8) :: Func
15         End Function Func
16     End Interface
17
18     X(:) = -23.0D0
19     CALL Migrad(Func, X, Ch)
20     Write(*,*)Ch
21     Write(*,*)X
22     Write(*,*)Tan(X(1)), Cos(X(2))
23

```

```

25  Stop
End Program TestAPI

27  Function Func(X)

29  Real (kind=8), Intent (in) :: X(:)
Real (kind=8) :: Func

31  Func = (X(1)-tan(X(1)))**2 + (X(2) - Cos(X(2)))**2

33  Return
35  End Function Func

```

## 6.3 Subroutine Misimplex(Func, X, Fval, [Bound], [Release], [logfile])

### 6.3.1 Description

The routine **Misimplex**, minimises the function of several variables **Func** using Minuit **SIMPLEX** minimizer. As an output you get the position of the minima in the vector **X(:)**, and the value of the function in the minima in the variable **Fval**.

From the minuit documentation:

This genuine multidimensional minimization routine is usually much slower than MIGRAD, but it does not use first derivatives, so it should not be so sensitive to the precision of the FCN calculations, and is even rather robust with respect to gross fluctuations in the function value. However, it gives no reliable information about parameter errors, no information whatsoever about parameter correlations, and worst of all cannot be expected to converge accurately to the minimum in a finite time. Its estimate of EDM is largely fantasy, so it would not even know if it did converge.

Several optional parameters can be used to put boundaries in the values of the parameters, or to specify a release order for the parameters.

### 6.3.2 Arguments

**Func:** The function we want to minimise. An interface like the following should be declared

```

Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func
  End Function Func
End Interface

```

**X(:):** Real double precision one dimensional array. As input an estimate of the position of the minima. As output the position of the minima.

**Fval**: Real double precision. Output. The value of the function at the minima.

**Bound(:)**: Real double precision one dimensional array. Optional. Parameter limits. The minimum value for parameter  $n$  is given by **Bound(2n-1)**, and the maximum value is given by **Bound(2n)**. If both limits are 0.0D0 the parameter has no limits.

**Release(0,:)**: Integer two dimensional array. Optional. The integer two dimensional array can be used to specify a release order of parameters. The first dimension of **Release(0,:)** contains the steps in which you want to release the parameters. The vector **Release(0,:)** contains the number of parameters released in each step. **Release(1,:)** contains the parameters released in the first step. **Release(2,:)** contains the parameters released in the second step, etc...

For example, the array (an \* means that the value of this element of the array is irrelevant).

$$R_{ij} = \begin{pmatrix} 2 & 2 & 3 & * \\ 1 & 2 & * & * \\ 3 & 4 & * & * \\ 5 & 6 & 7 & * \end{pmatrix} \quad (6.3)$$

Means that parameters 1,2 are free in the first step of the fit. Parameters (1,2,3,4) are free in the second step, and finally parameters (1,2,3,4,5,6,7) are released in the last step of the fit.

**logfile**: Character (len=\*). Optional. A file name where MINUIT will write some information about the minimisation process.

### 6.3.3 Example

Listing 6.3: Using minuit library to minimize a function.

```

1  Program TestAPI
3
4      USE MinuitAPI
5      USE Statistics
6      USE Constants
7
8      Integer, Parameter :: N = 2
9      Real (kind=8) :: X(N), Y(N), Ye(N), C(2), Ch
10
11     Interface
12         Function Func(X)
13             Real (kind=8), Intent (in) :: X(:)
14             Real (kind=8) :: Func
15         End Function Func
16     End Interface
17
18     X(:) = -23.0D0
19     CALL Misimplex(Func, X, Ch)
20     Write(*,*)Ch
21     Write(*,*)X

```



```

23   Write(*,*)Tan(X(1)), Cos(X(2))
25   Stop
27 End Program TestAPI

27 Function Func(X)

29   Real (kind=8), Intent (in) :: X(:)
31   Real (kind=8) :: Func

33   Func = (X(1)-tan(X(1)))**2 + (X(2) - Cos(X(2)))**2

35   Return
End Function Func

```

## 6.4 Subroutine Miseek(Func, X, Fval, [Bound], [Release], [logfile])

### 6.4.1 Description

The routine `Miseek`, minimises the function of several variables `Func` using Minuit `SEEK` minimizer. As an output you get the position of the minima in the vector `X(:)`, and the value of the function in the minima in the variable `Fval`.

From the minuit documentation:

We have retained this Monte Carlo search mainly for sentimental reasons, even though the limited experience with it is less than spectacular. The method now incorporates a Metropolis algorithm which always moves the search region to be centred at a new minimum, and has probability  $e^{-F/F_{\min}}$  of moving the search region to a higher point with function value  $F$ . This gives it the theoretical ability to jump through function barriers like a multidimensional quantum mechanical tunneler in search of isolated minima, but it is widely believed by at least half of the authors of Minuit that this is unlikely to work in practice (counterexamples are welcome) since it seems to depend critically on choosing the right average step size for the random jumps, and if you knew that, you wouldn't need Minuit.

Several optional parameters can be used to put boundaries in the values of the parameters, or to specify a release order for the parameters.

### 6.4.2 Arguments

**Func:** The function we want to minimise. An interface like the following should be declared

```

Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func
  End Function Func
End Interface

```

**X(:):** Real double precision one dimensional array. As input an estimate of the position of the minima. As output the position of the minima.

**Fval:** Real double precision. Output. The value of the function at the minima.

**Bound(:):** Real double precision one dimensional array. Optional. Parameter limits. The minimum value for parameter  $n$  is given by **Bound(2n-1)**, and the maximum value is given by **Bound(2n)**. If both limits are 0.0D0 the parameter has no limits.

**Release(0,:):** Integer two dimensional array. Optional. The integer two dimensional array can be used to specify a release order of parameters. The first dimension of **Release(0,:)** contains the steps in which you want to release the parameters. The vector **Release(0,:)** contains the number of parameters released in each step. **Release(1,:)** contains the parameters released in the first step. **Release(2,:)** contains the parameters released in the second step, etc...

For example, the array (an \* means that the value of this element of the array is irrelevant).

$$R_{ij} = \begin{pmatrix} 2 & 2 & 3 & * \\ 1 & 2 & * & * \\ 3 & 4 & * & * \\ 5 & 6 & 7 & * \end{pmatrix} \quad (6.4)$$

Means that parameters 1,2 are free in the first step of the fit. Parameters (1,2,3,4) are free in the second step, and finally parameters (1,2,3,4,5,6,7) are released in the last step of the fit.

**logfile:** Character (len=\*). Optional. A file name where MINUIT will write some information about the minimisation process.

### 6.4.3 Example

Listing 6.4: Using minuit library to minimize a function.

```

1  Program TestAPI
3
3  USE MinuitAPI
3  USE Statistics
5  USE Constants
7
7  Integer, Parameter :: N = 2
7  Real (kind=8) :: X(N), Y(N), Ye(N), C(2), Ch
9
9  Interface
11     Function Func(X)
11         Real (kind=8), Intent (in) :: X(:)
13         Real (kind=8) :: Func
13     End Function Func
15 End Interface
17
17 X(:) = -23.0D0

```

```

19  CALL Miseek(Func, X, Ch)
    Write(*,*)Ch
21  Write(*,*)X
    Write(*,*)Tan(X(1)), Cos(X(2))
23
    Stop
25 End Program TestAPI

27 Function Func(X)

29  Real (kind=8), Intent (in) :: X(:)
    Real (kind=8) :: Func

31  Func = (X(1)-tan(X(1)))**2 + (X(2) - Cos(X(2)))**2

33
    Return
35 End Function Func

```

## 6.5 Subroutine Miscan(Func, X, Fval, [Bound], [Release], [logfile])

### 6.5.1 Description

The routine `Miscan`, minimises the function of several variables `Func` using Minuit `SCAN` minimizer. As an output you get the position of the minima in the vector `X(:)`, and the value of the function in the minima in the variable `Fval`.

From the minuit documentation:

This is not intended to minimize, and just scans the function, one parameter at a time. It does however retain the best value after each scan, so it does some sort of highly primitive minimization.

Several optional parameters can be used to put boundaries in the values of the parameters, or to specify a release order for the parameters.

### 6.5.2 Arguments

**Func:** The function we want to minimise. An interface like the following should be declared

```

Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func
  End Function Func
End Interface

```

**X(:):** Real double precision one dimensional array. As input an estimate of the position of the minima. As output the position of the minima.

**Fval:** Real double precision. Output. The value of the function at the minima.

**Bound(:):** Real double precision one dimensional array. Optional. Parameter limits. The minimum value for parameter  $n$  is given by **Bound(2n-1)**, and the maximum value is given by **Bound(2n)**. If both limits are 0.0D0 the parameter has no limits.

**Release(0,:):** Integer two dimensional array. Optional. The integer two dimensional array can be used to specify a release order of parameters. The first dimension of **Release(0,:)** contains the steps in which you want to release the parameters. The vector **Release(0,:)** contains the number of parameters released in each step. **Release(1,:)** contains the parameters released in the first step. **Release(2,:)** contains the parameters released in the second step, etc...

For example, the array (an \* means that the value of this element of the array is irrelevant).

$$R_{ij} = \begin{pmatrix} 2 & 2 & 3 & * \\ 1 & 2 & * & * \\ 3 & 4 & * & * \\ 5 & 6 & 7 & * \end{pmatrix} \quad (6.5)$$

Means that parameters 1,2 are free in the first step of the fit. Parameters (1,2,3,4) are free in the second step, and finally parameters (1,2,3,4,5,6,7) are released in the last step of the fit.

**logfile:** Character (len=\*). Optional. A file name where MINUIT will write some information about the minimisation process.

### 6.5.3 Example

Listing 6.5: Using minuit library to minimize a function.

```

1  Program TestAPI
3
4      USE MinuitAPI
5      USE Statistics
6      USE Constants
7
8      Integer, Parameter :: N = 2
9      Real (kind=8) :: X(N), Y(N), Ye(N), C(2), Ch
10
11     Interface
12         Function Func(X)
13             Real (kind=8), Intent (in) :: X(:)
14             Real (kind=8) :: Func
15         End Function Func
16     End Interface
17
18     X(:) = -23.0D0
19     CALL Miscan(Func, X, Ch)
20     Write(*,*)Ch
21     Write(*,*)X

```

```
23   Write(*,*)Tan(X(1)), Cos(X(2))
25   Stop
27 End Program TestAPI
29 Function Func(X)
31   Real (kind=8), Intent (in) :: X(:)
33   Real (kind=8) :: Func
35   Func = (X(1)-tan(X(1)))**2 + (X(2) - Cos(X(2)))**2
37   Return
39 End Function Func
```



# Seven

---

## MODULE Linear

---

This is the documentation of the `MODULE Linear`, a set of `FORTRAN 90` routines to solve linear systems of equations. This module make use of the `MODULE NumTypes`, and `MODULE Error` so please read the documentation of these modules *before* reading this.

### 7.1 Subroutine `Pivoting(M,Ipiv,Idet)`

#### 7.1.1 Description

Permute the rows of  $M$  so that the biggest elements (in absolute value) of  $M$  are in the diagonal.

#### 7.1.2 Arguments

`M(:, :)`: Real or complex single or double precision two dimensional array. Initially it contains the matrix to permute, after calling the routine, it contains the permuted matrix. *Note that  $M$  is overwritten when calling this routine.*

`Ipiv(:)`: Integer one dimensional array. It returns the permutation of rows made to  $M$ .

`Idet`: Integer. If the number of permutations is odd, `Idet` = 1, if it is even `Idet` = -1

#### 7.1.3 Examples

Listing 7.1: Pivoting data of a matrix label

```
1 Program TestLinear
3   USE NumTypes
4   USE Linear
5
6   Integer, Parameter :: Nord = 4
7
8   Real (kind=DP) :: M(Nord,Nord), L(Nord,Nord), U(Nord,Nord), &
9     & Mcp(Nord,Nord)
10  Integer :: Ipiv(Nord), Iperm
11
```

```

13  ! Fill M of random numbers
    CALL Random_Number(M)
15
16  Write(*,*) 'Original M: '
17  Do I = 1, Nord
    Write(*, '(100ES10.3)')(M(I,J), J = 1, Nord)
19  End Do

21  CALL Pivoting (M, Ipiv, Iperm)
    Write(*,*) 'Permuted M: '
23  Do I = 1, Nord
    Write(*, '(100ES10.3)')(M(I,J), J = 1, Nord)
25  End Do

27  Stop
End Program TestLinear

```

## 7.2 Subroutine LU(M, Ipiv, Idet)

### 7.2.1 Description

Make the LU decomposition of matrix  $M$ . That is to say, given a matrix  $M$ , this function returns two matrix  $L$  and  $U$ , such that

$$M = LU \quad (7.1)$$

where  $L$  is lower triangular, and  $U$  upper triangular.

$$L = \begin{pmatrix} 1 & 0 & 0 & \dots \\ L_{21} & 1 & 0 & \dots \\ L_{31} & L_{32} & 1 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}; \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} & \dots \\ 0 & U_{22} & U_{23} & \dots \\ 0 & 0 & U_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (7.2)$$

The rows of  $M$  are permuted so that the biggest possible elements are on the diagonal (this makes the problem more stable). The two matrices  $L$  and  $U$  are returned *overwriting*  $M$ .

### 7.2.2 Arguments

**M(:, :):** Real or complex single or double precision two dimensional array. Initially it contains the matrix to decompose, after calling the routine, it contains  $L$  in its lower part, and  $U$  in its upper part. *Note that  $M$  is overwritten when calling this routine.*

**Ipiv(:):** Integer one dimensional array. It returns the permutation of rows made to  $M$ .

**Idet:** Integer. If the number of permutations is odd, **Idet** = 1, if it is even **Idet** = -1



## 7.2.3 Examples

Listing 7.2: Making the LU decomposition.

```

Program TestLinear
2
3   USE NumTypes
4   USE Linear
5
6   Integer, Parameter :: Nord = 4
7
8   Real (kind=DP) :: M(Nord,Nord), L(Nord,Nord), U(Nord,Nord), &
9       & Mcp(Nord,Nord)
10  Integer :: Ipiv(Nord), Iperm
11
12  ! Fill M of random numbers, and make a copy
13  CALL Random_Number(M)
14  Mcp = M
15  L = 0.0_DP
16  U = 0.0_DP
17
18  ! Make the LU decomposition and fill the matrices
19  ! L and U
20  CALL Lu(M, Ipiv, Iperm)
21
22  Do I = 1, Nord
23      L(I,I) = 1.0_DP
24      U(I,I) = M(I,I)
25      Do J = I+1, Nord
26          L(J,I) = M(J,I)
27          U(I,J) = M(I,J)
28      End Do
29  End Do
30
31  ! Now Make the product and see that it is the original matrix with
32  ! some rows permuted
33  Write(*,*) 'M: '
34  Do I = 1, Nord
35      Write(*, '(100ES10.3)')(Mcp(I,J), J = 1, Nord)
36  End Do
37
38  Write(*,*) 'L: '
39  Do I = 1, Nord
40      Write(*, '(100ES10.3)')(L(I,J), J = 1, Nord)
41  End Do
42  Write(*,*) 'U: '
43  Do I = 1, Nord
44      Write(*, '(100ES10.3)')(U(I,J), J = 1, Nord)
45  End Do
46
47  M = MatMul(L,U)
48  Write(*,*) 'LU (Same as M with some rows permuted): '
49  Do I = 1, Nord

```

```

50      Write(*, '(100ES10.3)')(M(I,J), J = 1, Nord)
      End Do
52
54      Stop
      End Program TestLinear

```

## 7.3 Subroutine LUsolve(M, b)

### 7.3.1 Description

Solves the linear system of equations

$$\begin{aligned}
 M_{11}x_1 + M_{12}x_2 + M_{13}x_3 + M_{14}x_4 + \dots &= b_1 \\
 M_{21}x_1 + M_{22}x_2 + M_{23}x_3 + M_{24}x_4 + \dots &= b_2 \\
 &\vdots
 \end{aligned}
 \tag{7.3}$$

### 7.3.2 Arguments

**M(:, :):** Real or complex single or double precision two dimensional array. The matrix of coefficients. *M is overwritten when solving the system.*

**b(:):** Real or complex single or double precision one dimensional array. The independent terms before calling the routine, and the solution of the linear system of equations after calling it. *Note that b is overwritten when calling this routine.*

### 7.3.3 Examples

Listing 7.3: Solving systems of linear equations.

```

1  Program TestLinear
3
4      USE NumTypes
5      USE Linear
6
7      Integer, Parameter :: Nord = 10
8
9      Real (kind=DP) :: M(Nord,Nord), L(Nord,Nord), U(Nord,Nord), &
10         & Mcp(Nord,Nord), b(Nord), bcp(Nord)
11      Integer :: I piv(Nord), Iperm
12
13      ! Fill M and b of random numbers, and make a copy of both
14      CALL Random_Number(M)
15      CALL Random_Number(b)
16      Mcp = M
17      bcp = b
18
19      ! Solve the linear system
20      CALL LUsolve(M,b)

```

```

21  ! Check that it is a solution:
    b = MatMul(Mcp,b)
23  Write(*,*) 'b: '
    Write(*, '(100ES10.3) ')(Abs(bcp(I)-b(I)), I = 1, Nord)
25
27  Stop
End Program TestLinear

```

## 7.4 Function Det(M)

### 7.4.1 Description

Computes the determinant of the matrix  $M$ .

### 7.4.2 Arguments

$M(:, :)$ : Real or complex, simple or double precision two dimensional array. The matrix whose determinant we want to know.

### 7.4.3 Output

The value of the determinant. Same precision as the input argument.

### 7.4.4 Examples

Listing 7.4: Computing the determinant of a matrix.

```

Program TestLinear
2
    USE NumTypes
4    USE Linear

6    Integer, Parameter :: Nord = 10

8    Real (kind=DP) :: M(Nord,Nord), L(Nord,Nord), U(Nord,Nord), &
        & Mcp(Nord,Nord), b(Nord), bcp(Nord)
10   Integer :: I piv(Nord), Iperm

12
14   ! Fill M of randoms numbers
    CALL Random_Number(M)

16   ! Now compute the determinant.
    Write(*, '(ES15.8) ') Det(M)
18
20   Stop
End Program TestLinear

```



# Eight

---

## MODULE NonNum

---

This is the documentation of the MODULE NonNum, a set of FORTRAN 90 routines to sort and search. This module make use of the MODULE NumTypes, and MODULE Error so please read the documentation of these modules *before* reading this.

### 8.1 Subroutine Swap(X,Ind1,Ind2)

#### 8.1.1 Description

Swaps elements Ind1 and Ind2 of the array X(:).

#### 8.1.2 Arguments

X(:): Integer, real single or real double precision one dimensional array. *Note that X is overwritten when calling this routine.*

Ind1, Ind2: Integer. The elements that we want to permute.

#### 8.1.3 Examples

Listing 8.1: Sorting data.

```
1 Program TestNN
3   USE NumTypes
3   USE NonNumeric
5
5   Integer, Parameter :: Nmax = 10
7   Integer :: Ima(Nmax), I
9
9   ! Fill Ima(:)
11  Forall (I=1:Nmax) Ima(I) = I
13
13  ! Plot the numbers
13  Do I = 1, Nmax
```

```

15      Write(*, '(1000I10) ')Ima(I)
      End Do

17      ! Swap first and last elemetns of Ima(:) and plot them.
19      CALL Swap(Ima, 1, Nmax)
      Write(*,*) '# With first and last elements permuted: '
21      Do I = 1, Nmax
          Write(*, '(1000I10) ')Ima(I)
23      End Do

25
27      Stop
      End Program TestNN

```

## 8.2 Subroutine Insrt(X[, Ipt])

### 8.2.1 Description

Sort the elements of  $X(:)$  in ascendant order<sup>1</sup>.

### 8.2.2 Arguments

$X(:)$ : Integer, real single or real double precision one dimensional array. Initially it contains unsorted numbers, and after calling the routine, it contains the sorted elements. *Note that X is overwritten when calling this routine.*

$Ipt(:)$ : Integer vector, Optional. It returns the permutation made to  $X(:)$  to sort it.

### 8.2.3 Examples

Listing 8.2: Sorting data.

```

1 Program TestNN

3     USE NumTypes
     USE NonNumeric

5

7     Integer, Parameter :: Nmax = 10
     Integer :: Ima(Nmax)
     Real (kind=DP) :: X(Nmax), Y(Nmax)

9

11    ! Fill X(:) with random data, and define Y(:)
     CALL Random_Number(X)
13    Y = Sin(12.34_DP*(X-0.5_DP))

15    ! Plot an unsorted data table
     Do I = 1, Nmax

```

<sup>1</sup>This routine uses *insertion sort*, and is much slower than `Qsort`. To sort more than 10 elements, use `Qsort` unless you know what you are doing.

```

17      Write(*, '(1000ES13.5) ')X(I), Y(I)
18      End Do
19
20      ! Sort them, and plot the table again. Same points, but this time
21      ! sorted
22      CALL Insrt(X, Ima)
23      Write(*,*) '# Again, this time sorted: '
24      Do I = 1, Nmax
25          Write(*, '(1000ES13.5) ')X(I), Y(Ima(I))
26      End Do
27
28
29      Stop
End Program TestNN

```

## 8.3 Subroutine Qsort(X[, Ipt])

### 8.3.1 Description

Sort the elements of  $X(:)$  in ascendant order.

### 8.3.2 Arguments

$X(:)$ : Integer, real single or real double precision one dimensional array. Initially it contains unsorted numbers, and after calling the routine, it contains the sorted elements. *Note that  $X$  is overwritten when calling this routine.*

$Ipt(:)$ : Integer vector, Optional. It returns the permutation made to  $X(:)$  to sort it.

### 8.3.3 Examples

Listing 8.3: Sorting data.

```

Program TestNN
2
3      USE NumTypes
4      USE NonNumeric
5
6      Integer, Parameter :: Nmax = 10
7      Integer :: Ima(Nmax)
8      Real (kind=DP) :: X(Nmax), Y(Nmax)
9
10
11      ! Fill X(:) with random data, and define Y(:)
12      CALL Random_Number(X)
13      Y = Sin(12.34_DP*(X-0.5_DP))
14
15      ! Plot an unsorted data table
16      Do I = 1, Nmax
17          Write(*, '(1000ES13.5) ')X(I), Y(I)
18      End Do

```

```

20  ! Sort them, and plot the table again. Same points, but this time
    ! sorted
22  CALL Qsort(X, Ima)
    Write(*,*) '# Again, this time sorted: '
24  Do I = 1, Nmax
        Write(*, '(1000ES13.5) ')X(I), Y(Ima(I))
26  End Do

28
    Stop
30 End Program TestNN

```

## 8.4 Function Locate( $X$ , $X_0$ [, $Iin$ ])

### 8.4.1 Description

Given a *sorted* vector of elements  $X(:)$ , and a point  $X_0$ , **Locate** returns the position  $n$  such that  $X(n) < X_0 < X(n+1)$ . If  $X_0$  is less than all the elements of  $X(:)$ , **Locate** returns 0, and if it is greater than all the elements of  $X(:)$ , it returns the number of elements of  $X(:)$

### 8.4.2 Arguments

$X(:)$ : Integer, real single or real double precision one dimensional *sorted* array.

$X_0$ : Integer, real single or real double precision number, but the same type as  $X(:)$ . Point that we want to locate in the sorted vector  $X(:)$ .

$Iin$ : Integer, Optional. Initial guess of the position.

### 8.4.3 Output

Integer. The position  $n$  such that

$$X(n) < X_0 < X(n+1)$$

### 8.4.4 Examples

Listing 8.4: Searching data position in an ordered list.

```

Program TestNN
2
    USE NumTypes
4    USE NonNumeric

6    Integer, Parameter :: Nmax = 100
    Integer :: Ima(Nmax), Idx
8    Real (kind=DP) :: X(Nmax), Y(Nmax), X0

10    ! Fill X(:) with random data, and set X0 to some arbitrary value.

```



```
12  CALL Random_Number(X)
    X0 = 0.276546754_DP
14
16  ! Sort X(:), find the position of X0, and plot the neighborr
    ! elements.
18  CALL Qsort(X)
    Idx = Locate(X, X0)
20  Write(*, '(1A,1ES33.25) ') 'Searched element: ', X0
    Write(*, '(1A,1ES33.25) ') 'Previous element in the list: ', X(Idx)
    Write(*, '(1A,1ES33.25) ') 'Next element in the list:      ', X(Idx+1)
22
24  Stop
    End Program TestNN
```



# Nine

---

## MODULE SpecialFunc

---

This is the documentation of the `MODULE SpecialFunc`, a set of FORTRAN 90 routines to compute the value of some functions. This module make use of the `MODULE NumTypes`, `MODULE Constants`, `MODULE Error` so please read the documentation of these modules *before* reading this.

### 9.1 Function GammaLn(X)

#### 9.1.1 Description

Compute  $\log(\Gamma(X))$ .

#### 9.1.2 Arguments

**x:** Double (DP) precision. The point in which we want to know the value of  $\Gamma(X)$ .

#### 9.1.3 Output

A real Double precision (DP).

#### 9.1.4 Examples

This program should write the factorial of the first 100 numbers.

Listing 9.1: Computing the Gamma Function.

```
1  Program TestSpecialFunc
2
3  USE NumTypes
4  USE SpecialFunc
5
6  Integer :: q
7
8
9  Do q = 1, 100
10   Write(*, '(1A13,1I4,1A3,1ES33.25) ') 'Factorial of: ', q, ' = ', &
      & exp(GammaLn(Real(q+1,kind=DP)))
```

```

12      End Do
14
16      Stop
End Program TestSpecialFunc

```

## 9.2 Function erf( $X$ )

### 9.2.1 Description

Compute values of the error function  $\text{erf}(X)$ . A high precision algorithm by W. J. Candy [2] is used.

### 9.2.2 Arguments

**x:** Single (SP) or double (DP) precision. The point in which we want to know the value of  $\text{erf}(X)$ .

### 9.2.3 Output

Same type as input.

### 9.2.4 Examples

This program should compute the  $\text{erf}(X)$  and its complementary in an given point.

Listing 9.2: Computing the Error Function.

```

Program TestSpecialFunc
2
  USE NumTypes
4  USE SpecialFunc

6  Real (kind=DP) :: X

8  Write(*,*) 'Enter a number: '
  Read(*,*)X
10  Write(*, '(ES33.25) ') erf(X)
  Write(*, '(ES33.25) ') erfc(X)
12
  Stop
14 End Program TestSpecialFunc

```

## 9.3 Function erfc( $X$ )

### 9.3.1 Description

Compute values of the complementary error function  $\text{erfc}(X)$ . A high precision algorithm by W. J. Candy [2] is used.

### 9.3.2 Arguments

**x:** Single (SP) or double (DP) precision. The point in which we want to know the value of  $\text{erfc}(X)$ .

### 9.3.3 Output

Same type as input.

### 9.3.4 Examples

This program should compute the  $\text{erf}(X)$  and its complementary in an given point.

Listing 9.3: Computing the complementary Error Function.

```

1  Program TestSpecialFunc
2
3  USE NumTypes
4  USE SpecialFunc
5
6  Real (kind=DP) :: X
7
8  Write(*,*) 'Enter a number: '
9  Read(*,*)X
10 Write(*, '(ES33.25) ') erf(X)
11 Write(*, '(ES33.25) ') erfc(X)
12
13 Stop
14 End Program TestSpecialFunc

```

## 9.4 Function inverf(X)

### 9.4.1 Description

Compute values of the inverse error function  $\text{inverf}(X)$ . A combination of a rational approximation by Peter John Acklam, and Halley's rational method is used to attain double precision.

### 9.4.2 Arguments

**x:** Single (SP) or double (DP) precision. The point in which we want to know the value of  $\text{inverf}(X)$ .

### 9.4.3 Output

Same type as input.

### 9.4.4 Examples

This program should use the  $\text{inverf}(X)$  function and test it is ok.

Listing 9.4: Computing the Inverse Error Function.

```

1  Program Test
2
3  USE NumTypes
4  USE SpecialFunc
5
6  Real (kind=DP) :: X = 0.6783457843_DP, Y
7
8  Write(*, '(ES33.25) ') inverf(-0.998_DP)
9  Write(*, '(ES33.25) ') inverf(-0.5_DP)
10 Write(*, '(ES33.25) ') inverf(0.988_DP)
11
12 Y = inverf(X)
13 Write(*,*) 'Same number two times!'
14 Write(*, '(ES33.25) ') X
15 Write(*, '(ES33.25) ') erf(Y)
16
17 Stop
18 End Program Test

```

## 9.5 Function Theta(i, z, tau[, Prec])

### 9.5.1 Description

Compute the value of the  $i^{\text{th}}$  Jacobi theta function ( $i = 1, 2, 3, 4$ ) with nome  $q = e^{i\pi\tau}$

$$\vartheta_i(z|\tau) \quad (9.1)$$

For a definition and properties of these functions take a look [3], here we will only say that following the conventions of the cited reference, our Theta functions have quasi-periods  $\pi$  and  $\tau\pi$ .

### 9.5.2 Arguments

- i:** Integer. Which theta function we want to compute.  $i$  must have one of the following values: 1, 2, 3, 4.
- z:** Complex Double Precision (DPC) or Complex Single Precision (SPC). The point in which we want to compute the Theta function.
- tau:** Complex, with the same precision as **z**. is the quasi period of the Theta function. must be in the upper half plane ( $\text{Im}(\tau) > 0$ ).
- Prec:** Real, Optional. If **z** is DPC (SPC), **Prec** must be double precision (single precision). An estimation of the desired precision of the result. The default value is  $1 \times 10^{-3}$

### 9.5.3 Output

If **z** is Double Precision Complex (SPC), the the result will be Double Precision Complex (SPC).

### 9.5.4 Examples

Listing 9.5: Computing the Jacobi Theta functions.

```

2  Program TestSpecialFunc
4  USE NumTypes
4  USE SpecialFunc
6  Complex (DPC) :: Z, tau
8
10 Z = Cmplx(0.546734, 2.76457643, kind=DPC)
10 tau = Cmplx(0.0_DP, 3.76387540_DP)
12 ! Check the quasi-periodicity of the Third
12 ! Jacobi Theta function.
14 Write(*,*) Theta(3, Z, tau)
14 Write(*,*) Theta(3, Z+Cmplx(PI_DP), tau)
16 Write(*,*) Theta(3, Z+PI_DP*tau, tau) * &
16         &exp(PI*IMAG_DPC*tau + 2.0_DP*UNITIMAG_DPC*Z)
18
20 Stop
End Program TestSpecialFunc

```

## 9.6 Function ThetaChar(a, b, z, tau[, Prec])

### 9.6.1 Description

Computes the value of the Theta function with Characteristics  $(a, b)$  and quasi-periods  $(\pi, \pi\tau)$  in the point  $z$ :

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (z|\tau) \quad (9.2)$$

### 9.6.2 Arguments

**a, b:** Complex or Real, Single or double precision. The two characteristics of the Theta function.

**z:** Complex (Single or Double precision). The point in the complex plane.

**tau:** Complex (Single or Double precision). The quasi-period of the theta function. Must have  $(\text{Im}(\tau) > 0)$ .

**Prec:** Real (Single or Double precision). Optional. An estimation of the desired precision of the value of the theta function.

### 9.6.3 Output

Complex Single or Double precision, the same as the input values.

### 9.6.4 Examples

Listing 9.6: Computing the Jacobi Theta functions with characteristics.

```

1  Program TestSpecialFunc
3
4      USE NumTypes
5      USE SpecialFunc
6
7      Real(kind=DP) :: Deriv, X1, X2
8      Complex (DPC) :: Wmas, Wmenos, Z, tau
9      Integer :: q, s
10
11
12      Z = Cmplx(0.546734, 2.76457643, kind=DPC)
13      tau = Cmplx(0.0_DP, 3.76387540_DP)
14
15      Write(*,*) 'Theta 1: '
16      Write(*,*) Theta(1, Z, tau)
17      Write(*,*) -ThetaChar(0.5_DP,0.5_DP, Z, tau)
18      Write(*,*) 'Theta 2: '
19      Write(*,*) Theta(2, Z, tau)
20      Write(*,*) ThetaChar(0.5_DP,0.0_DP, Z, tau)
21      Write(*,*) 'Theta 3: '
22      Write(*,*) Theta(3, Z, tau)
23      Write(*,*) ThetaChar(0.0_DP,0.0_DP, Z, tau)
24      Write(*,*) 'Theta 4: '
25      Write(*,*) Theta(4, Z, tau)
26      Write(*,*) ThetaChar(0.0_DP,0.5_DP, Z, tau)
27
28
29      Stop
30  End Program TestSpecialFunc

```

## 9.7 Function Hermite(n,x[, Dval])

### 9.7.1 Description

Returns the value of the  $n^{\text{th}}$  Hermite polynomial in the point  $X$ . If **Dval** is specified, the value of the Derivative of the  $n^{\text{th}}$  Hermite polynomial in the point  $X$  is also returned.

### 9.7.2 Arguments

**n:** Integer. Which Hermite polynomial wants to compute.

**x:** Real (Single or Double precision). The point in which we want to compute the Polynomial.

**Dval:** Real (Single or Double precision). Optional. If specified, it stores the value of the Derivative of the Polynomials.



### 9.7.3 Output

Real single or double precision (same as input). The value of the  $n^{\text{th}}$  Hermite Polynomial in the point X.

### 9.7.4 Examples

Listing 9.7: Computing the first 31 Hermite numbers.

```

1 Program TestSpecialFunc
2
3   USE NumTypes
4   USE SpecialFunc
5
6   Integer :: q
7
8
9   Write(*,*) 'The first 31 Hermite Numbers '
10  Write(*,*) 'http://www.research.att.com/~njas/sequences/A067994 '
11  Do q = 1, 31
12    Write(*, '(1I4,1ES33.25) ')q, Hermite(q, 0.0_DP)
13  End Do
14
15  Stop
16 End Program TestSpecialFunc

```

## 9.8 Function HermiteFunc(n, x[, Dval])

### 9.8.1 Description

Returns the value of the  $n^{\text{th}}$  Hermite function

$$\frac{1}{\sqrt{n!2^n\sqrt{\pi}}}e^{-x^2/2}H_n(x) \quad (9.3)$$

in the point X. If Dval is specified, the value of the Derivative of the  $n^{\text{th}}$  Hermite function in the point X is also returned.

### 9.8.2 Arguments

**n:** Integer. Which Hermite function wants to compute.

**x:** Real (Single or Double precision). The point in which we want to compute the Polynomial.

**Dval:** Real (Single or Double precision). Optional. If specified, it stores the value of the Derivative of the function.

### 9.8.3 Output

Real single or double precision (same as input). The value of the  $n^{\text{th}}$  Hermite function in the point X.

### 9.8.4 Examples

Listing 9.8: Compute the Hermite functions.

```

2  Program TestSpecialFunc
4      USE NumTypes
4      USE SpecialFunc
6      Real(kind=DP) :: Deriv, X1, X2, Sum
6      Complex (DPC) :: Wmas, Wmenos, Z, tau
8      Integer :: q, s
10
10     Write(*,*) 'A (really bad) proof of orthonormality: '
12     X1 = -10.0_DP
12     Sum = 0.0_DP
14     Do q = -1000, 1000
14         Sum = Sum + HermiteFunc(6,X1)**2
16         X1 = X1 + 0.01_DP
16     End Do
18
18     Write(*, '(1ES33.25) ') Sum*0.01_DP
20
20     Stop
22 End Program TestSpecialFunc

```

## 9.9 Function Basis(X1, X2, n, s, q, itau[, Prec])

### 9.9.1 Description

Return the value of the basis elements of the Hilbert space  $\mathcal{H}_q$  of quasi-periodic functions

$$|n, s\rangle = e^{i\frac{f}{2}x_1x_2} \sum_{k \in s+q\mathbb{Z}} e^{-u^2/2} H_n(u) e^{2\pi i k \frac{x_1}{l_1}} \quad n = 0, \dots, \infty; s = 1, \dots, q \quad (9.4)$$

defined in the appendix of [1] (look there for more details and properties).

### 9.9.2 Arguments

**X1,X2:** Real (Single or Double precision). The point in the Torus.

**n,s:** Integer. Specify which element of the basis.

**q:** Integer. Specify the Hilbert space  $\mathcal{H}_q$ .

**itau:** Real (Single or Double precision). Specify the ratio of quasi-periods:  $\text{itau} = l_2/l_1$ .

**Prec:** Real (Single or Double precision). Optional. An estimation of the desired precision.

### 9.9.3 Output

Complex single or double precision, depends of the input arguments.

### 9.9.4 Examples

Listing 9.9: Computing the bassi of a special Hilbert space (details in [1]).

```

2  Program TestSpecialFunc
4
6  USE NumTypes
4  USE SpecialFunc
6
6  Real(kind=DP) :: X1, X2
6  Complex (DPC) :: Wmas, Wmenos,
8  Integer :: I, q, s
10
12  Write(*,*) 'Looking at the quasi-periodicity properties:'
12  X1 = 0.97834D0
12  X2 = 0.873873D0
14  q = 4
14  s = 3
16  Do I = 0, 8
18      Wmas = Basis( X1, X2+1.0_DP, I, s, q, 1.0_DP, 1.0D-15) * &
18      & exp(PLIMAG_DPC*X1*q)
20      Wmenos = Basis( X1+1.0_DP, X2, I, s, q, 1.0_DP, 1.0D-15) * &
20      & exp(-PLIMAG_DPC*X2*q)
22      Write(*, '(1I3,2ES33.25)')I, Basis( X1, X2, I, s, q, 1.0_DP, 1.0D-15)
22      Write(*, '(1I3,2ES33.25)')I, Wmas
22      Write(*, '(1I3,2ES33.25)')I, Wmenos
24  End Do
26
28  Stop
28  End Program TestSpecialFunc

```

## 9.10 Function Factorial(N)

### 9.10.1 Description

Compute  $N!$ . Better (faster and more accurate for small numbers) than the use of `GammaLn` to compute the factorial of a number.

### 9.10.2 Arguments

**N:** Integer. The number to compute the factorial.

### 9.10.3 Output

A real Double precision (DP).

### 9.10.4 Examples

This program should write the factorial of the first 100 numbers.

Listing 9.10: Computing the factorial.

```

2  Program TestSpecialFunc
4
6  USE NumTypes
4  USE SpecialFunc
6
8  Integer :: q
8
10 Do q = 1, 100
10     Write(*, '(1A13,1I4,1A3,1ES33.25) ') 'Factorial of: ', q, ' = ', &
12         & Factorial(q)
12 End Do
14
16 Stop
16 End Program TestSpecialFunc

```

## 9.11 Function Legendre(l, m, X)

### 9.11.1 Description

Computes the value at  $X$  of the  $(l, m)$  Legendre polynomial  $P_l^m(X)$ .

### 9.11.2 Arguments

**l,m:** Integer. Specifies which Legendre polynomial we want to compute.

**x:** Real single or double precision. The point at which we want to compute the polynomial.

### 9.11.3 Output

A real single or double precision (same as **X**).

### 9.11.4 Examples

This program writes the values of the Legendre polynomials  $P_l^m(0.5)$  for  $0 \leq m \leq l$ .

Listing 9.11: Computing some legendre polynomials.

```

2  Program TestLeg
4
6  USE NumTypes
4  USE SpecialFunc
6
8  Real (kind=DP) :: Y
8
10 Read(*,*)L
10
12 Do I = 0, L

```

```

12      Y = Legendre(L, I, 0.50_DP)
13      Write(*,*)I, Y, Legendre(L, I, 0.5_SP)
14  End Do
15
16  Stop
End Program TestLeg

```

## 9.12 Function SphericalHarmonics(l, m, th,ph)

### 9.12.1 Description

Computes the value at  $(\theta, \phi)$  of the  $(l, m)$  spherical harmonic  $Y_l^m(\theta, \phi)$ .

### 9.12.2 Arguments

**l,m:** Integer. Specifies which spherical harmonic we want to compute.

**th,ph:** Real single or double precision.  $(\theta, \phi)$  coordinates of the point at which we want to compute the value of the spherical harmonic.

### 9.12.3 Output

A complex single or double precision (same as **th**).

### 9.12.4 Examples

This program checks the relation  $Y_l^m(\theta, \phi) = (-)^m Y_l^{*-m}(\theta, \phi)$  for  $\theta = 0.5, \phi = 0.35$ .

Listing 9.12: Computing some spherical harmonics.

```

1  Program TestSph
2
3
4      USE NumTypes
5      USE SpecialFunc
6
7      Complex (kind=DPC) :: Y
8
9      Read(*,*)L
10
11     Do I = 0, L
12         Y = SphericalHarmonic(L, I, 0.50_DP, 0.35_DP)
13         Write(*, '(1A,1I4,1A,1I4)') 'Spherical Harmonic l=', L, ' m=+', I
14         Write(*,*) ' +m ', Y
15         Write(*,*) ' -m ', SphericalHarmonic(L, -I, 0.5_DP, 0.35_DP)
16     End Do
17
18     Stop
19 End Program TestSph

```



# Ten

---

## MODULE Statistics

---

This is the documentation of the `MODULE Statistics`, a set of `FORTRAN 90` routines to perform statistical description of data. This module make use of the `MODULE NumTypes`, `MODULE Constants`, `MODULE Error` and `MODULE Linear` so please read the documentation of these modules *before* reading this.

### 10.1 Function Mean(X)

#### 10.1.1 Description

Compute the mean value of the numbers stored in `X(:)`.

#### 10.1.2 Arguments

`X(:)`: Double (DP) or simple (SP) precision one dimensional array. The values whose mean we want to compute.

#### 10.1.3 Output

A real double or simple precision (same type as the input). The mean of the values.

#### 10.1.4 Examples

Listing 10.1: Computing the Mean of a vector of numbers.

```
1 Program Tests
3   USE NumTypes
3   USE Error
5   USE Statistics
7   Integer, Parameter :: Nmax = 100
7   Real (kind=DP) :: X(Nmax)
9
9   CALL Random_Number(X)
11  Write(*, '(ES33.25) ') Mean(X)
```

```

13      Stop
      End Program Tests

```

## 10.2 Function Median(X)

### 10.2.1 Description

Compute the median value of the numbers stored in `X(:)`.

### 10.2.2 Arguments

`X(:)`: Double (DP) or simple (SP) precision one dimensional array. The values whose median we want to compute.

### 10.2.3 Output

A real double or simple precision (same type as the input). The median of the values.

### 10.2.4 Examples

Listing 10.2: Computing the Median of a vector of numbers.

```

      Program Tests
2
      USE NumTypes
4      USE Error
      USE Statistics
6
      Integer, Parameter :: Nmax = 5
8      Real (kind=SP) :: X(Nmax) = (/1.0, 1.0, 2.0, 4.0, 1.5/)
10
      Write(*, '(ES33.25) ') Median(X)
12
      Stop
14      End Program Tests

```

## 10.3 Function WMedian(X, w)

### 10.3.1 Description

Compute the weighted median of the numbers stored in `X(:)` with weights `w(:)`.

### 10.3.2 Arguments

`X(:)`: Double (DP) or simple (SP) precision one dimensional array. The values whose median we want to compute.

`w(:)`: Double (DP) or simple (SP) precision one dimensional array. The weights.



### 10.3.3 Output

A real double or simple precision (same type as the input). The weighted median of the values.

### 10.3.4 Examples

Listing 10.3: Computing the Weighted Median of a vector of numbers.

```

1 Program Tests
2
3   USE NumTypes
4   USE Error
5   USE Statistics
6
7   Integer, Parameter :: Nmax = 5
8   Real (kind=SP) :: X(Nmax) = (/1.0, 1.0, 2.0, 4.0, 1.5/)
9   Real (kind=SP) :: w(Nmax) = (/10.0, 2.0, 3.0, 4.0, 1.5/)
10
11   Write(*, '(ES33.25) ') WMedian(X, w)
12
13
14   Stop
15 End Program Tests

```

## 10.4 Function WPercentile(X, w, p)

### 10.4.1 Description

Compute the weighted percentile  $p$  of the numbers stored in  $X(:)$  with weights  $w(:)$ .

### 10.4.2 Arguments

$X(:)$ : Double (DP) or simple (SP) precision one dimensional array. The values whose median we want to compute.

$w(:)$ : Double (DP) or simple (SP) precision one dimensional array. The weights.

$p$ : Double (DP) or simple (SP) precision number. The percentile (should be between 0 and 100).

### 10.4.3 Output

A real double or simple precision (same type as the input). The weighted percentile  $p$  of the values.

### 10.4.4 Examples

Listing 10.4: Computing the Weighted Median in two ways.

```

1 Program Tests

```

```

3  USE NumTypes
   USE Error
5  USE Statistics

7  Integer, Parameter :: Nmax = 5
   Real (kind=SP) :: X(Nmax) = (/1.0, 1.0, 2.0, 4.0, 1.5/)
9  Real (kind=SP) :: w(Nmax) = (/10.0, 2.0, 3.0, 4.0, 1.5/)

11 Write(*, '(ES33.25) ') WMedian(X, w)
   Write(*, '(ES33.25) ') WPercentile(X, w, 50.0_DP)
13
15 Stop
End Program Tests

```

## 10.5 Subroutine WConfInt(X, w, Xmin, Xmax)

### 10.5.1 Description

Compute the weighted 16<sup>th</sup> and 84<sup>th</sup> percentiles of the numbers stored in `X(:)` with weights `w(:)`, and return the values in `Xmin` and `Xmax`.

### 10.5.2 Arguments

`X(:)`: Double (DP) or simple (SP) precision one dimensional array. The values whose median we want to compute.

`w(:)`: Double (DP) or simple (SP) precision one dimensional array. The weights.

`Xmin`: Double (DP) or simple (SP) precision number. Output. The 16<sup>th</sup> percentile.

`Xmax`: Double (DP) or simple (SP) precision number. Output. The 84<sup>th</sup> percentile.

### 10.5.3 Examples

Listing 10.5: Computing the 1 sigma confidence interval in two ways.

```

Program Tests

2  USE NumTypes
   USE Error
4  USE Statistics

6

   Integer, Parameter :: Nmax = 500
8  Real (kind=SP) :: X(Nmax), w(Nmax), X1, X2

10 CALL Normal(X)
   CALL Random_Number(w)

12
   Write(*, '(ES33.25) ') WPercentile(X, w, 16.0_DP)
14  Write(*, '(ES33.25) ') WPercentile(X, w, 84.0_DP)

```

```

16  CALL WConfInt(X, W, X1, X2)
    Write(*, '(ES33.25) ')X1, X2
18
    Stop
End Program Tests

```

## 10.6 Function Var(X)

### 10.6.1 Description

Compute the variance of a vector of numbers  $X(:)$

### 10.6.2 Arguments

$X(:)$ : Double (DP) or simple (SP) precision one dimensional array. The values whose variance we want to compute.

### 10.6.3 Output

A real double or simple precision (same type as the input). The variance of the values.

### 10.6.4 Examples

Listing 10.6: Computing the Variance of a set of numbers.

```

1  Program Tests
3
4  USE NumTypes
5  USE Error
6  USE Statistics
7
8  Integer, Parameter :: Nmax = 100, Npinta = 100, Npar = 4
9  Real (kind=DP) :: X(Nmax), Y(Nmax), Yer(Nmax), &
10     & Coef(Npar), Cerr(Npar), Corr, Xd(Nmax,2)
11
12  CALL Random_Number(X)
13  Write(*, '(ES33.25) ')Var(X)
14
15  Stop
17 End Program Tests

```

## 10.7 Function Stddev(X)

### 10.7.1 Description

Computes the standard deviation of the numbers stored in the vector  $X(:)$ .

### 10.7.2 Arguments

**x(:):** Double (DP) or simple (SP) precision one dimensional array. The values whose standard deviation we want to compute.

### 10.7.3 Output

Real Single or Double precision, the same as the input values. The standard deviation of the values.

### 10.7.4 Examples

Listing 10.7: Computing the standard deviation.

```

1 Program Tests
3   USE NumTypes
3   USE Error
5   USE Statistics
7   Integer, Parameter :: Nmax = 100, Npinta = 100, Npar = 4
7   Real (kind=DP) :: X(Nmax), Y(Nmax), Yer(Nmax), &
9       & Coef(Npar), Cerr(Npar), Corr, Xd(Nmax,2)
11
11  CALL Random_Number(X)
13  Write(*, '(ES33.25) ') Stddev(X)
15
15  Stop
17 End Program Tests

```

## 10.8 Function Moment(X, k)

### 10.8.1 Description

Returns the  $k^{th}$  moment of the values stored in the vector **x(:)**.

### 10.8.2 Arguments

**x(:):** Real (Single or Double precision). The numbers whose  $k^{th}$  moment we want to compute.

**k:** Integer. Which moment we want to compute.

### 10.8.3 Output

Real single or double precision. The  $k^{th}$  moment of the numbers.

### 10.8.4 Examples

Listing 10.8: Computing the  $k^{\text{th}}$  moment of a data set.

```

1 Program Tests
3   USE NumTypes
   USE Error
5   USE Statistics

7   Integer, Parameter :: Nmax = 100, Npinta = 100, Npar = 4
   Real (kind=DP) :: X(Nmax), Y(Nmax), Yer(Nmax), &
9       & Coef(Npar), Cerr(Npar), Corr, Xd(Nmax,2)

11
   CALL Random_Number(X)
13   Write(*,*) 'We should obtain the same numbers twice: '
   Write(*, '(ES33.25) ') Moment(X,2), Var(X)
15
   Stop
17 End Program Test

```

## 10.9 Subroutine Histogram(Val, Ndiv, Ntics, Vmin, Vmax, h)

### 10.9.1 Description

Given a set of points `Val(:)`, this routine makes `Ndiv` divisions between the minimum and the greatest value of `Val` (respectively returned in `Vmin` and `Vmax`), each of size `h` (also returned), and returns in the integer vector `Nticks(:)` the number of points that are in each interval.

### 10.9.2 Arguments

**Val(:):** Real (Single or Double precision) one dimensional array. The original values.

**Ndiv:** Integer. The number of divisions.

**Nticks:** Integer one dimensional array. `Ndiv(I)` Tells how many points of `Val(:)` are between `Vmin + (I - 1)h` and `Vmin + Ih`.

**Vmin, Vmax:** Real (Single or Double precision). The minimum and maximum values of `Val`.

**h:** Real (Single or Double precision). After calling the routine has the step of the division.

### 10.9.3 Examples

Listing 10.9: Making Histograms.

```

1 Program Tests
3   USE NumTypes
   USE Error

```

```

5  USE Statistics

7  Integer, Parameter :: Nmax = 500000, Npinta = 100, Npar = 4, Ndiv = 100
Real (kind=DP) :: X(Nmax), Y(Nmax), Yerr(Nmax), &
9      & Coef(Npar), Cerr(Npar), Corr, Xd(Nmax,2), &
      & Xmin, Xmax, h, Xac
11 Integer :: Ntics(Ndiv)

13 CALL Normal(X, 1.23_DP, 0.345_DP)
CALL Histogram(X, Ndiv, Ntics, Xmin, Xmax, h)

15
16 Do I = 1, Ndiv
17     Xac = Xmin + (I-1)*h
      Write(*, '(1ES33.25,1I)') Xac, Ntics(I)
19 End Do

21 Stop
End Program Tests

```

## 10.10 Subroutine LinearReg(X, Y, Yerr, [Func], Coef, Cerr, ChisqrV)

### 10.10.1 Description

Given a set of points  $X(:)$  and  $Y(:)$ , this routine performs a linear fit to a set of functions defined by **Func**.

$$Y = \sum_i a_i f_i(X)$$

This routine also performs multi-dimensional fitting, in which case the points are specified as  $X(:, :)$ , where the first argument tells which point, and the second which variable.

### 10.10.2 Arguments

**X(:, :)**: Real single or double precision one dimensional array (for a one dimensional fit) or two dimensional array (for a multidimensional fit). The independent variables. For a multidimensional fit, the first argument tells which point, and the second which variable. So the size of the array should be **X(Npoints, Ndim)**.

**Y(:)**: Real single or double precision one dimensional array. The dependent variable.

**Yerr(:, :)**: Real single or double precision one or two dimensional array. If a one dimensional array is inserted, they are the errors of the points (if you don't have them, you should put all of them to some non-zero value). If a two dimensional array is given, it is treated as the correlation matrix.

**Func**: Optional. This routine define the functions to fit. An interface like this should be provided

Interface

```

Function Func(Xx, i)

  USE NumTypes

  Real (kind=SP), Intent (in) :: Xx
  Integer, Intent (in) :: i
  Real (kind=SP) :: Func

End Function Func
End Interface

```

if you want to perform a one dimensional fitting, and like this

```

Interface
  Function Func(Xx, i)

    USE NumTypes

    Real (kind=SP), Intent (in) :: Xx(:)
    Integer, Intent (in) :: i
    Real (kind=SP) :: Func

  End Function Func
End Interface

```

if it is a multidimensional fitting. Since you are making a fitting to a function of the type

$$Y = \sum_i a_i f_i(X)$$

the values  $f_i(X)$  are given by this function as `Func(X, I)`. If the functions are not specified (i.e. you don't put this argument), a fit to a polynomial is made (this only work for one-dimensional fittings).

**Coef(:):** Real single or double precision one dimensional array. The parameters that you want to determine.

**Cerr(:):** Real single or double precision one dimensional array. The errors in the parameters.

**ChiSqr:** Real single or double precision. The  $\chi^2$  per degree of freedom of the fit.

### 10.10.3 Examples

Listing 10.10: Doing linear regressions.

```

Program Tests
2
  USE NumTypes
4  USE Error
  USE Statistics

```

```

6      Integer, Parameter :: Nmax = 200, Npinta = 100, Npar = 4, Ndiv = 100
8      Real (kind=DP) :: X(Nmax), Y(Nmax), Yer(Nmax), &
          & Coef(Npar), Cerr(Npar), Corr, Xd(Nmax,2), &
10         & Xmin, Xmax, h, Xac
12      Integer :: Ntics(Ndiv)
14
15      Interface
16         Function Fd(Xx, i)
17
18            USE NumTypes
19
20            Real (kind=DP), Intent (in) :: Xx(:)
21            Integer, Intent (in) :: i
22            Real (kind=DP) :: Fd
23
24         End Function Fd
25      End Interface
26
27      CALL Random_Number(Xd)
28      Xd(:, :) = 10.0_DP*(Xd(:, :) - 0.8_DP)
29
30      CALL Normal(Yer, 0.0_DP, 1.0E-3_DP)
31      Y(:) = 12.34_DP*Xd(:,1)*sin(Xd(:,2)) - 2.23_DP + &
          & 0.67_DP*Xd(:,1)**2*Xd(:,2) + 0.23_DP*Xd(:,1) + Yer(:)
32
33
34      CALL LinearReg(Xd, Y, Yer, Fd, Coef, Cerr, Corr)
35
36      ! This should print the adjusted parameters,
37      ! that have values: 12.34, -2.23, 0.67, 0.23
38      Do I = 1, Npar
39         Write(*, '(2ES33.25)') Coef(I), Cerr(I)
40      End Do
41
42      ! This prints the ChiSqr, that should be very
43      ! close to 1.
44      Write(*, '(1A,1ES33.25)') 'ChiSqr of the Fit: ', Corr
45
46
47      Stop
48      End Program Tests
49
50      ! *****
51      ! *
52      Function Fd(X, i)
53      ! *
54      ! *****
55
56      USE NumTypes
57
58      Real (kind=DP), Intent (in) :: X(:)

```



```

60  Integer, Intent (in) :: i
    Real (kind=DP) :: Fd

62  If (I==1) Then
    Fd = 1.0_DP
64  Else If (I==2) Then
    Fd = X(1)*sin(X(2))
66  Else If (I==3) Then
    Fd = X(1)**2*X(2)
68  Else If (I==4) Then
    Fd = X(1)
70  End If

72  Return
End Function FD

```

## 10.11 Subroutine NonLinearReg(X, Y, Yerr, Func, Coef, Cerr, ChisqrV)

### 10.11.1 Description

Given a set of points  $X(:)$  and  $Y(:)$ , this routine performs a non-linear fit to a set of functions defined by **Func**.

This routine also performs multi-dimensional fitting, in which case the points are specified as  $X(:, :)$ , where the first argument tells which point, and the second which variable.

This routine uses the Levenberg-Marquardt algorithm to perform the optimisation<sup>1</sup>.

### 10.11.2 Arguments

**X(:, :)**: Real single or double precision one dimensional array (for a one dimensional fit) or two dimensional array (for a multidimensional fit). The independent variables. For a multidimensional fit, the first argument tells which point, and the second which variable. So the size of the array should be  $X(Npoints, Ndim)$ .

**Y(:)**: Real single or double precision one dimensional array. The dependent variable.

**Yerr(:, :)**: Real single or double precision one or two dimensional array. If a one dimensional array is inserted, they are the errors of the points (if you don't have them, you should put all of them to some non-zero value). If a two dimensional array is given, it is treated as the correlation matrix.

**Func**: This routine define the functions to fit. An interface like this should be provided

```

Interface
  Subroutine Func(X, Cf, Valf, ValD)

```

```

  USE NumTypes

```

<sup>1</sup>[http://en.wikipedia.org/wiki/Levenberg-Marquardt\\_algorithm](http://en.wikipedia.org/wiki/Levenberg-Marquardt_algorithm)

```

      Real (kind=SP), Intent (in) :: X, Cf(:)
      Real (kind=SP), Intent (out) :: Valf, ValD(Size(Cf))

      End Subroutine Func
End Interface

```

if you want to perform a one dimensional fitting, and like this

```

Interface
  Subroutine Func(X, Cf, Valf, ValD)

    USE NumTypes

    Real (kind=SP), Intent (in) :: X(:), Cf(:)
    Real (kind=SP), Intent (out) :: Valf, ValD(Size(Cf))

  End Subroutine Func
End Interface

```

if it is a multidimensional fitting.

This routine returns the values of the function at  $X$  for some values of the parameters given in **Cf** in the variable **Valf**, and a vector with the derivatives (respect with the parameters) in **ValD**(:).

**Coef**(:): Real single or double precision one dimensional array. Output. The parameters that you want to determine.

**Cerr**(:): Real single or double precision one dimensional array. Output. The errors in the parameters.

**ChiSqr**: Real single or double precision. The  $\chi^2$  per degree of freedom of the fit.

### 10.11.3 Examples

In this example we will fit some generated data, with a Normal noise to the function

$$f(x_1, x_2; a, b) = \sin(ax_1) + bx_1x_2$$

We will generate the data with the values  $a = 2.0$  and  $b = 0.2$ , so our fitting routine *should* return this values within errors.

The derivatives of the function (repect the parameters  $a$  and  $b$ ), as well as the value of function are given by the user routine **Func**. The derivatives are:

$$\begin{aligned} \frac{\partial f(x_1, x_2; a, b)}{\partial a} &= x_1 \cos ax_1 \\ \frac{\partial f(x_1, x_2; a, b)}{\partial b} &= x_1x_2 \end{aligned}$$

Listing 10.11: Doing non linear regressions.

```

1  Program NLFit
3      USE NumTypes
3      USE Statistics
5
5      Interface
7          Subroutine Func(X, Cf, Valf, ValD)
9
9              USE NumTypes
11
11              Real (kind=DP), Intent (in) :: X(:), Cf(:)
11              Real (kind=DP), Intent (out) :: Valf, ValD(Size(Cf))
13
13              End Subroutine Func
15      End Interface
17
17      Integer, Parameter :: Np = 20, Ndim = 2
17      Real (kind=DP) :: X(Np, Ndim), Y(Np), Ye(Np), Co(2), Vd(2), Ce(2), Ch
19
19      ! First Fill the data
21      CALL Random_Number(X)
21      X = 2.0_DP*(X - 0.5_DP)
23      Co(1) = 2.0_DP
23      Co(2) = 0.2_DP
25      CALL Normal(Ye, 0.0_DP, 0.5_DP)
25      Do I = 1, Np
27          CALL Func(X(I,:), Co, Y(I), Vd)
27          Y(I) = Y(I) + Ye(I)
29      End Do
31
31      ! Now Perform the non linear fit
33      Co = 1.0_DP
33      CALL NonLinearReg(X, Y, Abs(Ye), Func, Co, Ce, Ch)
35      Do I = 1, Npar
35          Write(*, '(1A,100ES33.25)') 'Parameter and error: ', Co(I), Ce(I)
37      End Do
37      Write(*, '(1A,100ES33.25)') &
39      & 'Chi Square per degree of freedom of the Fit: ', Ch
41
41      Stop
43  End Program NLFit
45
45  Subroutine Func(X, Cf, Valf, ValD)
47
47      USE NumTypes
49
49      Real (kind=DP), Intent (in) :: X(:), Cf(:)
49      Real (kind=DP), Intent (out) :: Valf, ValD(Size(Cf))
51

```

```

53  Valf = Sin(Cf(1)*X(1)) + Cf(2)*X(1)*X(2)
    ValD(1) = X(1)*Cos(Cf(1)*X(1))
55  ValD(2) = X(1)*X(2)
57  End Subroutine Func

```

## 10.12 Subroutine SetLuxLevel(Ilevel)

### 10.12.1 Description

This function changes the behaviour of the “intrinsic” `RandomNumber` subroutine. To call the intrinsic FORTRAN 90 routine, you should set the Luxury level to zero. Other values makes the `RandomNumber` subroutine use the Marsaglia and Zaman algorithm modified by M. Lüscher [4]. This part of the code has some inspiration in the FORTRAN 90 implementation of Allan Miller.

**Note:** This routine affects the way all the other routines that use pseudo random number generators works. That is to say, all the routines of this module that follow this one.

### 10.12.2 Arguments

**Ilevel:** Integer. Set the way the random number generator works. We have the following options:

- **Ilevel = 0.** The default intrinsic FORTRAN 90 routine is used. This is usually considered a *bad*, non portable pseudo-random number generator, that you do not want to use when correlations are important.
- **Ilevel = 1.** This is the original Marsaglia and Zaman algorithm. Probably better than the intrinsic procedure, but still with large correlations.
- **Ilevel = 2.** The M. Lüscher modified version of the algorithm with  $p = 48$ . Still fail many tests, but a very good pseudo-number generator for many things.
- **Ilevel = 3.** The M. Lüscher modified version of the algorithm with  $p = 97$ . Theoretically still defective (fail the serial correlation test).
- **Ilevel = 4.** The M. Lüscher modified version of the algorithm with  $p = 218$ . Any theoretically possible correlation has a very small probability of being observed. A value very similar to this has been used for large scale high precision lattice gauge theory computations. This corresponds with the Lüscher recommended level 0.
- **Ilevel = 5.** The M. Lüscher modified version of the algorithm with  $p = 404$ . Good for anything you need the pseudo-random numbers (but cryptography, of course). It seems completely pointless to use values of  $p$  higher than this. This corresponds with the Lüscher recommended level 1. This is the **default value**.
- **Ilevel = 6.** The M. Lüscher modified version of the algorithm with  $p = 794$ . Good for anything you need the pseudo-random numbers (but cryptography, of course). Probably waste of computer resources. This corresponds with the Lüscher recommended level 2.
- Other values of **Ilevel** ( $> 24$ ). Sets the value of  $p$  in the M. Lüscher modified version of the algorithm. Don't use it, unless you know what you are doing.

### 10.12.3 Examples

Listing 10.12: Setting the Luxury level of the pseudo random number generator.

```

1 Program Tests
3   USE NumTypes
   USE Statistics
5
   Integer, Parameter :: NN = 70
7   Real (kind=SP) :: rr(NN)
9
   CALL SetLuxLevel(5)
   CALL Random_Number(rr)
11  Write(*,*) rr
13
   Stop
15 End Program Tests

```

## 10.13 Subroutine PutLuxSeed([ISeed(25)])

### 10.13.1 Description

Restarts the position in the Luxury pseudo-random number generator.

### 10.13.2 Arguments

**ISeed(25):** Integer one dimensional array of 25 elements. Optional. If present, restarts the generator from an output of `GetLuxSeed`. If not present, initialises the random number generator.

### 10.13.3 Examples

Listing 10.13: Using a previously saved point in the generating process.

```

1 Program Tests
3   USE NumTypes
   USE Statistics
5
   Integer, Parameter :: NN = 70
7   Real (kind=SP) :: rr(NN)
   Integer :: Sd(25)
9
   Open(Unit=69, File="seed.dat")
   Read(69,*) Sd
13  CALL PutLuxSeed(Sd)
   CALL Random_Number(rr)
15  Write(*,*) rr

```

```

17      Stop
19 End Program Tests

```

## 10.14 Subroutine GetLuxSeed(ISeed(25))

### 10.14.1 Description

Saves the position in the Luxury pseudo-random number generator.

### 10.14.2 Arguments

**ISeed(25):** Integer one dimensional array of 25 elements. Saves the position in the random number generator.

### 10.14.3 Examples

Listing 10.14: Saving a point in the generating process.

```

1 Program Tests
3   USE NumTypes
4   USE Statistics
5
6   Integer, Parameter :: NN = 70
7   Real (kind=SP) :: rr(NN)
8   Integer :: Sd(25)
9
10
11  CALL Random_Number( rr )
12  CALL GetLuxSeed(Sd)
13  Open(Unit=69, File="seed.dat")
14  Write(69,*)Sd
15
16
17  Stop
End Program Tests

```

## 10.15 Subroutine Normal(X, [Rm], [Rsig])

### 10.15.1 Description

Fills **X(:)** with numbers from a normal distribution with mean **Rm**, and standard deviation **Rsig**. The parameters **Rm** and **Rsig** are optional. If they are not given the mean will be 0, and the standard deviation 1.

### 10.15.2 Arguments

**X(:):** Real (Single or Double precision) one dimensional array. A vector that will be filled with numbers according to the normal distribution.

**Rm:** Real (Single or Double precision), Optional. The mean of the normal distribution. If not present the default value is 0.

**Rsig:** Real (Single or Double precision), Optional. The standard deviation of the normal distribution. If not present the default value is 1.

### 10.15.3 Examples

Listing 10.15: Obtaining numbers with a normal distribution.

```

1  Program Tests
2
3  USE NumTypes
4  USE Error
5  USE Statistics
6
7  Integer, Parameter :: Nmax = 100
8  Real (kind=DP) :: X(Nmax)
9
10
11 CALL Normal(X, 1.23_DP, 0.345_DP)
12 ! Now compute the mean and standard deviation of the data
13 Write(*,*) 'We should obtain 1.23 and 0.345: '
14 Write(*, '(ES33.25) ') Mean(X), Stddev(X)
15
16
17 Stop
18 End Program Tests

```

## 10.16 Subroutine Levy(X, $\alpha$ , [c], [ $\beta$ ], [ $\mu$ ])

### 10.16.1 Description

Fills **X(:)** with numbers from a Levy stable distribution with mean  $\mu$ , scale  $c$  and exponent  $\alpha$ . The probability distribution is defined via

$$\rho(x) = \frac{1}{\pi} \Re \int_{-\infty}^{+\infty} dt \exp \left\{ -it(x - \mu) - c|t|^\alpha \left[ 1 + i \frac{t\beta}{|t|} \tan \left( \frac{\pi\alpha}{2} \right) \right] \right\} \quad (10.1)$$

For  $\alpha = 1$  the distribution reduces to the Cauchy distribution. For  $\alpha = 2$  it is a Gaussian distribution with  $\sigma = \sqrt{2}c$  and mean  $\mu$ .

The algorithm only works for  $0 < \alpha \leq 2$ .

### 10.16.2 Arguments

**X(:):** Real (Single or Double precision) one dimensional array. A vector that will be filled with numbers according to the normal distribution.

$\alpha$ : Real (Single or Double precision). The characteristic exponent (or Levy index).

$c$ : Real (Single or Double precision), Optional. The scale factor.

$\beta$ : Real (Single or Double precision), Optional. The skewness parameter.

$\mu$ : Real (Single or Double precision), Optional. The translation factor.

### 10.16.3 Examples

Listing 10.16: Obtaining numbers with a Levy skew stable distribution.

```

1 Program Tests
2
3   USE NumTypes
4   USE Error
5   USE Statistics
6
7   Integer, Parameter :: Nmax = 100
8   Real (kind=DP) :: X(Nmax)
9
10
11  CALL Levy(X, 1.23_DP)
12  ! Now compute the mean and standard deviation of the data
13  Write(*,*) 'We should obtain 0.0 and something without sense!: '
14  Write(*, '(ES33.25) ') Mean(X), Stddev(X)
15
16
17  Stop
18 End Program Tests

```

## 10.17 Subroutine FishTipp(X, Rm, Rb)

### 10.17.1 Description

Fills  $X(:)$  with numbers from a Fisher-Tippett distribution with parameters<sup>2</sup>  $R_m$ , and  $2R_b^2$ .

### 10.17.2 Arguments

$X(:)$ : Real (Single or Double precision) one dimensional array. A vector that will be filled with numbers according to the normal distribution.

$R_m$ : Real (Single or Double precision).

$R_b$ : Real (Single or Double precision).

<sup>2</sup>More info about this distribution in the Wikipedia: [http://en.wikipedia.org/wiki/Fisher-Tippett\\_distribution](http://en.wikipedia.org/wiki/Fisher-Tippett_distribution)



### 10.17.3 Examples

Listing 10.17: Obtaining numbers with a Fisher-Tippet distribution.

```

1 Program Tests
2
3   USE NumTypes
4   USE Error
5   USE Statistics
6
7   Integer, Parameter :: Nmax = 100
8   Real (kind=DP) :: X(Nmax)
9
10
11  CALL FishTipp(X, 2.00_DP, 1.00_DP)
12  ! Now compute the mean and standard deviation of the data
13  Write(*,*) 'We should obtain 2.57721... and 1.2782...: '
14  Write(*, '(ES33.25) ') Mean(X), Stddev(X)
15
16
17  Stop
18 End Program Tests

```

## 10.18 Subroutine Laplace(X, Rm, Rb)

### 10.18.1 Description

Fills X(:) with numbers from a Laplace distribution with mean Rm, and variance  $2Rb^2$ .

### 10.18.2 Arguments

**X(:):** Real (Single or Double precision) one dimensional array. A vector that will be filled with numbers according to the Laplace distribution.

**Rm:** Real (Single or Double precision). The mean of the Laplace distribution.

**Rb:** Real (Single or Double precision). The width of the Laplace distribution (i.e. The variance is  $2Rb^2$ ).

### 10.18.3 Examples

Listing 10.18: Obtaining numbers with a Laplace distribution.

```

1 Program Tests
2
3   USE NumTypes
4   USE Error
5   USE Statistics
6
7   Integer, Parameter :: Nmax = 100
8   Real (kind=DP) :: X(Nmax)

```

```

10      CALL Laplace(X, 1.23_DP, 1.0_DP)
12      ! Now compute the mean and standard deviation of the data
13      Write(*,*) 'We should obtain 1.23 and sqrt(2): '
14      Write(*, '(ES33.25) ')Mean(X), Stddev(X)

16
17      Stop
18  End Program Tests

```

## 10.19 Subroutine/Function Irand([Irnd], N, M)

### 10.19.1 Description

If present, fills `Irnd(:)` with random integer numbers between `N` and `M` with an uniform distribution. If `Irnd(:)` is not present returns a integer random number between `N` and `M`.

### 10.19.2 Arguments

`Irnd(:)`: Integer, Optional. A vector that will be filled with integer numbers according to a uniform distribution.

`N`: Integer. The minimum number that we can obtain.

`M`: Integer. The maximum number that we can obtain.

### 10.19.3 Examples

Listing 10.19: Obtaining integer random numbers.

```

Program Tests
2
3      USE NumTypes
4      USE Error
5      USE Statistics
6
7      Integer, Parameter :: Nmax = 100
8      Integer :: Irnd(Nmax)
9
10
11     CALL Irand(Irnd, 0, 1)
12     ! Now compute the mean
13     Write(*,*) 'We should obtain 0.5: '
14     Write(*, '(ES33.25) ')Mean(Real(Irnd(:), kind=DP))
15
16
17     Stop
18 End Program Tests

```

## 10.20 Subroutine Permutation(Id<sub>x</sub>)

### 10.20.1 Description

Returns a random permutation of  $N$  elements. It uses the Knuth shuffle algorithm<sup>3</sup>.

### 10.20.2 Arguments

**Id<sub>x</sub>(:):** Integer one dimensional array. Output. The random permutation.

Listing 10.20: Obtaining a permutation.

```

1  Program Tests
2
3  USE NumTypes
4  USE Error
5  USE Statistics
6
7  Integer, Parameter :: Nmax = 10
8  Integer :: Id(Nmax)
9
10
11 CALL Permutation(Id)
12 Write(*, '(100I3)')(Id(I), I = 1, Nmax)
13
14 Stop
15
16 End Program Tests

```

## 10.21 Subroutine BootStrap(Ibt)

### 10.21.1 Description

Generates  $N_b$  bootstrap sequence of  $N$  numbers each. These bootstraps are returned in the two dimensional integer array **Ibt**(:,:) of size  $N \times N_b$ .

### 10.21.2 Arguments

**Ibt(:):** Integer. A two dimensional array of size  $N \times N_b$ , where  $N_b$  is the number of bootstraps, and  $N$  is the range of each bootstrap.

### 10.21.3 Examples

Listing 10.21: Resampling some data.

```

1  Program Tests
2
3  USE NumTypes
4  USE Error

```

<sup>3</sup>[http://en.wikipedia.org/wiki/Knuth\\_shuffle](http://en.wikipedia.org/wiki/Knuth_shuffle)

```

USE Statistics

6
Integer, Parameter :: Nmax = 8, Nbt = 5
8
Real (kind=DP) :: X(Nmax)
Integer :: Ib(Nmax, Nbt), I, J
10

12
CALL Random_Number(X)
! Generate 5 bootstraps
14
CALL BootStrap(Ib)

16
! Write the original sample, and the bootstraps
Write(*, '(1000ES33.25)')(X(J), J=1, Nmax)
18
Do I = 1, Nbt
    Write(*, '(1000ES33.25)')(X(Ib(J)), J=1, Nmax)
20
End Do

22
Stop
End Program Tests

```

## 10.22 Subroutine SaveBstrp(Ibt, Filename)

### 10.22.1 Description

Saves the bootstrap stored in `Ibt` and saves it in the file `Filename`.

### 10.22.2 Arguments

`Ibt(:)`: Integer. A two dimensional array of size  $N \times N_b$ , where  $N_b$  is the number of bootstraps, and  $N$  is the range of each bootstrap.

`Filename`: Character (len=\*). A file name to save the resampling data.

### 10.22.3 Examples

Listing 10.22: Reading the resampling info.

```

1 Program Tests

3
USE NumTypes
USE Error
5
USE Statistics

7
Integer, Parameter :: Nmax = 8, Nbt = 5
Integer :: Ib(Nmax, Nbt)
9

11
! Generate 5 bootstraps
CALL BootStrap(Ib)
13
! Save it

```

```

15  SaveBstrp(Ibt, 'example.bst')
17  Stop
End Program Tests

```

## 10.23 Subroutine ReadBstrp(Ibt, Filename)

### 10.23.1 Description

Reads the bootstrap stored in the file `Filename`, and returns it in `Ibt`.

### 10.23.2 Arguments

`Ibt(:)`: Integer. A two dimensional array of size  $N \times N_b$ , where  $N_b$  is the number of bootstraps, and  $N$  is the range of each bootstrap.

`Filename`: Character (len=\*). A file name to read the resampling data.

### 10.23.3 Examples

Listing 10.23: Saving the resampling info.

```

Program Tests
2
  USE NumTypes
4  USE Error
  USE Statistics
6
  Integer, Parameter :: Nmax = 8, Nbt = 5
8  Integer :: Ib(Nmax, Nbt)
10
  ! Read a saved Bootstrap.
12  ReadBstrp(Ibt, 'example.bst')
14
  Stop
End Program Tests

```

## 10.24 Subroutine EstBstrp(Data, Ibt, Func, Val, Err[, Rest])

### 10.24.1 Description

Estimates using the Bootstrap method the average and error of an estimator given as a user supplied function.

### 10.24.2 Arguments

**Data(:):** Double precision Real. A one dimensional array with the original sampling.

**ibt(:,):** Integer two dimensional array. The bootstrap that we want to use to make the estimation.

**Func:** A user supplied function that returns the value of the estimator. An interface block of the following type should be defined.

```

Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func

  End Function Func
End Interface

```

**Val:** Double precision real. Output. The value of the estimation of the parameter.

**Err:** Double precision real. Output. An estimation of the error in the estimation of the parameter.

**Rest:** Double precision real one dimensional array (same dimension as the number of bootstraps in **Ibt**). Output. The value of the estimator for each resampling.

### 10.24.3 Examples

Listing 10.24: Estimating the average.

```

1  Program Tests
3
3  USE NumTypes
3  USE Error
5  USE Statistics
7
7  Integer, Parameter :: Nmax = 100, Nbt = 50
7  Integer :: Ib(Nmax, Nbt)
9  Real (kind=DP) :: Avg, Err, Data(Nmax), Rest(Nbt)
11
11 Interface
11   Function F(X)
13     USE NumTypes
15
15     Real (kind=DP), Intent (in) :: X(:)
15     Real (kind=DP) :: F
17
17   End Function F
19 End Interface

```

```

21  ! Read a saved Bootstrap, and the data from a file
    ReadBstrp(Ibt, 'example.bst')
23  Open (Unit=22, File="data.dat")
    Read(22,*)Data
25  Close(22)

27  ! And estimate the average
    CALL EstBstrp(Data, Ibt, F, Avg, Err, Rest)
29

31  ! Print the Average of each resampling
    Do I = 1, Nbt
        Write(*,*)I, Rest(I)
33  End Do

35  Stop
End Program Tests

37
Function F(X)
39  USE NumTypes
    USE Statistics
41

43  Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: F
45
47  F = Mean(X)

End Function F

```

## 10.25 Subroutine BstrpConfInt(Data, Ibt, alpha, Func, dmin, dpls))

### 10.25.1 Description

Gives an Confidence interval for the estimator given as the user supplied function **Func**, such that

$$\mathcal{P}(dmin < Func(Data) < dpls) = 1 - 2\alpha$$

### 10.25.2 Arguments

**Data(:):** Double precision Real. A one dimensional array with the original sampling.

**ibt(:,):** Integer two dimensional array. The bootstrap that we want to use to make the estimation.

**alpha:** Double precision real. The level of the confidence interval.

**Func:** A user supplied function that returns the value of the estimator. An interface block of the following type should be defined.

```

Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func

  End Function Func
End Interface

```

**dmin:** Double precision real. Output. The lower limit of the confidence interval.

**dpls:** Double precision real. Output. The higher limit of the confidence interval.

### 10.25.3 Examples

Listing 10.25: Giving a confidence interval.

```

1 Program Tests
3   USE NumTypes
4   USE Error
5   USE Statistics
7   Integer, Parameter :: Nmax = 1000, Nb = 10000, Ndiv = 50
8   Real (kind=DP) :: Rdata(Nmax), Rmean(Nb), avg, Xmin, Xmax, h, Xac,&
9     & err, dmin, dpls
10  Integer :: Id(Nmax), Ib(Nb, Nmax), Ntics(Ndiv)
11
12  Interface
13    Function F(X)
14      USE NumTypes
15
16      Real (kind=DP), Intent (in) :: X(:)
17      Real (kind=DP) :: F
18
19    End Function F
20  End Interface
21
22  CALL Normal(Rdata)
23
24  avg = Mean(Rdata)
25  ! Now create the resamples
26  CALL Bootstrap(Ib)
27
28  CALL EstBstrp(Rdata, Ib, F, avg, Err, Rmean)
29  Write(*,*) '#', avg, Err, Mean(Rdata)
30
31  CALL BstrpConfInt(Rdata, Ib, 0.1_DP, F, dmin, dpls)
32  Write(*,*) 'Interval: ', dmin, dpls
33  Write(*,*) Avg - Dmin, Dpls - Avg
34  Write(*,*) (dpls - dmin)/2.0_DP, 1.64485_DP/Sqrt(Real(Nmax, kind=DP))

```



```

35      Stop
37
38 End Program Tests
39
40 Function F(X)
41     USE NumTypes
42     USE Statistics
43
44     Real (kind=DP), Intent (in) :: X(:)
45     Real (kind=DP) :: F
46
47     F = Mean(X)
48
49     Return
50 End Function F

```

## 10.26 Function Prop\_Error(X, Dx, Func[, N])

### 10.26.1 Description

Being **Func** a function of one or several variables, and **X** a point known with accuracy given by **Dx**, this routine propagate the errors in the position of the point, to obtain the accuracy of **F(X)**.

### 10.26.2 Arguments

**X[(:)]:** Double (DP) or simple (SP) precision one dimensional array or number. The value(s) of the variables.

**Dx[(:)]:** Double (DP) or simple (SP) precision one dimensional array or number. The value(s) of the error in the variables.

**Func:** A user supplied function. The function of the variables whose errors you want to propagate. An interface block of the following type should be defined.

```

Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func

  End Function Func
End Interface

```

**N:** Integer, Optional. The number of samples used to propagate the error. The default value is 1000.

### 10.26.3 Output

A real double or simple precision (same type as the input). An estimate of the accuracy of the value  $F(X)$ .

### 10.26.4 Examples

Here we compare our method of propagating errors, with the traditional propagation of errors formula for the case of the function  $f(x, y) = x^2 + y^2$ .

Listing 10.26: Propagating errors in a function.

```

2  Program Tests
4  USE NumTypes
4  USE Statistics
6  Real (kind=DP) :: Xm(2), Em(2)
8  Interface
10     Function F2(X)
10         USE NumTypes
12         Real (kind=DP), Intent (in) :: X(:)
12         Real (kind=DP) :: F2
14     End Function F2
14 End Interface
16
18 Xm(1) = 1.23_DP
18 Xm(2) = 1.62_DP
20 Em(1) = 0.02_DP
20 Em(2) = 0.03_DP
22 Write(*,*) 'Value of the function at Xm: ', F2(Xm)
22 Write(*,*) ' Error estimate: ', Prop_Error(Xm, Em, F2)
24 Write(*,*) ' Error estimate by hand: ', &
24     & Sqrt((Em(1)/Xm(1))**2 + (Em(2)/Xm(2))**2)*F2(Xm)
26 Stop
28 End Program Tests
30
32 Function F2(X)
32     USE NumTypes
34     Real (kind=DP), Intent (in) :: X(:)
34     Real (kind=DP) :: F2
36     F2 = X(1)**2 + X(2)**2
38     Return
38 End Function F2

```

## 10.27 Subroutine MCIntegration(X1, X2, Func, Val, Err[, Tol])

### 10.27.1 Description

Uses the Monte Carlo method to stimate the value of the integral of the several variables function **Func** between the limits (**X1**(1), **X1**(2), ..., **X1**(Ndim)) and (**X2**(1), **X2**(2), ..., **X2**(Ndim)). The value of the integral is returned in **Val**, and an estimation of the error in **Err**. The desired precision can be introduced via the optional argument **Tol**.

### 10.27.2 Arguments

**X1(:)**: Double or simple precision Real. A one dimensional array with the lower limits of the integrals.

**X2(:)**: Double or simple precision Real. A one dimensional array with the upper limits of the integrals.

**Func**: A user suplied function that returns the value of the function to integrate. An interface block of the following type should be defined.

```
Interface
  Function Func(X)
    USE NumTypes

    Real (kind=DP), Intent (in) :: X(:)
    Real (kind=DP) :: Func

  End Function Func
End Interface
```

**Val**: Double or simple precision real (same as input). Output. An estimation of the value of the integral.

**Err**: Double or simple precision real (same as input). Output. An estimation of the error.

**Tol**: Double or simple precision real. Optional. An estimation of the desired precision.

### 10.27.3 Examples

Listing 10.27: Integrating a two dimensional function.

```
1 MODULE ExtData
3   USE NumTypes
5   Real (kind=DP) :: Sig = 2.345763450_DP
6   Real (kind=DP) :: X1 = -0.324563562345643650_DP, X2 = 1.623473_DP, &
7     & Y1 = 1.34764574570_DP, Y2 = 5.3476347634_DP
```

```

9      End MODULE ExtData
11
12      Program MonteCarlo
13
14          USE NumTypes
15          USE Error
16          USE Statistics
17
18          USE ExtData
19
20
21          IMPLICIT NONE
22
23          Real (kind=DP) :: Res, Err
24
25          Interface
26              Function F(X)
27                  USE NumTypes
28
29                  Real (kind=DP), Intent (in) :: X(:)
30                  Real (kind=DP) :: F
31              End Function F
32          End Interface
33
34          CALL MCIntegration((/X1,Y1/), (/X2,Y2/), F, Res, Err, 1.0E-4_DP)
35          Write(*, '(1A,100ES33.25)') '# Value of the integral: ', Res, Err
36
37
38          Stop
39      End Program Monte
40
41      Function F(X)
42
43          USE NumTypes
44          USE ExtData
45
46          Real (kind=DP), Intent (in) :: X(:)
47          Real (kind=DP) :: F
48
49
50          F = exp(-1.0_DP/(2.0_DP*Sig**2) * (Sum(X(:)**2)))*Sin(X(1)*X(2))
51
52          Return
53      End Function F

```

# Eleven

---

## MODULE Polynomial

---

This is the documentation of the `MODULE Polynomial`, a set of `FORTTRAN 90` routines to work with polynomials. This module make use of the `MODULE NumTypes`, `MODULE Constants`, `MODULE Error` and `MODULE Linear` so please read the documentation of these modules *before* reading this.

### 11.1 Type `Pol`

#### 11.1.1 Description

A new data type `Pol` is defined to work with polynomials. This type has two components: The coefficients of the polynomial, and the degree.

#### 11.1.2 Components

`Coef(:):` Real double precision one dimensional array.

`dg:` Integer. The degree of the polynomial.

#### 11.1.3 Examples

A small example showing how to define a polynomial.

Listing 11.1: Defining a polynomial.

```
1 Program TestPoly
2
3   USE NumTypes
4   USE Error
5   USE Polynomial
6
7   Type (Pol) :: P1
8
9   Stop
10 End Program TestPoly
```

## 11.2 Type CmplxPol

### 11.2.1 Description

A new data type `CmplxPol` is defined to work with complex polynomials. This type has two components: The coefficients of the polynomial, and the degree.

All the routines, operators, etc. . . defined in this module works for complex as well as for real polynomials.

### 11.2.2 Components

`Coef(:)`: Complex double precision one dimensional array.

`dg`: Integer. The degree of the polynomial.

### 11.2.3 Examples

A small example showing how to define a polynomial of complex coefficients.

Listing 11.2: Defining a polynomial.

```

1 Program TestPoly
2
3   USE NumTypes
4   USE Error
5   USE Polynomial
6
7   Type (CmplxPol) :: P1
8
9   Stop
10 End Program TestPoly

```

## 11.3 Assignment

### 11.3.1 Description

You can directly assign one defined polynomial (complex or real) to another, or to an array of real numbers, that are interpreted as the coefficients.

### 11.3.2 Examples

Listing 11.3: Assigning polynomials.

```

1 Program TestPoly
2
3   USE NumTypes
4   USE Error
5   USE Polynomial
6
7   Integer, Parameter :: Deg = 4
8   Real (kind=DP) :: Hcoef(Deg+1)

```

```

10  Type (Pol) :: Hermite4
12  ! The fourth Hermite polynomial is  $x^4 - 6x^2 + 3$ , so
12  ! we first assign the values of the coefficients.
14  Hcoef = 0.0_DP
14  Hcoef(1) = 3.0_DP
16  Hcoef(3) = -6.0_DP
16  Hcoef(5) = 1.0_DP
18  Hermite4 = Hcoef
20  ! Now Show what we have in our data type:
22  Do I = 0, Hermite4%deg
22      Write(*, '(1I5,ES33.25)') I, Hermite4%Coef(I)
24  End Do
26  Stop
26  End Program TestPoly

```

## 11.4 Operator +

### 11.4.1 Description

You can naturally sum Pol or CmplxPol data types.

### 11.4.2 Examples

Listing 11.4: Adding polynomials.

```

Program TestPoly
2
3  USE NumTypes
4  USE Error
5  USE Polynomial
6
7  Integer, Parameter :: Deg = 4
8  Real (kind=DP) :: Hcoef(Deg+1)
9  Type (Pol) :: Hermite4, Hermite3, Sum
10
11  ! The Third Hermite polynomial is  $x^3 - 3x$ , so
12  ! we first assign the values of the coefficients.
14  Hcoef = 0.0_DP
14  Hcoef(2) = -3.0_DP
16  Hcoef(4) = 1.0_DP
18  Hermite3 = Hcoef(1:4)
20
21  ! The fourth Hermite polynomial is  $x^4 - 6x^2 + 3$ , so
22  ! we first assign the values of the coefficients.
24  Hcoef = 0.0_DP
24  Hcoef(1) = 3.0_DP

```

```

24   Hcoef(3) = -6.0_DP
      Hcoef(5) = 1.0_DP

26   Hermite4 = Hcoef

28   ! Now Add the two polynomials, and show the result.
      Sum = Hermite3 + Hermite4
30   Do I = 0, Sum%deg
      Write(*, '(1I5, ES33.25) ') I, Sum%Coef(I)
32   End Do

34   Stop
End Program TestPoly

```

## 11.5 Operator -

### 11.5.1 Description

You can subtract Pol or CmplxPol data types.

### 11.5.2 Examples

Listing 11.5: Subtracting polynomials.

```

1  Program TestPoly

3   USE NumTypes
   USE Error
5   USE Polynomial

7   Integer, Parameter :: Deg = 4
   Real (kind=DP) :: Hcoef(Deg+1)
9   Type (Pol) :: Hermite4, Hermite3, Sum

11  ! The Third Hermite polynomial is  $x^3 - 3x$ , so
   ! we first assign the values of the coefficients.
13  Hcoef = 0.0_DP
   Hcoef(2) = -3.0_DP
15  Hcoef(4) = 1.0_DP

17  Hermite3 = Hcoef(1:4)

19  ! The fourth Hermite polynomial is  $x^4 - 6x^2 + 3$ , so
   ! we first assign the values of the coefficients.
21  Hcoef = 0.0_DP
   Hcoef(1) = 3.0_DP
23  Hcoef(3) = -6.0_DP
   Hcoef(5) = 1.0_DP

25  Hermite4 = Hcoef

27

```



```

29      ! Now Subtract the two polynomials, and show the result.
      Sum = Hermite3 - Hermite4
      Do I = 0, Sum%deg
31          Write(*, '(1I5,ES33.25)')I, Sum%Coef(I)
      End Do
33
      Stop
35 End Program TestPoly

```

## 11.6 Operator \*

### 11.6.1 Description

You can naturally multiply `Pol` data types, `Pol` data types with double precision real numbers, `CmplxPol` data types and `CmplxPol` data types with real or complex numbers.

### 11.6.2 Examples

Listing 11.6: Computing the product of two polynomials.

```

1  Program TestPoly
3
4      USE NumTypes
5      USE Error
6      USE Polynomial
7
8      Integer, Parameter :: Deg = 4
9      Real (kind=DP) :: Hcoef(Deg+1)
10     Type (Pol) :: Hermite4, Hermite3, Sum
11
12     ! The Third Hermite polynomial is x^3 - 3x, so
13     ! we first assign the values of the coefficients.
14     Hcoef = 0.0_DP
15     Hcoef(2) = -3.0_DP
16     Hcoef(4) = 1.0_DP
17
18     Hermite3 = Hcoef(1:4)
19
20     ! The fourth Hermite polynomial is x^4 - 6x^2 + 3, so
21     ! we first assign the values of the coefficients.
22     Hcoef = 0.0_DP
23     Hcoef(1) = 3.0_DP
24     Hcoef(3) = -6.0_DP
25     Hcoef(5) = 1.0_DP
26
27     Hermite4 = Hcoef
28
29     ! Now multiply the two polynomials, and show the result.
30     Sum = Hermite3 * Hermite4
31     Do I = 0, Sum%deg
32         Write(*, '(1I5,ES33.25)')I, Sum%Coef(I)

```

```

33     End Do
34     Stop
35 End Program TestPoly

```

## 11.7 Subroutine Init(P, Dgr)

### 11.7.1 Description

Allocate memory space for the coefficients of a `Pol` or a `CmplxPol` type.

### 11.7.2 Arguments

**P:** Type `Pol` or `CmplxPol`. The polynomial that you want to allocate space for.

**Dgr:** Integer. The degree of the polynomial.

### 11.7.3 Examples

Listing 11.7: Initialising a polynomial data type.

```

1 Program TestPoly
2
3     USE NumTypes
4     USE Error
5     USE Polynomial
6
7     Integer, Parameter :: Deg = 4
8     Real (kind=DP) :: Hcoef(Deg+1)
9     Type (Pol) :: Hermite4, Hermite3, Sum
10
11
12     ! An alternative way of setting the third Hermite
13     ! polynomial.
14     CALL Init(Hermite3, 3)
15     Hermite3%Coef(0) = 0.0_DP
16     Hermite3%Coef(1) = -3.0_DP
17     Hermite3%Coef(2) = 0.0_DP
18     Hermite3%Coef(3) = 1.0_DP
19     Hermite3%dg = 3
20
21
22     Stop
23 End Program TestPoly

```

## 11.8 Function Degree(P)

### 11.8.1 Description

Returns the degree of the polynomial P.

### 11.8.2 Arguments

P: Type Pol or CmplxPol. The polynomial whose degree we want to know.

### 11.8.3 Output

Integer. The degree of the polynomial P.

### 11.8.4 Examples

Listing 11.8: Returns the degree of a polynomial.

```

1 Program TestPoly
3   USE NumTypes
3   USE Error
5   USE Polynomial
7   Integer, Parameter :: Deg = 4
7   Real (kind=DP) :: Hcoef(Deg+1), X
9   Type (Pol) :: Hermite4, Hermite3, Sum
11  ! The Third Hermite polynomial is  $x^3 - 3x$ , so
11  ! we first assign the values of the coefficients.
13  Hcoef = 0.0_DP
13  Hcoef(2) = -3.0_DP
15  Hcoef(4) = 1.0_DP
17  Hermite3 = Hcoef(1:4)
19  ! The fourth Hermite polynomial is  $x^4 - 6x^2 + 3$ , so
19  ! we first assign the values of the coefficients.
21  Hcoef = 0.0_DP
21  Hcoef(1) = 3.0_DP
23  Hcoef(3) = -6.0_DP
23  Hcoef(5) = 1.0_DP
25  Hermite4 = Hcoef
27  ! Now Mutiply the two polynomials, and show the result.
29  Sum = Hermite3 * Hermite4
31  ! Show the degree of the product. It should be 4+3=7.
31  Write(*,*) Degree(Sum)
33
35  Stop
End Program TestPoly

```

## 11.9 Function Value(P, X)

### 11.9.1 Description

Computes the value of the polynomial P in the point X.

### 11.9.2 Arguments

**P:** Type `Pol` or `CmplxPol`. The polynomial.

**X:** Real double precision if P is of type `Pol` and Complex double precision if P is `CmplxPol`.  
The point in which you want to compute the value.

### 11.9.3 Output

Real double precision. The value of the polynomial P in the point X.

### 11.9.4 Examples

Listing 11.9: Computes the values of a polynomial at some points.

```

Program TestPoly
2
3   USE NumTypes
4   USE Error
5   USE Polynomial
6
7   Integer, Parameter :: Deg = 4
8   Real (kind=DP) :: Hcoef(Deg+1), X
9   Type (Pol) :: Hermite4, Hermite3, Sum
10
11  ! The Third Hermite polynomial is x^3 - 3x, so
12  ! we first assign the values of the coefficients.
13  Hcoef = 0.0_DP
14  Hcoef(2) = -3.0_DP
15  Hcoef(4) = 1.0_DP
16
17  Hermite3 = Hcoef(1:4)
18
19  ! The fourth Hermite polynomial is x^4 - 6x^2 + 3, so
20  ! we first assign the values of the coefficients.
21  Hcoef = 0.0_DP
22  Hcoef(1) = 3.0_DP
23  Hcoef(3) = -6.0_DP
24  Hcoef(5) = 1.0_DP
25
26  Hermite4 = Hcoef
27
28  ! Now Mutiply the two polynomials, and show the result.
29  Sum = Hermite3 * Hermite4
30
31  ! Compute the value of the product in some point in two

```

```

32  ! different ways.
    X = 9.34564_DP
34  Write(*, '(ES33.25) ') Value(Sum, X)
    Write(*, '(ES33.25) ') Value(Hermite3, X)*Value(Hermite4, X)
36
38  Stop
End Program TestPoly

```

## 11.10 Function Deriv(P)

### 11.10.1 Description

Computes the derivative of the polynomial P.

### 11.10.2 Arguments

P: Type Pol or CmplxPol. The polynomial whose derivative we want to compute.

### 11.10.3 Output

Type Pol. Another polynomial: the derivative of P.

### 11.10.4 Examples

Listing 11.10: Computing the derivative of a polynomial.

```

1  Program TestPoly
3
   USE NumTypes
   USE Error
   USE Polynomial
5
7  Integer, Parameter :: Deg = 4
   Real (kind=DP) :: Hcoef(Deg+1), X
9  Type (Pol) :: Hermite4, Hermite3, Res, Sum
11
   ! The Third Hermite polynomial is x^3 - 3x, so
   ! we first assign the values of the coefficients.
13  Hcoef = 0.0_DP
   Hcoef(2) = -3.0_DP
15  Hcoef(4) = 1.0_DP
17
   Hermite3 = Hcoef(1:4)
19
   ! The fourth Hermite polynomial is x^4 - 6x^2 + 3, so
   ! we first assign the values of the coefficients.
21  Hcoef = 0.0_DP
   Hcoef(1) = 3.0_DP
23  Hcoef(3) = -6.0_DP
   Hcoef(5) = 1.0_DP

```

```

25 Hermite4 = Hcoef
27
28 ! Now compute the derivative of Hermite4
29 Res = Deriv(Hermite4)
30
31 ! From the recursion relation of the Hermite polynomials
32 ! we should obtain twice the same number:
33 X = 7.346582_DP
34 Write(*, '(ES33.25) ') Value(Res, X)
35 Write(*, '(ES33.25) ') 4.0_DP*Value(Hermite3, X)
36
37 Stop
38 End Program TestPoly

```

## 11.11 Function Integra(P[, Cte])

### 11.11.1 Description

Computes the integral of the polynomial P. If Cte is present then it is used as *integration constant*.

### 11.11.2 Arguments

**P:** Type Pol or CmplxPol. The polynomial whose integral we want to compute.

**Cte:** Optional. Real double precision if P is of type Pol and Complex double precision if P is CmplxPol. If not present, the default value is 0.

### 11.11.3 Output

Type Pol. Another polynomial: the integral of P.

### 11.11.4 Examples

Listing 11.11: Computing the integral of a polynomial.

```

1 Program TestPoly
2
3   USE NumTypes
4   USE Error
5   USE Polynomial
6
7   Integer, Parameter :: Deg = 4
8   Real (kind=DP) :: Hcoef(Deg+1), X
9   Type (Pol) :: Hermite4, Hermite3, Res, Sum
10
11 ! The Third Hermite polynomial is x^3 - 3x, so
12 ! we first assign the values of the coefficients.
13 Hcoef = 0.0_DP

```

```

15   Hcoef(2) = -3.0_DP
    Hcoef(4) = 1.0_DP

17   Hermite3 = Hcoef(1:4)

19   ! The fourth Hermite polynomial is  $x^4 - 6x^2 + 3$ , so
    ! we first assign the values of the coefficients.
21   Hcoef = 0.0_DP
    Hcoef(1) = 3.0_DP
23   Hcoef(3) = -6.0_DP
    Hcoef(5) = 1.0_DP

25   Hermite4 = Hcoef

27   ! Now compute the derivative of Hermite4
29   Res = Integra(Hermite3, 3.0_DP/4.0_DP)

31   ! From the recursion relation of the Hermite polynomials
    ! we should obtain twice the same number:
33   X = 7.346582_DP
    Write(*, '(ES33.25) ') Value(Res, X)
35   Write(*, '(ES33.25) ') 0.25_DP*Value(Hermite4, X)

37
39   Stop
End Program TestPoly

```

## 11.12 Function InterpolValue(X, Y, Xo)

### 11.12.1 Description

Computes the value of the interpolation polynomial that pass trough  $(X(:), Y(:))$  in the point  $Xo$ .

### 11.12.2 Arguments

$X(:), Y(:)$ : Real double precision or Complex double precision one dimensional arrays. Specify the points at which the interpolation polynomial should pass.

$Xo$ : Same type as  $X(:)$  and  $Y(:)$ . The point at which you want to compute the interpolation polynomial.

### 11.12.3 Output

Real double precision if input is real and complex double precision if input is complex. The value of the interpolation polynomial in  $Xo$ .

### 11.12.4 Examples

Listing 11.12: Compute values of the Interpolation polynomial.

```

1  Program TestPoly
3      USE NumTypes
4      USE Error
5      USE Polynomial
7
8      Integer, Parameter :: Deg = 4, Np = 7
9      Real (kind=DP) :: Hcoef(Deg+1), X, Xp(Np), Yp(Np)
10     Type (Pol) :: Hermite4, Hermite3, Res, Sum
11
12     CALL Random_Number(Xp)
13     Yp = 3.347234_DP*Xp - 2.475875_DP*Xp**3 - 7.23467_DP*Xp**4 + &
14         & 1.47854_DP*Xp**6
15
16     ! Now we compute the value of the interpolation polynomial
17     ! at X, and compare it with the real value of the Polynomial
18     X = -1.23899843_DP
19     Write(*, '(ES33.25) ') InterpolValue(Xp, Yp, X)
20     Write(*, '(ES33.25) ') 3.347234_DP*X - 2.475875_DP*X**3 - &
21         & 7.23467_DP*X**4 + 1.47854_DP*X**6
22
23     Stop
24 End Program TestPoly

```

## 11.13 Function Interpol(X, Y)

Computes the interpolation polynomial that pass trough (X(:), Y(:)). **Note that using this function can be very unstable.**

### 11.13.1 Arguments

X(:), Y(:): Real double precision or Complex double precision one dimensional arrays. Specify the points at which the interpolation polynomial should pass.

### 11.13.2 Output

Type Pol if input is real, and CmplxPol if input is complex. The interpolation polynomial.

### 11.13.3 Examples

Listing 11.13: Computes the interpolation polynomial.

```

1  Program TestPoly
3      USE NumTypes
4      USE Error
5      USE Polynomial

```



```

7  Integer, Parameter :: Deg = 4, Np = 7
   Real (kind=DP) :: Hcoef(Deg+1), X, Xp(Np), Yp(Np)
9  Type (Pol) :: Hermite4, Hermite3, Res, Sum

11
   CALL Random_Number(Xp)
13  Yp = 3.347234_DP*Xp - 2.475875_DP*Xp**3 - 7.23467_DP*Xp**4 + &
      & 1.47854_DP*Xp**6

15
   ! Now we compute the interpolation polynomial
17  ! at X, and compare it with the real value of the Polynomial
   X = -1.23899843_DP
19  Res = Interpol(Xp,Yp)
   Write(*, '(ES33.25) ') Value(Res, X)
21  Write(*, '(ES33.25) ') 3.347234_DP*X - 2.475875_DP*X**3 - &
      & 7.23467_DP*X**4 + 1.47854_DP*X**6

23
25  Stop
End Program TestPoly

```

## 11.14 Subroutine Spline(X, Y, Ypp0, YppN, PolS)

### 11.14.1 Description

Compute the cubic spline interpolation polynomial that pass trough (X(:), Y(:)).

### 11.14.2 Arguments

X(:), Y(:): Real double precision one dimensional arrays. Specify the points at which the cubic spline interpolation polynomial should pass.

Ypp0, YppN: The values of the second derivatives of the cubic spline interpolation polynomial in the first and last points.

PolS(:): Type Pol one dimensional array. Returns the N-1 cubic interpolation polynomials.

### 11.14.3 Examples

Listing 11.14: Computes the cubic spline interpolation polynomial.

```

Program TestPoly

2  USE NumTypes
   USE Error
4  USE Polynomial
   USE NonNumeric

6

8  Integer, Parameter :: Deg = 4, Np = 7
   Real (kind=DP) :: Hcoef(Deg+1), X, Xp(Np), Yp(Np)

```

```

10  Type (Pol) :: Hermite4, Hermite3, Res, Sum, Spl(Np-1)

12

13  CALL Random_Number(Xp)
14  ! Order Xp
15  CALL Qsort(Xp)
16  Yp = 3.347234_DP*Xp - 2.475875_DP*Xp**3 - 7.23467_DP*Xp**4 + &
      & 1.47854_DP*Xp**6

18

19  ! Now we compute the interpolation polynomial
20  ! at X, and compare it with the real value of the Polynomial, and
21  ! the value of the spline cubic interpolation polynomial.
22  X = 0.23899843_DP
23  Res = Interpol(Xp,Yp)
24  CALL Spline(Xp, Yp, 0.0_DP, 0.0_DP, Spl)
25  Write(*, '(ES33.25) ') Value(Res, X)
26  Write(*, '(ES33.25) ') Value(Spl(Locate(Xp, X)), X)
27  Write(*, '(ES33.25) ') 3.347234_DP*X - 2.475875_DP*X**3 - &
28      & 7.23467_DP*X**4 + 1.47854_DP*X**6

30

31  Stop
32 End Program TestPoly

```

# Twelve

---

## MODULE Root

---

This is the documentation of the `MODULE Root`, a set of `FORTRAN 90` routines to compute roots of functions. This module make use of the `MODULE NumTypes`, `MODULE Constants` and `MODULE Error` so please read the documentation of these modules *before* reading this.

### 12.1 Subroutine `RootPol(a, b, [c, d], z1, z2, [z3, z4])`

#### 12.1.1 Description

Returns the complex roots of a polynomial of degree 2, 3 or 4.

#### 12.1.2 Arguments

**a, b, c, d:** The coefficients of the polynomial. The meaning of the coefficieents **a,b,c,d** depends on the degree of the polynomial:

$$\begin{aligned}P(x) &= x^2 + ax + b \\P(x) &= x^3 + ax^2 + bx + c \\P(x) &= x^4 + ax^3 + bx^2 + cx + d\end{aligned}$$

**z1,z2,z3,z4:** Complex simple or double precision. The roots of the polynomial.

#### 12.1.3 Examples

Listing 12.1: Computing roots of polynomials.

```
2 Program TestRoot
3
4   USE NumTypes
5   USE Error
6   USE Root
7
8   Real (kind=DP) :: a, b, c, d
9   Complex (kind=DPC) :: z1, z2, z3, z4, ac, bc, cc, dc
```

```

10  CALL Random_Number(a)
12  CALL Random_Number(b)
14  CALL Random_Number(c)
16  CALL Random_Number(d)
18  CALL RootPol(a,b,z1,z2)
19  Write(*,'(3ES20.12)')Z1, Abs(z1**2 + a*z1 + Cmplx(b,kind=DPC))
20  Write(*,'(3ES20.12)')Z2, Abs(z2**2 + a*z2 + Cmplx(b,kind=DPC))
22
23  CALL RootPol(a,b,c,z1,z2,z3)
24  Write(*,*)
25  Write(*,'(3ES20.12)')Z1, Abs(z1**3+a*z1**2+b*z1+Cmplx(c,kind=DPC))
26  Write(*,'(3ES20.12)')Z2, Abs(z2**3+a*z2**2+b*z2+Cmplx(c,kind=DPC))
27  Write(*,'(3ES20.12)')Z3, Abs(z3**3+a*z3**2+b*z3+Cmplx(c,kind=DPC))
29
30  ac = Cmplx(a,kind=DPC)
31  bc = Cmplx(b,a,kind=DPC)
32  cc = Cmplx(c,kind=DPC)
33  dc = Cmplx(d,kind=DPC)
34  CALL RootPol(ac,bc,z1,z2)
35  Write(*,*)
36  Write(*,'(3ES20.12)')Z1, Abs(z1**2 + ac*z1 + Cmplx(bc,kind=DPC))
37  Write(*,'(3ES20.12)')Z2, Abs(z2**2 + ac*z2 + Cmplx(bc,kind=DPC))
38  CALL RootPol(ac,bc,cc,dc,z1,z2,z3,z4)
39  Write(*,*)
40  Write(*,'(3ES20.12)')Z1, Abs(z1**4+ac*z1**3+bc*z1**2+cc*z1+dc)
41  Write(*,'(3ES20.12)')Z2, Abs(z2**4+ac*z2**3+bc*z2**2+cc*z2+dc)
42  Write(*,'(3ES20.12)')Z3, Abs(z3**4+ac*z3**3+bc*z3**2+cc*z3+dc)
43  Write(*,'(3ES20.12)')Z4, Abs(z4**4+ac*z4**3+bc*z4**2+cc*z4+dc)
45
46  Stop
47  End Program TestRoot

```

## 12.2 Function Newton( $X_0$ , Fnew, [Tol])

### 12.2.1 Description

Compute a root of the function defined by the routine Fnew.

### 12.2.2 Arguments

**$X_0$ :** Real simple or double precision. An initial guess of the position of the root.

**Fnew:** The function whose root we want to compute. It is defined as a subroutine that returns the value of the function and of its derivative. If it is an external function, an interface block like this should be defined

```

Interface
  Subroutine FNew( $X_0$ , F, D)

```

```

USE NumTypes

Real (kind=DP), Intent (in) :: Xo
Real (kind=DP), Intent (out) :: F, D
End Subroutine FNew
End Interface

```

where F is the value of the function in Xo, and D the value of the derivative in Xo. If the arguments are of simple precision, a similar interface should be provided, where the arguments of Fnew are of single precision.

**Tol:** Real single or double precision. Optional. An estimation of the desired accuracy of the position of the root.

### 12.2.3 Output

Real single or double precision. The position of the root.

### 12.2.4 Examples

Listing 12.2: Computing roots of non-linear functions with the Newton method.

```

Program TestRoot
2
  USE NumTypes
4  USE Error
  USE Root
6
  Real (kind=DP) :: a, b, c, d, X
  Complex (kind=DPC) :: z1, z2, z3, z4, ac, bc, cc, dc
8
10
  Interface
12    Subroutine FNew(Xo, F, D)
14      USE NumTypes
16      Real (kind=DP), Intent (in) :: Xo
        Real (kind=DP), Intent (out) :: F, D
18    End Subroutine FNew
  End Interface
20
22  ! Compute the value such that cos(x) = x
  X = Newton(0.0_DP, Fnew, 1.0E-10_DP)
24  Write(*, '(1A,ES33.25)') 'Point: ', X
  Write(*, '(1A,ES33.25)') 'Value of Cos: ', Cos(X)
26
28  Stop

```

```

30 End Program TestRoot
31 ! *****
32 ! *
33 Subroutine FNew(Xo, F, D)
34 ! *
35 ! *****
36
37     USE NumTypes
38
39     Real (kind=DP), Intent (in)  :: Xo
40     Real (kind=DP), Intent (out) :: F, D
41
42     F = Xo - Cos(Xo)
43     D = 1.0_DP + Sin(Xo)
44
45     Return
46 End Subroutine FNew

```

## 12.3 Function Bisec(a, b, Fbis, [Tol])

### 12.3.1 Description

Compute the root of the function defined by **Fbis**.

### 12.3.2 Arguments

**a, b:** Real single or double precision. Initial points, such that **Fbis(a)Fbis(b) < 0**.

**Fbis:** The function whose root we want to compute. It is defined as a function that returns the value of the function. If it is an external function, an interface block like this should be defined

```

Interface
  Function F(X)

      USE NumTypes

      Real (kind=DP), Intent (in) :: X
      Real (kind=DP) :: F
  End Function F
End Interface

```

where **F** is the value of the function in **X**. If the arguments are of simple precision, a similar interface should be provided, where the arguments of **F** are of single precision.

**Tol:** Real single or double precision. Optional. An estimation of the desired accuracy of the position of the root.

### 12.3.3 Output

Real single or double precision. The position of the root of Fbis.

### 12.3.4 Examples

Listing 12.3: Computing roots with the bisection method.

```

1 Program TestRoot
3   USE NumTypes
4   USE Error
5   USE Root
7   Real (kind=DP) :: a, b, c, d, X
8   Complex (kind=DPC) :: z1, z2, z3, z4, ac, bc, cc, dc
9
10  Interface
11    Function Fbis(X)
12
13      USE NumTypes
14
15      Real (kind=DP), Intent (in) :: X
16      Real (kind=DP) :: Fbis
17    End Function Fbis
18  End Interface
19
20  ! Compute the value such that cos(x) = x
21  X = Bisec(0.0_DP, 1.1_DP, Fbis, 1.0E-10_DP)
22  Write(*, '(1A,ES33.25)') 'Point: ', X
23  Write(*, '(1A,ES33.25)') 'Value of Cos: ', Cos(X)
24
25
26  Stop
27 End Program TestRoot
28
29 ! *****
30 ! *
31 Function FBis(X)
32 ! *
33 ! *****
34
35 USE NumTypes
36
37 Real (kind=DP), Intent (in) :: X
38 Real (kind=DP) :: FBis
39
40
41 FBis = X - Cos(X)
42
43 Return
44 End Function FBis

```





# Thirteen

---

## MODULE Fourier

---

This is the documentation of the `MODULE Fourier`, a set of `FORTRAN 90` routines to work with Fourier series. This module make use of the `MODULE NumTypes` and the `MODULE Constants` so please read the documentation of these modules *before* reading this.

### 13.1 Type `Fourier_Serie`

#### 13.1.1 Description

A new data type `Fourier_Serie` is defined to work with Fourier series. This type has two components: The modes, and the number of modes.

#### 13.1.2 Components

`Coef(:)`: Complex double precision one dimensional array. The modes.

`Nterm`: Integer. The number of terms of the Fourier series.

#### 13.1.3 Examples

A small example showing how to define a Fourier serie.

Listing 13.1: Defining a Fourier serie.

```
1 Program TestFourier
2
3   USE NumTypes
4   USE Constants
5   USE Fourier
6
7   Type (Fourier_Serie) :: Ff
8
9   Stop
10 End Program TestPoly
```

## 13.2 Type Fourier\_Serie\_2D

### 13.2.1 Description

A new data type `Fourier_Serie_2D` is defined to work with two dimensional Fourier series. This type has two components: The modes, and the number of modes.

### 13.2.2 Components

`Coef(:, :)`: Complex double precision two dimensional array. The modes.

`Nterm`: Integer. The number of terms of the Fourier series.

### 13.2.3 Examples

A small example showing how to define a polynomial.

Listing 13.2: Defining a two-dimensional Fourier serie.

```

1 Program TestFourier
2
3     USE NumTypes
4     USE Constants
5     USE Fourier
6
7     Type (Fourier_Serie_2D) :: Ff
8
9     Stop
10 End Program TestPoly

```

## 13.3 Assignment

### 13.3.1 Description

You can directly assign one defined Fourier series (one or two dimensional) to another.

### 13.3.2 Examples

This example uses the `Init_Serie` subroutine. For details of the usage of this function look at the section (13.8), page (130).

Listing 13.3: Assigning Fourier series.

```

1 Program TestFourier
2
3     USE NumTypes
4     USE Constants
5     USE Fourier
6
7     Type (Fourier_Serie) :: FS1, FS2
8
9     CALL Init_Serie(FS1, 20)
10    CALL Init_Serie(FS2, 20)

```

```

12  FS1%Coef( 1) = Cmplx(1.0_DP, 0.5_DP, kind=DPC)
    FS1%Coef(-1) = Cmplx(1.0_DP, 0.7_DP, kind=DPC)
14
16  FS2 = FS1
18  Write(*, '(2ES33.25) ')FS2%Coef( 1)
    Write(*, '(2ES33.25) ')FS2%Coef(-1)
20
    Stop
End Program TestFourier

```

## 13.4 Operator +

### 13.4.1 Description

You can naturally sum one or two dimensional Fourier series. If they have different sizes, it is assumed that the non defined modes of the short Fourier Series are zero.

### 13.4.2 Examples

This example uses the `Init_Serie` subroutine. For details of the usage of this function look at the section (13.8), page (130).

Listing 13.4: Adding Fourier series.

```

1  Program TestFourier
3
4  USE NumTypes
5  USE Constants
6  USE Fourier
7
8  Type (Fourier_Serie_2D) :: FS1, FS2, FS3
9  Integer :: Nt
10
11 Nt = 4
12 CALL Init_Serie(FS1, Nt)
13 CALL Init_Serie(FS2, Nt)
14
15 FS1%Coef( 1,1) = Cmplx(1.0_DP, 0.5_DP, kind=DPC)
16 FS1%Coef(-1,1) = Cmplx(1.0_DP, 0.7_DP, kind=DPC)
17
18 FS2%Coef( 1,1) = Cmplx(-1.0_DP, 4.5_DP, kind=DPC)
19 FS2%Coef(-1,1) = Cmplx(-1.0_DP, -6.78745_DP, kind=DPC)
20
21 FS3 = FS1 + FS2
22 Write(*, '(2ES33.25) ')FS3%Coef( 1,1)
23 Write(*, '(2ES33.25) ')FS3%Coef(-1,1)
24
25 Stop
End Program TestFourier

```

## 13.5 Operator -

### 13.5.1 Description

You can naturally subtract one or two dimensional Fourier series. If they have different sizes, it is assumed that the non defined modes of the short Fourier Series are zero.

### 13.5.2 Examples

Listing 13.5: Subtracting Fourier series.

```

Program TestFourier
2
  USE NumTypes
4  USE Constants
  USE Fourier
6
  Type (Fourier_Serie) :: FS1, FS2, FS3
8  Integer :: Nt
10
  Nt = 4
  CALL Init_Serie(FS1, Nt)
12
  FS1%Coef( 1) = Cmplx(1.0_DP, 0.5_DP, kind=DPC)
14  FS1%Coef(-1) = Cmplx(1.0_DP, 0.7_DP, kind=DPC)
16
  FS2 = FS1
18
  FS3 = FS1 - FS2
  Write(*, '(2ES33.25) ')FS3%Coef( 1)
20  Write(*, '(2ES33.25) ')FS3%Coef(-1)
22
  Stop
End Program TestFourier

```

## 13.6 Operator \*

### 13.6.1 Description

You can naturally multiply one or two dimensional Fourier series, in which case the convolution of the Fourier Modes is performed. If they have different sizes, it is assumed that the non defined modes of the short Fourier Series are zero.

### 13.6.2 Examples

Listing 13.6: Computing the convolution of Fourier series.

```

1 Program TestFourier
3
  USE NumTypes
  USE Constants

```

```

5  USE Fourier
7  Type (Fourier_Serie) :: FS1, FS2, FS3
   Integer :: Nt
9
   Nt = 4
11 CALL Init_Serie(FS1, Nt)
13 FS1%Coef( 1) = Cmplx(1.0_DP, 0.5_DP, kind=DPC)
   FS1%Coef(-1) = Cmplx(1.0_DP, 0.7_DP, kind=DPC)
15
   FS2 = FS1
17
   FS3 = FS1 * FS2
19 Write(*, '(2ES33.25) ')FS3%Coef( 0)
21
   Stop
End Program TestFourier

```

## 13.7 Operator \*\*

### 13.7.1 Description

You can naturally compute the integer power of a one or two dimensional Fourier series, in which case the convolution of the Fourier modes with themselves are performed a certain number of times.

### 13.7.2 Examples

Listing 13.7: "Exponentiating" Fourier series.

```

Program TestFourier
2
   USE NumTypes
   USE Constants
   USE Fourier
4
6
   Type (Fourier_Serie) :: FS1, FS2, FS3
   Integer :: Nt
8
   Nt = 4
   CALL Init_Serie(FS1, Nt)
12  CALL Init_Serie(FS2, Nt)
   CALL Init_Serie(FS3, Nt)
14
   FS1%Coef( 1) = Cmplx(1.0_DP, 0.5_DP, kind=DPC)
16  FS1%Coef(-1) = Cmplx(1.0_DP, 0.7_DP, kind=DPC)
18
   FS3%Coef(0) = Cmplx(1.0_DP, 0.0_DP, kind=DPC)
20
   FS2 = FS1**8

```

```

22      Do I = 1, 8
          FS3 = FS3 * FS1
      End Do

24      Write(*, '(2ES33.25) ')FS2%Coef( 0)
26      Write(*, '(2ES33.25) ')FS3%Coef( 0)

28      Stop
End Program TestFourier

```

## 13.8 Subroutine Init\_Serie(FS,Ns)

### 13.8.1 Description

Allocate memory space for the modes of a one or two dimensional Fourier series.

### 13.8.2 Arguments

**FS:** Type `Fourier_Serie` or type `Fourier_Serie_2D`. The Fourier series that you want to allocate space for.

**Ns:** Integer. The number of modes.

### 13.8.3 Examples

Any of the examples of some of the previous sections are also good examples of the use of the `Init_Serie` subroutine. Here we simply repeat one of them.

Listing 13.8: Initialising a Fourier series.

```

1  Program TestFourier

3      USE NumTypes
      USE Constants
5      USE Fourier

7      Type (Fourier_Serie) :: FS1, FS2, FS3
      Integer :: Nt

9      Nt = 4
11     CALL Init_Serie(FS1, Nt)

13     FS1%Coef( 1) = Cmplx(1.0_DP, 0.5_DP, kind=DPC)
      FS1%Coef(-1) = Cmplx(1.0_DP, 0.7_DP, kind=DPC)
15

17     FS2 = FS1

19     FS3 = FS1 * FS2
      Write(*, '(2ES33.25) ')FS3%Coef( 0)

21     Stop
End Program TestFourier

```

## 13.9 Function Eval\_Serie(FS, X, [Y], Tx, [Ty])

### 13.9.1 Description

Compute the value of the Fourier series FS with periods Tx,Ty at the point X,Y.

### 13.9.2 Arguments

**FS:** Type `Fourier_Serie` or type `Fourier_Serie_2D`. The Fourier series that you want to evaluate.

**X,Y:** Real double precision. The point in which you want to evaluate the Fourier series. If **FS** is a two dimensional Fourier series, then **Y** must be present.

**Tx,Ty:** Real double precision. The period(s). If **FS** is a two dimensional Fourier series, then **Ty** must be present.

### 13.9.3 Output

Real double precision. The value of the function defined by the modes in **FS** at the point (X[,Y]).

### 13.9.4 Examples

Listing 13.9: Evaluating a Fourier series at a point.

```

1  Program TestFourier
2
3  USE NumTypes
4  USE Constants
5  USE Fourier
6
7  Type (Fourier_Serie) :: FS1, FS2, FS3
8  Integer :: Nt
9
10 Nt = 4
11 CALL Init_Serie(FS1, Nt)
12 CALL Init_Serie(FS2, Nt)
13 CALL Init_Serie(FS3, Nt)
14
15 FS1%Coef( 1) = Cmplx(1.0_DP, 0.5_DP, kind=DPC)
16 FS1%Coef(-1) = Cmplx(1.0_DP, 0.7_DP, kind=DPC)
17
18 FS2 = FS1**2
19
20 FS3 = FS1*FS2
21
22 Write(*, '(2ES33.25)') Eval_Serie(FS1,0.12_DP,1.0_DP) * &
23                               & Eval_Serie(FS2,0.12_DP,1.0_DP)
24 Write(*, '(2ES33.25)') Eval_Serie(FS3,0.12_DP,1.0_DP)
25
26
```

```

28      Stop
End Program TestFourier

```

## 13.10 Function Unit(FS, Ns)

### 13.10.1 Description

Allocate memory space for the modes of a one or two dimensional Fourier series and sets the zero mode equal to 1.

### 13.10.2 Arguments

**FS:** Type `Fourier_Serie` or type `Fourier_Serie_2D`. The Fourier series that you want to allocate space for.

**Ns:** Integer. The number of modes.

### 13.10.3 Examples

Listing 13.10: Obtaining a constant Fourier series.

```

Program TestFourier
2
  USE NumTypes
4  USE Constants
  USE Fourier
6
  Type (Fourier_Serie) :: FS1, FS2, FS3
8  Integer :: Nt
10
  Nt = 4
  CALL Init_Serie(FS1, Nt)
12  CALL Init_Serie(FS2, Nt)
  CALL Init_Serie(FS3, Nt)
14
16  FS1%Coef( 1) = Cmplx(1.0_DP, 0.5_DP, kind=DPC)
  FS1%Coef(-1) = Cmplx(1.0_DP, 0.7_DP, kind=DPC)
18
  CALL Unit(FS2, Nt)
20
  FS3 = FS1*FS2
22
  Write(*, '(2ES33.25)') Eval_Serie(FS1,0.12_DP,1.0_DP)
24  Write(*, '(2ES33.25)') Eval_Serie(FS3,0.12_DP,1.0_DP)
26
  Stop
End Program TestFourier

```



## 13.11 Function DFT(Data, Is)

### 13.11.1 Description

Compute the Discrete Fourier Transform of the values stored in the complex array **Data**. If **Is** is present and is set to -1, the inverse Discrete Fourier Transform is performed. The direct Fourier transform is defined as

$$\tilde{f}(k) = \sum_{n=0}^N f_n e^{\frac{2\pi i n k}{N}} \quad \forall k \in \left[-\frac{N}{2}, \frac{N}{2}\right]$$

the inverse one is defined as

$$\tilde{f}(k) = \frac{1}{N} \sum_{n=0}^N f_n e^{\frac{-2\pi i n k}{N}} \quad \forall k \in \left[-\frac{N}{2}, \frac{N}{2}\right]$$

### 13.11.2 Arguments

**Data(:, :)**: One or two dimensional double precision complex array. The data whose Discrete Fourier Transform we want to compute.

**Is**: Integer. Optional. A flag to tell if we want to compute the direct or the inverse Fourier transform.

### 13.11.3 Output

Type **Fourier\_Serie** if **Data(:)** is one dimensional, and type **Fourier\_Serie\_2D** if **Data(:, :)** is two dimensional.

### 13.11.4 Examples

This example compute the discrete Fourier transform of  $f(x_i) = \sin(x_i)$ .

Listing 13.11: Computing the Discrete Fourier Transform.

```

1 Program TestFourier
3   USE NumTypes
3   USE Constants
5   USE Fourier
7   Integer, Parameter :: Nmax=20
7   Type (Fourier_Serie) :: FS1, FS2, FS3
9   Complex (kind=DPC) :: Data(Nmax), X
9   Integer :: Nt
11
11  Do I = 1, Nmax
13    X = Cmplx(TWOPLDP*I/Nmax)
13    Data(I) = Sin(X)
15  End Do
17  FS1 = DFT(Data)

```

```

19  Write(*, '(1A,2ES33.25) ') 'Mode k= 1: ', FS1%Coef( 1)
    Write(*, '(1A,2ES33.25) ') 'Mode k=-1: ', FS1%Coef(-1)
21  Write(*, '(ES33.25) ') Sum(Abs(FS1%Coef(:)))

23  Stop
End Program TestFourier

```

## 13.12 Function Conjg(FS)

### 13.12.1 Description

Computes the Fourier modes that correspond to the conjugate function. This means: If the modes of FS are  $\tilde{f}(k)$ , this function returns a Fourier series with modes  $\tilde{f}(-k)$ .

### 13.12.2 Arguments

**FS:** Type `Fourier_Serie` or type `Fourier_Serie_2D`. The Fourier series whose conjugate you want to compute.

### 13.12.3 Output

Type `Fourier_Serie` if FS is of type `Fourier_Serie`, and type `Fourier_Serie_2D` if FS is of Type `Fourier_Serie_2D`.

### 13.12.4 Examples

Listing 13.12: Computing the Conjugate Fourier Series.

```

Program TestFourier

2  USE NumTypes
4  USE Constants
   USE Fourier

6

   Integer, Parameter :: Nmax=20
8  Type (Fourier_Serie) :: FS1, FS2, FS3
   Complex (kind=DPC) :: Data(Nmax), X
10 Integer :: Nt

12 Do I = 1, Nmax
    X = Cmplx(TWOPLDP*I/Nmax, kind=DPC)
14    Data(I) = Sin(X) + Cmplx(0.0_DP, I*2.0_DP, kind=DPC)
   End Do

16
   FS1 = DFT(Data)

18
   Write(*, '(2ES33.25) ') Eval_Serie(FS1, 0.23_DP, 1.0_DP)
20 Write(*, '(2ES33.25) ') Eval_Serie(Conjg(FS1), 0.23_DP, 1.0_DP)

```

```

22      Stop
24 End Program TestFourier

```

## 13.13 Subroutine Save\_Serie(FS, File)

### 13.13.1 Description

Write the Fourier series **FS** to the file **File**.

### 13.13.2 Arguments

**FS**: Type `Fourier_Serie` or type `Fourier_Serie_2D`. The Fourier series that you want to store in a file.

**File**: Character string of arbitrary length. The name of the file in which you want to save **FS**.

### 13.13.3 Examples

Listing 13.13: Saving a Fourier Serie in a file.

```

Program TestFourier
2
3   USE NumTypes
4   USE Constants
5   USE Fourier
6
7   Integer, Parameter :: Nmax=20
8   Type (Fourier_Serie) :: FS1, FS2, FS3
9   Complex (kind=DPC) :: Data(Nmax), X
10  Integer :: Nt
11
12  Do I = 1, Nmax
13      X = Cmplx(TWOPLDP*I/Nmax, kind=DPC)
14      Data(I) = Sin(X) + Cmplx(0.0_DP, I*2.0_DP, kind=DPC)
15  End Do
16
17  FS1 = DFT(Data)
18
19  CALL Save(FS1, 'datamodes.dat')
20
21  Stop
22 End Program TestFourier

```

## 13.14 Subroutine Read\_Serie(FS, File)

### 13.14.1 Description

Reads the Fourier series **FS** stored in the file **File**.

### 13.14.2 Arguments

**FS:** Type `Fourier_Serie` or type `Fourier_Serie_2D`. The name of the Fourier series data type in which you want to store that data.

**File:** Character string of arbitrary length. The name of the file in which the saved series is.

### 13.14.3 Examples

Listing 13.14: Reading a Fourier series from a file.

```

1 Program TestFourier
2
3   USE NumTypes
4   USE Constants
5   USE Fourier
6
7   Integer, Parameter :: Nmax=20
8   Type (Fourier_Serie) :: FS1, FS2, FS3
9   Complex (kind=DPC) :: Data(Nmax), X
10  Integer :: Nt
11
12  Do I = 1, Nmax
13    X = Cmplx(TWOPLDP*I/Nmax, kind=DPC)
14    Data(I) = Sin(X) + Cmplx(0.0_DP, I*2.0_DP, kind=DPC)
15  End Do
16
17  FS1 = DFT(Data)
18
19  CALL Save_Serie(FS1, 'datamodes.dat')
20  CALL Read_Serie(FS2, 'datamodes.dat')
21
22  Write(*, '(ES33.25)') Sum(Abs(FS1%Coef(:) - FS2%Coef(:)))
23
24  Stop
25
26 End Program TestFourier

```

# Fourteen

---

## MODULE Time

---

The `MODULE Time` is a module to provide access to date and time properties.

### 14.1 Type `tm`

#### 14.1.1 Description

A new data type, called `tm` is defined. It has some properties common with the same derived type defined in the C standard library. The components of the type specify a time: Day, year, month, hour, etc. . .

#### 14.1.2 Components

**hour:** Integer. Hour of the day [0-23].

**min:** Integer, Minutes after the hour [0-59].

**sec:** Integer. Seconds after the minute [0-59].

**msec:** Integer. Milliseconds after the second [0-999].

**year:** Integer. Year.

**month:** Integer. Month of the year [0-11].

**mday:** Integer. Day of the month [1-31].

**wday:** Integer. Day of the week since Sunday [0-6].

#### 14.1.3 Example

A small example defining a `tm` data type.

Listing 14.1: Defining a Time data type.

```
2 Program Test
4   USE NumTypes
   USE Time
```

```
6   Type (tm) :: Oneday
8   OneDay%hour = 12
9   OneDay%min  = 0
10  OneDay%sec  = 0
11  OneDay%amday = 10
12  OneDay%mon  = 0
13  OneDay%year = 2007
14  OneDay%yday = 3
16  Stop
End Program Test
```

## 14.2 Function `gettextime()`

### 14.2.1 Description

The function `gettextime()` returns the current time and date in a `type tm` data type.

### 14.2.2 Arguments

This function has no arguments.

### 14.2.3 Output

Type `tm`, containing all the information about the date and time.

### 14.2.4 Example

A small program that prints the current year.

Listing 14.2: Obtaining the current date and time.

```
1 Program Test
3   USE NumTypes
4   USE Time
5
6   Type (tm) :: Oneday
7
8   Oneday = gettextime()
9
10  Write(*,*) 'Current year: ', Oneday%year
11
12  Stop
13 End Program Test
```

## 14.3 Function isleap(Nyr)

### 14.3.1 Description

The function `isleap(Nyr)` returns `.true.` if `Nyr` is a leap year, and `.false.` otherwise. Note that the leap years are different in the Julian and Gregorian calendars. In this code the Gregorian calendar is supposed valid *after* 1582<sup>1</sup>.

### 14.3.2 Arguments

**Nyr:** Integer. The year.

### 14.3.3 Output

Logical. `.true.` if `Nyr` is a leap year, and `.false.` otherwise.

### 14.3.4 Example

A small program that tell us if the current year is leap.

Listing 14.3: Are we in a leap year?.

```

1 Program Test
3   USE NumTypes
4   USE Time
5
6   Type (tm) :: Oneday
7
8   Oneday = gettime()
9
10  If (isleap(Oneday%year)) Then
11    Write(*,*) 'We are in a leap year.'
12  Else
13    Write(*,*) 'We are not in a leap year.'
14  End If
15
16  Stop
17 End Program Test

```

## 14.4 Function asctime(t)

### 14.4.1 Description

The function `asctime`, returns a 24 length character string from a type `tm` data type, containing the date and time, in a similar way that the function `asctime` of the C standard library, for example:

<sup>1</sup>For more details, take a look at

[http://en.wikipedia.org/wiki/Gregorian\\_calendar](http://en.wikipedia.org/wiki/Gregorian_calendar)

Wed Jan 10 19:15:49 2007

### 14.4.2 Arguments

**t:** Type `tm`. A Type `tm` data type containing the date and time.

### 14.4.3 Output

Character (len=24). A 24 length character string with the format `Www Mmm dd hh:mm:ss yyyy`, where `Www` is the weekday, `Mmm` the month in letters, `dd` the day of the month, `hh:mm:ss` the time, and `yyyy` the year.

### 14.4.4 Example

A small program that prints the current time.

Listing 14.4: Printing current date/time.

```
1 Program Test
3     USE NumTypes
   USE Time
5
7     Write(*, '(1A) ') asctime(gettime())
9
   Stop
11 End Program Test
```

## 14.5 Function Day\_of\_Week(Day, Month, Year)

### 14.5.1 Description

The function `Day_of_Week(Day, Month, Year)`, returns the day of the week since sunday (sunday is 0), of the date that correspond to the input `Day, Month, Year`.

### 14.5.2 Arguments

**Day:** Integer. The day of the month [1-31].

**Month:** Integer. The month of the year [0-11].

**Year:** Integer. The year.

### 14.5.3 Output

Integer. The day of the week since sunday, thus a number between 0 and 6, with 0 corresponding to sunday.



#### 14.5.4 Example

A small program that prints the date and time of the first of january of 1900.

Listing 14.5: Day of week of the first of January 1900.

```
1 Program Test
3   USE NumTypes
   USE Time
5
   Type (tm) :: Oneday
7
   Oneday%hour = 12
   Oneday%min  = 0
   Oneday%sec  = 0
11  Oneday%anday = 1
   Oneday%mon  = 0
13  Oneday%year = 1900
15
   Oneday%wday = Day_of_Week(Oneday%anday, Oneday%mon, Oneday%year)
17
   Write(*,*) asctime(Oneday)
19
   Stop
End Program Test
```



---

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# Index

---

- Subroutine abort([routine], msg), 6
- Function asctime(t), 139
- Function Basis(X1, X2, n, s, q, itau[,  
Prec]) , 70
- Function Bisec(a, b, Fbis, [Tol]), 122
- Subroutine BootStrap(Ibt), 95
- Subroutine Bracket(X1, X2, X3, Func), 29
- Subroutine BstrpConfInt(Data, Ibt, alpha,  
Func, dmin, dpls), 99
- Function Conjg(FS), 134
- Subroutine ConjGrad(X, F, Fd[, Tol]) ,  
33
- Function Day\_of\_Week(Day, Month, Year),  
140
- Function Degree(P), 110
- Function Deriv(P), 113
- Function Det(M), 55
- Function DFT(Data, Is), 133
- Function erf(X), 64
- Function erfc(X), 64
- Subroutine EstBstrp(Data, Ibt, Func, Val,  
Err), 97
- Function Euler(Init, Xo, Xfin, Feuler,  
[Tol]), 21
- Function Eval\_Serie(FS, X, [Y], Tx, [Ty])  
, 131
- Function Factorial(N), 71
- Subroutine FishTipp(X, Rm, Rb), 92
- Function GammaLn(X), 63
- Subroutine GetLuxSeed([ISeed]), 90
- Function gettime(), 138
- Function Hermite(n,x[, Dval]), 68
- Function HermiteFunc(n, x[, Dval]), 69
- Subroutine Histogram(Val, Ndiv, Ntics,  
Vmin, Vmax, h), 81
- Subroutine Init(P, Dgr), 110
- Subroutine Init\_Serie(FS,Ns), 130
- Subroutine Insrt(X[, Ipt]), 58
- Function Integra(P, Cte), 114
- Function Interpol(X, Y), 116
- Function InterpolValue(X, Y, Xo), 115
- Function IntQuadrilateral(P1,P2,P3,P4,Fval)  
, 26
- Function inverf(X), 65
- Subroutine/Function Irand([Irnd], N, M),  
94
- Function isleap(Nyr), 139
- Subroutine Laplace(X, Rm, Rb), 93
- Function Legendre(l, m, X), 72
- Subroutine Levy(X,  $\alpha$ , [c], [ $\beta$ ], [ $\mu$ ]),  
91
- Subroutine LinearReg(X, Y, Yerr, [Func],  
Coef, Cerr, ChisqrV), 82
- Subroutine LineSrch(X, Func[, Tol]), 31
- Function Locate(X, Xo[, Iin]), 60
- Subroutine LU(M, Ipiv, Idet), 52
- Subroutine LUsolve(M, b), 54
- Function MaxPosition(FVal, IpX, IpY), 37
- MCIntegrationMCIntegration(X1, X2, Func,  
Val, Err[, Tol]), 103
- Function Mean(X), 75
- Function Median(X), 76
- Subroutine Migrad(Func, X, Fval, [Bound],[Release],  
[logfile]), 41
- Subroutine Minimize(Func, X, Fval, [Bound],[Release],  
[logfile]), 39

- Subroutine Miscan(Func, X, Fval, [Bound], [Release], [logfile]), 47
- Subroutine Miseek(Func, X, Fval, [Bound], [Release], [logfile]), 45
- Subroutine Misimplex(Func, X, Fval, [Bound], [Release], [logfile]), 43
- Function Moment(X, k), 80
- Function Newton(Xo, Fnew, [Tol]), 120
- Subroutine NonLinearReg(X, Y, Yerr, Func, Coef, Cerr, ChisqrV), 85
- Subroutine Normal(X, [Rm], [Rsig]), 90
- Subroutine Permutation(Idx), 95
- Subroutine perror([routine], msg), 5
- Subroutine Pivoting(M, Ipiv, Idet), 51
- Function Prop\_Error(X, Dx, F[, N]), 101
- Subroutine PutLuxSeed([ISeed]), 89
- Subroutine Qsort(X[, Ipt]), 59
- Subroutine Read\_Serie(FS, File), 135
- Subroutine ReadBstrp(Ibt, Filename), 97
- Function Rgnkta(Init, Xo, Xfin, Feuler, [Tol]), 24
- Subroutine RootPol(a, b, [c, d], z1, z2, [z3, z4]), 119
- Subroutine Save\_Serie(FS, File), 135
- Subroutine SaveBstrp(Ibt, Filename), 96
- Subroutine SetLuxLevel(Ilevel), 88
- Function Simpson(a, b, Func, [Tol]), 11
- Function SimpsonAb(a, b, Func, [Tol]), 14
- Function SimpsonInfDw(a, Func, [Tol]), 17
- Function SimpsonInfUp(a, Func, [Tol]), 15
- Function SimpsonSingDw(a, b, Func, [Tol], gamma), 20
- Function SimpsonSingUp(a, b, Func, [Tol], gamma), 18
- Function SphericalHarmonics(l, m, th, ph), 73
- Subroutine Spline(X, Y, Ypp0, YppN, Pols), 117
- Function Stddev(X), 79
- Function Step(X, FStep[, Tol]), 35
- Subroutine Swap(X, Ind1, Ind2), 57
- Function Theta(i, z, tau[, Prec]), 66
- Function ThetaChar(a, b, z, tau[, Prec]), 67
- Type tm, 137
- Function Trapecio(a, b, Func, [Tol]), 9
- Function TrapecioAb(a, b, Func, [Tol]), 12
- Function Unit(FS, Ns), 132
- Function Value(P, X), 112
- Function Var(X), 79
- Subroutine WConfInt(X, w, Xmin, Xmax), 78
- Function WMedian(X, w), 76
- Function WPercentile(X, w, p), 77