TRIBHUVAN UNIVERSITY

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/ Science Full Marks: 60
Microprocessor (CSC 162) Pass Marks: 24
Time: 3 hours.

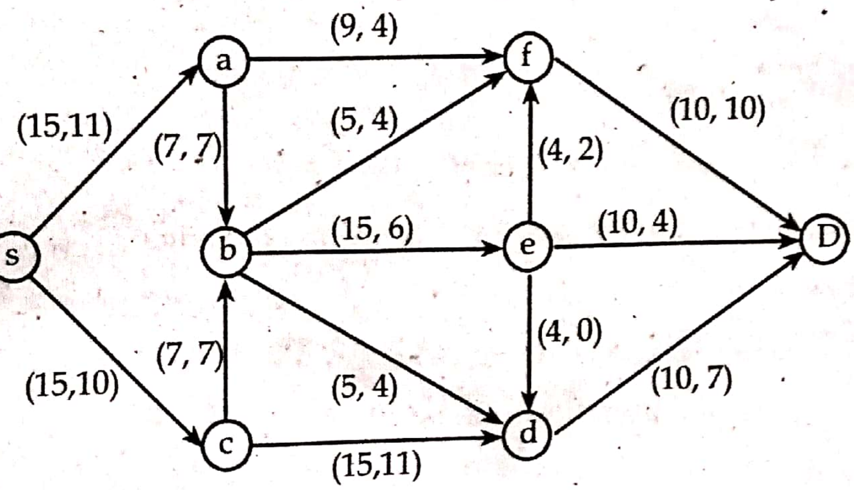
TU QUESTIONS-ANSWERS 2075

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **(2x10=20)**

1. **What is S-D cut? For the following network flow find the maximal flow from S to D.** **[2 + 8]**



Ans: In a flow network, an **s-t cut** is a **cut** that requires the source 's' and the sink 't' to be in different subsets, and it consists of edges going from the source's side to the sink's side. The capacity of an **s-t cut** is defined by the sum of the capacity of each edge in the **cut-set**.

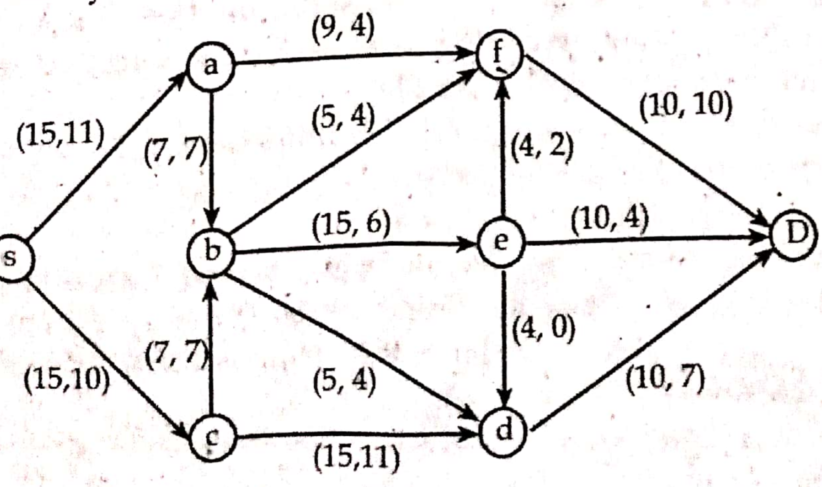


Fig: residual graph

Taking augmented path:

S → a → f → e → D

We can increase the flow by 2

Saturating the edge f → e

Taking augmented path

S → a → f → b → e → D

We can increase the flow by 2 again saturating the edge S → a

Taking augmented path

S → C → d → D

We can increase the flow by 3 saturating the edge a D

Again,

Taking augmented path

S → C → d → b → e → D

We can increase the flow by 1 saturating the edge C → d

Here,

No from s to D can be increased any more.

∴ The maximum flow will be 29.

Since,

Total outpoint flow (15 + 14 = 29) is equal to

Total incoming flow at D (10 + 9 + 10) = 29

2. Consider a set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. What will be the computer representation for set containing the number which are multiple of 3 not exceeding 6? Describe injective, surjective and bijective function with example. [2 + 8]

Ans: Assume that the universal set U is finite (and of reasonable size so that the number of elements of U is not larger than the memory size of the computer being used).

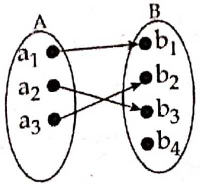
First, specify an arbitrary ordering of the elements of U, for instance $a_1, a_2, \ldots, a_n$. Represent a subset A of U with the bit string of length n, where the i^{th} bit in this string is 1 if a_i belongs to A and is 0 if a_i does not belong to A.

Let set A represent the no which are multiple of 3 not exceeding 6.

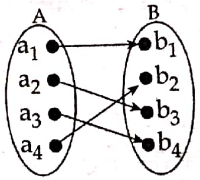
i.e. A = {3, 6}

So computer representation bit string for A is 0010010000.

- **Definition:** A function f is said to be one-to-one or injective (or an injection) if $\forall$ x and y in the domain of f, $f(x) = f(y) \Rightarrow x = y$
- Intuitively, an injection simply means that each element in the range has at most one preimage (antecedent).
- It is useful to think of the contra positive of this definition.
- $x \neq y \Rightarrow f(x) \neq f(y)$



- Is this function
  - It this a function
    - One-to-one (injective)? Why Yes, no, b_i has 2 preimages
    - Onto (surjective)? Why? No, b_4 has no preimage
- **Definition:** A function f : A → B is called onto or surjective (or an surjection) if $\forall b \in \exists a \in A$ with f(a) = b
- Intuitively, a surjection means that every elements in the codomain is mapped into (i.e., it is an image, has an antecedent)
- Thus, the range is the sameas the codemain



- Is this a function
  - One-to-one (injective)? Thus, it is a bijection or a
  - Onto (surjective)? One-to-one correspondence

3. Compute the following values.

a. 3 mod 4 **b.** 7 mod 5 **c.** – 5 mod 3

d. 11 mod 5 **e.** – 8 mod 6

Write down recursive algorithm to find the value of b^n and prove its correctness using induction. [5 + 5]

Ans: 3 mod 4= 3

7 mod 5= 2

-5 mod 3=1

11 mod 5=1

-8mod 6=4

Recursive algorithm to find b^n

An algorithm is called recursive algorithm if it solves a problem by reducing it to an instances of same problem with smaller input.

```
procedure powerN(int base, int n)
if (n <0) then
return ("Illegal Power Argument")
if (n == 0) then
return1
else
return base * powerN(base, n - 1)
end powerN
```

Section B (Short Answer Questions)

Attempt any EIGHT questions: **[8 × 5 = 40]**

4. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 2$. **[5]**

Ans: Let c_1 and c_2 be real number. Suppose that $r^2 - c_1r - c_2 = 0$ has two distinct roots r_1 and r_2. Then the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2}$. if and only if $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ for n = 0, 1, 2. . . Where α_1 and α_2 are constants.

The characteristic equation of the recurrence relation is

$r^2 - 5r + 6 = 0$

Solving we get the root $r_1 = 1$ and $r_2 = 6$

Hence, the sequence $\{a_n\}$ is a Solution to the recurrence relation if any only if

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$

For some constant α_1 and α_2

From initial condition, it flows that

$a_0 = 1 = \alpha_1 + \alpha_2$ . . . (i)

$a_1 = 2 = \alpha_1 .1 + \alpha_2 . 6$ . . . (i) ($\because c_1 = \alpha_1 r_1 + \alpha_2 + \alpha_2 r_2$)

Solving equation (i) and (ii), we get

$\alpha_1 = \frac{1}{5}$ and $\alpha_2 = \frac{4}{5}$

Therefore, the solution to the recurrence relation and initial condition is the sequence $\{a_n\}$ with.

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$

$$\therefore \quad a_n = \frac{1}{5}(1)^n + \frac{4}{5}(6)^n.$$

5. Find the value of x such that $x \equiv 1 \pmod 5$ and $x \equiv 2 \pmod 7$ using Chinese remainder theorem. **[5]**

Ans: **Theorem:** (Chinese Remainder Theorem). Let $m_1, m_2, \ldots m_r$ be a collection of pair wise relatively prime integers. Then the system of simultaneous congruences.

$x \equiv a_1 \pmod{m_1}$

$x \equiv a_2 \pmod{m_2}$

⋮

$x \equiv a_r \pmod{m_r}$

has a unique solution modulo $M = m_1m_2, \ldots m_r$, for any given integers $a_1, a_2, \ldots a_r$.

Proof of CRT. Put $M = m_1 \ldots m_r$ and for each k = 1, 2, r, r let $M_k = \frac{M}{m_k}$. then $\gcd(M_k, m_k) = 1$ for all k. Let y_k be an inverse of M_k modulo m_k, for each k. Then by definition of inverse we have $M_ky_k \equiv 1 \pmod{m_k}$. Let

$$x = a_1M_1 + a_2M_2y_2 + \ldots + a_rM_ry_r$$

Then x is a simultaneous solution to all of the congruences. Since the moduli $m_1, \ldots, m_r$ are pairwise relatively prime, any two simultaneous solutions to the system must be congruent modulo. Thus the solution is a unique congruence class modulo M, and the value of x computed above is in that class.

$x = 1 \pmod 5$ $\qquad$ $x = 2 \pmod 7$

Here,

$a_1 = 1$ $\qquad$ $m_1 = 5$

$a_2 = 2$ $\qquad$ $m_2 = 7$

M $= m_1 \times m_2 = 5 \times 7 = 35$

$M_1 = M/m_1 = \frac{35}{5} = 7$

$M_2 = M/m_1 = \frac{35}{7} = 5$

Finding y_k

$7 \times y_1 = 1 \pmod 5$ $\qquad$ $5 \times y_2 = 1 \pmod 7$

Now,

$7 = 5 \times 1 + 2$

$5 = 2 \times 2 + 1$

$2 = 1 \times 2 + 0$

or, Using extended Euclidean also

$1 = 5 + 2(-2)$

$1 = 5(3) + 4(-2)$ $\qquad$ ($\therefore$ after subtraction)

or, $y_1 = -2 \bmod 5 = 3$

$y_2 = 3$

Finally,

x $= a_1 m_1, y_1 + a_2M_2y_2 = 1. 7. 3 + 2. 5. 3 = 21 + 30 \pmod M$

$= 51 \pmod{35} = 16$

6. **Prove that $5^n - 1$ is divisible by 4 using mathematical induction.** [5]

Ans: Let $P(n) = 5^n - 1$

1. **Basic step:**
   For n = 1, we have
   P (1) = 5 − 1 = 4 divisible by 4.
2. **Indicutive Hypothesis**
   Assume P (K) is true for n = k
   i.e. $P(k) = 5^k - 1$ is true.
3. **Inductive step:**
   $P(k+1) = 5^{k+1} - 1 = 5^k \times 5 - 1 = 5 \times 5^{k-1} = (4 \times 5^k + 5^k) - 1$
   $= (4 \times 5^k) + (5^k - 1)$

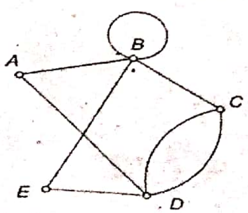
   Here,
   4×5^k is divisible by 4
   Also,
   $5^k - 1$ is also divisible by 4
   From Inductive hypothesis.
   Therefore, P (k + 1) is true.
   Hence proved.

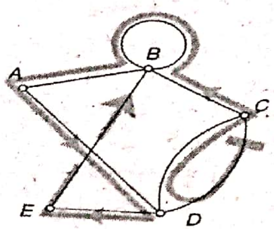
7. Let A="Aldo is Italian" and B="Bob is English". Formalize the following sentences into proposition.
   Aldo isn't Italian
   Aldo is Italian while Bob is English
   If Aldo is Italian then Bob is not English
   Aldo is Italian or if Aldo isn't Italian then Bob is English
   Either Also is Italian and Bob is English, or neither Aldo is Italian nor Bob is English.

Ans: (a) $\neg A$ (b) $A \vee B$
(c) $A \rightarrow \neg B$ (d) $A \vee (\neg A \rightarrow B)$
(e) $(A \wedge B) \vee \neg(A \vee B)$

8. **Define Euler path and Hamilton path with example. Draw the Hasse diagram for the divisibility relation on the set {1, 2, 5, 8, 16, 32} and find the maximal, minimal, greatest and least element if exist.** [2+3=5]

Ans: An Euler path in G is a simple path containing every edge of G. It starts and ends at different vertices.





Another Euler Path: CBCBBADEB

The criterion for Euler Paths
The inescapable conclusion ("based on reason alone!")

If a graph G has an Euler path, then it must have exactly two odd vertices.

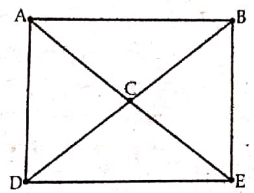
Or to put it another way.

If the number of odd vertices in G is anything other than 2, then G cannot have an Euler path.

Hamiltonian Path – A simple path in a graph that passes through every vertex exactly once is called a Hamiltonian path. Unlike Euler paths and circuits, there is no simple necessary and sufficient criteria to determine if there are any Hamiltonian paths or circuits in a graph

Example: Hamilton Path.

- Graph (a) shown has Hamilton path A, B, C, D, E, D. The graph also has Hamilton path C, B, A, D, E. Can you find some others?



In order theory, a Hasse diagram is a type of mathematical diagram used to represent a finite partially ordered set, in the form of a drawing of its transitive reduction.

A Hass diagram is a graphical representation of the relation of elements of partially ordered set (poset) with an implied upward orientation. A point is drawn for each element of the partially ordered set (poset) and joined with the line segment according to the following rules:

- If p < q in the poset, then the point corresponding to p appears lower in the drawing than the point corresponding to q.
- The two points p and q will be joined by line segment iff p is related to q.

To draw a Hasses diagram, provided set must be a poset

A poset or partially ordered set A is a pair, $(B, \leq)$ of a set B whose elements are called the vertices of A and obeys following rules.

1. Reflexivity $\rightarrow p \leq p \ \forall \ p \in B$
2. Anti-symmetric $\rightarrow p \leq q$ and $q \leq p$ iff $p = q$
3. Transitivity $\rightarrow$ if $p \leq q$ and $q \leq r$ then $p \leq r$

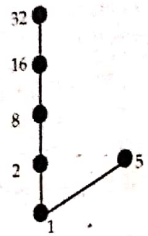
For Regular Hasses Diagram.

- Maximal elements are those which are not succeeded by another element.
- Minimal elements are those which are not preceded by another element.
- Greatest element (if it exists) is the element succeeding all other elements.
- Least element is the element that precedes all other elements.

let the set is A

i.e. A = {(1 ≤ 2), (1 ≤ 5), (1 ≤ 8), (1 ≤ 16), (1 ≤ 32), (1 ≤ 8), (1 ≤ 16), (1 ≤ 32), (1 ≤ 16), (8 ≤ 32), (16 ≤ 32)}

So, Hasse diagram will be.



Maximal element = 32
Minimal element = 1
Greatest element = 32
Least element = 1

9. What does primality testing means? Describe how little fermat's theorem test for prime numbers with suitable example.

Ans: A primality test is a test to determine whether or not a given number is prime, as opposed to actually decomposing the number into its constituent prime factors (which is known as prime factorization).

Primality tests come in two varieties: deterministic and probabilistic. Deterministic tests determine with absolute certainty whether a number is prime. Examples of deterministic tests include the Lucas-Lehmer test and elliptic curve primality proving. Probabilistic tests can potentially (although with very small probability) falsely identify a composite number as prime (although not vice versa). However, they are in general *much* faster than deterministic tests. Numbers that have passed a probabilistic prime test are therefore properly referred to as probable primes until their primality can be demonstrated deterministically.

Fermat's Little Theorem: If p is prime and a is an integer not divisible by p, then

$a^{p-1} \equiv 1 \pmod{p}$.

Furthermore, for every integer a we have

$a^{p} \equiv a \pmod{p}$.

Remark: Fermat's little theorem tells us that if $a \in z_p$, then $a^{p-1} = 1$ in Z_p

Example: Find 7^{222} and 11.

Solution: We can fermat's little theorem to evaluate 7^{222} mod 11 rather than using the fast modular exponentiaion algorithm. By Fermat's little theorem we know that $7^{10} \equiv \pmod{11}$, so $(7^{10})^k \equiv 1 \pmod{11}$ for every positive integer k. To take advantage of this last congruence, we divide the exponent 222 by 10, findings that 222 = 22. 10 + 2. We know see that

$7^{222} + 7^{22 \cdot 10 + 2} = (7^{10})\ 7^2 \equiv (1)^{22} \cdot 49 \equiv 5 \pmod{11}$

If follow that 7^{222} mod 11 = 5

Example 9 Illustrated how we can use Fermat's little theorem compute a^n mod p, which p is prime and p × a : First, we use the division algorithm to find the quotient q and remainder r when n is divided by p − 1, so that n = q (p − 1) + r where 0 ≤ r < p − 1. It follows that $a^n = a^{q(p-1)+r} = (a^{p-1})^q a^r \equiv 1qa^r \equiv a^{r \pmod{p}}$. Hence, to find a^n mod p, we only need to compute a^r mod p. We will take advantage of the simplification many times in our study of number theory.

10. List any two applications of conditional probability. You have 9 families you would like to invite to a wedding. Unfortunately, you can only invite 6 families. How many different sets of invitations could you write?

Ans: Two application of conditional probability

- Drawing a 2nd Ace from a deck of cards given we got the initial Ace.
- Finding the probability of liking Harry Potter given we know the individual prefers fiction.

Since order doesn't matter while selecting families, we use combination for the solution.

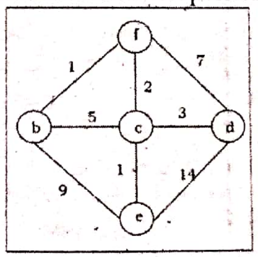
9C6 = 84

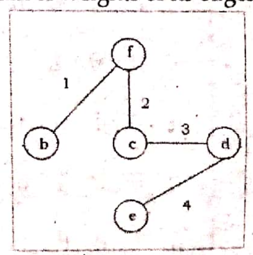
11. Design spanning tree and minimum spanning tree. Mention the condition of two graph for being isomorphic with example .

Ans: Let G be a simple graph. A spanning tree of G is a sub-graph of G that is a tree containing every vertex of G.

A simple graph with a spanning tree must be connected, because there is a path in the spanning tree between any two vertices. The converse is also true; that is, every connected simple graph has a spanning tree

A minimum spanning tree in a connected weighted graph is a spanning tree that has the smallest possible sum of weights of its edges.



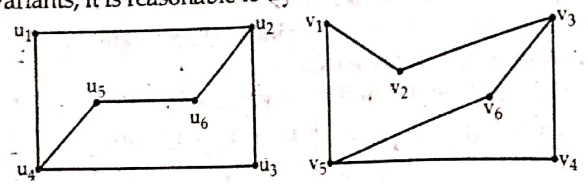


Two algorithms
1. prism's algorithm and
2. Kruskal algorithm are used to find the minimum spanning tree.

The simple graphs G1 = (V1, E1) and G2 = (V2, E2) are *isomorphic* if there exists a one- to - one and onto function *f* from V1 to V2 with the property that *a* and *b* are adjacent in G1 if and only if *f (a)* and *f* (*b*) are adjacent inG2, for all *a* and *b* in *V*1. Such a function *f* is called an *isomorphism*. *Two simple graphs that are not isomorphic are called *no isomorphic*.

Example: Determine whether the graphs G and H displayed in figure are isomorphic.

Solution: Both G and H have six vertices and seven edges. Both have four vertices of degree two and two vertices of degree three. It is also easy to see the sub-graphs of G and H consisting of all vertices of degree two and the edges connecting them are isomorphic (as the reader should verify). Because G' and H agree with respect to these invariants, it is reasonable to try to find an isomorphism f.



Graph G and H

We now will define a function f and then determine whether it is an isomorphism. Because deg $(u_1)_2$ and because u_1 is not adjacent to any other vertex of degree two, the image of u_1 must be either v_4 or v_6. [If we found that this choice did not lead to isomorphism, we would then try $f(u_1) = u_4$] Because u_2 is adjacent to u_1, the possible images of u_2 are v_3 digress as a guide, we set $f(u_3) = u_4$. $f(u_5) = v_5$, $f(u_4) = v_1$ and $f(u_6) = v_2$. We now have a one-to-one correspondence between the vertex set of G and the vertex set of H. Namely, $f(u_1) = v_6$, $f(u_2) = v_3$, $f(u_3) = v_4$, $f(u_4) = v_5$, $f(u_5) = v_1$, $f(u)_6 = v_2$. To see whether f preserves edges, we examine the adjacency matrix of G.

$$A_G = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{array} \begin{array}{c} \begin{array}{cccccc} u_1 & u_2 & u_2 & u_4 & u_5 & u_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

And the adjacency matrix of H with the rows and columns labeled by the images of the corresponding vertices in G.

$$A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_2 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Because $A_G = A_H$, it follows that f preserves edges. We conclude that f is an isomorphism, So G and H are isomorphic. Note that if f turned out not to be an isomorphism, we would not have established that G and H are not isomorphic, because another correspondence of the vertices in G and H may be an isomorphism.

12. **Prove that the product xy is odd if and only if both x and y are odd integers.**

Ans: The product xy is odd if and only if both x and y are odd.

Proof:

Part 1

If x and y are both odd then xy is odd.

Proof: By definition a = 2n + 1 and b = 2m + 1 for n, m integers.

Now consider the product ab = (2n + 1)(2m + 1)

= 4nm +2n +2m +1 = 2(2nm +n + m) + 1 = 2k

Where k = 2nm + n + m is an integer. Therefore the product xy is odd by definition of odd.

Part 2

If xy is odd then x and y are each odd.

Proof: (by contradiction)

Given that xy is odd assume that x and y are not both odd. If x and y are not both even then we must consider two cases

Case 1: Let x be even and y is odd. By definition we have x = 2n and y = 2m + 1 where n, m are integers.

Consider xy = (2n)(2m +1)= 4nm + 2n = 2(2nm +n) which is even.

This contradicts that xy is in fact odd.

Case 2: Let x and y both be even. By definition x = 2n and y = 2m for n, m integers.

Consider the product xy = (2n)(2m) = 2(2nm) = 2k for k = 2nm is an integer. There we have that xy is even which is a contradiction to the fact that xy is odd.

Therefore by the two cases above x and y must both be odd.

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Pass Marks: 24

Microprocessor (CSC 162)

Time: 3 hours.

TU QUESTIONS-ANSWERS 2076

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The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **(2x10=20)**

1. **State pigeonhole principle. Solve the recurrence relation $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$ with initial conditions $a_0=1$, $a_1 = 3$, $a_2=7$.**

Ans: If n pigeonholes are occupied by n+1 or more pigeons, then at least one pigeonhole is occupied by greater than one pigeon. **Generalized pigeonhole principle is:** - If n pigeonholes are occupied by kn+1 or more pigeons, where k is a positive integer, then at least one pigeonhole is occupied by k+1 or more pigeons.

Numerical part,

We have recurrence relation as,

$a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$ with initial conditions $a_0=1$, $a_1 = 3$, $a_2=7$

Now the characteristic equation of this recurrence relation is

$r^3 - 3r^2 + 3r - 1 = 0$

or, $r^3 - r^2 - 2r^2 + 2r + r - 1 = 0$

or, $r^2(r-1) - 2r(r-1) + (r-1) = 0$

or, $(r-1)(r^2 - 2r + 1) = 0$

or, $(r-1)(r^2 - r - r + 1) = 0$

or, $(r-1)[r(r-1) - 1(r-1)] = 0$

or, $(r-1)(r-1)(r-1) = 0$

⇨ $r = 1, 1, 1$

Because $r^3 - 3r^2 + 3r - 1 = (r-1)^3$, there is a single root $r = 1$ of multiplicity three of the characteristic equation. By Theorem of roots of non-linear equation with more than two roots are distinct, the solutions of this recurrence relation are of the form

$a_n = \alpha_{1,0}(1)^n + \alpha_{1,1}n(1)^n + \alpha_{1,2}n^2(1)^n$

To find the constants $\alpha_{1,0}$, $\alpha_{1,1}$ and $\alpha_{1,2}$ use the initial conditions. This gives

$a_0 = 1 = \alpha_{1,0}$

$a_1 = 3 = \alpha_{1,0} + \alpha_{1,1} + \alpha_{1,2}$

$a_2 = 7 = \alpha_{1,0} + 2\alpha_{1,1} + 4\alpha_{1,2}$

The simultaneous solution of these three equations is $\alpha_{1,0} = 1$, $\alpha_{1,1} = 3$, and $\alpha_{1,2} = -2$

Hence, the unique solution to this recurrence relation and the given initial conditions is the sequence $\{a_n\}$ with

$a_n = (1 + 3n - 2n^2)(1)^n$.

2. **Find the value of x such that $x = 1 \pmod 5$, $x = 1 \pmod 7$, $x = 3 \pmod{11}$ and $x = 0 \pmod 7$ using Chinese remainder theorem.**

Ans: We have,

$x \equiv 1 \pmod 5$

$x \equiv 1 \pmod 7$

$x \equiv 3 \pmod{11}$

We have $a_1 = 1$, $a_2 = 1$, $a_3 = 3$

$m_1=5$, $m_2=7$, $m_3=11$

$x = (M_1x_1a_1 + M_2x_2a_2 + M_3x_3a_3) \pmod M$

$M = m_1.m_2.m_3 = 5 \times 7 \times 11 = 385$

$$M_1 = \frac{M}{M1} = \frac{385}{5} = 77$$

$$M_2 = \frac{M}{m_2} = \frac{385}{7} = 55$$

$$M_3 = \frac{M}{m3} = \frac{385}{11} = 35$$

$M_1 x_1 = 1 \pmod{m_1}$

Or, $77 x_1 = 1 \pmod 5$

Or, $2 x_1 = 1 \pmod 5$

Multiplying both side by 3 since we need to make coefficient of x_1 to 1,

$6 x_1 = 3 \pmod 5$

Or, $1 x_1 = 3$

⇨ $x_1 = 3$

Also, $M_2 x_2 = 1 \pmod{m_2}$

Or, $55 x_2 = 1 \pmod 7$

Or, $6 x_2 = 1 \pmod 7$

Multiplying both side by 6 since we need to make coefficient of x_1 to 1,

$36 x_2 = 6 \pmod 7$

Or, $1 x_2 = 6$

⇨ $x_2 = 6$

Also, $M_3 x_3 = 1 \pmod{m_3}$

Or, $35 x_3 = 1 \pmod{11}$

Or, $2 x_3 = 1 \pmod{11}$

Multiplying both side by 6 since we need to make coefficient of x_1 to 1,

$12 x_3 = 6 \pmod{11}$

Or, $1 x_3 = 6$

⇨ $x_3 = 6$

$$x = (M_1x_1a_1 + M_2x_2a_2 + M_3x_3a_3) \pmod{M}$$
$$= (77 \times 3 \times 1 + 55 \times 6 \times 1 + 35 \times 6 \times 3) \pmod{385}$$
$$= (231 + 330 + 630) \pmod{385}$$
$$= 1191 \pmod{385}$$
$$= 36 \text{ Ans.}$$

Also we can test the solution as,

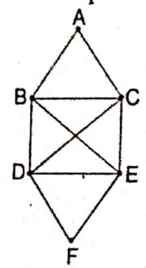
36 (mod 5) = 1

36 (mod 7) = 1

36 (mod 11) = 3

3. Define Eular circuit with suitable example. Find the maximal flow s to t from the given network flow.

Ans: An Euler path, in a graph or multi-graph, is a walk through the graph which uses every edge exactly once. An Euler circuit is an Euler path which starts and stops at the same vertex.



D, E, B, C, A, B, D, C, E, F, D

F-augmenting path are:

{s – 1 – 4 – t}, {s – 2 – 4 – t}, {s – 3 – 4 – t}, {s – 3 – 5 – t}

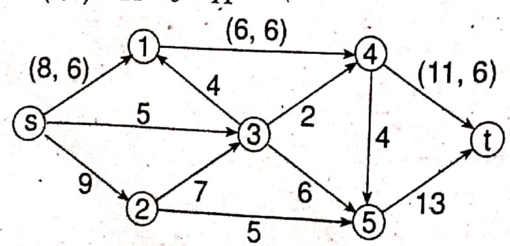
{s – 2 – 3 – 5 – t}, {s – 3 – 1 – 4 – t} etc

For f-augmenting path {S – 1 – 4 – t}

Slack of (5, 1) = 8 – 0 = 8

Slack of (1, 4) = 6 – 0 = 6 (min)

Slack of (4, t) = 11 – 0 = 11

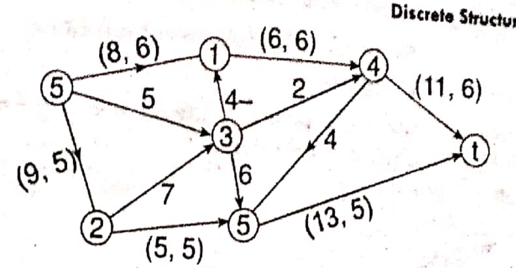


Also, for f-augmented path (s – 2 – 5 – t)

Slack of (s, 2) = 9 – 0 = 9

Slack of (2, 5) = 3 – 0 = 5 (min)

Slack of (5, t) = 3 – 0 = 13

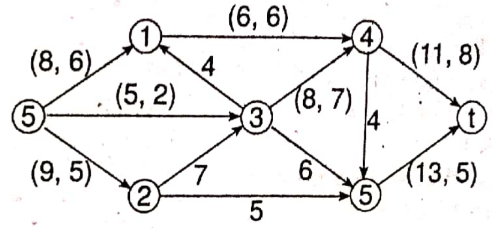


Also, for f-augmented path {s – 3 – 4 – t}

Slack of (s, 3) = 5 – 0 = 5

Slack of (3, 4) = 2 – 0 = 2 (min)

Slack of (4, t) = 11 – 6 = 5

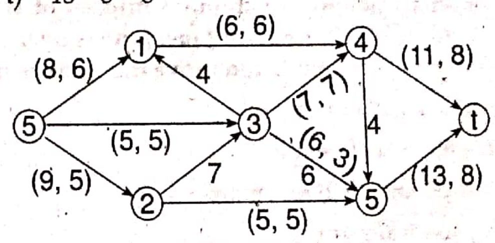


Also, for f-augmented path {s – 3 – 5 – t}

Slack of (s, 3) = 5 – 2 = 3 (min)

Slack of (3, 5) = 6 – 0 = 6

Slack of (5, t) = 13 – 5 = 8



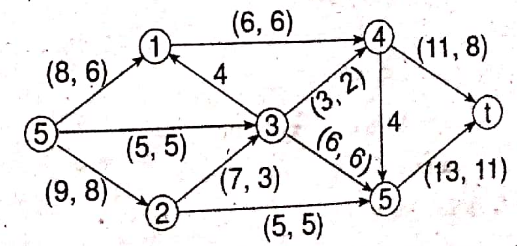
For f-augmented path (s – 2 – 3 – 5 - t)

Slack of (s, 2) = 9 – 5 = 4

Slack of (2, 3) = 7 – 0 = 7

Slack of (3, 5) = 6 – 3 = 3 (min)

Slack of (5, t) = 13 – 8 = 5



Now there are no possible f-augmented path thus, max flow = 6 + 5 + 8 = 8 + 11 = 19

Section B (Short Answer Questions)

Attempt any EIGHT questions: **[8 × 5 = 40]**

4. **Prove that for every positive integer $n \geq 1$, n^2+n is even integer using mathematical induction.**

Ans: **Basic step.** Let n = 1.

$n^2 + n$

$1^2 + 1$

$1 + 1 = 2$

∴ $n^2 + n$ is even if n = 1. [2 is an even number.]

Inductive step: Let n = k.

Assume that $k^2 + k$ is even.

Let n = k + 1.

$n^2 + n = (k + 1)^2 + (k + 1)$
$= (k^2 + 2k + 1) + (k + 1)$
$= k^2 + 2k + 1 + k + 1$
$= k^2 + k + 2k + 2$

From inductive hypothesis $k^2 + k$ is even

Hence $n^2 + n$ = even + 2k + 2
= even + 2 (k+1)

Also 2 (k+1) is even for every k>0

Hence $n^2 + n$ is even.

5. **All over smart people are stupid. Children of stupid people are naughty. John is a children of Jane. Jane is over smart. Represent these statements in FOPL and prove that John is naughty.**

Ans: Let x = total number of people

y = total children of people

OS (x) = x people are over smart

S(x) = x people are stupid

C (y, x) = y children of x people

n (y) = y children are naughty

OS (jane) = Jane is overs mart

n(john) = john is naughty

Promises:

1. $\forall x$ (OS → S(X))
2. $\forall x, y$ (S (X) ∧ C (y, x) → n(y))
3. OS (Jane)
4. C (john, Jane)

Conclusion: n (John)

	Steps	Reason
1	OS (Jane)	Promises
2	$\forall x$ (OS → S(X))	Promises
3	OS (Jane) → S (Jane)	Promises
4	S (Jane)	Universal instantiation (2)
5	C (John, Jane)	Modus Pones (1 & 3)
6	$\forall x\, y$ (S (X) ∧ C(y, x) → n(y))	Promises
7	S (Jane) ∧ C (John, Jane) → n (John)	Promises
8	C (Jane, Jane) → n (John)	Universal Instantiation (6)
	n (John)	Simplification
		Modus Pones (5, 1)

6. **Which of the following are posets?**

a. **(Z, =)**

b. **(Z, ≠)**

c. **(Z, ⊆)**

Ans: a) **(Z, =):** This is a poset. The only ordered pairs we will have in this relation is (a, a) for all a ∈ Z. This would mean that the relation is reflexive, anti-symmetric, and transitive.

b) **(Z, 6=):** This is not a poset because it is not reflexive. We cannot have the order pair (a, a) for all a ∈ Z. This relation is also not anti-symmetric and not transitive.

c) **(Z, ≥):** This is a poset. For reflexive, we can have the ordered pair (a, a) for all a ∈ Z. This is also anti-symmetric because consider the ordered pair (a, b) and a 6= b, this would mean that a > b. If this is the case, then b > a is not true and you cannot have (b, a). This is also transitive because if a > b, b > c, and a 6= b 6= c. Then it follows that a > c for all a, b, c ∈ Z.

7. **Define reflexive closure and symmetric closure. Find the remainder when $4x^2 - x + 3$ is divided by x + 2 using remainder theorem.**

Ans: **Reflexive closure:** Let R be a relation on A. The reflexive closure of R, denoted r(R), is R U Δ.

The reflexive closure of a relation R on A is obtained by adding (a, a) to R for each a ∈ A.

Example:

Let R be the relation on the set {0, 1, 2, 3} containing the ordered pairs(0, 1),(1, 1),(1, 2),(2, 0),(2, 2),(3, 0).

Find the reflexive closure of R.

We need to add (a, a) in R to make a reflexive closure.

{(0, 0),(0, 1),(1, 1),(1, 2),(2, 0),(2, 2),(3, 0),(3, 3)}

Symmetric closure

The symmetric closure of a relation R on a set A is defined as the smallest symmetric relation s(R) on A that contains R. The symmetric closure s(R) is obtained by adding the elements (b, a) to the relation R for each pair (b, a) ε R.

Example: Let R be the relation on the set {0, 1, 2, 3} containing the ordered pairs(0, 1),(1, 1),(1, 2),(2, 0),(2, 2),(3, 0). Find the symmetric closure of R

We need to add (b, a) for each (a, b) in R to make a symmetric closure.

{(0, 1),(0, 2),(0, 3),(1, 0),(1, 1),(1, 2),(2, 0),(2, 1),(2, 2),(3, 0)}

Numerical portion,

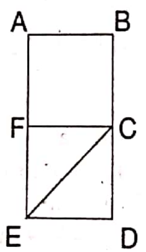
$$\begin{array}{r} 4x-9 \\ x+2\,\overline{)\,4x^2-x+3} \\ \underline{4x^2+8x} \\ -9x+3 \\ \underline{-9x-18} \\ 21 \end{array} \text{— Remainder}$$

8. **Define Eular path and Hamilton path. Give examples of both Eular and Hamilton path.**

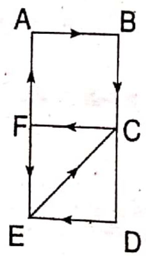
Ans: Euler Path

A path that travels through every edge of a connected graph once and only once and starts and ends at different vertices is called Euler path.

Example:



One Euler path for the above graph is F, A, B, C, F, E, C, D, E as shown below.



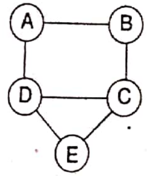
This Euler path travels every edge once and only once and starts and ends at different vertices. This graph cannot have an Euler circuit since no Euler path can start and end at the same vertex without crossing over at least one edge more than once.

Hamiltonian path

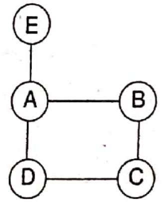
A Hamiltonian path also visits every vertex once with no repeats, but does not have to start and end at the same vertex.

A Hamiltonian path, also called a Hamilton path, is a graph path between two vertices of a graph that visits each vertex exactly once. If a Hamiltonian path exists whose endpoints are adjacent, then the resulting graph cycle is called a Hamiltonian cycle (or Hamiltonian cycle). A graph that possesses a Hamiltonian path is called a traceable graph.

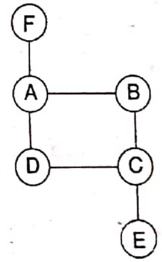
Example



Hamiltonian path= ABCDE



Hamiltonian path= EABCD



Hamiltonian path does not exist

9. **How many 3 digits numbers can be formed from the digits 1,2,3,4 and 5 assuming that:**

a. **Repetitions of digits are allowed**

b. **Repetitions of digits are not allowed**

Ans: When Repetitions of digits are allowed

Here, Total number of digits = 5

Let 3-digit number be XYZ.

Now the number of digits available for X = 5,

As repetition is allowed,

So the number of digits available for Y and Z will also be 5 (each).

Thus, The total number of 3-digit numbers that can be formed
= 5×5×5
= 125.

When Repetitions of digits are not allowed

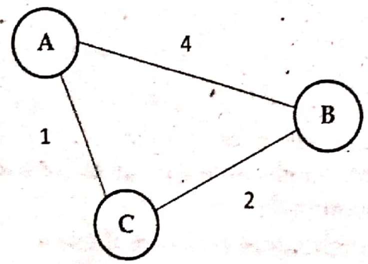
Here, Total number of digits = 5
Let 3-digit number be XYZ.
Now the number of digits available for X = 5,
As repetition is not allowed,
So the number of digits available for Y = 4 (As one digit has already been chosen at X),
Similarly, the number of digits available for Z = 3.
Thus, The total number of 3-digit numbers that can be formed
= 5×4×3
= 60.

10. What is minimum spanning tree? Explain Kruskal's algorithm for finding minimum spanning tree.

Ans: Minimum spanning tree

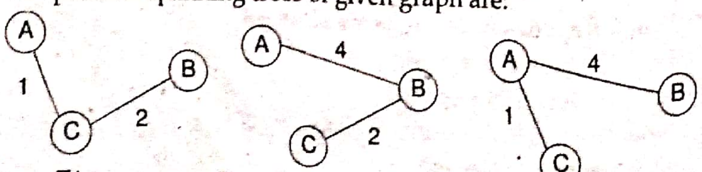
A minimum spanning tree in a connected weighted graph is a spanning tree that has the smallest possible total weights of its edges out of all possible spanning trees. In other words in a weighted graph, a minimum spanning tree is a spanning tree that has minimum weight than all other spanning trees of the same graph. In real-world situations, this weight can be measured as distance, congestion, traffic load or any arbitrary value denoted to the edges.

Example: Find MST of given graph



Solution:

The possible spanning trees of given graph are:



T1 T2 T3

Total weight of T1=3
Total weight of T2=6
Total weight of T3=5
Since T1 has minimum weight out of all possible spanning trees of given graph hence, tree T1 acts as minimum spanning tree of given graph.

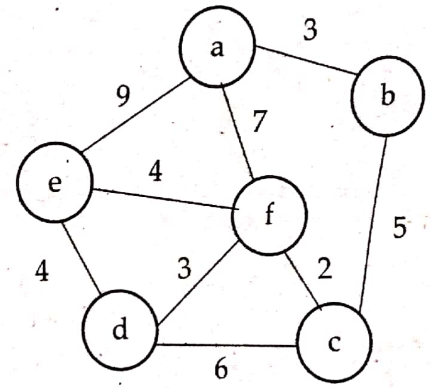
Kruskal's algorithm

Kruskal's algorithm is a minimum spanning tree algorithm that takes a graph as input and finds the subset of the edges of that graph which:

- form a tree that includes every vertex
- has the minimum sum of weights among all the trees that can be formed from the graph

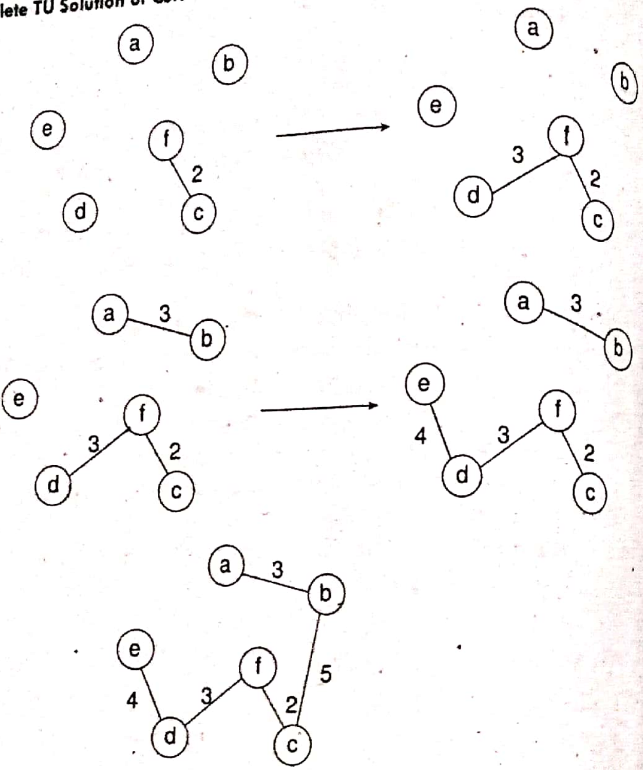
It is the procedure for producing a minimum spanning tree of a given weighted graph that successively adds edges of least weight that are not already in the tree such that no edges produce a simple circuit when it is added.

Example: Find minimum spanning tree of given graph by using Kruskal's algorithm.

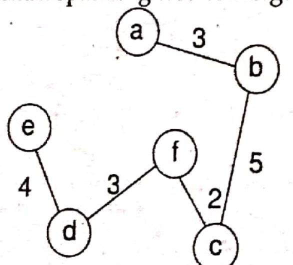


Edges in sorted order are given below:

Edges	Cost	Action
(f, c)	2	Accept
(f, d)	3	Accept
(a, b)	3	Accept
(d, e)	4	Accept
(b, c)	5	Accept
(d, c)	6	Reject
(a, f)	7	Reject
(a, e)	9	Reject



The edges (d, c), (a, f) and (a, e) forms cycle so discard these edges from the tree. Thus the minimum spanning tree of weight 17 is;



11. **List any two applications of graph coloring theorem. Prove that "A tree with n vertices has n-1 edges"**

Ans: The graph coloring problem has huge number of applications.

1. **Making Schedule or Time Table:** Suppose we want to make am exam schedule for a university. We have list different subjects and students enrolled in every subject. Many subjects would have common students (of same batch, some backlog students, etc.). How do we schedule the exam so that no two exams with a common student are scheduled at same time? How many minimum time slots are needed to schedule all exams? This problem can be represented as a graph where every vertex is a subject and an edge between two vertices mean there is a common student. So this is a graph coloring problem where minimum number of time slots is equal to the chromatic number of the graph.

2. **Sudoku:** Sudoku is also a variation of Graph coloring problem where every cell represents a vertex. There is an edge between two vertices if they are in same row or same column or same block.

A tree with 'n' vertices has 'n-1' edges. If it has one more edge extra than 'n-1', then the extra edge should obviously has to pair up with two vertices which leads to form a cycle. Then, it becomes a cyclic graph which is a violation for the tree graph.

12. **Define ceiling and floor function. Why do we need Inclusion - Exclusion principle? Make it clear with suitable example.**

Ans: **Ceiling Function:** The Ceiling Math Function is classified under Trigonometry Functions and Excel Math. Floor ceil enables returning a Number that is rounded up to the closest enough Integer or multiple of significance. It is a Function where the smallest successive Integer is returned successfully. This is to say; the Ceiling Function of a real Number 'p' is the least Integer that is greater than or equals to (≥) the given Number 'p'. Mathematically, the Ceiling Function is thus described as:

$F(x) = \text{minimum } \{a \in Z ; a \geq p\}$

Example: Determine the Ceiling value of 4.8.

Solution:

Given, $x = 4.8$

As we can notice, the Integers greater than 4.8 are 5, 6,7,8,9...and so on.

The closest enough Integer here is 5.

Therefore, $f(4.8) = \lceil 4.8 \rceil = 5$

Floor Function

Floor Function is the reverse Function of the Ceiling Function. It provides us with the largest nearest Integer or multiple of significance of the specified value. The formula used to find the Floor value for any given value is as below and is denoted by:

$f(x) = \lfloor x \rfloor$ = Highest Nearest Integer of specified value

Example: Find the Floor value of 4.8.

Ans: Given, $x = 4.8$

If we observe, the Number of Integers less than 4.8 is 4, 3,2,1,0, and -1,-2 and so on. So, the largest Integer will be 4.

$f(4.8) = \lfloor 4.8 \rfloor = 4$

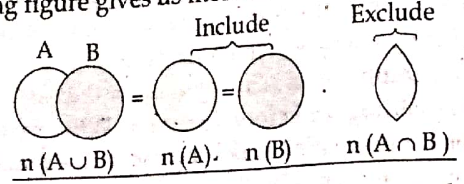
Principle of Inclusion and Exclusion

Principle of Inclusion and Exclusion is an approach which derives the method of finding the number of elements in the union of two finite sets. This is used for solving combinations and probability problems when it is necessary to find a counting method, which makes sure that an object is not counted twice.

Consider two finite sets A and B. We can denote the Principle of Inclusion and Exclusion formula as follows.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Here n(A) denotes the cardinality of set A, n(B) denotes the cardinality of set B and $n(A \cap B)$ denotes the cardinality of $(A \cap B)$. We have included A and B and excluded their common elements. The following figure gives us more idea on this.

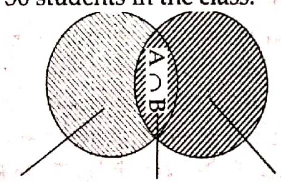


Example 1: In a discrete mathematics class every student is a major in computer science or mathematics, or both. The number of students having computer science as a major (possibly along with students having mathematics) is 25; the number of students having mathematics as a major (possibly along with computer science) is 13; and the number of students majoring in both computer science and mathematics is 8. How many students are in this class?

Solution: Let A be the set of students in the class majoring in computer science and B be the set of students in the class majoring in mathematics. Then $A \cap B$ is the set of students in the class who are joint mathematics and computer science majors. Because every student in the class is majoring in either computer science or mathematics (or both), it follows that the number of students in the class is $|A \cup B|$. Therefore,

$|A \cup B| = |A| + |B| - |A \cap B| = 25 + 13 - 8 = 30$

Therefore, there are 30 students in the class.



$|A| = 25$ $|A \cap B| = 8$ $|B| = 13$

TRIBHUVAN UNIVERSITY

Institution of Science and Technology

Bachelor Level/ First Year/ Second Semester/ Science Full Marks: 60

Microprocessor (CSC 162) Pass Marks: 24

Time: 3 hours.

TU QUESTIONS-ANSWERS 2078

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Section A (Long Answer Question Section)

Attempt any TWO questions. **(2x10=20)**

1. Prove that for all integers x and y, if $x^2 + y^2$ is even then $x + y$ is even. Using induction prove that $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3 = n^2(n+1)^2/4$

Ans: Let's given that,

$x^2 + y^2$ is even then 2xy is also even,

Thus $x^2 + y^2 + 2xy$ is even

Or, $(x+y)^2$ is even

$\Rightarrow$ $x + y$ is even

Second part,

Let P (n): $1^3 + 2^3 + 3^3 + 4^4 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

For n = 1,

L.H.S. = $1^3 = 1$

R.H.S. = $\left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{1 \times 2}{2}\right)^2 = (1)^2 = 1$

Hence, L.H.S. = R.H.S.

$\therefore$ P (n) is true for n = 1

Assume that P (k) is true

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + k^3 = \left(\frac{k(k+1)}{2}\right)^2 \quad \ldots (1)$$

We will prove that P (k + 1) is true.

$$1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \left(\frac{(k+1)(k+1)+1}{2}\right)^2$$

$$1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$$

We have to prove p(k + 1) from P (k) i.e. (2) from (1) ... (2)

$$= \frac{(k+1)^2 (k^2 + 4k + 4)}{4}$$

$$= \frac{(k+1)^2 (k^2 + 2k + 2k + 4)}{4}$$

$$= \frac{(k+1)^2 (k(k+2) 2(k+2))}{4}$$

$$= \frac{(k+1)^2 ((k+2) (k+2))}{4}$$

$$= \frac{(k+1)^2 (k+2)^2}{4}$$

$$= \left(\frac{(k+1)(k+2)}{2}\right)^2$$

Thus, $1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2$ i.e. P(k + 1) is true whenever P(k) is true

∴ By the principle of mathematical induction, P(n) is true for n where n is a natural number

From (1)

$$1^3 + 2^3 + 3^3 + 4^3 \ldots + k^3 = \left(\frac{k(k+1)}{2}\right)^2$$

Adding $(k + 1)^3$ both sides,

$$1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{2^2} + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$

$$= \frac{(k+1)^2 + (k^2 + 4k + 4)}{4}$$

$$= \frac{(k+1)^2 + (k^2 + 2k + 2k + 4)}{4}$$

$$= \frac{(k+1)^2 + (k(k^2 + 2) + 2(k+2))}{4}$$

$$= \frac{(k+1)^2 + ((k+2)(k+2))}{4}$$

$$= \frac{(k+1)^2 (k+2)^2}{4}$$

$$= \left(\frac{(k+1)(k+2)}{2}\right)^2$$

Thus, $1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \left(\frac{(k+1)(k+2)^2}{2}\right)^2$

i.e. P (k + 1) is true whenever P (k) is true

∴ By the principle mathematics indication, (P) is true for n, where n is a natural number

2. **State division and reminder algorithm. Suppose that the domain of the propositional function P(x) consists of the integer 0. 1, 2, 3 and 4. Write out each of the following propositions using disjunctions, conjunctions and negations.**

   a. **∃x P(x)**
   b. **∀x P(x)**
   c. **∃x ¬P(x)**
   d. **∀x ¬P(x)**
   e. **¬∃x P(x)**
   f. **¬∀x P(x)**

Ans: Division algorithm

The division algorithm for integers states that given any two integers a and b, with b > 0, we can find integers q and r such that 0 < r < b and a = bq + r.

The numbers q and r should be thought of as the quotient and remainder that result when b is divided into a. Of course the remainder r is non-negative and is always less that the divisor, b.

Examples:

If a = 9 and b = 2, then q = 4 and r = 1.

Reminder algorithm

The Remainder Theorem is useful for evaluating polynomials at a given value of x, though it might not seem so, at least at first blush.

The Remainder Theorem starts with an unnamed polynomial p(x), where "p(x)" just means "some polynomial p whose variable is x". Then the Theorem talks about dividing that polynomial by some linear factor x – a, where a is just some number. Then, as a result of the long polynomial division, you end up with some polynomial answer q(x) (the "q" standing for "the quotient polynomial") and some polynomial remainder r(x).

As a concrete example of p, a, q, and r, let's look at the polynomial p(x) = x3 – 7x – 6, and let's divide by the linear factor x – 4 (so a = 4):

$$
\begin{array}{r l}
 & x^2 + 4x + 9 \\
x-4 \,) & x^3 + 0x^2 - 7x - 6 \\
 & -x^3 + 4x^2 \\
\hline
 & 4x^2 - 7x - 6 \\
 & -4x^2 + 16x \\
\hline
 & 9x - 6 \\
 & -9x + 36 \\
\hline
 & 30
\end{array}
$$

So we get a quotient of q(x) = x2 + 4x + 9 on top, with a remainder of r(x) = 30.

Numerical part,

a) ∃xP(x)= P(0)v P(1)vP(2)vP(3)vP(4)

b) ∀xP(x)= P(0)∧P(1)∧P(2)∧P(3)∧P(4)

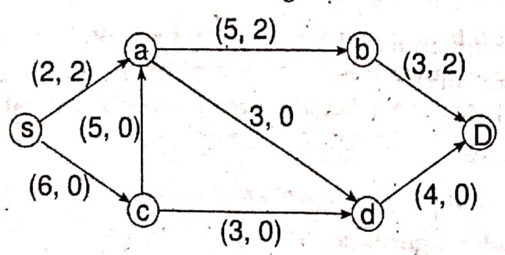
c) ∃x ¬P(x)= ¬P(0)v ¬P(1)v ¬P(2)v ¬P(3)v ¬P(4))

d) ∀x ¬P(x)= ¬P(0)∧ ¬P(1)∧ ¬P(2)∧ ¬P(3)∧ ¬P(4)

e) ¬∃xP(x)=¬(P(0)vP(1)vP(2)vP(3)vP(4))

f) ¬ ∀xP(x)= ¬(P(0)∧P(1)∧P(2)∧P(3)∧P(4))

3. **List all the necessary conditions for the graph to be isomorphic with an example. Find the maximum flow from the node SOURCE to SINK in the following network flow.**



Ans: given graphs are isomorphic if they have:

- Equal number of vertices.
- Equal number of edges.
- Same degree sequence
- Same number of circuit of particular length

Solution:

At first listing f-augmenting paths as

{S→a→b→D},{S→c→d→D},{S→a→d→D},{S→a→c→d→D}, {S→c→a→b→D}, ,{S→c→a→d→D}, {S→c→d→a→b→D}

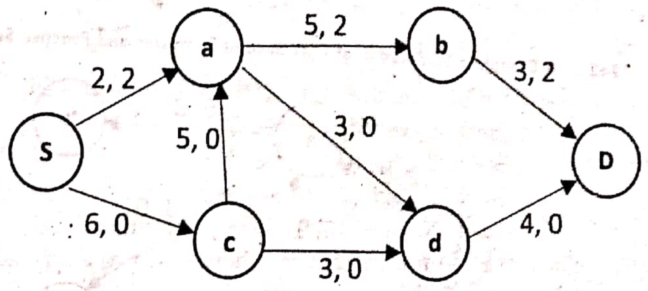
Step 1: In the f-augmenting path {S→a→b→D},

Slack of edge (S, a)=2-0=2 (minimum)

Slack of edge (a, b)=5-0=5

Slack of edge (b, D)=3-0=3

Since the minimum slick is 2 hence add 2 to every edge's flow of the path {S→a→b→D},



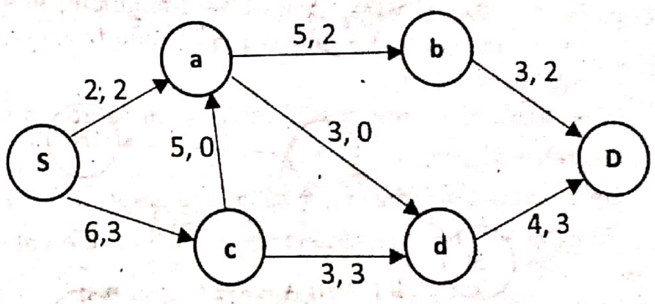
Step 2: In the f-augmenting path {S→c→d→D},

Slack of edge (S, c) =6-0=6

Slack of edge (c, d) =3-0=3 (minimum)

Slack of edge (d, D) =4-0=4

Since the minimum slick is 3 hence add 3 to every edge of the path {S→c→d→D},



Step 3: In the f-augmenting path {S→a→d→D},

No change

Step 4: In the f-augmenting path {S→a→c→d→D},

No change

Step 5: In the f-augmenting path {S→c→a→b→D},

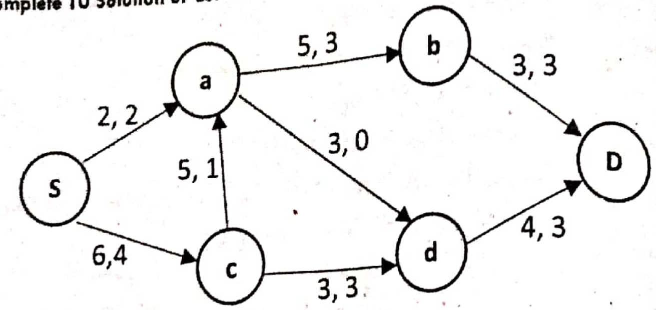
Slack of edge (S, c) =6-3=3

Slack of edge (c, a) =5-0=5

Slack of edge (a, b) =5-2=3

Slack of edge (b, D) =3-2=1 (minimum)

Since the minimum slick is 1 hence add to every edge of the path {S→c→a→b→D},



Step 6: In the f-augmenting path {S→c→a→d→D},

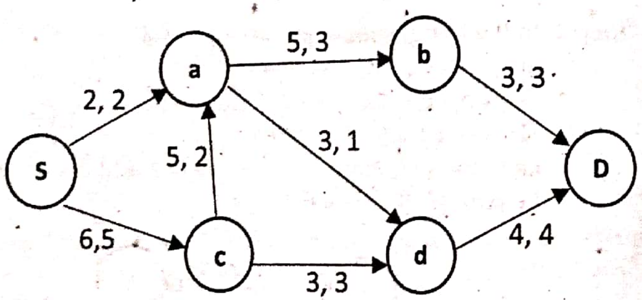
Slack of edge (S, c) =6-4=2

Slack of edge (c, a) =5-1=4

Slack of edge (a, d) =3-0=3

Slack of edge (d, D) =4-3=1 (minimum)

Since the minimum slick is 1 hence add to every edge of the path {S→c→a→d→D},



Now there is no any possible path form source to sink without saturated edge

Hence maximum flow of given network graph is 3+4=7

Hence flow out from source=5+2=7=flow in to the sink=3+4=7.

Section B (Short Answer Questions)

Attempt any EIGHT questions: **[8 × 5 = 40]**

4. What is the coefficient of x^2 in $(1+x)^{11}$? Describe how relation can be represented using matrix.

Ans: $(1+x)^{11} = [{}^{11}C_0 \cdot 1^{11}. x^0]+ [{}^{11}C_1 \cdot 1^{10}. x^1]+ [{}^{11}C_2 \cdot 1^9. x^2]+[{}^{11}C_3 \cdot 1^8. x^3]+[{}^{11}C_4 \cdot 1^7. x^4]+]+[{}^{11}C_5 \cdot 1^6. x^5]+]+[{}^{11}C_6 \cdot 1^5. x^6]+ [{}^{11}C_7 \cdot 1^4. x^7]+ [{}^{11}C_8 \cdot 1^3. x^8]+ [{}^{11}C_9 \cdot 1^2. x^9]+ [{}^{11}C_{10} \cdot 1^1. x^{10}]+ [{}^{11}C_{11} \cdot 1^0. x^{11}]$

Coefficient of $x^2 = {}^{11}C_2 \cdot 1^9. x^2$

$= 11 * 10/2\, x^2$

$= 55\, x^2$

Thus Coefficient of x^2 = 55 Ans.

Adjacency Matrix

Let A={a1,a2,...,am} and B={b1,b2,...,bn} be finite sets of cardinality m and n, respectively. Let r be a relation from A into B. Then r can be represented by the m×n matrix R defined by,

$$R_{ij} = \begin{cases} 1 & \text{if } a_i r b_j \\ 0 & \text{otherwise} \end{cases}$$

Where, R is called the adjacency matrix (or the relation matrix) of r

Example:

Let A={2,5,6} and let r be the relation {(2,2),(2,5),(5,6),(6,6)} on A. Since r is a relation from A into the same set A (the B of the definition), we have a1=2, a2=5, and a3=6, while b1=2, b2=5, and b3=6. Next, since

2r2, we have R11=1

2r5, we have R12=1

5r6, we have R23=1

6r6, we have R33=1

All other entries of R are zero, so

$$R = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

5. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ with initial conditions $a_0=1$, $a_1=4$.

Ans: The characteristic equation of the recurrence relation is,

$r^2 - 5r + 6 = 0$

or, $r^2 - 3r - 2r + 6 = 0$

or, $r(r-3) - 2(r-3) = 0$

or, $(r-3)(r-2) = 0$

$\Rightarrow$ $r = 3, 2$

Its roots are r = 3 and r = 2.

Hence, the sequence {an} is a solution to the recurrence relation if and only if $a_n = \alpha_1 3^n + \alpha_2 2^n$

For n=0,

$a_0 = \alpha_1 3^n + \alpha_2 2^n$

or, $1 = \alpha_1 3^0 + \alpha_2 2^0$

$\Rightarrow$ $\alpha_1 + \alpha_2 = 1$ ....(1)

For n=1,

$a_1 = \alpha_1 3^n + \alpha_2 2^n$

or, $4 = \alpha_1 3^1 + \alpha_2 2^1$

$\Rightarrow$ $3\alpha_1 + 2\alpha_2 = 4$ ....(2)

By solving equation 1 and 2 we get,

$\alpha_1 = 2$ and $\alpha_2 = -1$

Hence, the solution to the recurrence relation and initial conditions is the sequence {an} with

$a_n = 2 \cdot 3^n + (-1).\ 2^n$

6. Prove that if n is positive integer, then n is odd if and only if 5n+6 is odd.

Ans: We will use a direct proof on "If n is odd, then 5n + 6 is odd". Assume n is odd, so n = 2k + 1

For some integer k.

Then, 5n+6 = 5(2k+1)+6 = 10k+5+6 = 10k+11 = 2(5k+5)+1

Thus, 5n+6 is odd.

We now must prove the converse, "If 5n + 6 is odd, then n is odd."

For this, we will use proof by contrapositive.

So the statement becomes "If n is not odd, then 5n+6 is not odd."

Assume that n is not odd, so n = 2k for some integer k.

Then $5n + 6 = 5(2k) + 6$

$= 10k + 6$

$= 2(5k + 3)$

Thus, 5n + 6 is not odd.

Hence we have proven that n is odd if and only if 5n + 6 is odd.

7. Describe preposition. Consider the argument "John, a student in this class knows how to write program in C. Everyone who know how to write program in C can get a high paying job. Therefore, someone in this class can get high paying job'. Now explain which rules of inferences are need for each step.

Ans: A proposition is a collection of declarative statements that has either a truth value "true" or a truth value "false". A propositional consists of propositional variables and connectives. We denote the propositional variables by capital letters (A, B, etc). The connectives connect the propositional variables.

Some examples of Propositions are given below:

1) "Man is Mortal", it returns truth value "TRUE"

2) "12 + 9 = 3 – 2", it returns truth value "FALSE"

Numerical,

C(x) = "x is in this class"

J(x) = "x knows how to write programs in C"

H(x) = "x can get a high paying job"

Premise 1 C(John)

Premise 2 J(John)

Premise 3 $\forall x(J(x) \rightarrow H(x))$

Conclude $\exists x(C(x) \wedge H(x))$

Step	Reason	
1	$\forall x(J(x) \rightarrow H(x))$	Premise 3
2	$J(John) \rightarrow H(John)$	Universal Instantiation from (1)
3	J(John)	Premise 2
4	H(John)	Modus Ponens from (2) and (3)
5	C(John)	Premise 1
6	$C(John) \wedge H(John)$	Conjunction from (4) and (5)
∴	$\exists x(C(x) \wedge H(x))$	Existential generalization from (6)

8. Show that if there are 30 students in a class, then at least two have same names that begin with the same letter. Explain the Pascal's triangle.

Ans: There are total number of students N=30

No of letters=26

Now, according to pigeonhole principle, $\lceil 30/26 \rceil = 2$

Pascal's triangle, in algebra, a triangular arrangement of numbers that gives the coefficients in the expansion of any binomial expression, such as $(x + y)^n$. It is named for the 17th-century French mathematician Blaise Pascal, but it is far older. Chinese mathematician Jia Xian devised a triangular representation for the coefficients in the 11th century.

Pascals Triangles Formula

The formula to fill the number in the nth column and mth row of Pascal's triangle we use the Pascals triangle formula. The formula requires the knowledge of the elements in the (n–1)th row, and (m – 1)th and nth columns.

The formula for the pascal's triangle is:

$$\binom{n}{m} = \binom{n-1}{m-1} + \binom{n-1}{m}$$

Where

- n is the number of rows, and m is the number of elements.
- n is the non-negative integer, and
- $0 \le m \le n$.

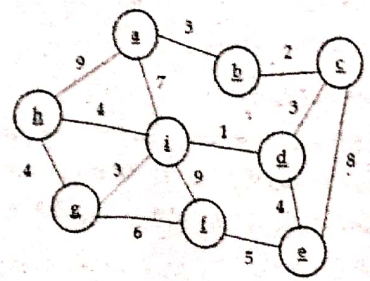
Example:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1

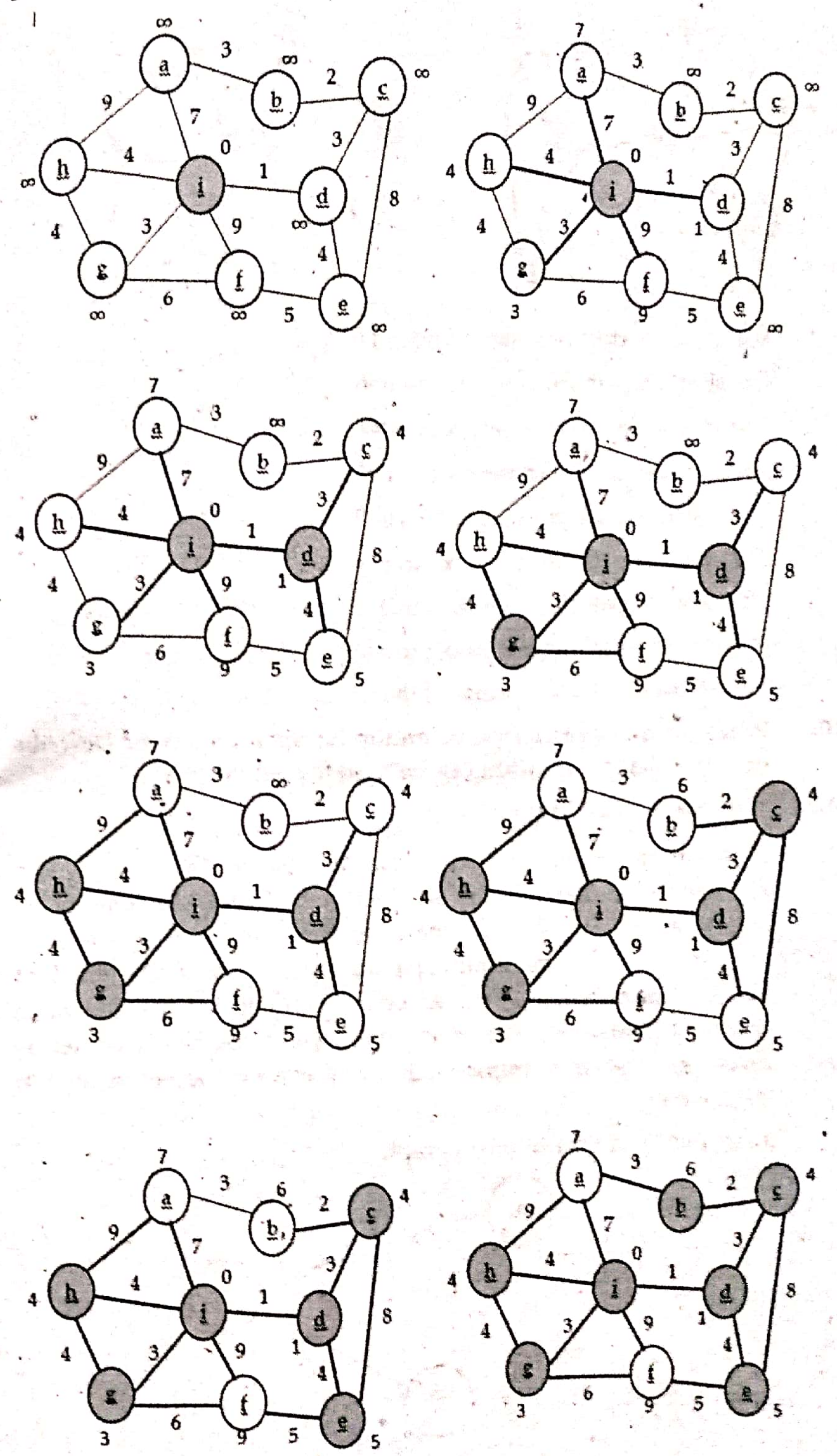
9. Illustrate the Dijkstra's Algorithm to find the shortest path from source node to destination node with an example.

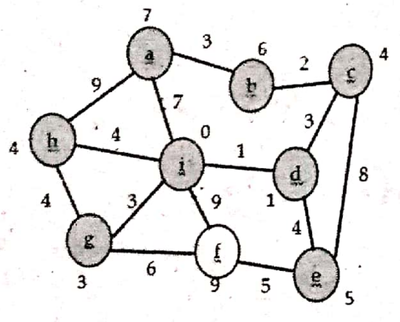
Ans: Dijkstra's algorithm works using greedy approach, as we will see later. Dijkstra's algorithm finds the shortest path from one vertex v0 to each other vertex in a digraph. When it has finished, the length of the shortest distance from v_0 to v is stored in the vertex v, and the shortest path from v_0 to v is recorded in the back pointers of v and the other vertices along that path.

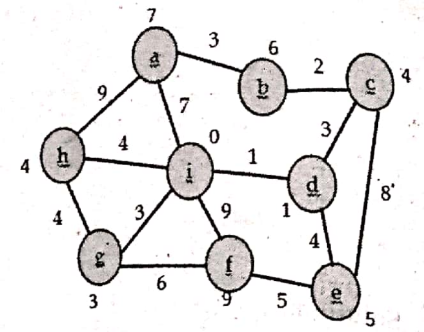
Example Find the shortest paths from the source node i to all other vertices using Dijkstra's algorithm.



Solution:







Thus, the shortest path from vertex i to a=7

The shortest path from vertex i to b=6

The shortest path from vertex i to c=4

The shortest path from vertex i to d=1

The shortest path from vertex i to e=5

The shortest path from vertex i to f=9

The shortest path from vertex i to g=3

The shortest path from vertex i to h=4

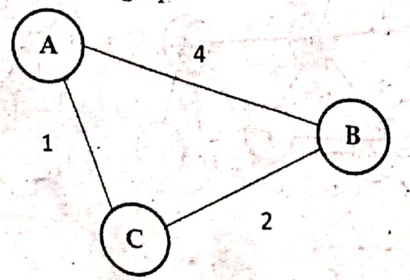
The shortest path from vertex i to i=0

10. **What are the significance of minimum Spanning Tree? Describe how Kruskal's algorithm can be used to find the MST.**

Ans: Minimum spanning tree

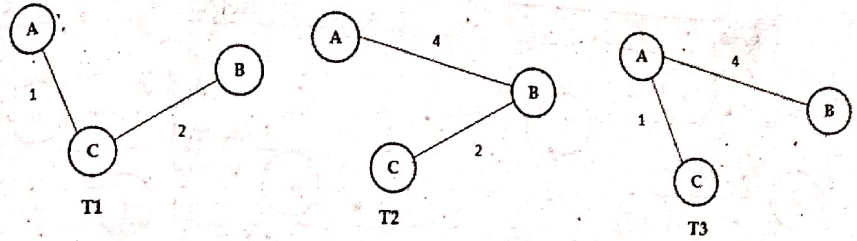
A minimum spanning tree in a connected weighted graph is a spanning tree that has the smallest possible total weights of its edges out of all possible spanning trees. In other words in a weighted graph, a minimum spanning tree is a spanning tree that has minimum weight than all other spanning trees of the same graph. In real-world situations, this weight can be measured as distance, congestion, traffic load or any arbitrary value denoted to the edges.

Example: Find MST of given graph



Solution:

The possible spanning trees of given graph are:



Total weight of T1=3

Total weight of T2=6

Total weight of T3=5

Since T1 has minimum weight out of all possible spanning trees of given graph hence, tree T1 acts as minimum spanning tree of given graph.

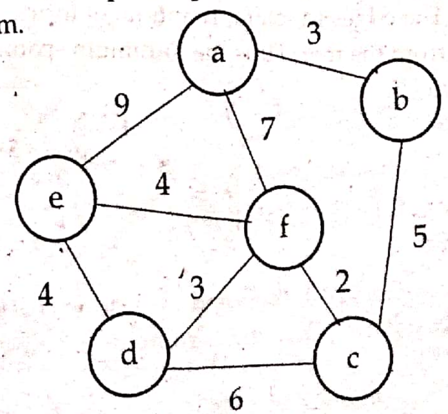
Kruskal's algorithm

Kruskal's algorithm is a minimum spanning tree algorithm that takes a graph as input and finds the subset of the edges of that graph which:

- form a tree that includes every vertex
- has the minimum sum of weights among all the trees that can be formed from the graph

It is the procedure for producing a minimum spanning tree of a given weighted graph that successively adds edges of least weight that are not already in the tree such that no edges produce a simple circuit when it is added.

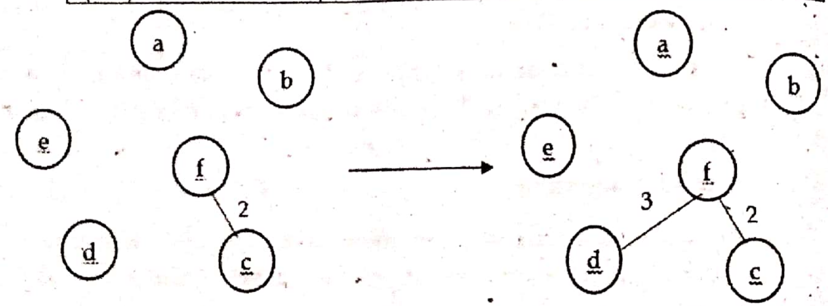
Example: Find minimum spanning tree of given graph by using Kruskal's algorithm.

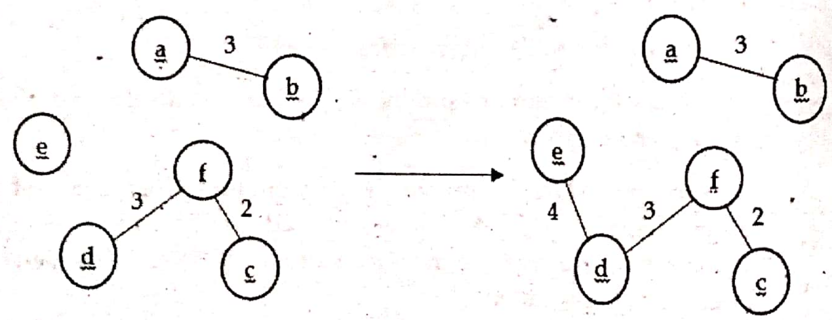


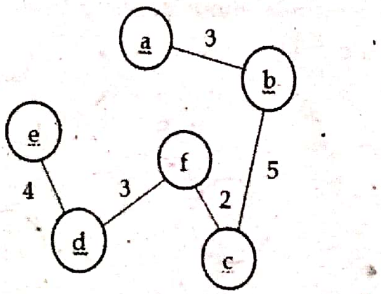
Solution:

Edges in sorted order are given below:

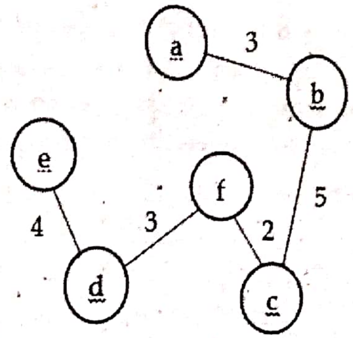
Edges	Cost	Action
(f, c)	2	Accept
(f, d)	3	Accept
(a, b)	3	Accept
(d, e)	4	Accept
(b, c)	5	Accept
(d, c)	6	Reject
(a, f)	7	Reject
(a, e)	9	Reject







The edges (d, c), (a, f) and (a, e) forms cycle so discard these edges from the tree. Thus the minimum spanning tree of weight 17 is;



11. Define zero-one matrix. Explain the types of function.

Ans: A zero-one matrix has entries that are either zero or one. Such matrices are often used to encode a relation between two sets of objects; a one at (i, j) means the object i is related to the object j and a zero means they are not. For an undirected graph, a one at (i, j) means that vertex i is joined to vertex j.

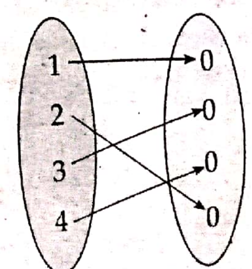
Types of Functions–Based on Set Elements

This type of classification of function depends on the number of relationship amongst the elements in the domain and the codomain. The different types of functions depending on the set elements are as discussed below.

One-One Function or Injective Function

The one-to-one function is also termed an injective function. Here each element of the domain possesses a different image or co-domain element for the assigned function.

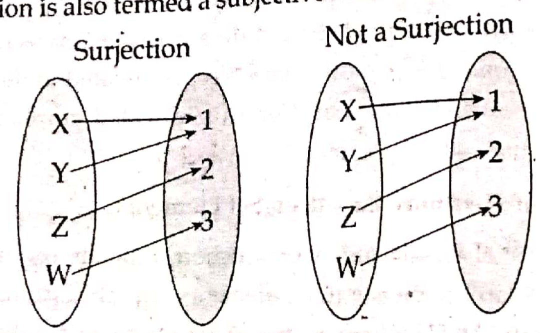
A function f: A → B is declared to be a one-one function, if different components in A have different images or are associated with different elements in B.



One to One

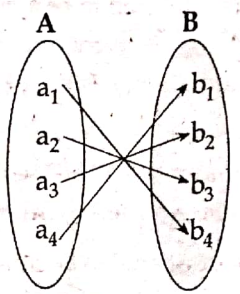
One Function or Surjective Fuction

A function f: A → B is declared to be an onto function if each component in B has at least one pre-image in A. i.e. If range of function f = co-domain of function f, then f is onto. The onto function is also termed a subjective function.



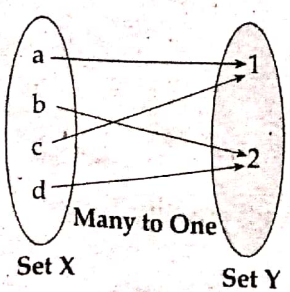
Bijective Function or One One and Onto Function

A function t: A → B is declared to be a bijective function if it is both one-one and onto function. In other words, we can say that every element of set A is related to a different element in set B, and there is not a single element in set B which has been left out to be connected to set A.



Many-one Function

Any function f: A → B is said to be many-one if two (or more than two) distinct components in A have identical images in B. In a many to one function, more than one element owns the same co-domain or image



Into Function

Any function f: A → B is said to be a into function if there exists at least one elements in B which does not have a pre-image in A. i.e., If the Range of function f ⊂ Co-domain of function f, then f is into

12. **Represent any three set operations using Venn-diagram. Give a recursive defined function to find the factorial of any given positive integer.**

Ans: **Set intersection**

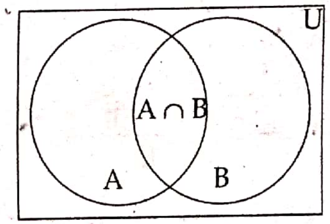
Let A and B be the two sets. The intersection of A and B is the set of all those elements which belong to both A and B.

Now we will use the notation A ∩ B (which is read as 'A intersection B') to denote the intersection of set A and set B.

Thus, A ∩ B = {x : x ∈ A and x ∈ B}.

Clearly, x ∈ A ∩ B

⇒ x ∈ A and x ∈ B



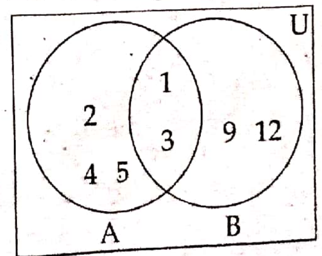
A ∩ B

Example. If A = {1, 2, 3, 4, 5} and B = {1, 3, 9, 12}. Find A ∩ B using venn diagram.

Solution:

According to the given question we know, A = {1, 2, 3, 4, 5} and B = {1, 3, 9, 12}

Now let's draw the Venn diagram to find A intersection



A ∩ B = (1, 3)

Therefore, from the Venn diagram we get A ∩ B = {1, 3}

Set Union operation

Let A and B be the two sets. The union of A and B is the set of all those elements which belong either to A or to B or both A and B.

Now we will use the notation A U B (which is read as 'A union B') to denote the union of set A and set B.

Thus, A U B = {x : x ∈ A or x ∈ B}.

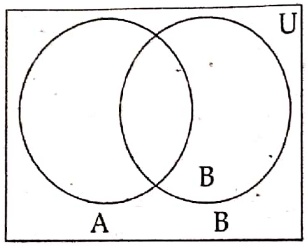
Clearly, x ∈ A U B

⇒ x ∈ A or x ∈ B

Similarly, if x ∉ A U B

⇒ x ∉ A or x ∉ B

Therefore, the shaded portion in the adjoining figure represents A U B.



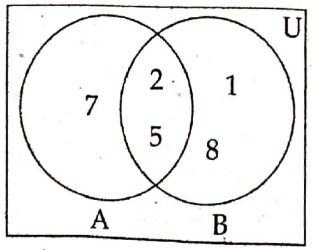
A ∪ B

Example: If A = {2, 5, 7} and B = {1, 2, 5, 8}. Find A U B using venn diagram.

Solution:

According to the given question we know, A = {2, 5, 7} and B = {1, 2, 5, 8}

Now let's draw the Venn diagram to find A union B.

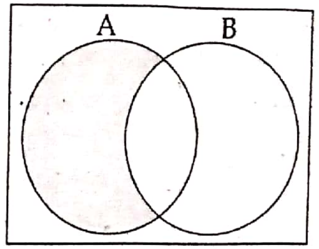


A ∪ B = {1, 2, 5, 7, 8}

Therefore, from the Venn diagram we get A U B = {1, 2, 5, 7, 8}

Set difference operation

The set A−B consists of elements that are in A but not in B. For example if A = {1, 2, 3} and B = {3, 5}, then A−B = {1, 2}. In Figure below, A−B is shown by the shaded area using a Venn diagram.



Factorial of a non-negative integer, is multiplication of all integers smaller than or equal to n. For example factorial of 6 is 6*5*4*3*2*1 which is 720.

Factorial can be calculated using following recursive formula.

n! = n * (n-1)!

n! = 1 if n = 0 or n = 1