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Numerical Methods 2 Coursework report

The governing equation, for this system, can be obtained in the following way:

$$\hat{F} = -k\nabla \mathrm{T} \quad (1)$$
$$\nabla \cdot \hat{\mathrm{F}} = 0 \quad (2)$$

Combine these two equations together:

$$\nabla \cdot (-\mathrm{k}\nabla \mathrm{T}) = 0$$
$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} \cdot \left(-k \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} T \right) = 0$$
$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} \cdot \left(-k \left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \right) \right) = 0$$
$$-k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0$$
$$-k\nabla^2 T = 0$$

To find finite difference approximations in a form appropriate for simultaneous over relaxation:

$$\left(\frac{\partial^2 u}{\partial x^2} \right) \approx \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2}$$
$$\therefore \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \approx \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}$$
$$0 \approx -k \left(\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} \right)$$

Assuming $\Delta \mathrm{x} = \Delta y$:

$$0 \approx -k \left(\frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j}}{\Delta x^2} \right)$$

With the finite difference approximation for the system it is now time to find the boundary conditions for each boundary. In this system we can find flux at all boundaries, other than the bottom boundary, by equating flux out of the radiator with the appropriate component of the flux within the radiator. There are 16 boundaries on the radiator but only 5 different types of boundaries that need to be solved for. These are the bottom boundary (Dirlecht), left hand vertical boundaries (Robin), right hand vertical boundaries (Robin), bottom horizontal boundaries (Robin) and top horizontal boundaries (Robin).

For the bottom Dirlecht boundary:

$$T = 200°C \ which\ is\ kept\ constant$$
$$a_{i,j} = b_{i,j} = c_{i,j} = d_{i,j} = 0$$
$$e_{i,j} = 1$$
$$f_{i,j} = 200$$

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For left hand vertical boundaries:

$$\underline{n} = boundary\ normal$$
$$\hat{F} \cdot \underline{n} = F_n$$
$$F_n = F_{out}$$

Here flux moves from right to left, so flux is negative

$$k\frac{\partial T}{\partial x} = h\left(T_{i,j} - T_w\right)$$
$$k\frac{T_{i+1,j} - T_{i,j}}{\Delta x} = h\left(T_{i,j} - T_w\right)$$
$$T_{i+1,j} - T_{i,j} = \frac{h\Delta x}{k}\left(T_{i,j} - T_w\right)$$
$$T_{i+1,j} - T_{i,j}\left(1 + \frac{h\Delta x}{k}\right) = -\frac{h\Delta x}{k}T_w$$
$$a_{i,j} = 1$$
$$b_{i,j} = c_{i,j} = d_{i,j} = 0$$
$$e_{i,j} = -\left(1 + \frac{h\Delta x}{k}\right)$$
$$f_{i,j} = -\frac{h\Delta x}{k}T_w$$

For right hand vertical boundaries flux moves from left to right, so flux is positive:

$$-k\frac{\partial T}{\partial x} = h\left(T_{i,j} - T_w\right)$$
$$-k\frac{T_{i,j} - T_{i-1,j}}{\Delta x} = h\left(T_{i,j} - T_w\right)$$
$$T_{i,j} - T_{i-1,j} = -\frac{h\Delta x}{k}\left(T_{i,j} - T_w\right)$$
$$-T_{i-1,j} + T_{i,j}\left(1 + \frac{h\Delta x}{k}\right) = \frac{h\Delta x}{k}T_w$$
$$b = -1$$
$$e = \left(1 + \frac{h\Delta x}{k}\right)$$
$$f = \frac{h\Delta x}{k}T_w$$

For bottom horizontal boundaries flux moves from top to bottom, so flux is negative:

$$\frac{\partial T}{\partial y} = h\left(T_{i,j} - T_w\right)$$
$$k\frac{T_{i,j+1} - T_{i,j}}{\Delta x} = h\left(T_{i,j} - T_w\right)$$
$$T_{i,j+1} - T_{i,j} = \frac{h\Delta x}{k}\left(T_{i,j} - T_w\right)$$
$$T_{i,j+1} - T_{i,j}\left(1 + \frac{h\Delta x}{k}\right) = -\frac{h\Delta x}{k}T_w$$
$$c_{i,j} = 1$$
$$a_{i,j} = b_{i,j} = d_{i,j} = 0$$

$$e_{i,j} = -\left(1 + \frac{h\Delta x}{k}\right)$$
$$f_{i,j} = -\frac{h\Delta x}{k} T_w$$

For top horizontal boundaries flux moves from bottom to top, so flux is positive:

$$-k\frac{\partial T}{\partial y} = h\left(T_{i,j} - T_w\right)$$
$$-k\frac{T_{i,j} - T_{i,j-1}}{\Delta x} = h\left(T_{i,j} - T_w\right)$$
$$T_{i,j} - T_{i,j-1} = -\frac{h\Delta x}{k}\left(T_{i,j} - T_w\right)$$
$$-T_{i,j-1} + T_{i,j}\left(1 + \frac{h\Delta x}{k}\right) = \frac{h\Delta x}{k} T_w$$
$$d = -1$$
$$e = \left(1 + \frac{h\Delta x}{k}\right)$$
$$f = \frac{h\Delta x}{k} T_w$$

Using this I was able to produce a mask which allowed iteration over all internal points and implement appropriate boundary conditions at all boundaries. At points where no calculation is required, the mask has a value of zero ensuring that the system is the correct shape.

```
imshow(transpose(mask), origin='lower')
```

```
<matplotlib.image.AxesImage at 0x7f959751f7b8>
```

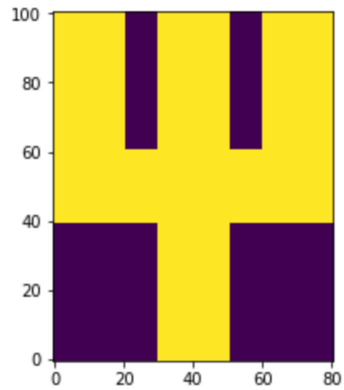

Figure 1: Mask used for my program.

Following this, in order to calculate temperature, it was important to ensure that all constants were in the same units. To do this I used a Δx = 0.01, k = 25 and a h = 100. In addition, for my final plot I used a critical residual of 0.001as this is a suitable value to get an accurate plot.

After 8300 iterations I produced a plot which showed temperature at all points in the system. Using this simulation, we can look at the effects of changing thermal conductivity and the external heat transfer coefficient in the system. Following from this, I can see that when I increase thermal conductivity by an order of magnitude, temperature within the material diffuses further into the material. Conversely, when external heat transfer coefficient is increased by an order of magnitude, heat transfer is inhibited, and temperature is unable to diffuse through the system. When thermal conductivity is doubled and external heat transfer is halved, temperature increases further into the system but not as much as when thermal conductivity is increased by an order of magnitude. This is expected as when the external heat transfer coefficient is increased heat is allowed to leave the system at a greater rate at the boundaries which causes a decrease in temperature of the radiator meaning the system is more efficient in losing heat. In addition, when the thermal conductivity of the material is increased by an order of magnitude, heat will travel through the material easier meaning that the temperature profile will be higher towards the top of the material. If conductivity is low then heat will not be able to diffuse through the material and so temperature will be very low towards the top of the system as seen in figure 2c.

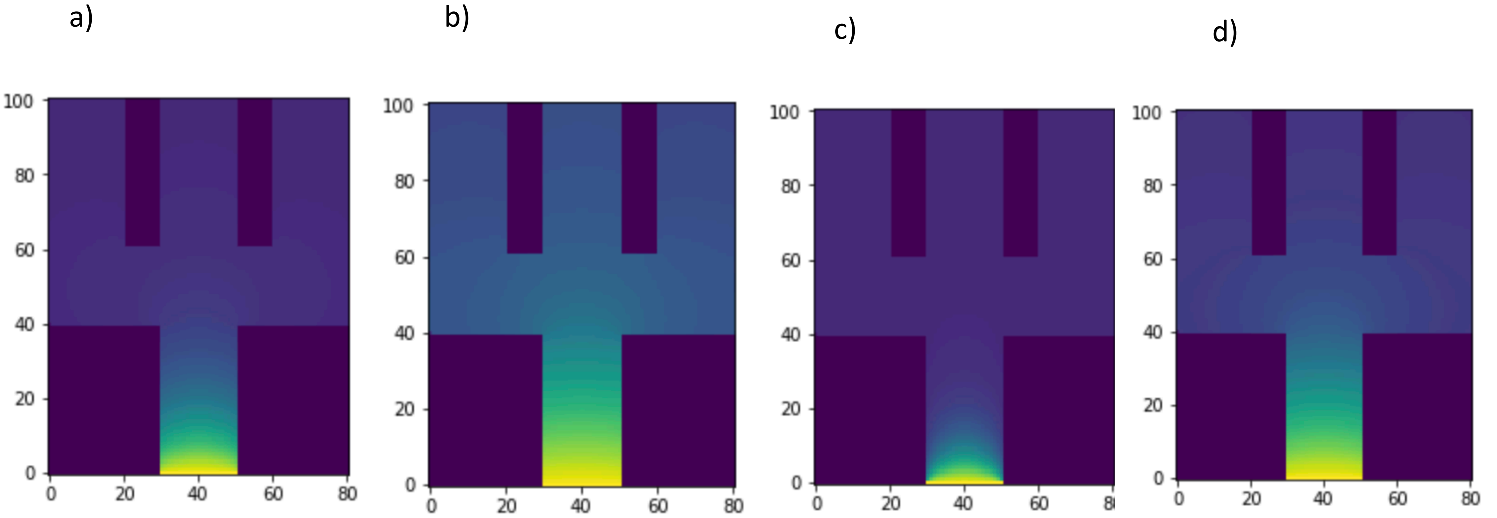

Figure 2: Comparing variations in temperature with different thermal conductivity and external heat transfer coefficient. a) k = 25 $Wm^{-1}K^{-1}$ and h = 100 $Wm^{-2}K^{-1}$. b) k = 250 $Wm^{-1}K^{-1}$ and h = 100 $Wm^{-2}K^{-1}$. c) k = 25 $Wm^{-1}K^{-1}$ and h = 1000 $Wm^{-2}K^{-1}$. d) k = 50 $Wm^{-1}K^{-1}$ and h = 50 $Wm^{-2}K^{-1}$.

Furthermore, we can look at the heat flux within the radiator to view where flux is greatest.

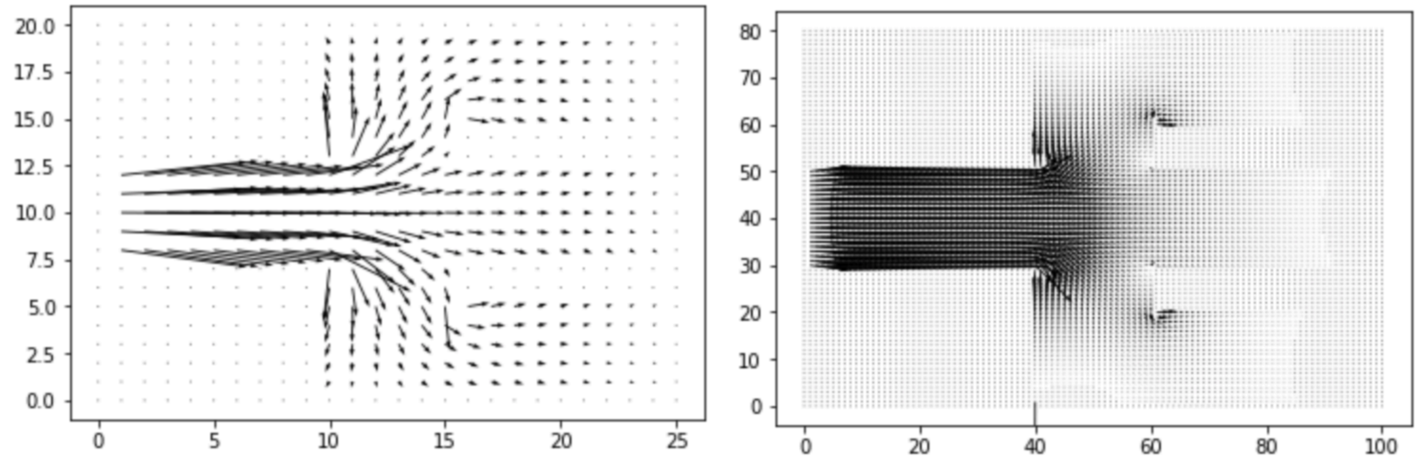

Figure 3: Looking at heat flux within the radiator to view where heat flux is greatest and where heat flux is least.

When we look at the heat flux diagrams within the radiator, we can see that heat flux is zero in areas that there is little temperature. An example of this is at the ends of the top of the radiator. In addition, we can also clearly see that heat flux is strongest at the bottom of the radiator and has both an x and y component to it.

```
The total flux into the radiator is -136 W
The total flux out of the radiator is -319 W
```

In order to find the total amount of heat lost by the radiator, I have found the sum of the appropriate component of the fluxes at each boundary. This means that at vertical boundaries I have summed the x components and at y boundaries I have summed the y components. When doing this, it can be seen that there is a large difference between the flux in and flux out. This could be due to error produced by the model during the iteration process meaning that heat is lost within the system rather than at the boundaries. A potential solution to this could be using a smaller critical resolution but the downside to this would be the fact that it would take significantly longer to compute a solution in doing so.

Using the same program I produced to model the heat flow within the original radiator, I produced my own shape with the intention of producing a radiator which releases heat better.

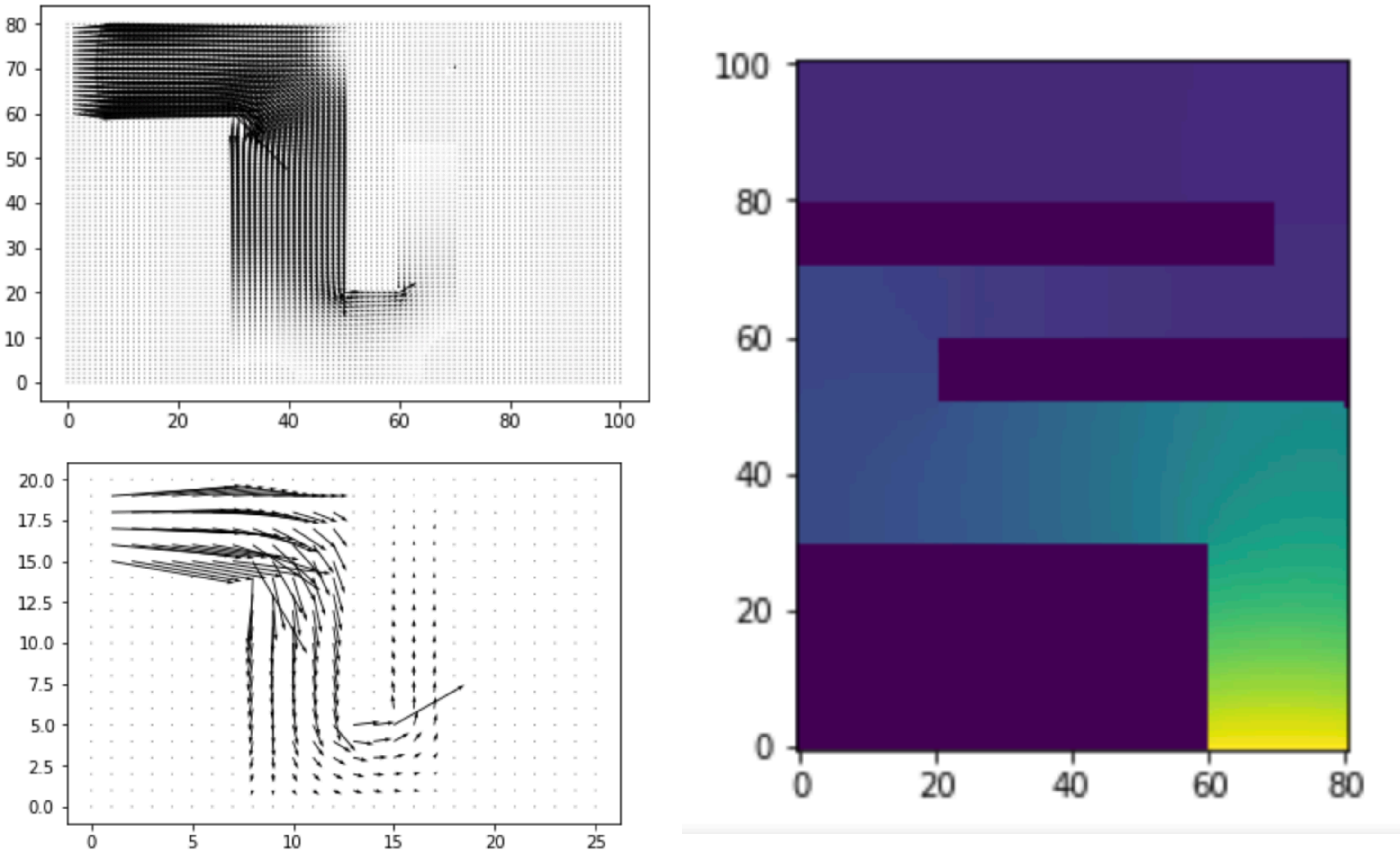

Figure 4: Looking at temperature profile and flux within newly produced radiator. k = 250 $Wm^{-1}K^{-1}$ and h = 100 $Wm^{-2}K^{-1}$.

When compared to the original shape of radiator with the same coefficients (fig 2b) you can see that at the base of my radiator there is a much higher temperature profile and heat flux. The idea with this design is to create a larger surface area for heat to be lost and therefore, provide heat to surroundings. Due to the larger surface area at the base, a lot of heat is lost during this and the majority of the heat within the radiator is lost there.