Day 1: Introduction to Machine Learning

Summer STEM: Machine Learning

Department of Electrical and Computer Engineering
NYU Tandon School of Engineering
Brooklyn, New York

August 3, 2020

Outline

NYU TANDON SCHOOL OF ENGINEERING

Huaijiang

Alireza

Armand

Hao

Tell the class about yourself

- Write down the following information:
  - Name
  - Grade
  - In which city/town are you currently living?
  - What do you want to get out of this class?
  - What is your favourite movie?
  - What is the IMDB score of this movie!
  - What is the category of this movie? (thriller/drama/action, etc)
  - Rate your coding experience from 1 (no experience) to 5 (plenty of experience)!
- Share your answers with the class!
- We'll visualize this dataset using Python tomorrow!
  - Link to excel sheet here

Outline

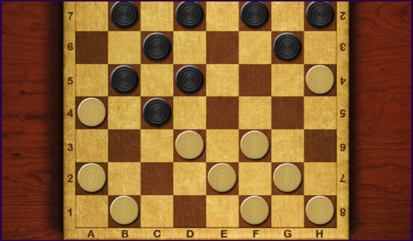
NYU TANDON SCHOOL OF ENGINEERING

Machine Learning

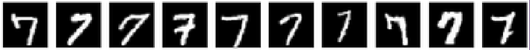
- Most recent exciting technology
- We use these algorithms dozens of times a day
  - Search Engine
  - Recommendations
- Machine Learning is an important component to achieve artificial general intelligence
- Practice is the key to learn machine learning

Definition

- Machine Learning is a field of study that gives computers the ability to learn without being explicitly programmed.



Example: Digit Recognition



- Challenges with expert approach
  - Simple expert rule breaks down in practice
  - Difficult to translate our knowledge into code
- Machine Learning approach
  - Learned systems do very well on image recognition problems

```
def classify(image):
    ...
    nv = count_vert_lines(image)
    nh = count_horiz_lines(image)
    ...

    if (nv == 1) and (nh == 1):
        digit = 7
    ...

    return digit
```

Example: CIFAR 10

Machine Learning Problem Pipeline

1. Formulate the problem: regression, classification, or others?
2. Gather and visualize the data
3. Design the model and the loss function
4. Train your model
    - (a) Perform feature engineering
    - (b) Construct the design matrix
    - (c) Choose regularization techniques
    - (d) Tune hyper-parameters using a validation set
    - (e) If the performance is not satisfactory, go back to step (a)
5. Evaluate the model on a test set

Outline

Course Outline

- Day 1: Introduction to ML
- Day 2: Linear Regression
- Day 3: Overfitting and Generalization
- Day 4: Classification and Logistic Regression
- Day 5: Mini Project
- Day 6: Neural Networks
- Day 7: Convolutional Neural Networks
- Day 8: Deep Generative Models
- Day 9: Final Project
- Day 10: Social Impacts of ML and Final Project Presentations

Course Format, Website, Resources

- Course Website: `https://github.com/asarmadi/summer_ml_s3`
  - Github: share collections of documents, repositories of code
  - Contains lecture slides, code notebooks, and datasets
  - Slides and demo code posted before lecture, solutions to the lab posted after
- After-class discussion: Piazza. Links to class recordings will also be posted there.
- We strongly encourage programming in Python via Google Colab.
- We'll give additional resources at the end of each day based on student interest

Outline

Vectors

- A **vector** is an ordered list of numbers or symbols
  - Ex:

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 8 \\ 6 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

Vectors

- Vectors of the same size may be added together, element-wise
  - Ex: $\mathbf{u} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

    $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 3+1 \\ (-1)+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$
- Vectors may be scaled by a number, element-wise
  - Ex: $3\mathbf{v} = \begin{bmatrix} 3 \times 1 \\ 3 \times 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

Vectors

- Norm of a vector (L2 Norm)
  - Ex: If $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\|\mathbf{v}\| = \sqrt{1^2 + 2^2} = \sqrt{5}$
- Inner product: sum of element-wise products of two vectors
  - Ex: $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 3 \times 1 + (-1) \times 2 = 3 - 2 = 1$
  - Gives the angle between two vectors $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \, \|\mathbf{v}\|}$

Vectors

- If $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$
  $\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{bmatrix}$
- For any real number α, $\alpha\mathbf{u} = \begin{bmatrix} \alpha u_1 \\ \alpha u_2 \\ \vdots \\ \alpha u_n \end{bmatrix}$

NYU TANDON SCHOOL OF ENGINEERING

Vectors

- inner product :

$$\mathbf{u} \cdot \mathbf{v} = u_1 \times v_1 + u_2 \times v_2 + \cdots + u_n \times v_n = \Sigma_{i=1}^{n} u_i \times v_i$$

- norm :

$$\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2} = \sqrt{\Sigma_{i=1}^{n} u_i^2}$$

- Squared norm :

$$\|\mathbf{u}\|^2 = u_1^2 + u_2^2 + \cdots + u_n^2 = \Sigma_{i=1}^{n} u_i^2$$

NYU TANDON SCHOOL OF ENGINEERING

Exercise: Vectors

Let $\mathbf{p} = \begin{bmatrix} 3 \\ 2 \\ 9 \\ 4 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 1 \\ 9 \\ 0 \\ 3 \end{bmatrix}$, calculate

- $3\mathbf{q} + 2\mathbf{p}$
- $\mathbf{q} \cdot \mathbf{q}$ and $\|\mathbf{q}\|^2$
- $\mathbf{p} \cdot \mathbf{q}$ and $\|\mathbf{p}\| \, \|\mathbf{q}\|$
- $\left\| \frac{\mathbf{p}}{\|\mathbf{p}\|} \right\|$

NYU TANDON SCHOOL OF ENGINEERING

Matrices

- A **matrix** is a rectangular array of numbers or symbols arranged in rows and columns. We can conceptualize it as a collection of vectors.
  - Ex: 2 by 2 matrix, $M = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$
- Matrices of the same shape may be added together, element-wise
  - Ex: $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 8 \\ 7 & 11 \end{bmatrix}, A + B = \begin{bmatrix} 1 & 9 \\ 9 & 12 \end{bmatrix}$
- Matrices may be scaled, element-wise
  - Ex: $\alpha B = \begin{bmatrix} 0 & 8\alpha \\ 7\alpha & 11\alpha \end{bmatrix}$, where α is a scalar

Exercise: Matrices

- $\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 9 & 2 \\ -7 & 6 \\ 3 & 1 \end{bmatrix} = ?$
- $2 \begin{bmatrix} 1 & 9 \\ 3 & -2 \end{bmatrix} = ?$

Vectors and Matrices

- We may consider a vector as a matrix
  - **Row Vector**: shape (1 x N)
    Ex: $\mathbf{v} = \begin{bmatrix} 1 & 2 \end{bmatrix}$
  - **Column Vector**: shape (N x 1)
    Ex: $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
  - We'll consider vectors as column vectors by default

Matrices

Example :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

- What is the shape of A ?

NYU TANDON SCHOOL OF ENGINEERING

Matrices

Example :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

- What is the shape of A ?

$$(2 \times 5)$$

Matrices

General case :

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & & & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nm} \end{bmatrix}$$

- What is the shape of A ?

Matrices

General case :

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & & & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nm} \end{bmatrix}$$

- What is the shape of A ?

$$n \times m$$

Matrices

General case :

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & & & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nm} \end{bmatrix}$$

- What is the shape of A ?

$$n \times m$$

- A_{ij} is the element at the i^{th} row and j^{th} column

Matrices

Example :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

- $A_{13} =?$
- $A_{21} =?$
- $A_{24} =?$

Matrices

- Two matrices, A and B, can be multiplied together provided their shapes meet the criteria :
- **Criteria:** # cols of A must equal the # rows of B
  - Ex : $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$
  - Shape of A : (2×3), Shape of B : (3×2)

Matrices

- Two matrices, A and B, can be multiplied together provided their shapes meet the criteria :
- **Criteria:** # cols of A must equal the # rows of B
  - Ex : $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$
  - Shape of A : (2×3), Shape of B : (3×2)
- Result is a matrix with shape (# rows A $\times$ # cols B)
  - Ex : If $C = AB$ then, C is of shape (2×2) :
    $C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

Matrices

- **To sum up :**
  If A is of shape $(M \times K)$ and B of shape $(K \times N)$,
  We can define $C = AB$, and C will be of shape $(M \times N)$

Matrices

- If $C = AB$ then, $(C)_{ij} = \sum_{k=1}^{N} A_{ik} B_{kj}$
- Inner product of the i-th row of A and the j-th column of B

Matrices

- If $C = AB$ then, $(C)_{ij} = \sum_{k=1}^{N} A_{ik} B_{kj}$
- Inner product of the i-th row of A and the j-th column of B
- Ex : $C = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$$C_{11} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = -6 \quad C_{12} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 3$$

$$C_{21} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = 6 \quad C_{22} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 0$$

NYU TANDON SCHOOL OF ENGINEERING

Matrices

- $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} =?$
- $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} =?$
- $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} =?$
- In general, $AB \neq BA$

NYU TANDON SCHOOL OF ENGINEERING

Matrices

- **Transpose**: A^T swaps the rows and columns of matrix A
- Ex: $\begin{bmatrix} 2 \\ 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$
- $\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} =?$
- $(AB)^T = B^T A^T$

Exercises: Matrix Multiplication

- $X = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ $Y = \begin{bmatrix} 3 & 1 \\ 0 & -1 \\ -2 & 3 \end{bmatrix}$ $Z = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$
- Calculate XY, YX, $Z^T Y$

Matrices

- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ?$
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = ?$

Matrices

- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ?$
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = ?$
- $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$
- For any matrix of shape $n \times n$, $AI_n = I_nA = A$

Matrix Inverse

- Analogy: Reciprocal of a number $\frac{1}{a}a = 1$
- Matrix inverse only defined for square matrix (# rows = # cols)

$$A^{-1}A = AA^{-1} = I = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Matrix Inverse

- Hard to compute by hand, but for 2 by 2 matrix, it is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Matrix Inverse

- Hard to compute by hand, but for 2 by 2 matrix, it is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- The matrix inverse does not always exist. Can you tell when that is the case for 2 by 2 matrices based on the formula given above?

Matrix Inverse

When is matrix inverse useful? We can use it to solve systems of linear equations!

- Consider the following equations

$$\begin{cases} x + 2y = 5 \\ 3x + 5y = 13 \end{cases}$$

- In matrix-vector form

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

NYU TANDON SCHOOL OF ENGINEERING

Matrix inverse

$$\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}5\\13\end{bmatrix}$$

$$\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}^{-1}\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}^{-1}\begin{bmatrix}5\\13\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}^{-1}\begin{bmatrix}5\\13\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}^{-1}\begin{bmatrix}5\\13\end{bmatrix}$$

NYU TANDON SCHOOL OF ENGINEERING

Outline

NYU TANDON SCHOOL OF ENGINEERING

Setting Up Python

- Google Colab
  - Interactive programming online
  - No installation
  - Free GPU for 12 hours
- Your task:
  - Register a Google account and set up Google Colab
  - Run `print('hello world!')`
  - Open the notebook `demo_python_basics.ipynb` from the Github repo.

NYU TANDON SCHOOL OF ENGINEERING

Outline

Python Basics

- Program
  - We write operations to be executed on variables
- Variables
  - Referencing and interacting with items in the program
- If-Statements
  - Conditionally execute lines of code
- Functions
  - Reuse lines of code at any time

Python Basics

- Lists
  - Store an ordered collection of data
- Loops
  - Conditionally re-execute code
- Strings
  - Words and sentences are treated as lists of characters
- Classes (advanced)
  - Making your own data-type. Functions and variables made to be associated with it too.

Lab: Python Basics

- Write a function to find the second largest number in a list (Hint: use `sort()`)
- Define a class `Student`
- Use the `__init__()` function to assign the values of two attributes of the class: `name` and `grade`
- Define a function `study()` with an argument `time` in minutes. When calling this function, it should be printed "(the student's name) has studied for (time) minutes"

Outline

NYU TANDON SCHOOL OF ENGINEERING

Demo and Exercises: NumPy

A Python package is a collection of code people wrote for other users to run directly. Today, we learn how to use the package NumPy for linear algebra.

- Open `demo_vectors_matrices.ipynb`
- Your task: use NumPy functions to compute the exercises we did earlier this morning. Compare the results.

Homework

- Matrices exercises on Piazza
- Tutorial Video on Matplotlib

These slides have been modified from the original slides provided through the courtesy of Nikola, Akshaj, Aishwarya, and Jack.