Day 1: Introduction to Machine Learning

Summer STEM: Machine Learning

Department of Electrical and Computer Engineering
NYU Tandon School of Engineering
Brooklyn, New York

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Outline

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Tell the class about yourself

- Write down the following information:
  - Name
  - Grade
  - In which city/town are you currently living?
  - What is your favourite movie?
  - What is the IMDB score of this movie!
  - What is the category of this movie? (thriller/drama/action, etc)
  - Rate your coding experience from 1 (no experience) to 5 (plenty of experience)!
- Share your answers with the class!
- We'll visualize this dataset using Python tomorrow!
  - Link to excel sheet here

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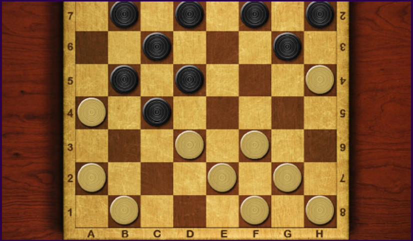
Outline

Machine Learning

- Most recent exciting technology
- We use these algorithms dozens of times a day
  - Search Engine
  - Recommendations
- Machine Learning is an important component to achieve artificial general intelligence
- Practice is the key to learn machine learning

Definition

- Machine Learning is a field of study that gives computers the ability to learn without being explicitly programmed.



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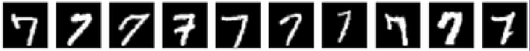
Example: Digit Recognition



- Challenges with expert approach
  - Simple expert rule breaks down in practice
  - Difficult to translate our knowledge into code
- Machine Learning approach
  - Learned systems do very well on image recognition problems

```
def classify(image):
    ...
    nv = count_vert_lines(image)
    nh = count_horiz_lines(image)
    ...

    if (nv == 1) and (nh == 1):
        digit = 7
    ...

    return digit
```

Example: CIFAR 10

Machine Learning Problem Pipeline

1. Formulate the problem: regression, classification, or others?
2. Gather and visualize the data
3. Design the model and the loss function
4. Train your model
   - (a) Perform feature engineering
   - (b) Construct the design matrix
   - (c) Choose regularization techniques
   - (d) Tune hyper-parameters using a validation set
   - (e) If the performance is not satisfactory, go back to step (a)
5. Evaluate the model on a test set

Outline

Course Outline

- Day 1: Introduction to ML
- Day 2: Linear Regression
- Day 3: Overfitting and Generalization
- Day 4: Classification and Logistic Regression
- Day 5: Mini Project
- Day 6: Neural Networks
- Day 7: Convolutional Neural Networks
- Day 8: Deep Generative Models
- Day 9: Final Project
- Day 10: Social Impacts of ML and Final Project Presentations

Course Format, Website, Resources

- Course Website:
  https://github.com/asarmadi/tandon_summer2021_ml
  - Github: share collections of documents, repositories of code
  - Contains lecture slides, code notebooks, and datasets
  - Slides and demo code posted before lecture, solutions to the lab posted after
- After-class discussion: Campuswire. Links to class recordings will also be posted there.
- We strongly encourage programming in Python via Google Colab.
- We'll give additional resources at the end of each day based on student interest

Outline

Vectors

- A **vector** is an ordered list of numbers or symbols
  - Ex:

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \ \mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 8 \\ 6 \end{bmatrix}, \ \mathbf{w} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

Vectors

- Vectors of the same size may be added together, element-wise
  - Ex: $\mathbf{u} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
    $\mathbf{u} + \mathbf{v} = \begin{bmatrix} 3+1 \\ (-1)+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$
- Vectors may be scaled by a number, element-wise
  - Ex: $3\mathbf{v} = \begin{bmatrix} 3 \times 1 \\ 3 \times 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

Vectors

- Norm of a vector (L2 Norm)
  - Ex: If $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\|\mathbf{v}\| = \sqrt{1^2 + 2^2} = \sqrt{5}$
- Inner product: sum of element-wise products of two vectors
  - Ex: $\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}^T \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 3 \times 1 + (-1) \times 2 = 3 - 2 = 1$
  - Gives the angle between two vectors $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \, \|\mathbf{v}\|}$

Vectors

- If $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

  $$\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{bmatrix}$$

- For any real number α, $\alpha\mathbf{u} = \begin{bmatrix} \alpha u_1 \\ \alpha u_2 \\ \vdots \\ \alpha u_n \end{bmatrix}$

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Vectors

- inner product :

$$\mathbf{u} \cdot \mathbf{v} = u_1 \times v_1 + u_2 \times v_2 + \cdots + u_n \times v_n = \Sigma_{i=1}^{n} u_i \times v_i$$

- norm :

$$\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2} = \sqrt{\Sigma_{i=1}^{n} u_i^2}$$

- Squared norm :

$$\|\mathbf{u}\|^2 = u_1^2 + u_2^2 + \cdots + u_n^2 = \Sigma_{i=1}^{n} u_i^2$$

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Exercise: Vectors

Let $\mathbf{p} = \begin{bmatrix} 3 \\ 2 \\ 9 \\ 4 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 1 \\ 9 \\ 0 \\ 3 \end{bmatrix}$, calculate

- $3\mathbf{q} + 2\mathbf{p}$
- $\mathbf{q} \cdot \mathbf{q}$ and $\|\mathbf{q}\|^2$
- $\mathbf{p} \cdot \mathbf{q}$ and $\|\mathbf{p}\| \, \|\mathbf{q}\|$

Matrices

- A **matrix** is a rectangular array of numbers or symbols arranged in rows and columns. We can conceptualize it as a collection of vectors.
  - Ex: 2 by 2 matrix, $M = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$
- Matrices of the same shape may be added together, element-wise
  - Ex: $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 8 \\ 7 & 11 \end{bmatrix}, A + B = \begin{bmatrix} 1 & 9 \\ 9 & 12 \end{bmatrix}$
- Matrices may be scaled, element-wise
  - Ex: $\alpha B = \begin{bmatrix} 0 & 8\alpha \\ 7\alpha & 11\alpha \end{bmatrix}$, where α is a scalar

Exercise: Matrices

- $\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 9 & 2 \\ -7 & 6 \\ 3 & 1 \end{bmatrix} = ?$
- $2 \begin{bmatrix} 1 & 9 \\ 3 & -2 \end{bmatrix} = ?$

Vectors and Matrices

- We may consider a vector as a matrix
  - **Row Vector**: shape (1 x N)
    Ex: $\mathbf{v} = \begin{bmatrix} 1 & 2 \end{bmatrix}$
  - **Column Vector**: shape (N x 1)
    Ex: $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
  - We'll consider vectors as <u>column vectors by default</u>

Matrices

Example :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

- What is the shape of A ?

Matrices

Example :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

- What is the shape of A ?

$$(2 \times 5)$$

Matrices

General case :

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & & & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nm} \end{bmatrix}$$

- What is the shape of A ?

Matrices

General case :

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1m} \\ A_{21} & A_{22} & \dots & A_{2m} \\ \vdots & & & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nm} \end{bmatrix}$$

- What is the shape of A ?

$$n \times m$$

Matrices

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- What is the shape of A ?

$$n \times m$$

- A_{ij} is the element at the i^{th} row and j^{th} column

Matrices

Example :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

- $A_{13} =?$
- $A_{21} =?$
- $A_{24} =?$

Matrices

- Two matrices, A and B, can be multiplied together provided their shapes meet the criteria :
- **Criteria:** # cols of A must equal the # rows of B
  - Ex : $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$
  - Shape of A : (2×3), Shape of B : (3×2)

Matrices

- Two matrices, A and B, can be multiplied together provided their shapes meet the criteria :
- **Criteria:** # cols of A must equal the # rows of B
  - Ex : $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$
  - Shape of A : (2×3), Shape of B : (3×2)
- Result is a matrix with shape (# rows A $\times$ # cols B)
  - Ex : If $C = AB$ then, C is of shape (2×2) : $C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

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Matrices

- **To sum up :**
  If A is of shape $(M \times K)$ and B of shape $(K \times N)$,
  We can define $C = AB$, and C will be of shape $(M \times N)$

Matrices

- If $C = AB$ then, $(C)_{ij} = \sum_{k=1}^{K} A_{ik} B_{kj}$
- Inner product of the i-th row of A and the j-th column of B

Matrices

- If $C = AB$ then, $(C)_{ij} = \sum_{k=1}^{K} A_{ik} B_{kj}$
- Inner product of the i-th row of A and the j-th column of B
- Ex : $C = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$

$$C_{11} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = -6 \quad C_{12} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 3$$

$$C_{21} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 0 \\ 3 \end{bmatrix} = 6 \quad C_{22} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 0$$

Matrices

- $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = ?$
- $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = ?$
- $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = ?$
- In general, $AB \neq BA$

Matrices

- **Transpose**: A^T swaps the rows and columns of matrix A
- Ex: $\begin{bmatrix} 2 \\ 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$
- $\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = ?$
- $(AB)^T = B^T A^T$

Exercises: Matrix Multiplication

- $X = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}$ $Y = \begin{bmatrix} 3 & 1 \\ 0 & -1 \\ -2 & 3 \end{bmatrix}$ $Z = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$
- Calculate XY, YX, $Z^T Y$

Matrix Inverse

- Analogy: Reciprocal of a number $\frac{1}{a}a = 1$
- Matrix inverse only defined for square matrix (# rows = # cols)

$$A^{-1}A = AA^{-1} = I = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Matrix Inverse

- Hard to compute by hand, but for 2 by 2 matrix, it is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Matrix Inverse

- Hard to compute by hand, but for 2 by 2 matrix, it is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- The matrix inverse does not always exist. Can you tell when that is the case for 2 by 2 matrices based on the formula given above?

Matrix Inverse

When is matrix inverse useful? We can use it to solve systems of linear equations!

- Consider the following equations

$$\begin{cases} x + 2y = 5 \\ 3x + 5y = 13 \end{cases}$$

- In matrix-vector form

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

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Matrix inverse

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 13 \end{bmatrix}$$

Outline

Setting Up Python

- Google Colab
  - Interactive programming online
  - No installation
  - Free GPU for 12 hours
- Your task:
  - Register a Google account and set up Google Colab
  - Run `print('hello world!')`
  - Open the notebook `demo_python_basics.ipynb` from the Github repo.

Outline

Python Basics

- Program
  - We write operations to be executed on variables
- Variables
  - Referencing and interacting with items in the program
- If-Statements
  - Conditionally execute lines of code
- Functions
  - Reuse lines of code at any time

Python Basics

- Lists
  - Store an ordered collection of data
- Loops
  - Conditionally re-execute code
- Strings
  - Words and sentences are treated as lists of characters
- Classes (advanced)
  - Making your own data-type. Functions and variables made to be associated with it too.

Lab: Python Basics

- Write a function to find the second largest number in a list (Hint: use `sort()`)
- Define a class `Student`
- Use the `__init__()` function to assign the values of two attributes of the class: `name` and `grade`
- Define a function `study()` with an argument `time` in minutes. When calling this function, it should be printed "(the student's name) has studied for (time) minutes"

Outline

Demo and Exercises: NumPy

A Python package is a collection of code people wrote for other users to run directly. Today, we learn how to use the package NumPy for linear algebra.

- Open `demo_vectors_matrices.ipynb`
- Your task: use NumPy functions to compute the exercises we did earlier this morning. Compare the results.

Homework

- Matrices exercises on Campuswire
- Tutorial Video on Matplotlib

These slides have been modified from the original slides provided through the courtesy of Nikola, Akshaj, Aishwarya, and Jack.