Introduction to Cryptography

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What is cryptography (alt. cryptology)?

Definition

- The study of techniques to allow secure communication and data storage in presence of attackers

Features provided

- **Confidentiality**: data can be accessed only by chosen entities
- **Integrity/freshness**: detect/prevent tampering or replays
- **Authenticity**: data and their origin are certified
- **Non-repudiation**: data creator cannot repudiate created data
- **Advanced features**: proofs of knowledge/computation

Cryptographic prehistory

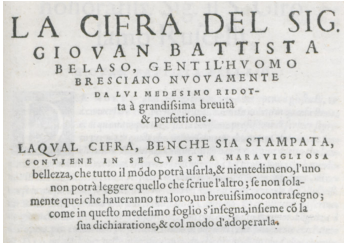
As old as written communication

- Born for commercial (recipe for lacquer on clay tablets) or military (Spartans) uses
- Designed by humans, for human computers
- Algorithms computed by hand, with pen and paper

Ancient view

Original approach

- A battle of wits between
  - cryptographers: ideate a secret method to obfuscate a text
  - cryptanalysts: figure out the method, break the "cipher"
- Bellaso (1553) [1] separates the encryption method from the key


LA CIFRA DEL SIG.
GIOVAN BATTISTA
BELASO, GENTIL'HVOMO
BRESCIANO NVOVAMENTE
DA LVI MEDESIMO RIDOT-
ta à grandiſsima breuità
& perfettione.

LAQVAL CIFRA, BENCHE SIA STAMPATA,
CONTIENE IN SE QVESTA MARAVIGLIOSA
bellezza, che tutto il mõdo potrà uſarla, & nientedimeno, l'uno
non potrà leggere quello che ſcriue l'altro; ſe non ſola-
mente quei che haueranno tra loro, un breuiſsimo contraſegno;
come in queſto medeſimo foglio s'inſegna, inſieme cõ la
ſua dichiaratione, & col modo d'adoperarla.

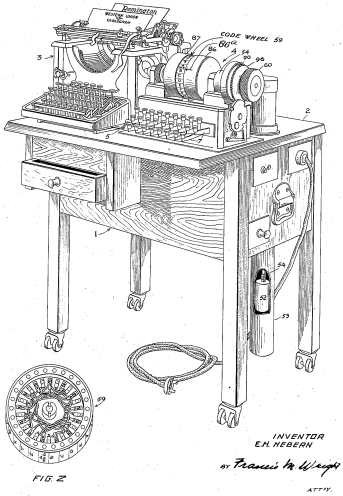
1883 - Kerchoff's six principles for a good cipher (apparatus)

1. It must be practically, if not mathematically, unbreakable
2. It should be possible to make it public, even to the enemy
3. The key must be communicable without written notes and changeable whenever the correspondants want
4. It must be applicable to telegraphic communication
5. It must be portable, and should be operable by a single person
6. Finally, given the operating environment, it should be easy to use, it shouldn't impose excessive mental load, nor require a large set of rules to be known

Cryptographic modern history

The advent of the machines

- Mechanical computation changes cryptography
  - First rotor machine in 1917 by Ed Hebern
  - Design "popularized" in WWII by German Enigma
- Cryptanalysist at Bletchley park (Turing among them) credited for a decisive effort in winning the war by Eisenhower



Moving into the modern age

An end to the battle of wits

- Shannon (1949) [4] - Proves that a mathematically unbreakable cipher exists
- Nash (1955) [2] - Argues that computationally secure ciphers are ok
  - Considers a cipher with a finite, λ bit long, key
  - Conjecture: if "*parts of the key interact complexly [...] in the determination of their effects on the ciphertext*", the attacker effort to break the cipher would be $\mathcal{O}(2^{\lambda})$
  - The owner of the key takes $\mathcal{O}(\lambda^2)$ to compute the cipher
  - The computational gap is unsurmountable for large λ

Not mathematically unbreakable but impractical

Outline of the topics

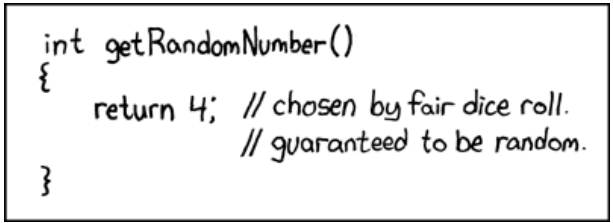
In this course

- Definitions of ciphers as components with functionalities
- How to obtain confidentiality, integrity, data/origin authentication
- An overview of protocols (combinations of ciphers)
- Goal: be able to use cryptographic components properly

In Cryptography and Architectures for Computer Security

- Design and cryptanalysis techniques for ciphers
- Cryptographic protocols and their inner workings
- Efficient, side channel attack resistant implementations
- Goal: be able to engineer cryptographic components

Before we start...



https://xkcd.com/221/

A word on randomness

- Randomness (in this course) characterizes a generative process
- Stating: "00101 is a random string" actually makes little sense

Definitions

Data

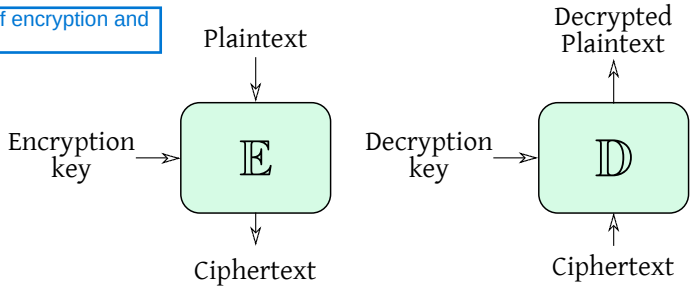
- Plaintext space **P**: set of possible messages $\texttt{ptx} \in \mathbf{P}$
  - Old times: words in some human-readable alphabet, modern times $\{0,1\}^l$
- Ciphertext space **C**: set of possible ciphertext $\texttt{ctx} \in \mathbf{C}$
  - Usually $\{0,1\}^{l'}$, not necessarily $l = l'$ (ciphertexts may be larger)
- Key space **K**: set of possible keys
  - $\{0,1\}^\lambda$, keys with special formats are derived from bitstrings

Definitions

Functions

- Encryption function $\mathbb{E} : \mathbf{P} \times \mathbf{K} \to \mathbf{C}$
- Decryption function $\mathbb{D} : \mathbf{C} \times \mathbf{K} \to \mathbf{P}$
  - Correctness: for all $\mathtt{ptx} \in \mathbf{P}$, we need $k, k' \in \mathbf{K}$ s.t. $\mathbb{D}(\mathbb{E}(\mathtt{ptx}, k), k') = \mathtt{ptx}$

The cipher is the pair of encryption and decryption functions



Providing confidentiality

Goal

- Prevent anyone not authorized from being able to understand data

Possible attacker models

- The attacker simply eavesdrops (ciphertext only attack)
- The attacker knows a set of possible plaintexts
  - Limit case: the attacker chooses the set of plaintexts
- The attacker may tamper with the data and observe the reactions of a decryption-capable entity
  - Limit case: the attacker sees the actual decrypted value

Perfectly secure cipher

Definition

- In a *perfect cipher*, for all $\texttt{ptx} \in \mathbf{P}$ and $\texttt{ctx} \in \mathbf{C}$,
  $\Pr(\text{ptx sent} = \texttt{ptx}) = \Pr(\text{ptx sent} = \texttt{ptx} \mid \text{ctx sent} = \texttt{ctx})$
- In other words: seeing a ciphertext $c \in \mathbf{C}$ gives us *no information* on what the plaintext corresponding to c could be

Question

- The definition is not constructive! Does a perfect cipher exist?
  - If yes, what does it look like?

Perfectly secure cipher

Theorem (Shannon 1949)

Any symmetric cipher $\langle \mathbf{P}, \mathbf{K}, \mathbf{C}, \mathbb{E}, \mathbb{D} \rangle$ with $|\mathbf{P}| = |\mathbf{K}| = |\mathbf{C}|$ is perfectly secure if and only if

↳ encryption and decryption keys are the same

- every key is used with probability $\frac{1}{|\mathbf{K}|}$
- a unique key maps a given plaintext into a given ciphertext:
  $\forall(\mathtt{ptx}, \mathtt{ctx}) \in \mathbf{P} \times \mathbf{C},\ \exists! k \in \mathbf{K}$ s.t. $\mathbb{E}(\mathtt{ptx}, k) = \mathtt{ctx}$

A simple working example

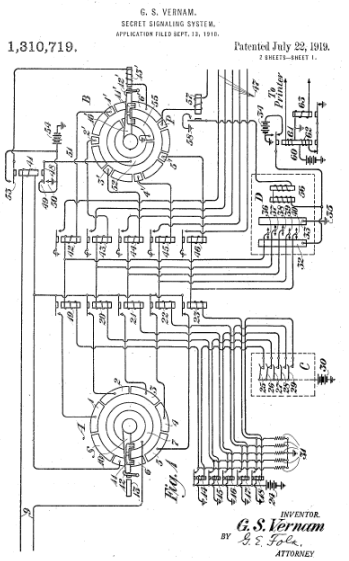
- Assume $\mathbf{P}, \mathbf{K}, \mathbf{C}$ to be set of binary strings. The encryption function draws a *uniformly random, fresh* key k out of $\mathbf{K}$ each time it is called and computes $\mathtt{ctx} = \mathtt{ptx} \oplus k$

xor

Anticausal implementations

A concrete apparatus

- Gilbert Vernam actually patented a telegraphic machine implementing $\texttt{ptx} \oplus k$ on Baudot code in 1919
- Joseph Mauborgne suggested the use of a random tape containing k
- Using Vernam's encrypting machine with Mauborgne's suggestion implements a perfect cipher


G. S. VERNAM.
SECRET SIGNALING SYSTEM.
APPLICATION FILED SEPT. 13, 1918.

1,310,719. Patented July 22, 1919.

2 SHEETS—SHEET 1.

Printer

Fig. 1

INVENTOR.
G. S. Vernam
BY
E. E. Yoke
ATTORNEY

Perfectly secure $\neq$ perfectly usable

In diplomatic contexts keys were physically transported by ambassadors.
However, human generation of random keys is impractical (biases, laziness etc...)

Key storage/management

- storing key material and changing keys is a nightmare
- perfect cipher broken in practice due to key theft/reuse
- generating random keys was also an issue (and caused breaks)

Photo courtesy of Cryptomuseum.com

Computationally secure cryptography

A more practical assumption

- Build a cipher so that a successful attack is also able to solve a hard computational problem efficiently
  - Solve a generic nonlinear Boolean simultaneous equation set [> NP hard]
  - Factor large integers / find discrete logarithms
  - Decode a random code/find shortest lattice vector

Can we avoid assumptions?

- Open question: prove an exponential lower bound for the time taken to solve a hard problem, which has efficiently verifiable solution
  - it would shift from a computational security assumption to a theorem
  - ... and prove $P \neq NP$ as a corollary

Proving computational security

Outline of the method

1. Define the ideal attacker behaviour
2. Assume a given computational problem is hard
3. Prove that any non ideal attacker solves the hard problem

How to represent attacker and properties?

- Attacker represented as a program able to call given libraries
- Libraries implement the cipher at hand
- Define the security property as answering to a given question
- The attacker wins the game if it breaks the security property more often than what is possible through a random guess

Cryptographically Safe Pseudorandom Number Generators

rand() from C is not really pseudorandom, that is, it not cryptographically safe

Motivation and assumption

- We want to use a finite-length key and a Vernam cipher
  - We somehow need to "expand" the key
- We assume that the attacker can only perform poly(λ) computations

Definition

A CSPRNG is a deterministic function $\text{PRNG}: \{0,1\}^{\lambda} \to \{0,1\}^{\lambda+l}$ whose output cannot be distinguished from an uniform random sampling of $\{0,1\}^{\lambda+l}$ in $\mathcal{O}(\text{poly}(\lambda))$. l is the CSPRNG stretch.

string of lenght lambda

CSPRNG should not be invertible

We are using a cryptographic library that have an encrypt() function, which needs us to generate a key. Now, if we don't use a CSPRNG but only a PRNG it is a risk

CSPRNGs in practice

How to generate random numbers? ask OS, since it can access physical measures and derive randomness

Existence

- In practice, we have only candidate CSPRNGs
  - We have no proof that a function PRNG exists
  - Proving that a CSPRNG exists implies directly P≠NP

Practical constructions

- Building a CSPRNG "from scratch" is possible, but it is not the way they are commonly built (not efficient)
- Practically built with another building block: PseudoRandom Permutations (PRPs)
  - defined starting from PseudoRandom Functions (PRFs)

Random Functions (RFs)

Randomly drawing a function

- Consider the set $\mathbf{F} = \{f : \{0,1\}^{in} \rightarrow \{0,1\}^{out}; in, out \in \mathbb{N}\}$
- A uniformly randomly sampled $f \xleftarrow{\$} \mathbf{F}$ can be encoded by a 2^{in} entries table, each entry out bit wide. $|\mathbf{F}| = (2^{out})^{2^{in}}$

Toy example $in = 2, out = 1$

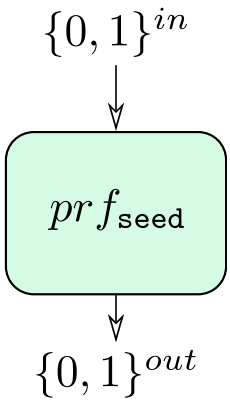
- $\mathbf{F} = \{f : \{0,1\}^2 \rightarrow \{0,1\}^1\}$ is the set of the 16 Boolean functions w/ two inputs
- Each one is represented by a 4-entry truth table
- Intuitively, the functions are 16 as there are $2^4 = 16 = (2^1)^{2^2}$ tables

for each of the (2^in) entry, I have 2^out (that is, each possible output) possible outputs.

Pseudorandom Functions (PRFs)

Definition

- A function $prf_{\texttt{seed}} : \{0,1\}^{in} \rightarrow \{0,1\}^{out}$ taking an input and a λ bits seed.
- The entire $prf_{\texttt{seed}}$ is described by the value of the seed
- It cannot be told apart from a random $f \in \{f : \{0,1\}^{in} \rightarrow \{0,1\}^{out}\}$ in poly(λ)
- That is, if they give you $a \in \{f : \{0,1\}^{in} \rightarrow \{0,1\}^{out}\}$, you can't tell which one of the following is true
  - $a = prf_{\texttt{seed}}(\cdot)$ with $\texttt{seed} \xleftarrow{\$} \{0,1\}^{\lambda}$
  - $b \xleftarrow{\$} \mathbf{F}$, where $\mathbf{F} = \{f : \{0,1\}^{in} \rightarrow \{0,1\}^{out}\}$



Pseudorandom Permutations (PRPs)

Pseudorandom Permutation definition

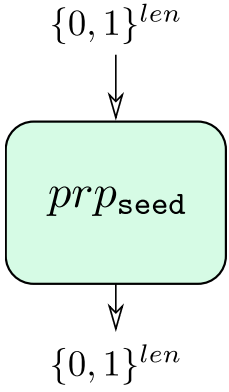
- A bijective PRF: $prf_{\texttt{seed}} : \{0,1\}^{len} \rightarrow \{0,1\}^{len}$

Wrapping your mind around it

- It is uniquely identified by the value of the seed
- It is not possible to tell apart in poly(λ) from a RF
- It's a permutation of all the possible $\{0,1\}^{len}$ strings

Operatively speaking

- acts on a block of bits outputs another one of the same size
- the output "looks unrelated" to the input
- its action is fully identified by the seed
  - Useful to think of the seed as a key



Real world PRPs

The issue

- No formally proven PRP exists, yet
  - again, its existence would imply P$\neq$NP

Typical construction

1. Compute a small bijective Boolean function f of input and key
2. Compute f again between the previous output and the key
3. Repeat 2 until you're satisfied

In the real world...

Practical solution: public scrutiny

- Modern PRPs are the outcome of public contests
- Cryptanalytic techniques provide ways (=poly(λ) tests) to detect biases in their outputs: good designs are immune

PRPs a.k.a. Block ciphers

- Concrete PRPs go by the historical name of block ciphers
- Considered broken if, with less than 2^{λ} operations, they can be told apart from a PRP, e.g., via:
  - Deriving the input corresponding to an output without the key
  - Deriving the key identifying the PRP, or reducing the amount of plausible ones
  - Identifying non-uniformities in their outputs
- The key length λ is chosen to be large enough so that computing 2^{λ} guesses is not practically feasible

Quantifying computational unfeasibility

2^ bit of the key

Energy needed to break block ciphers independently on the time

Boolean operations vs. energy to bring from 20 °C $\rightarrow$ 100 °C

- 2^{65} op.s $\approx$ an Olympic swimming pool
- 2^{80} op.s $\approx$ the annual rainfall on the Netherlands
- 2^{114} op.s $\approx$ all water on Earth

Practically acceptable unfeasibility

- Legacy level security: at least 2^{80} Boolean operations
- 5 to 10 years security: at least 2^{128} Boolean operations
- Long term security: at least 2^{256} Boolean operations

Widespread block ciphers

Advanced Encryption Standard (AES)

- 128 bit block, three key lengths: 128, 192 and 256 bits
- Selected after a 3 years public contest in 2000-10-2 by NIST out of 15 candidates, re-standardized by ISO/IEC
- ARMv8 and AMD64 include dedicated instructions accelerating its computation (hitting 3+ GB/s)

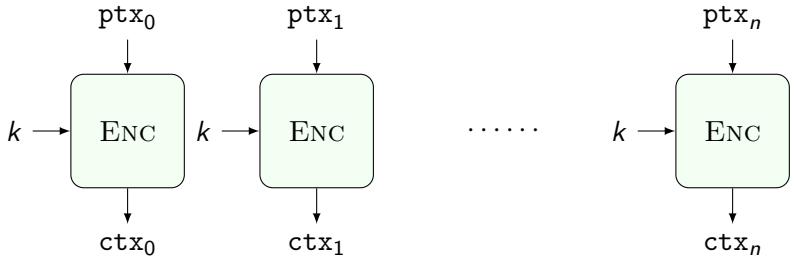
Data Encryption Algorithm (DEA, a.k.a. DES)

- Legacy standard by NIST (1977), the key is too short (56b)
- Patch via triple encryption, $\lambda = 112$ equivalent security
- Still found in some legacy systems, officially deprecated

An Electronic CodeBook (ECB)

A first attempt to encryption with PRPs

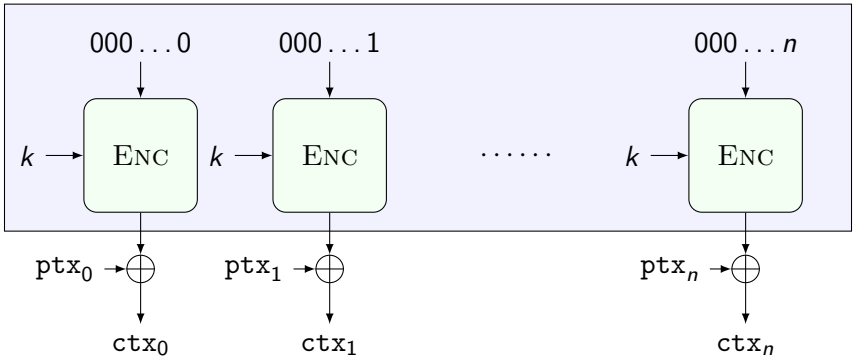
- It's ok to encrypt a plaintext $\leqslant$ block size with a block cipher
- An extension to multiple blocks could be split-and-encrypt
  - Is it good (equivalent to Vernam fed with a CSPRNG)?



Counter (CTR) mode

Getting it right

- The boxed construction is provably a PRNG if Enc is a PRP
- There is nothing special in the starting point of the counters



Raising the requirements

Confidentiality achieved ... for CoA

- Up to now, the attacker knew only ciphertext material

Confidentiality against Chosen Plaintext Attacks (CPAs)

- Our attacker knows a set of plaintexts which can be encrypted
- He wants to understand which one is being encrypted
- Ideal attacker: cannot tell which plaintext was encrypted out of two *he* chose (having the same length)
- Feels strange, but it happens with:
  - management data packets in network protocols (e.g., ICMP)
  - telling apart a encrypted commands to a remote host

Achieving CPA Security

No deterministic encryption

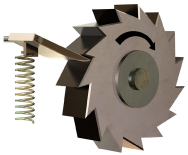
- The CTR mode of operation is insecure against CPA
  - The encryption is deterministic: same ptxs $\rightarrow$ same ctx

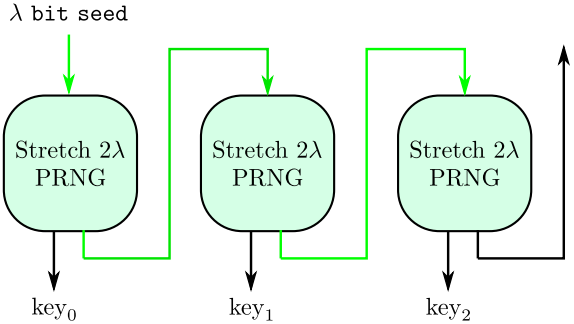
Decryptable nondeterministic encryption

1. Rekeying: change the key for each block with a ratchet
2. Randomize the encryption: add (removable) randomness to the encryption (change mode of employing PRP)
3. Numbers used ONCE (NONCEs): in the CTR case, pick a NONCE as the counter starting point. NONCE is public

Getting it right

- The construction takes the name from the mechanical component: it is not possible to roll-back the procedure once you delete the value carried by green arrows

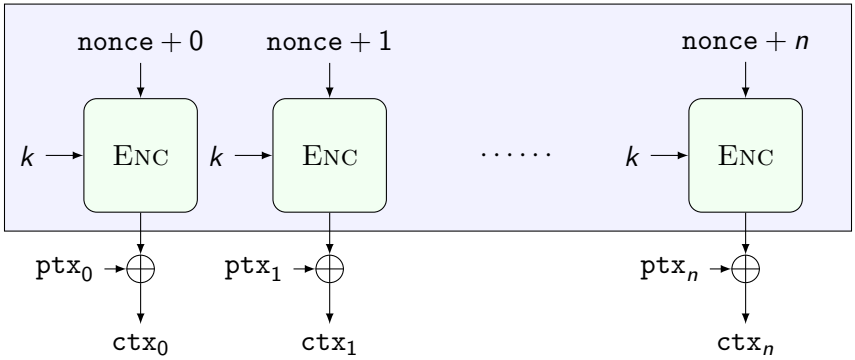




CPA-Secure Counter (CTR) mode

Getting it right

- Picking the counter start as a NONCE generates different bitstreams to be `xor`-ed with the `ptx` each time
- The same plaintext encrypted twice is turned into two different, random-looking, ciphertexts



Malleability and active attackers

Malleability

- Making changes to the ciphertext (not knowing the key) maps to predictable changes in the plaintext
  - Think about AES-CTR and AES-ECB
- Can be creatively abused to build decryption attacks
- Can be turned into a feature (homomorphic encryption)

How to avoid malleability

- Design an intrinsically non malleable scheme (non trivial)
- Add a mechanism ensuring data integrity (against attackers)

Providing data integrity

Confidentiality $\not\Rightarrow$ Integrity

- Up to now our encryption schemes provide confidentiality
- Changes in the ciphertext are undetected (at best)

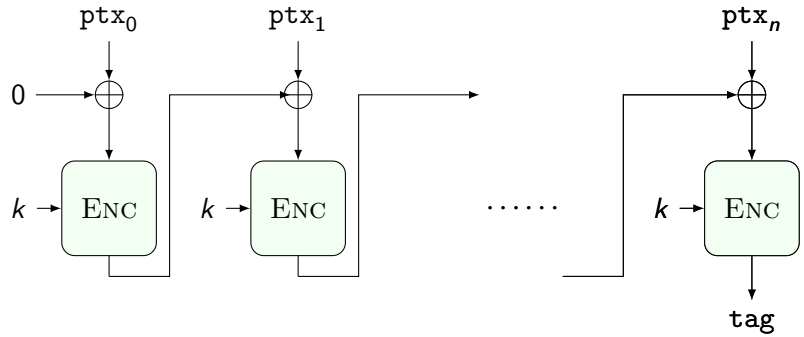
Message Authentication Codes (MAC) Should be called Message Integrity codes

- Add a small piece of information (`tag`) allowing us to test for the message integrity of the encrypted message itself
  - Adding it to the plaintext and then encrypting is not good
- Nomenclature misleads: MACs do not provide data authentication

Definition

- A MAC is constituted by a pair of functions:
  - COMPUTE_TAG(string,key): returns the tag for the input string
  - VERIFY_TAG(string,tag,key): returns `true` or `false`
- Ideal attacker model:
  - knows as many message-tag pairs as he wants
  - cannot forge a valid tag for a message for which he does not know it already
  - forgery also includes tag splicing from valid messages
- N.B. the tag creating entity and the verifying entity must both know the same secret key
  - The tag verifier is able to create a valid tag too
  - ... and there goes the non-repudiation property

How to build a MAC? the CBC-MAC



Building a MAC with a PRP (block cipher)

- The CBC-MAC is secure for prefix free messages (why?) anguage of strings where noone is prefix of another
- Encrypting the tag once more fixes (provably) the issue

Practical MAC uses

Browser cookies

- HTTP cookies are a "note to self" for the HTTP server[a]
- The note should not be tampered between server reads
- Solution: server runs COMPUTE_TAG(cookie,k) and stores both the (cookie,tag)

COOKIES' SENDER AND RECEIVER IS THE WEB SERVER, BUT THEY ARE STORED IN THE CLIENT.

[a] You can find a two slides cookies refresher at slides 40-41 of `https://polimi365-my.sharepoint.com/:b:/r/personal/10032133_polimi_it/Documents/FCI/2-Livello_Applicativo_v2020.pdf`

Later in the course

- Mitigating SYN-based denial of service attacks (SYN Cookies)
- Time-based two-factor authentication mechanisms (TOTP/HOTP)

Compressing for the sake of efficiency

Testing integrity

- Testing the integrity of a file requires us to compare it bit by bit with an intact copy or read it entirely to compute a MAC
- It would be fantastic to test only short, fixed length strings independently from the file size, representing the file itself
  - Major roadblock: there is a lower bound to the number of bits to encode a given content without information loss
- Can we build something close to the ideal scenario?

Cryptographic hashes

A pseudo-unique labeling function

- A cryptographic hash is a function $H : \{0,1\}^* \rightarrow \{0,1\}^l$ for which the following problems are computationally hard
  1. given $d = H(s)$ find s (1st preimage) (cannot invent it)
  2. given $s, d = H(s)$ find $r \neq s$ with $H(r) = d$ (2nd preimage)
  3. find $r, s; r \neq s$, with $H(s) = H(r)$ (collision)
- Ideal behaviour of a concrete cryptographic hash:
  1. finding 1st preimage takes $\mathcal{O}(2^d)$ hash computations guessing s
  2. finding 2nd preimage takes $\mathcal{O}(2^d)$ hash comp.s guessing r
  3. finding a collision takes $\approx \mathcal{O}(2^{\frac{d}{2}})$ hash computations
- The output bitstring of a hash is known as a digest

BIRTHDAY PARADOX = PEOPLE → INPUT STRINGS, PIGEONHOLE → DIGEST } IF #STRING = $\sqrt{\#\text{DIGESTS}}$ => 50% THAT THERE IS A COLLISION (?)

Concrete hash functions

What to use

- SHA-2 was privately designed (NSA), $d \in \{256, 384, 512\}$
- SHA-3 followed a public design contest (similar to AES), selected among ≈ 60 candidates, $d \in \{256, 384, 512\}$
- Both currently unbroken and widely standardized (NIST, ISO)

What not to use

- SHA-1: $d = 160$, collision-broken [6] (obtainable in $\approx 2^{61}$ op.s)
- MD-5: horribly broken [7]. Collisions in 2^{11}, public tools online [5]
  - In particular, collisions with arbitrary input prefixes in $\approx 2^{40}$

Uses for hash functions

Pseudonymized match

- Store/compare hashes instead of values (e.g., Signal contact discovery)

MACs

- Building MACs: generate tag hashing together the message and a secret string, verify tag recomputing the same hash
- A field-proven way of combining message and secret is HMAC
  - Standardized (RFC 2104, NIST FIPS 198)
  - Uses a generic hash function as a plug-in, combination denoted as HMAC-hash_name
  - HMAC-SHA1 (!), HMAC-SHA2 and HMAC-SHA3 are ok

Forensic use

- Write down only the hash of the disk image you obtained in official documents

Game changing ideas

Features we would like to have

- Agreeing on a short secret over a public channel
- Confidentially sending a message over a public authenticated channel without sharing a secret with the recipient
- Actual data authentication

NEED TO EXCHANGE KEYS WITHOUT A PRIORI SECURE CHANNEL

Solution: asymmetric cryptosystems

- Before 1976: rely on human carriers / physical signatures
- DH key agreement (1976) / Public key encryption (1977)
- Digital signatures (1977)

A cautionary note on security margins

Computational hardness

- Up to now, enumeration of the secret parameter was the best possible attack
- This is ok for modern block ciphers $\rightarrow$ best attack: $\mathcal{O}(2^{\lambda})$
- Asymmetric cryptosystems rely on hard problems for which bruteforcing the secret parameter is not the best attack
  - Factoring a λ bit number takes $\mathcal{O}\left(e^{k(\lambda)^{\frac{1}{3}}(\log(\lambda))^{\frac{2}{3}}}\right)$
- Comparing bit-sizes of the security parameters instead of actual complexities is really wrong
- Concrete bit-sizes for λ depending on the cipher: `www.keylength.org`

The Diffie-Hellman key agreement

Goal

- Make two parties share secret value w/ only public messages

Attacker model

- Can eavesdrop anything, but not tamper
- The Computational Diffie-Hellman assumption should hold

CDH Assumption

- Let $(\mathbf{G}, \cdot) \equiv \langle g \rangle$ be a finite cyclic group, and two numbers a, b sampled unif. from $\{0, \ldots, |\mathbf{G}| - 1\}$ ($\lambda = len(a) \approx \log_2 |\mathbf{G}|$)
- given g^a, g^b finding g^{ab} costs more than $\text{poly}(\log |\mathbf{G}|)$
- Best current attack approach: find either b or a (discrete log problem)

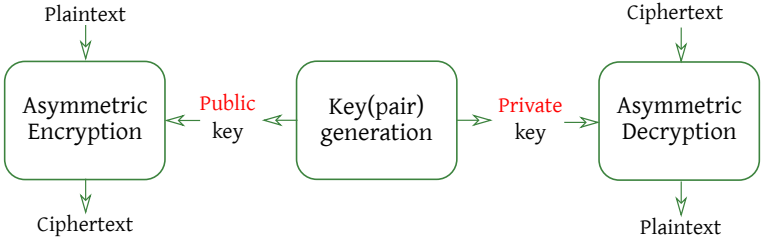
Key agreement between Alice and Bob

- Alice: picks $a \xleftarrow{\$} \{0, \ldots, |\mathbf{G}| - 1\}$, sends g^a to Bob
- Bob: picks $b \xleftarrow{\$} \{0, \ldots, |\mathbf{G}| - 1\}$, sends g^b to Alice
- Alice: gets g^b from Bob and computes $(g^b)^a$
- Bob: gets g^a from Bob and computes $(g^a)^b$
- $(\mathbf{G}, \cdot)$ is commutative $\rightarrow (g^b)^a = (g^a)^b$, we're done!

Groups used in practice

- A subgroup $(\mathbf{G}, \cdot)$ of $(\mathbb{Z}_n^*, \cdot)$ (integers mod n), breaking CDH takes $\min\left(\mathcal{O}\left(e^{k(\log(n))^{\frac{1}{3}}(\log(\log(n)))^{\frac{2}{3}}}\right), \mathcal{O}(2^{\frac{\lambda}{2}})\right)$
- EC points w/ dedicated addition, breaking CDH takes $\mathcal{O}(2^{\frac{\lambda}{2}})$

Public Key Encryption



Components

- *Different* keys are employed in encryption/decryption
- It is computationally hard to:
  - Decrypt a ciphertext without the *private* key
  - Compute the *private* key given only the *public* key

Widespread Asymmetric encryption ciphers

Rivest, Shamir, Adleman (RSA), 1977

- 2048 to 4096 bit message-and key-sizes
- Patented after the invention, patent now expired
- No ciphertext expansion
- Incidentally, the encryption with a fixed key is a PRP

Elgamal encryption scheme, 1985

- Either kbit range keys, or 100's of bits keys, depending on the variant
- Not encumbered by patents
- The ciphertext is twice the size of the plaintext
- Widely used as an RSA alternative where patents were a concern

Key Encapsulation

Assumption

- A public channel between Alice and Bob is available
- For the moment, the attacker model is "eavesdrop only"

Sharing a secret without agreement

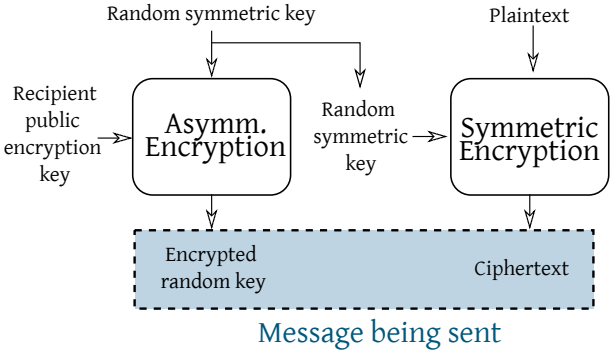
- Alice: generates a keypair (k_{pri}, k_{pub}), sends to Bob
- Bob: gets $s \xleftarrow{\$} \{0,1\}^{\lambda}$, encrypts it with k_{pub}, sends ctx to Alice
- Alice: decrypts ctx with k_{pri}, recovers s
- Note: Bob alone decides the value of the shared secret s
  - Repeat the procedure with swapped roles and combine the two secrets to achieve similar guarantees to a key agreement

Efficiency considerations

Can't I skip a step?

- Employing an asymmetric cryptosystem Bob encrypts a text for Alice without the need of sharing a secret beforehand
- In principle, Bob and Alice could employ *only* an asymmetric cryptosystem to communicate (encrypting directly the plaintext, not a key)
- In practice this approach would be extremely inefficient
  - Asymmetric cryptosystems are from $10\times$ to $1000\times$ slower than their symmetric counterparts

Best of both worlds



Hybrid encryption schemes

- Asymmetric schemes to provide key transport/agreement
- Symmetric schemes to encrypt the bulk of the data
- All modern secure transport protocols built around this idea

Authenticating data

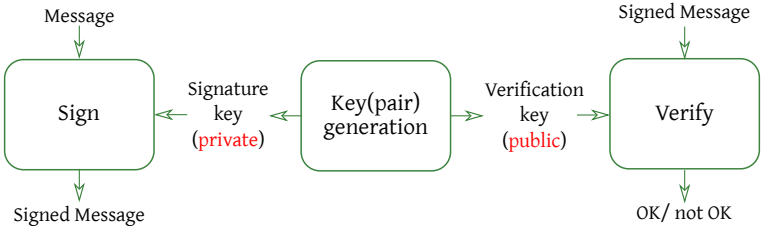
Motivations

- To build a secure hybrid encryption scheme we need to be sure that the public key the sender uses is the one of the recipient
- We'd like to be able to verify the authenticity of a piece of data without a pre-shared secret

Digital signatures

- Provide strong evidence that data is bound to a specific user
- No shared secret is needed to check (validate) the signature
- Proper signatures cannot be repudiated by the user
- They are asymmetric cryptographic algorithms
  - formally proven that you cannot get non repudiation otherwise

Digital signatures



Computationally hard problems

- Sign a message without the signature key
  - this includes splicing signatures from other messages
- Compute the signature key given only the verification key
- Derive the signature key from signed messages

Widespread Signature schemes

The object we use for signing is the same (?) we use for encrypting, however they do different things, we must be careful understanding which one we are using because there is the risk to decript instaed of signing

Rivest, Shamir, Adleman (RSA), 1977

- Unique case: the same hard-to-invert function to build an asymmetric encryption scheme and a signature (different message processing!)
- Signing definitely slower than verification ($\approx 300\times$)
- Standardized in NIST DSS (FIPS-184-4)

Digital Signature Standard (DSA)

- Derived from tweaking signature schemes by Schnorr and Elgamal
- Also standardized in NIST DSS (FIPS-184-4)
- Signature and verification take roughly the same time

Digital signature uses

Digital signatures are used to authenticate data and users

Authenticating digital documents

- For performance reasons, sign the hash of the document instead of the document
  - Signature properties now guaranteed only if both signature and hash algorithms are not broken

Authenticating users

- Alternative to password-based login to a system
  - The server has the user's public verification key (e.g. deposited at account creation)
  - The server asks the client to sign a long randomly generated bitstring (challenge)
  - If the client returns a correctly signed challenge, it has proven its identity to the server

Horror story 1

The importance of being static

- 2002-09-09: several pieces of software performing digital signatures with legal value in Italy allowed to sign MS Word documents *containing macros*
  - Macros allow to dynamically change the displayed text in a document according to, e.g., the current date
- Striking mismatch between what was thought to be signed (the visualized document) and the actual signed object (a program-document blend)
- Current standard for digital signatures on human-intended documents (PAdES,CAdES) target PDF and XML formats
  - n.b.: PDFs may embed Javascript, PDF/A do not

pdf format that prevents having code in it

The public key binding problem

Cautionary note

- Both in asymmetric encryption and digital signatures, the public key must be bound to the correct user identity
- If public keys are not authentic:
  - A MITM attack is possible on asymmetric encryption
  - Anyone can produce a signature on behalf of anyone else
- The public key authenticity is guaranteed with... another signature
  - We need someone to sign the public-key/identity pair
  - We need a format to distribute signed pairs

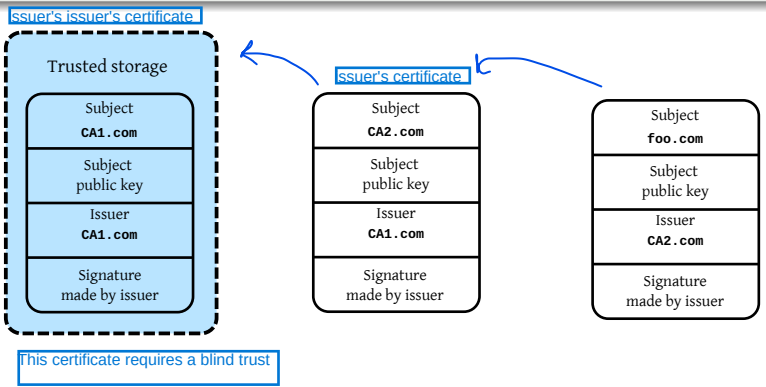
Digital certificates

- They bind a public key to a given identity, which is:
  - for humans: an ASCII string
  - for machines: either the CNAME or IP address
- They specify the intended use for the public key contained
  - Avoids ambiguities when a key format is ok for both an encryption and a signature algorithm
- They contain a time interval in which they are valid
- Most widely deployed format is described in ITU X.509

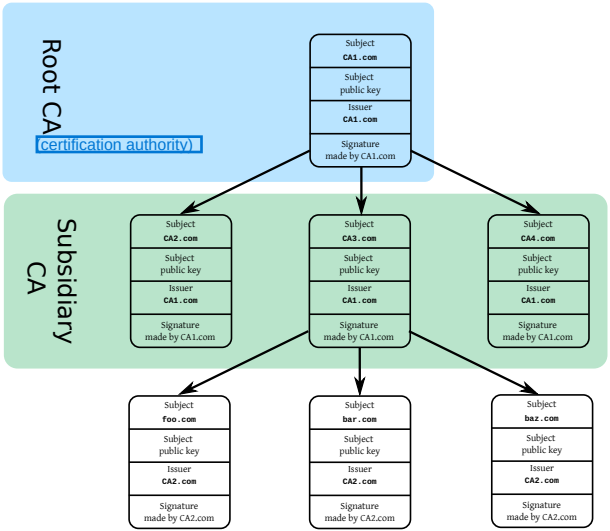
Certification authorities

Who signs the certificates

- The certificate signer is a trusted third party, the CA
- The CA public key is authenticated... with another certificate
- Up to a self-signed certificate which has to be trusted a priori



Certification authorities hierarchy

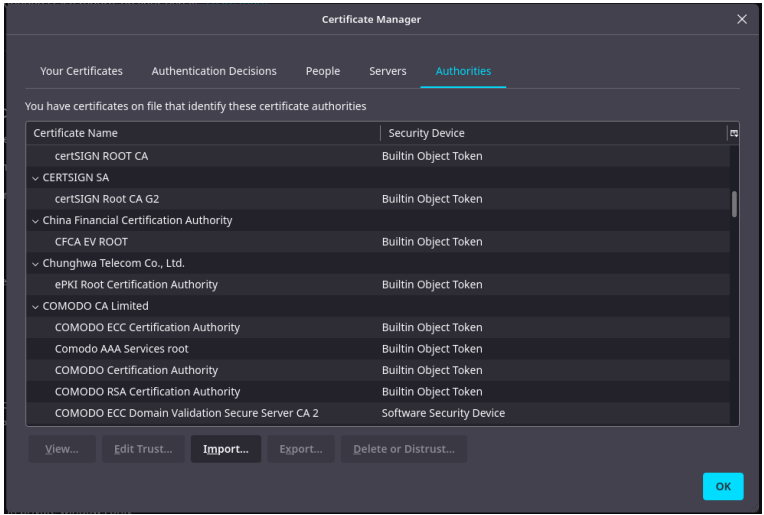


Horror story 2

Trust with care

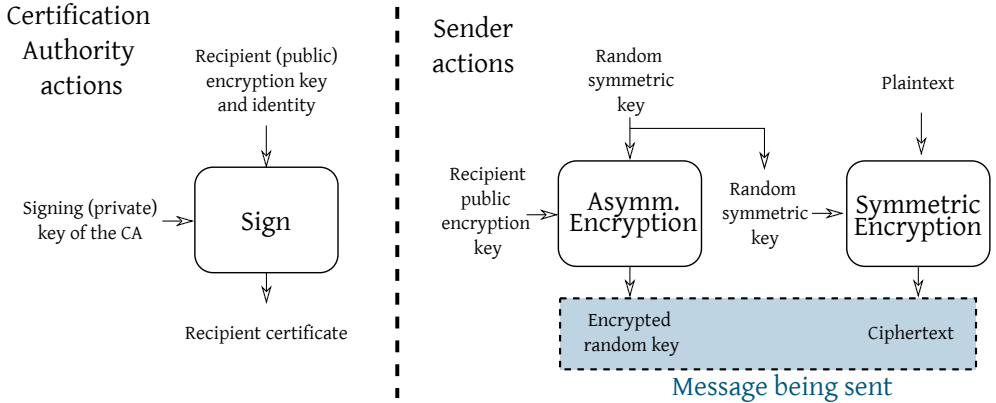
- 2003-02-20: Firma&Cifra was the digital signature application by PostECom
- When presented with any certificate bundled with a signed document, it considered the certificate authentic and added it to its trusted storage
- Signature forgery as easy as: 1) create your own CA certificate, 2) sign your target-user certificate, 3) sign the document
- Take away point: which certificates reside in your (applications') trusted storage determine who you trust

A recent browser certificate storage



Browser does key encapsulation for the public key using digital signature

Putting it all together



This way of communicating is the mainstay of modern secure comm. protocols (TLS, OpenVPN, IPSec)

Directions in modern cryptography

Issues to solve, features to realize

- What if we have a quantum computer?
  - Some computationally hard problems are no longer hard
  - Move away from cryptosystems based on factoring/dlog
  - Alternatives available and being standardized (2022-04)
- What if we want to compute on encrypted data? cloud computing without decrypting data
  - Yes, but it's moderately-to-horribly inefficient
- What if the attacker has physical access to the device computing the cipher (or some way of remotely measure it)
  - Take into account side channel information in the attacker model

Fundamentals of Information Theory

What is Shannon's information theory?

- Shannon's [3] way to mathematically frame communication
- A way to quantify information

What do we need this for (in this course)?

- Quantitatively frame "luck" and "guessing"

Basic Definitions

Basics

- A communication takes place between two endpoints
  - sender: made of an information source and an encoder
  - receiver: made of an information destination and a dencoder
- Information is carried by a channel in the form of a sequence of symbols of a finite alphabet

Transmitting and receiving

Losing uncertainty = Acquiring information

- The receiver gets information only through the channel
  - it will be uncertain on what the next symbol is, until the symbol arrives
  - thus we model the sender as a random variable
- Acquiring information is modeled as getting to know an outcome of a random variable $\mathcal{X}$
  - the amount of information depends on the distribution of $\mathcal{X}$
  - intuitively: the closer is $\mathcal{X}$ to a uniform distribution, the higher the amount of information I get from knowing an outcome
- Encoding maps each outcome as a finite sequence of symbols
  - More symbols should be needed when more information is sent

Measuring uncertainty: Entropy

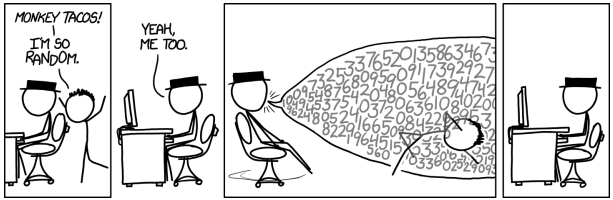
Desirable properties

- Non negative measure of uncertainty
- "combining uncertainties" should map to adding entropies

Definition

- Let $\mathcal{X}$ be a discrete r.v. with n outcomes in $\{x_0, \ldots, x_{n-1}\}$ with $\Pr(\mathcal{X} = x_i) = p_i$ for all $0 \leqslant i \leqslant n$
- The entropy of $\mathcal{X}$ is $H(\mathcal{X}) = \sum_{i=0}^{n-1} -p_i \log_b(p_i)$
- The measurement unit of entropy depends on the base b of the logarithm: typical case for $b = 2$ is bits

Examples



https://xkcd.com/1210/

By analyzing password leaks we get a distribution of the password used by people, thus we can understand how difficult is for an attacker to guess a password by running the entropy formula

$\mathcal{X}$: Uniformly random 6 letters word

- $\mathcal{X}$ is a sequence of 6 unif. random letters ($6^{26} \approx 3.1 \cdot 10^7$)
  - $H(\mathcal{X}) \approx \sum_{i=0}^{3.1 \cdot 10^7} -\frac{1}{3.1 \cdot 10^7} \log_b(\frac{1}{3.1 \cdot 10^7}) \approx 28.2b$
- $\mathcal{X}$ is a uniform pick from 6-letters English words
  - $H(\mathcal{X}) \approx \sum_{i=0}^{6300} -\frac{1}{6300} \log_b(\frac{1}{6300}) \approx 12.6b$

Shannon's noiseless coding theorem

Statement (informal)

It is possible to encode the outcomes n of i.i.d. random variables, each one with entropy $H(\mathcal{X})$, into no less than $nH(\mathcal{X})$ bits per outcome. If $< nH(\mathcal{X})$ bits are used, some information will be lost.

Consequences

- Arbitrarily compression of bitstrings is impossible without loss
  - Cryptographic hashes must discard some information
- Guessing a piece of information ($=$ one outcome of $\mathcal{X}$) is at least as hard as guessing a $H(\mathcal{X})$ bit long bitstring
  - overlooking for a moment the effort of decoding the guess

Min-Entropy

It's a different score of a random variable

A practical mismatch

- It is possible to have distributions with the same entropies

Plucking low-hanging fruits

- We define the min-entropy of $\mathcal{X}$ as $H_\infty(\mathcal{X}) = -\log(\max_i p_i)$
- Intuitively: it's the entropy of a r.v. with uniform distribution, where the probability of each outcome is $(\max_i p_i)$
- Guessing the most common outcome of $\mathcal{X}$ is at least as hard as guessing a $H_\infty(\mathcal{X})$ bit long bitstring

Example

A very biased r.v.

$$\text{Consider } \mathcal{X}: \begin{cases} \mathcal{X} = 0^{128} & \text{with } \Pr \frac{1}{2} \\ \mathcal{X} = a, a \in 1\{0,1\}^{127} & \text{with } \Pr \frac{1}{2^{128}} \end{cases}$$

half of the times I have zero

the other half I have a string uniformly distributed

Intuition and quantification

- Predicting an outcome shouldn't be too hard: just say 0^{128}
- $H(\mathcal{X}) = \frac{1}{2}(-\log_2(\frac{1}{2})) + 2^{127}\frac{1}{2^{128}}(-\log_2(\frac{1}{2^{128}})) = 64.5\text{b}$ Information content I get is 65 bit
- $H_\infty(\mathcal{X}) = -\log_2(\frac{1}{2}) = 1\text{b}$ one single string
- Min-entropy tells us that guessing the most common output is as hard as guessing a single bit string

For an uniform distribution the min-entropy equals the entropy

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