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Calculus of Complex Zonotopes

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Abstract

Abstract goes here....

Contents

List of Algorithms

Notation

Notation

- $\mathbb{N}$ the set of *natural* numbers
- $\mathbb{Q}$ the set of *rational* numbers
- $\mathbb{R}$ the set of *real* numbers
- $\mathbb{B}$ the set $\{0, 1\}$
- $\ldots$

Abbreviations

- BDT : Binary Decision Tree
- DNF : Disjunctive Normal Form
- PBNF : Pseudo-Boolean Normal Form

CHAPTER 1

Introduction

Intro....some citation [Ang87]

CHAPTER 2

Preliminaries

CHAPTER 3

Complex Zonotopes

In the previous chapter, we described the set representation called simple zonotopes, a sub-class of polytopes, which are geometrically Minkowski sums of line segments. Simple zonotopes are represented as a linear combination of real-valued vectors, where the combining co-efficients are bounded inside real-valued intervals. The advantage of simple zonotopes in reachability analysis of linear systems is that they are closed under matrix multiplication and Minkowski sum operations and these can be efficiently. However, for linear hybrid systems, the possibly complex-valued eigenstructure of matrices, i.e., having both real and imaginary parts, can be very useful in computing invariants and stability verification, as we shall discuss later. In this respect, this chapter introduces a new set representation called *complex zonotope*, which extends simple zonotopes by having complex-valued generators and complex-valued combining coefficients, and allows specification of invariants based on the complex eigenstructure. The real projections of complex zonotopes describe a richer class of sets than simple zonotopes, which are Minkowski sums of some ellipsoids along with line segments. Still, like simple zonotopes, complex zonotopes are also closed under matrix multiplications and Minkowski sums and these can be computed efficiently.

This chapter is organized as follows. PUT ORGANIZATION.

3.1 Motivation

When the eigenstructure of a linear system is real, it can be used to specify a real zonotopic positive invariant for the system. This is based on the following result.

Proposition 3.1. *Let us consider a simple zonotope centered at the origin $\mathcal{Z}\left(V,0\right)$, where the column vectors of $V \in \mathbb{M}_{m\times n}\left(\mathbb{R}\right)$ are the eigenvectors of a matrix $A \in \mathbb{M}_{n\times n}\left(\mathbb{R}\right)$ such that $AV = V\mathcal{D}\left(()\,\mu\right)$ where $\mu \in \mathbb{R}^m$ is a vector of eigenvalues* *corresponding to the eigenvectors among the columns of V. Then*

$$A\left(\mathcal{Z}\left(V,0\right)\right)=\mathcal{Z}\left(V\mathcal{D}\left(\mu\right),0\right).$$

If $\|\mu\|_{\infty}\leq 1$, then $A\left(\mathcal{Z}\left(V,0\right)\right)\subseteq\mathcal{Z}\left(V,0\right)$.

Proof. We have $A\left(\mathcal{Z}\left(V,0\right)\right)=\mathcal{Z}\left(AV,0\right)$ according to Proposition **??**. Next $\mathcal{Z}\left(AV,0\right)=\mathcal{Z}\left(V\mathcal{D}\left(\mu\right),0\right)$. This proves the first part of the Proposition. For the second part, we are given that $\|\mu\|_{\infty}\leq 1$. Let $y\in A\left(\mathcal{Z}\left(V,0\right)\right)=\mathcal{Z}\left(V\mathcal{D}\left(\mu\right),0\right)$. So, we can write y as $y=V\mathcal{D}\left(\mu\right)\delta : \|\delta\|_{\infty}\leq 1$. Let $\epsilon=\mathcal{D}\left(\mu\right)\delta$. Then $\|\epsilon\|_{\infty}\leq\|\mu\|_{\infty}\|\delta\|_{\infty}\leq 1$. So, $y=V\epsilon$ where $\|\epsilon\|_{\infty}\leq 1$, and hence $y\in\mathcal{Z}\left(V,0\right)$. Therefore, $A\left(\mathcal{Z}\left(V,0\right)\right)\subseteq\mathcal{Z}\left(V,0\right)$. □

The above proposition means that we can easily specify a positively invariant simple zonotope for a stable discrete time linear system with real eigenstructure by having only the (real) eigenvectors of the linear system as the generators of the zonotope.

[INCLUDE FIGURE]

CHAPTER 4

Chapter Title

CHAPTER 5

Conclusion

5.1 Summary

5.2 Future work

APPENDIX A

Appendix

Appendix goes here

A.1 Title

some citation [Ang87]

Bibliography

[Ang87] Dana Angluin. Learning regular sets from queries and counterexamples. *Information and Computation*, 75(2):87–106, 1987. (*Cited on pages 1 and 11*.)