ECON 6511: Econometrics
Midterm Exam
Points: 60, Time: 210 minutes

Instructions

- Use 3 or more decimal places unless otherwise stated
- No notes or cellphones

Section A: Multiple-choice (15 points)

1. The model: $y_t = \beta_0 + \beta_1 c_t + u_t$, $t = 1, 2, \ldots, n$, is an example of a(n):
   (a) autoregressive conditional heteroskedasticity model.
   (b) static model.
   (c) finite distributed lag model.
   (d) infinite distributed lag model.

2. Which of the following is an assumption on which time series regression is based?
   (a) A time series process follows a model that is nonlinear in parameters.
   (b) In a time series process, no independent variable is a perfect linear combination of the others.
   (c) In a time series process, at least one independent variable is a constant.
   (d) For each time period, the expected value of the error u_t, given the explanatory variables for all time periods, is positive.

3. If an explanatory variable is strictly exogenous it implies that:
   (a) changes in the lag of the variable does not affect future values of the dependent variable.
   (b) the variable is correlated with the error term in all future time periods.
   (c) the variable cannot react to what has happened to the dependent variable in the past.
   (d) the conditional mean of the error term given the variable is zero.

4. A study which observes whether a particular occurrence (e.g. a new policy) influences some outcome is referred to as a(n):
   (a) event study.
   (b) exponential study.

(c) laboratory study.

(d) comparative study.

5. A covariance stationary time series is weakly dependent if:

   (a) the correlation between the independent variable at time t and the dependent variable at time $t + h$ goes to ∞ as $h \to 0$.

   (b) the correlation between the independent variable at time t and the dependent variable at time $t + h$ goes to 0 as $h \to \infty$.

   (c) the correlation between the independent variable at time t and the independent variable at time $t + h$ goes to ∞ as $h \to 0$.

   (d) the correlation between the independent variable at time t and the independent variable at time $t + h$ goes to 0 as $h \to \infty$.

6. The model $y_t = e_t + \beta_1 e_{t-1} + \beta_2 e_{t-2}$, $t = 1, 2, \ldots$, where e_t is an i.i.d. sequence with zero mean and variance σ_e^2 represents a(n):

   (a) static model.

   (b) moving average process of order one.

   (c) moving average process of order two.

   (d) autoregressive process of order two.

7. The model $y_t = y_{t-1} + e_t$, $t = 1, 2, \ldots$ represents a:

   (a) AR(2) process.

   (b) MA(1) process.

   (c) random walk process.

   (d) random walk with a drift process.

8. If a process is said to be integrated of order one, or I(1), then:

   (a) it is covariance stationary

   (b) averages of such processes already satisfy the standard limit theorems

   (c) the first difference of the process is weakly dependent

   (d) it does not have a unit root

9. In the presence of serial correlation:

   (a) estimated standard errors remain valid.

(b) estimated test statistics remain valid.

(c) estimated OLS values are not BLUE.

(d) estimated variance does not differ from the case of no serial correlation.

10. Which of the following identifies an advantage of first differencing a time-series?

    (a) First differencing eliminates most of the serial correlation.

    (b) First differencing eliminates most of the heteroskedastcicty.

    (c) First differencing eliminates most of the multicollinearity.

    (d) First differencing eliminates the possibility of spurious regression.

11. A regression model exhibits unobserved heterogeneity if

    (a) there are unobserved factors affecting the dependent variable that change over time

    (b) there are unobserved factors affecting the dependent variable that do not change over time

    (c) there are unobserved factors which are correlated with the observed independent variables

    (d) there are no unobserved factors affecting the dependent variable

12. Ordinary least squares estimation is subject to (heterogeneity) bias if

    (a) the regression model exhibits heteroskedasticty

    (b) the unobserved effect is correlated with the observed explanatory variables

    (c) the regression model includes a lagged dependent variable

    (d) the explanatory variables do not change over time

13. What should be the degrees of freedom (*df*) for fixed effects estimation if the data set includes N cross sectional units over T time periods and the regression model has k independent variables?

    (a) $N - kT$

    (b) $NT - k$

    (c) $NT - N - k$

    (d) $N - T - k$

14. The general approach to obtaining fully robust standard errors and test statistics in the context of panel data is known as

(a) confounding

(b) differencing

(c) clustering

(d) attenuating

15. Which of the following assumptions is needed for the usual standard errors to be valid when differencing with more than two time periods?

    (a) The regression model exhibits heteroskedasticty.

    (b) The differenced idiosyncratic error or Δu_{it} is uncorrelated over time.

    (c) The unobserved factors affecting the dependent variable are time-constant.

    (d) The regression model includes a lagged independent variable.

Section B: Written Answer (45 points)

1. **(20 points)** Consider the following model that relates wages to productivity:

$$\log(hrwage_t) = \beta_0 + \beta_1 \log(outphr_t) + u_t,$$

where the variable *hrwage* is average hourly wage in the U.S. economy, and the variable *outphr* is output per hour. The data are annual data between 1947 and 1987 with $n = 41$. The output from this regression and many other specifications are included below.

(a) Interpret (with one perfect sentence) what the estimate of β_1 in the above model means.

(b) **Formally** test the hypothesis that $\beta_1 = 1$ against a two-sided alternative, i.e. test whether productivity growth is fully passed onto workers in the form of higher wages.

(c) Included in the attached output are regressions of both $\log(hrwage)$ and $\log(output)$ regressed on a simple linear time trend, t. What do these regressions suggest about the likely bias in the estimate of β_1 above? Explain.

(d) Consider a second specification that includes a time trend:

$$\log(hrwage_t) = \beta_0 + \beta_1 \log(outphr_t) + \beta_2 t + u_t$$

Interpret the coefficients on $\log(outphr_t)$ and t. Are they both statistically significant?

(e) Explain how you would linearly de-trend the variable $\log(hrwage)$, i.e. remove its time trend?

(f) After removing time-trends, suppose the first order autocorrelations of $\log(hrwage)$ and $\log(outphr)$ are $\hat{\rho} = 0.967$ and $\hat{\rho} = 0.945$ respectively. What do these findings imply? What is an appropriate remedy?

(g) Consider a model in first differences:

$$\Delta\log(hrwage_t) = \beta_0 + \beta_1 \Delta\log(outphr_t) + \Delta u_t$$

where $ghrwage = \Delta\log(hrwage_t)$ and $goutphr = \Delta\log(outphr_t)$ in the Stata output. Interpret the coefficient on $\Delta\log(outphr_t)$. **Formally** test the hypothesis that $\beta_1 = 1$ against a two-sided alternative.

(h) Consider a final model:

$$ghrwage_t = \beta_0 + \beta_1 goutphr_t + \beta_2 goutphr_{t-1} + u_t$$

that includes one lag of output growth. Is the lagged value statistically significant?

(i) Test $\beta_1 + \beta_2 = 1$ against a two-sided alternative.

$$ghrwage_t = \beta_0 + \theta goutphr_t + \beta_2(goutphr_{t-1} - goutphr_t) + u_t$$

(j) Does $goutphr_{t-2}$ need to be in the model? Explain.

```
. reg lhrwage loutphr

      Source |       SS       df       MS              Number of obs =      41
-------------+------------------------------           F(  1,    39) =  311.95
       Model |  .956010689     1  .956010689           Prob > F      =  0.0000
    Residual |  .119521723    39  .00306466            R-squared     =  0.8889
-------------+------------------------------           Adj R-squared =  0.8860
       Total |  1.07553241    40  .02688831            Root MSE      =  .05536

------------------------------------------------------------------------------
     lhrwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     loutphr |   .6891012    .039016    17.66   0.000     .6101839    .7680185
       _cons |  -1.534398   .1716633    -8.94   0.000     -1.88162   -1.187176
------------------------------------------------------------------------------
```

```
. reg loutphr t

      Source |       SS       df       MS              Number of obs =      41
-------------+------------------------------           F(  1,    39) =  800.97
       Model |  1.91976939     1  1.91976939           Prob > F      =  0.0000
    Residual |   .09347595    39  .002396819           R-squared     =  0.9536
-------------+------------------------------           Adj R-squared =  0.9524
       Total |  2.01324534    40  .050331133           Root MSE      =  .04896

------------------------------------------------------------------------------
     loutphr |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           t |   .0182881   .0006462    28.30   0.000     .0169811    .0195951
       _cons |   4.010183   .0155758   257.46   0.000     3.978678    4.041688
------------------------------------------------------------------------------
```

```
. reg lhrwage t

      Source |       SS       df       MS              Number of obs =      41
-------------+------------------------------           F(  1,    39) =  109.61
       Model |   .79327836     1   .79327836           Prob > F      =  0.0000
    Residual |  .282254052    39  .007237283           R-squared     =  0.7376
-------------+------------------------------           Adj R-squared =  0.7308
       Total |  1.07553241    40   .02688831           Root MSE      =  .08507

------------------------------------------------------------------------------
     lhrwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           t |   .0117559   .0011229    10.47   0.000     .0094847    .0140272
       _cons |   1.246799   .0270657    46.07   0.000     1.192053    1.301544
------------------------------------------------------------------------------
```

```
. reg lhrwage loutphr t

      Source |       SS       df       MS              Number of obs =      41
-------------+------------------------------           F(  2,    38) =  641.22
       Model |  1.04458064     2  .522290318           Prob > F      =  0.0000
    Residual |  .030951776    38   .00081452           R-squared     =  0.9712
-------------+------------------------------           Adj R-squared =  0.9697
       Total |  1.07553241    40   .02688831           Root MSE      =  .02854

------------------------------------------------------------------------------
     lhrwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     loutphr |   1.639639   .0933471    17.56   0.000     1.450668    1.828611
           t |    -.01823   .0017482   -10.43   0.000     -.021769   -.0146909
       _cons |  -5.328454   .3744492   -14.23   0.000    -6.086487   -4.570421
------------------------------------------------------------------------------
```

```
. reg ghrwage goutphr

      Source |       SS       df       MS              Number of obs =      40
-------------+------------------------------           F(  1,    38) =   21.77
       Model |  .006255013     1  .006255013           Prob > F      =  0.0000
    Residual |   .01091799    38  .000287316           R-squared     =  0.3642
-------------+------------------------------           Adj R-squared =  0.3475
       Total |  .017173003    39  .000440333           Root MSE      =  .01695

------------------------------------------------------------------------------
     ghrwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     goutphr |    .809316   .1734537     4.67   0.000     .4581773    1.160455
       _cons |  -.0036621     .00422    -0.87   0.391    -.0122051    .0048808
------------------------------------------------------------------------------
```

```
. reg ghrwage goutphr goutph_1

      Source |       SS       df       MS              Number of obs =      39
-------------+------------------------------           F(  2,    36) =   17.51
       Model |  .008456778     2  .004228389           Prob > F      =  0.0000
    Residual |  .008692555    36   .00024146           R-squared     =  0.4931
-------------+------------------------------           Adj R-squared =  0.4650
       Total |  .017149333    38  .000451298           Root MSE      =  .01554

------------------------------------------------------------------------------
     ghrwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     goutphr |   .7283641   .1672223     4.36   0.000     .3892217    1.067507
    goutph_1 |   .4576351   .1656126     2.76   0.009     .1217571     .793513
       _cons |   -.010425   .0045439    -2.29   0.028    -.0196404   -.0012096
------------------------------------------------------------------------------
```

```
. gen new_var = goutph_1 - goutphr
(2 missing values generated)

. reg ghrwage goutphr new_var

      Source |       SS       df       MS              Number of obs =      39
-------------+------------------------------           F(  2,    36) =   17.51
       Model |  .008456778     2  .004228389           Prob > F      =  0.0000
    Residual |  .008692555    36   .00024146           R-squared     =  0.4931
-------------+------------------------------           Adj R-squared =  0.4650
       Total |  .017149333    38  .000451298           Root MSE      =  .01554

------------------------------------------------------------------------------
     ghrwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     goutphr |   1.185999   .2031416     5.84   0.000     .7740089     1.59799
     new_var |   .4576351   .1656126     2.76   0.009     .1217571     .793513
       _cons |   -.010425   .0045439    -2.29   0.028    -.0196404   -.0012096
------------------------------------------------------------------------------
```

```
. reg ghrwage goutphr goutph_1 goutph_2

      Source |       SS       df       MS              Number of obs =      38
-------------+------------------------------           F(  3,    34) =   12.04
       Model |  .008048615     3  .002682872           Prob > F      =  0.0000
    Residual |  .007575626    34  .000222813           R-squared     =  0.5151
-------------+------------------------------           Adj R-squared =  0.4724
       Total |  .015624241    37  .000422277           Root MSE      =  .01493

------------------------------------------------------------------------------
     ghrwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     goutphr |   .7464273   .1615037     4.62   0.000     .4182123    1.074642
    goutph_1 |   .3740461   .1665312     2.25   0.031     .0356139    .7124783
    goutph_2 |   .0653486   .1597067     0.41   0.685    -.2592144    .3899116
       _cons |  -.0112838   .0048409    -2.33   0.026    -.0211217   -.0014458
------------------------------------------------------------------------------
```

2. **(15 points)** Consider the following model that relates traffic deaths per 100 million miles to two policies that attempt to reduce drunk driving. The two laws are *open container laws* – which make it illegal for travel with an open alcoholic drink and *administrative per se laws* – which enable courts to suspend driver licenses immediately after an arrest. Consider a cross-sectional model relating the death rate (*dthrte*) in a state to dummy variables indicating whether the state has any of these laws (*open* and *admin*):

$$dthrte_i = \beta_0 + \beta_1 open_i + \beta_2 admin_i + u_i.$$

This equation is estimated separately for 1985 where *open*85 and *admin*85 are the dummy variables for 1985. Results are provided below.

(a) How do you interpret the coefficients on *open*85 and *admn*85? Do the signs of the coefficients make sense? Explain.

(b) Are the estimates likely to be unbiased? Explain.

(c) Consider an alternative unobserved effects model using panel data with $T = 2$ periods:

$$dthrte_{it} = \beta_0 + \beta_1 d2_t + \beta_2 open_{it} + \beta_3 admin_{it} + a_i + u_{it}.$$

where $d2_t$ is a dummy variable equal to 1 in the second period. List two factors that might be included in a_i.

(d) Explain how we can obtain unbiased or at least consistent estimates of β_1 and β_2 assuming we have data from states before and after these laws were passed.

(e) Write down the Stata code that you would use to generate the change in *dthrte* between two periods assuming the data was organized so that you had one observation per state and period.

(f) Results from the following model:

$$\Delta dthrte_i = \beta_0 + \beta_1 \Delta open_i + \beta_2 \Delta admin_i + \Delta u_i.$$

using two years of data (1985 and 1990) are included below. In 1985, 19 states had open container laws, while 22 states had such laws in 1990. In 1985, 21 states had per se laws; the number had grown to 29 by 1990. Interpret the estimated coefficient on *copen*, which is equivalent to $\Delta open_i$ in the above model. Interpret the estimate of β_0.

(g) Does the sign on *cadmin* make sense? Is it statistically significant?

(h) What do we need to assume for the estimates in part (f) to be "correct?"

```
. reg dthrte85 open85 admn85

      Source |       SS       df       MS              Number of obs =      51
-------------+------------------------------           F(  2,    48) =    1.66
       Model |  1.17595166     2  .58797583           Prob > F      =  0.2013
    Residual |  17.0240489    48  .354667685           R-squared     =  0.0646
-------------+------------------------------           Adj R-squared =  0.0256
       Total |  18.2000005    50  .36400001           Root MSE      =  .59554

------------------------------------------------------------------------------
    dthrte85 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      open85 |  -.2131043   .1725004    -1.24   0.223    -.5599395    .1337309
      admn85 |    .229711   .1694617     1.36   0.182    -.1110146    .5704366
       _cons |   2.684805   .1257889    21.34   0.000     2.431889     2.93772
------------------------------------------------------------------------------
```

```
. reg cdthrte copen cadmn

      Source |       SS       df       MS              Number of obs =      51
-------------+------------------------------           F(  2,    48) =    3.23
       Model |  .762579785     2  .381289893           Prob > F      =  0.0482
    Residual |  5.66369475    48  .117993641           R-squared     =  0.1187
-------------+------------------------------           Adj R-squared =  0.0819
       Total |  6.42627453    50  .128525491           Root MSE      =   .3435

------------------------------------------------------------------------------
     cdthrte |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       copen |  -.4196787   .2055948    -2.04   0.047    -.8330547   -.0063028
       cadmn |  -.1506024   .1168223    -1.29   0.204    -.3854894    .0842846
       _cons |  -.4967872   .0524256    -9.48   0.000    -.6021959   -.3913784
------------------------------------------------------------------------------
```

3. **(15 points)** Consider the following model of rental prices in one of 64 college towns for the years 1980 and 1990. The idea is to see whether a stronger presence of students affects rental rates. The unobserved effects model is:

$$\log(rent_{it}) = \beta_0 + \delta_0 y90_t + \beta_1 \log(pop_{it}) + \beta_2 \log(avginc_{it}) + \beta_3 pctstu_{it} + a_i + u_{it},$$

where *pop* is city population, *avginc* is average income, and *pctstu* is student population as a percentage (0 to 100) of city population (during the school year). Results by pooled OLS, random effects, and fixed effects are given below.

(a) Using pooled OLS, what is the interpretation of the 1990 dummy variable? What does the estimate of β_3 imply?

(b) Are the standard errors for the pooled OLS estimates likely to be valid? Explain.

(c) What does the estimate of β_3 using fixed effects imply? Why is it so different to the pooled OLS estimate?

(d) List one factor that might be contained in a_i and explain how it might be correlated with *pctstu*.

(e) What does the estimate of β_3 under random effects imply about the likely magnitude of $\hat{\theta}$ (the quasi demeaning parameter)?

(f) Which model do you prefer and why?

(g) Suppose you estimated the model using OLS but included dummy variables for each city as opposed to time demeaning or quasi demeaning the data. What would you expect to get for your estimate of β_3?

	Pooled OLS	Fixed Effects	Random Effects
log(*pop*)	0.041	0.072	0.056
	(0.023)	(0.088)	(0.029)
log(*avginc*)	0.571**	0.310**	0.447**
	(0.053)	(0.066)	(0.052)
pctstu	0.005**	0.011**	0.005**
	(0.001)	(0.004)	(0.001)
*y*90	0.262**	0.386**	0.323**
	(0.035)	(0.037)	(0.029)
Constant	-0.569	1.409	0.445
	(0.535)	(1.167)	(0.566)
Observations	128	128	128
R^2	0.861	0.977	

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$

Formulae

- For simple regression model: $\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$
- Estimated slope parameter when regression equation passes through origin: $\frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$
- $SSR = \sum_{i=1}^{n} \hat{u}_i^2$
- $SSE = \sum_{i=1}^{n}(\hat{y}_i - \overline{y})^2$
- $SST_j = \sum_{i=1}^{n}(x_{ij} - \overline{x}_j)^2$
- $R^2 = \frac{SSE}{SST} = 1 - \frac{SSR}{SST}$
- For simple regression model: $se(\hat{\beta}_1) = \frac{\hat{\sigma}}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}}$
- $\widehat{Var}(\hat{\beta}_j) = \frac{\hat{\sigma}^2}{SST_j(1 - R_j^2)}$
- t statistic: $t_{\hat{\beta}_j} \equiv \frac{\hat{\beta}_j - a_j}{se(\hat{\beta}_j)} \sim t_{n-k-1} = t_{df}$
- 95% confidence interval: $P\Big(\hat{\beta}_j - c_{0.05} \cdot se(\hat{\beta}_j) \leq \beta_j \leq \hat{\beta}_j + c_{0.05} \cdot se(\hat{\beta}_j)\Big) = 0.95$
- F statistic $= \frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-k-1)}$
- R^2 form of F statistic $= \frac{(R_{ur}^2 - R_r^2)/q}{(1 - R_{ur}^2)/(n-k-1)}$
- $\overline{R}^2 = 1 - \frac{SSR/(n-k-1)}{SST/(n-1)}$
- Chow test statistic: $F = \frac{[SSR_P - (SSR_1 + SSR_2)]/(k+1)}{(SSR_1 + SSR_2)/[n - 2(k+1)]}$

Cumulative Areas under the Standard Normal Distribution

z	0	1	2	3	4	5	6	7	8	9
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148

(*continued*)

z	0	1	2	3	4	5	6	7	8	9
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Examples: If Z ~ Normal(0,1), then $P(Z \leq -1.32) = .0934$ and $P(Z \leq 1.84) = .9671$.
Source: This table was generated using the Stata® function normprob.

Critical Values of the t Distribution

	Significance Level				
1-Tailed:	**.10**	**.05**	**.025**	**.01**	**.005**
2-Tailed:	**.20**	**.10**	**.05**	**.02**	**.01**
Degrees of Freedom					
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
90	1.291	1.662	1.987	2.368	2.632
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

Examples: The 1% critical value for a one-tailed test with 25 *df* is 2.485. The 5% critical value for a two-tailed test with large (> 120) *df* is 1.96.
Source: This table was generated using the Stata® function invttail.

F Values for $\alpha = 0.05$

d_2 \ d_1	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.3	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
inf	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88