Problem Set 6&CA7 Solution

Please make your own determination on partial credit.

1. Problem 11 of Chapter 12, 13th edition, page 524

Question a = 8 points; Question b = 2 points

11. a. H_0: Type of ticket purchased is independent of the type of flight

H_a: Type of ticket purchased is not independent of the type of flight

Expected Frequencies:

e_{11} = 35.59 e_{12} = 15.41
e_{21} = 150.73 e_{22} = 65.27
e_{31} = 455.68 e_{32} = 197.32

Ticket	Flight	Observed Frequency (f_i)	Expected Frequency (e_i)	Chi–square $(f_i - e_i)^2 / e_i$
First	Domestic	29	35.59	1.22
First	International	22	15.41	2.82
Business	Domestic	95	150.73	20.61
Business	International	121	65.27	47.59
Full Fare	Domestic	518	455.68	8.52
Full Fare	International	135	197.32	19.68
	Totals:	920		$\chi^2 = 100.43$

Degrees of freedom = $(r-1)(c-1) = (3-1)(2-1) = 2$

Using the χ^2 table with $df = 2$, $\chi^2 = 100.43$ shows the p–value is less than .005.

Using Excel or Minitab, the p–value corresponding to $\chi^2 = 100.43$ is .0000.

p–value $\leq$ .05, reject H_0. Conclude that the type of ticket purchased is not independent of the type of flight. We can expect the type of ticket purchased to depend upon whether the flight is domestic or international.

b. Column Percentages

Type of Ticket	Type of Flight: Domestic	International
First Class	4.5%	7.9%
Business Class	14.8%	43.5%
Economy Class	80.7%	48.6%

A higher percentage of first class and business class tickets are purchased for international flights compared to domestic flights. Economy class tickets are purchased more for domestic flights. The first class or business class tickets are purchased for more than 50% of the international flights; 7.9% + 43.5% = 51.4%.

2. Problem 23 of Chapter 12, 13th edition, page 535

10 points for this problem

23. Expected frequencies: 20% each $n = 60$

$e_1 = 12, e_2 = 12, e_3 = 12, e_4 = 12, e_5 = 12$

Observed frequencies: $f_1 = 5, f_2 = 8, f_3 = 15, f_4 = 20, f_5 = 12$

$$\chi^2 = \frac{(5-12)^2}{12} + \frac{(8-12)^2}{12} + \frac{(15-12)^2}{12} + \frac{(20-12)^2}{12} + \frac{(12-12)^2}{12}$$
$$= 11.50$$

$k - 1 = 4$ degrees of freedom

Using the χ^2 table with $df = 4$, $\chi^2 = 11.50$ shows the p–value is between .01 and .025.

Using Excel or Minitab, the p–value corresponding to $\chi^2 = 11.50$ is .0215.

p–value $\leq .05$; reject H_0. Conclude the largest companies differ in performance from the 1000 companies. In general, the largest companies did not do as well as others. 15 of 60 companies (25%) are in the middle group and 20 of 60 companies (33%) are in the next lower group. These both are greater than the 20% expected. Relative few large companies are in the top A and B categories.

3. Problem 9 of Chapter 13, 13th edition, page 561
10 points for this problem

9.

	50°	60°	70°
Sample Mean	33	29	28
Sample Variance	32	17.5	9.5

$\bar{\bar{x}} = (33 + 29 + 28)/3 = 30$

$\text{SSTR} = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 = 5(33 - 30)^2 + 5(29 - 30)^2 + 5(28 - 30)^2 = 70$

$\text{MSTR} = \text{SSTR} /(k - 1) = 70 /2 = 35$

$\text{SSE} = \sum_{j=1}^{k} (n_j - 1)s_j^2 = 4(32) + 4(17.5) + 4(9.5) = 236$

$\text{MSE} = \text{SSE} /(n_T - k) = 236 /(15 - 3) = 19.67$

$F = \text{MSTR} /\text{MSE} = 35 /19.67 = 1.78$

Using F table (2 degrees of freedom numerator and 12 denominator), p-value is greater than .10

Using Excel or Minitab the p-value corresponding to $F = 1.78$ is .2104.

Because p-value $> \alpha = .05$, we cannot reject the null hypothesis that the mean yields for the three temperatures are equal.

4. Problem 31 of Chapter 13, 13th edition, page 581

10 points for this problem

31. Factor A is method of loading and unloading; Factor B is type of ride.

		Factor B Roller Coaster	Screaming Demon	Log Flume	Factor A Means
Factor A	Method 1	$\bar{x}_{11} = 42$	$\bar{x}_{12} = 48$	$\bar{x}_{13} = 48$	$\bar{x}_{1.} = 46$
	Method 2	$\bar{x}_{21} = 50$	$\bar{x}_{22} = 48$	$\bar{x}_{23} = 46$	$\bar{x}_{2.} = 48$
Factor B	Means	$\bar{x}_{.1} = 46$	$\bar{x}_{.2} = 48$	$\bar{x}_{.3} = 47$	$\bar{\bar{x}} = 47$

Step 1

$$\text{SST} = \sum_i \sum_j \sum_k \left(x_{ijk} - \bar{\bar{x}}\right)^2 = (41-47)^2 + (43-47)^2 + \cdots + (44-47)^2 = 136$$

Step 2

$$\text{SSA} = br\sum_i \left(\bar{x}_{i.} - \bar{\bar{x}}\right)^2 = 3\,(2)\,[\,(46-47)^2 + (48-47)^2\,] = 12$$

Step 3

$$\text{SSB} = ar\sum_j \left(\bar{x}_{.j} - \bar{\bar{x}}\right)^2 = 2\,(2)\,[\,(46-47)^2 + (48-47)^2 + (47-47)^2\,] = 8$$

Step 4

$$\text{SSAB} = r\sum_i \sum_j \left(\bar{x}_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{\bar{x}}\right)^2 = 2\,[\,(41-46-46+47)^2 + \cdots + (44-48-47+47)^2\,] = 56$$

Step 5

SSE = SST – SSA – SSB – SSAB = 136 – 12 – 8 – 56 = 60

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Factor A	12	1	12	12/10 = 1.2	.3153
Factor B	8	2	4	4/10 = .4	.6870
Interaction	56	2	28	28/10 = 2.8	.1384
Error	60	6	10		
Total	136	11			

Using F table for Factor A (1 degree of freedom numerator and 6 denominator), p-value is greater than .10

Using Excel or Minitab, the p-value corresponding to $F = 1.2$ is .3153.

Because p-value $> \alpha = .05$, Factor A is not significant.

Using F table for Factor B (2 degrees of freedom numerator and 6 denominator), p-value is greater than .10

Using Excel or Minitab, the p-value corresponding to $F = .4$ is .6870.

Because p-value $> \alpha = .05$, Factor B is not significant.

Using F table for Interaction (2 degrees of freedom numerator and 6 denominator), p-value is greater than .10

Using Excel or Minitab, the p-value corresponding to $F = 2.8$ is .1384.

Because p-value $> \alpha = .05$, Interaction is not significant.