Economics 6400: Econometrics

Lecture 9: Final exam revision

CSU, East Bay

November 28, 2017

Question 1

1. The following table contains the sales prices and assessed values for three houses (both in thousands of dollars):

i	*price* (y_i)	*assess* (x_i)
1	300	320
2	200	190
3	260	250

2. Show that the relationship between *price* and *assess* is:

$$\widehat{price} = 59.8425 + 0.7638 \; assess$$

i	y_i	x_i	$y_i - \overline{x}$	$x_i - \overline{x}$	$(y_i - \overline{y})(x_i - \overline{x})$	$(x_i - \overline{x})^2$
1	300	320				
2	200	190				
3	260	250				

Question 1

3. Suppose you are interested in testing whether the assessed housing price is a rational valuation. Test whether the slope parameter is equal to 1 at the 5% level against a two-sided alternative.
4. What is the adjusted R^2?
5. Graph/plot the sample regression function you derived in part (a). Be sure to label any intercept(s).
6. Suppose you added the variable, square footage (*sqrft*) to model. The model becomes:

   $$price = \beta_0 + \beta_1 assess + \beta_2 sqrft + u.$$

   Derive an expression that relates the coefficient on *assess* from the first model, $\tilde{\beta}_1$, to the coefficient on *assess* from this new model, $\hat{\beta}_1$. Show your workings. What is necessary for $\tilde{\beta}_1$ to be unbiased estimate of β_1?

Question 2

- The model to be estimated is:

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 exper + u.$$

where

  - *wage* is hourly wage
  - *educ* is education in years
  - *exper* is experience in years

1. **Interpret** the coefficient on *educ*. Be specific.
2. **Formally** test the null hypothesis that the coefficient on *educ* is equal to 0 against a two-sided alternative at the 10% level.
3. Suppose you wish to test whether the returns to an extra year of education and an extra year of experience are the same against a two-sided alternative. Using the Stata output, what is the estimate of $\theta = \beta_1 - \beta_2$? State the null and two-sided alternative hypotheses in terms of θ.

Question 2

4. Derive an equation that you can use to estimate θ and its standard error.
5. Using the Stata output, can you reject the null hypothesis from part (c) at the 5% level? Explain.

Now consider an alternative model in which we add $exper^2$:

$$\log(wage) = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + u.$$

6. Conduct a test for the overall significance of the regression.
7. **Formally** test the null hypothesis that $\log(wage)$ is linearly related to *exper* against the alternative that the relationship is quadratic at the 10% level.

Question 2

8. Using the Stata output, at what level does additional experience actually lower predicted log(*wage*)? Indicate when this would be a concern.
9. What is your prediction of log(*wage*) for someone with 12 years of education and 17 years of experience? Compute this by hand and indicate that it is (approximately) equal to the appropriate figure in the attached Stata output. What is the 90% confidence for this prediction? Use the second model for this.

Question 2 Stata output

	(1)	(2)	(3)	(4)	(5)
	log(wage)	log(wage)	log(wage)	log(wage)	log(wage)
educ	0.0979**	0.0876**		0.0904**	
	(0.0076)	(0.0073)		(0.0075)	
exper	0.0103**		-0.0876**	0.0410**	
	(0.0016)		(0.0073)	(0.0052)	
$educ + exper$		0.0103**	0.0979**		
		(0.0016)	(0.0076)		
$exper^2$				-0.0007**	
				(0.0001)	
$educ - 12$					0.0904**
					(0.0075)
$exper - 17$					0.0410**
					(0.0052)
$exper^2 - 289$					-0.0007**
					(0.0001)
Constant	0.2169*	0.2169*	0.2169*	0.1280	1.7033**
	(0.1086)	(0.1086)	(0.1086)	(0.1059)	(0.0296)
Observations	526	526	526	526	526
R^2	0.2493	0.2493	0.2493	0.3003	0.3003

Standard errors in parentheses.

Question 3

- The model to be estimated is:

$$wage = \beta_0 + \beta_1 female + \beta_2 exper + u$$

where

  - *wage* is hourly wage in dollars
  - *female* $= 1$ if the person is female, and 0 if the person is male
  - *exper* $=$ years of experience

1. Using the Stata output, interpret the coefficient on *female*, $\widehat{\beta}_1$. Be specific.
2. What percentage of the variation in *wage* is explained by variation in experience and gender? Compute the adjusted R^2 for this regression (it has been blanked out from the regression output). Why is it so close to the regular (unadjusted) R^2?

Question 3

Now consider adding the interaction *female* $\cdot$ *exper* to the model:

$$wage = \beta_0 + \beta_1 female + \beta_2 exper + \beta_3 female \cdot exper + u$$

3. Interpret the new coefficient on *female*, $\widehat{\beta}_1$. Be specific.
4. Using the Stata output, graph the estimated returns to experience for females and males (i.e. two separate lines).
5. What is the return to one extra year of experience for men? What is the return to one extra year of experience for women? How do these answers relate to your graph in part (d)?
6. Formally test the hypothesis that the return to experience is the same for men and women at the 5% level.
7. Test for equality of the parameters of the above equation for men and women using a Chow test at the 5% level.

Question 3

9. Suppose you replaced *female* $\cdot$ *exper* with *female* $\cdot$ (*exper* $- 10$). What would your interpretation of $\hat{\beta}_1$ be now?
10. Explain two approaches for conducting a test for the presence of heteroskedasticity in the first model?

Question 3 output

```
. reg wage female exper

      Source |       SS           df       MS      Number of obs   =       526
-------------+----------------------------------   F(2, 523)       =     37.51
       Model |  898.161983         2  449.080991   Prob > F        =    0.0000
    Residual |  6262.25231       523  11.9737138   R-squared       =    0.1254
-------------+----------------------------------   Adj R-squared   =    0.1221
       Total |  7160.41429       525  13.6388844   Root MSE        =    3.4603

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   -2.48142   .3022793    -8.21   0.000     -3.07525   -1.887589
       exper |   .0269163   .0111369     2.42   0.016     .0050379    .0487948
       _cons |   6.626882   .2862475    23.15   0.000     6.064546    7.189218
------------------------------------------------------------------------------
```

Question 3 output

```
. gen female_x_exper=female*exper

. reg wage female exper female_x_exper

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(  3,   522) =   27.31
       Model |  971.286312     3  323.762104           Prob > F      =  0.0000
    Residual |  6189.12798   522   11.856567           R-squared     =  0.1356
-------------+------------------------------           Adj R-squared =  0.1307
       Total |  7160.41429   525  13.6388844           Root MSE      =  3.4433

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -1.546547   .4818603    -3.21   0.001     -2.49317   -.5999231
       exper |   .0536048   .0154372     3.47   0.001     .0232782    .0839314
female_x_exper | -.0550699    .022175   -2.48   0.013     -.098633   -.0115068
       _cons |   6.158275   .3416741    18.02   0.000      5.48705    6.829501
------------------------------------------------------------------------------
```

Question 3 output

```
. reg wage exper if female==0

      Source |       SS       df       MS              Number of obs =     274
-------------+------------------------------           F(  1,   272) =    8.48
       Model |  142.965414     1  142.965414           Prob > F      =  0.0039
    Residual |  4583.4113    272  16.8507769           R-squared     =  0.0302
-------------+------------------------------           Adj R-squared =  0.0267
       Total |  4726.37672   273  17.3127352           Root MSE      =   4.105

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       exper |   .0536048   .0184034     2.91   0.004     .0173736     .089836
       _cons |   6.158275   .4073264    15.12   0.000     5.356362    6.960189
------------------------------------------------------------------------------
```

Question 3 output

```
. reg wage exper if female==1

      Source |       SS       df       MS              Number of obs =     252
-------------+------------------------------           F(  1,   250) =    0.02
       Model |  .100430445     1  .100430445           Prob > F      =  0.9006
    Residual |  1605.71668   250   6.4228667           R-squared     =  0.0001
-------------+------------------------------           Adj R-squared = -0.0039
       Total |  1605.81711   251  6.39767771           Root MSE      =  2.5343

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       exper |  -.0014651   .0117168    -0.13   0.901    -.0245413     .021611
       _cons |   4.611729   .2500797    18.44   0.000     4.119197     5.10426
------------------------------------------------------------------------------
```

Question 3 output

```
. reg wage exper

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(  1,   524) =    6.77
       Model |  91.2751351     1  91.2751351           Prob > F      =  0.0096
    Residual |  7069.13916   524  13.4907236           R-squared     =  0.0127
-------------+------------------------------           Adj R-squared =  0.0109
       Total |  7160.41429   525  13.6388844           Root MSE      =   3.673

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       exper |   .0307219   .0118111     2.60   0.010      .007519    .0539247
       _cons |   5.373305   .2569919    20.91   0.000     4.868444    5.878166
------------------------------------------------------------------------------
```

t table

Critical Values of the t Distribution

		Significance Level				
	1-Tailed:	.10	.05	.025	.01	.005
	2-Tailed:	.20	.10	.05	.02	.01
Degrees of Fre	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.499
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.350	1.771	2.160	2.650	3.012
	14	1.345	1.761	2.145	2.624	2.977
	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845

Next week

- Final exam. . . good luck!