Problem Set: Probability Distribution

1. Consider a random experiment in which we roll two fair, six-sided dices. Let random variable X be the sum of the two numbers resulted from a roll. Let Y be the absolute value of the difference between the two numbers. Answer the following questions. Simply provide your answers and there is no need to show your work in THIS problem.
   1. **Use a joint probability table to represent the joint probability distribution of X and Y**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| fXY(x,y) | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 | Total |
| x = 2 | 1/36 | 0/36 | 0/36 | 0/36 | 0/36 | 0/36 | 1/36 |
| x = 3 | 0/36 | 2/36 | 0/36 | 0/36 | 0/36 | 0/36 | 2/36 |
| x = 4 | 1/36 | 0/36 | 2/36 | 0/36 | 0/36 | 0/36 | 3/36 |
| x = 5 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 0/36 | 4/36 |
| x = 6 | 1/36 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 5/36 |
| x = 7 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 2/36 | 6/36 |
| x = 8 | 1/36 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 5/36 |
| x = 9 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 0/36 | 4/36 |
| x = 10 | 1/36 | 0/36 | 2/36 | 0/36 | 0/36 | 0/36 | 3/36 |
| x = 11 | 0/36 | 2/36 | 0/36 | 0/36 | 0/36 | 0/36 | 2/36 |
| x = 12 | 1/36 | 0/36 | 0/36 | 0/36 | 0/36 | 0/36 | 1/36 |
| Total | 6/36 | 10/36 | 8//36 | 6/36 | 4/36 | 2/36 | 1 |

* 1. **What is the pmf of Y, fY(y)?**

Consider values of y and their probabilities:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Y | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| fY(y) | 6/36 | 10/36 | 8/36 | 6/36 | 4/36 | 2/36 |

∑fY(y) = P(Y=y) = 6/36 + 10/36 + 8/36 + 6/36 + 4/36 + 2/36

= 36/36

= 1

* 1. **E(Y) = μ =∑ y. fY(y)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Y | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| fY(y) | 6/36 | 10/36 | 8/36 | 6/36 | 4/36 | 2/36 |
| Y \* fY(y) | 0 | 10/36 | 16/36 | 18/36 | 16/36 | 10/36 |

E(Y) = ∑ y. fY(y) = 0 + 10/36 + 16/36 + 18/36 + 16/36 + 10/36

= 70/36

= 1.9444

* 1. **Var(Y) = ∂2 = ∑ fY(y) \* (y - μ)2**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Y** | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| **fY(y)** | 6/36 | 10/36 | 8/36 | 6/36 | 4/36 | 2/36 |
| **Y - μ** | -64/36 | -60/36 | -62/36 | -64/36 | -66/36 | -68/36 |
| **(y - μ)2** | 3.160 | 2.777 | 2.966 | 3.160 | 3.361 | 3.567 |
| **fY(y) \* (y - μ)2** | 0.526 | 0.771 | 0.659 | 0.526 | 0.373 | 0.198 |

Var(Y) = ∂2 = ∑ fY(y) \* (y - μ)2 = 0.526 + 0.771 + 0.659 + 0.526 + 0.373 + 0.198

= 3.053

* 1. **Covariance (X, Y) =**

E(Y) = ∑ y. fY(y) = 0 + 10/36 + 16/36 + 18/36 + 16/36 + 10/36

= 70/36

= 1.9444

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Y** | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| **Y - μ** | -64/36 | -60/36 | -62/36 | -64/36 | -66/36 | -68/36 |

E(X) = μ =∑ x. fx(x)

= (2/36) + (6/36) + (12/36) + (20/36) + (30/36) + (42/36) + (40/36) + (36/36) +

(30/36) + (22/36) + (12/36)

= 252/36

= 7

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **X** | x = 2 | x = 3 | x = 4 | x = 5 | x = 6 | x = 7 |
| **f(x)** | 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 |
| **X - μ** | -250/36 | -249/36 | -248/36 | -247/36 | -246/36 | -245/36 |
| **X** | x = 8 | x = 9 | X = 10 | x = 11 | x = 12 |  |
| **f(x)** | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 |  |
| **X - μ** | -244/36 | -243/36 | -242/36 | -241/36 | -240/36 |  |

Taking non-zero values of f(xi, yi) :

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| fXY(x,y) | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| x = 2 | 1/36 |  |  |  |  |  |
| x = 3 |  | 2/36 |  |  |  |  |
| x = 4 | 1/36 |  | 2/36 |  |  |  |
| x = 5 |  | 2/36 |  | 2/36 |  |  |
| x = 6 | 1/36 |  | 2/36 |  | 2/36 |  |
| x = 7 |  | 2/36 |  | 2/36 |  | 2/36 |
| x = 8 | 1/36 |  | 2/36 |  | 2/36 |  |
| x = 9 |  | 2/36 |  | 2/36 |  |  |
| x = 10 | 1/36 |  | 2/36 |  |  |  |
| x = 11 |  | 2/36 |  |  |  |  |
| x = 12 | 1/36 |  |  |  |  |  |

**Covariance (X, Y) =** [-250/36\*-64/36\*1/36] + [-249/36\*-60/36\*2/36] + [-248/36\*-64/36\*1/36] + [-248/36\*-62/36\*2/36] + [-247/36\*-60/36\*2/36] + [-247/36\*-64/36\*2/36] + [-246/36\*-64/36\*1/36] + [-246/36\*-62/36\*2/36] + [-246/36\*-66/36\*2/36] + [-245/36\*-60/36\*2/36] + [-245/36\*-64/36\*2/36] + [-245/36\*-68/36\*2/36] + [-244/36\*-64/36\*1/36] + [-244/36\*-62/36\*2/36] + [-244/36\*-66/36\*2/36] + [-243/36\*-60/36\*2/36] + [-243/36\*-64/36\*2/36] + [-242/36\*-64/36\*1/36] + [-242/36\*-62/36\*2/36] + [-241/36\*-60/36\*2/36] + [-240/36\*-64/36\*1/36]

= [16000 + 29880 + 15872 + 30752 + 29640 + 31616 + 31488 + 30504 + 32472 + 29400 + 31360 + 33320 + 15616 + 30256 + 32208 + 29160 + 31104 + 15488 + 30008 + 28920 + 15360] / [46656]

= 570424 / 46656

= 12.2261

1. **Compute the coefficient of correlation between X and Y.**
2. **Are X and Y independent? Why or why not?**
3. **Based on your answers to questions f and g, what conclusion(s) can you draw?**