**Problem Set: Probability Distribution**

1. Consider a random experiment in which we roll two fair, six-sided dices. Let random variable X be the sum of the two numbers resulted from a roll. Let Y be the absolute value of the difference between the two numbers. Answer the following questions. Simply provide your answers and there is no need to show your work in THIS problem.
   1. **Use a joint probability table to represent the joint probability distribution of X and Y**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| fXY(x,y) | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 | Total fx(x) |
| x = 2 | 1/36 | 0/36 | 0/36 | 0/36 | 0/36 | 0/36 | 1/36 |
| x = 3 | 0/36 | 2/36 | 0/36 | 0/36 | 0/36 | 0/36 | 2/36 |
| x = 4 | 1/36 | 0/36 | 2/36 | 0/36 | 0/36 | 0/36 | 3/36 |
| x = 5 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 0/36 | 4/36 |
| x = 6 | 1/36 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 5/36 |
| x = 7 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 2/36 | 6/36 |
| x = 8 | 1/36 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 5/36 |
| x = 9 | 0/36 | 2/36 | 0/36 | 2/36 | 0/36 | 0/36 | 4/36 |
| x = 10 | 1/36 | 0/36 | 2/36 | 0/36 | 0/36 | 0/36 | 3/36 |
| x = 11 | 0/36 | 2/36 | 0/36 | 0/36 | 0/36 | 0/36 | 2/36 |
| x = 12 | 1/36 | 0/36 | 0/36 | 0/36 | 0/36 | 0/36 | 1/36 |
| Total fy(y) | 6/36 | 10/36 | 8//36 | 6/36 | 4/36 | 2/36 | 1 |

* 1. **What is the pmf of Y, fY(y)?**

Consider values of y and their probabilities:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Y | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| fY(y) | 6/36 | 10/36 | 8/36 | 6/36 | 4/36 | 2/36 |

∑fY(y) = P(Y=y) = 6/36 + 10/36 + 8/36 + 6/36 + 4/36 + 2/36

= 36/36

= 1

* 1. **E(Y) = μ =∑ y. fY(y)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Y | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| fY(y) | 6/36 | 10/36 | 8/36 | 6/36 | 4/36 | 2/36 |
| Y \* fY(y) | 0 | 10/36 | 16/36 | 18/36 | 16/36 | 10/36 |

E(Y) = ∑ y. fY(y) = 0 + 10/36 + 16/36 + 18/36 + 16/36 + 10/36

= 70/36

= 1.9444

* 1. **Var(Y) =** **2 = ∑ fY(y) \* (y - μ)2**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Y** | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| **fY(y)** | 6/36 | 10/36 | 8/36 | 6/36 | 4/36 | 2/36 |
| **Y - μ** | -1.944 | -0.944 | 0.056 | 1.056 | 2.056 | 3.056 |
| **(y - μ)2** | 3.779 | 0.891 | 0.003 | 1.115 | 4.227 | 9.339 |
| **fY(y) \* (y - μ)2** | 0.104 | 0.247 | 0.001 | 0.185 | 0.469 | 0.518 |

Var(Y) = 2 = ∑ fY(y) \* (y - μ)2 = 0.104 + 0.247 + 0.001 + 0.185 + 0.469 + 0.518

= 1.525

* 1. **Covariance (X, Y) =**

E(Y) = ∑ y. fY(y) = 0 + 10/36 + 16/36 + 18/36 + 16/36 + 10/36

= 70/36

= 1.9444

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Y** | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| **Y - μ** | -1.944 | -0.944 | 0.056 | 1.056 | 2.056 | 3.056 |

E(X) = μ =∑ x. fx(x)

= (2/36) + (6/36) + (12/36) + (20/36) + (30/36) + (42/36) + (40/36) + (36/36) +

(30/36) + (22/36) + (12/36)

= 252/36

= 7

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **X** | x = 2 | x = 3 | x = 4 | x = 5 | x = 6 | x = 7 |
| **f(x)** | 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 |
| **X - μ** | -5 | -4 | -3 | -2 | -1 | 0 |
| **X** | x = 8 | x = 9 | X = 10 | x = 11 | x = 12 |  |
| **f(x)** | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 |  |
| **X - μ** | 1 | 2 | 3 | 4 | 5 |  |

Taking non-zero values of f(xi, yi) :

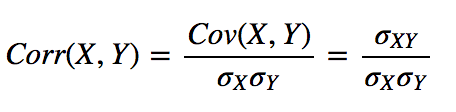
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| fXY(x,y) | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| x = 2 | 1/36 |  |  |  |  |  |
| x = 3 |  | 2/36 |  |  |  |  |
| x = 4 | 1/36 |  | 2/36 |  |  |  |
| x = 5 |  | 2/36 |  | 2/36 |  |  |
| x = 6 | 1/36 |  | 2/36 |  | 2/36 |  |
| x = 7 |  | 2/36 |  | 2/36 |  | 2/36 |
| x = 8 | 1/36 |  | 2/36 |  | 2/36 |  |
| x = 9 |  | 2/36 |  | 2/36 |  |  |
| x = 10 | 1/36 |  | 2/36 |  |  |  |
| x = 11 |  | 2/36 |  |  |  |  |
| x = 12 | 1/36 |  |  |  |  |  |

**Covariance (X, Y) =** [-5\*-1.944\*1/36] + [-4\*-0.944\*2/36] + [-3\*-1.944\*1/36] + [-3\*0.056\*2/36] + [-2\*-0.944\*2/36] + [-2\*1.056\*2/36] + [-1\*-1.944\*1/36] + [-1\*0.056\*2/36] + [-1\*2.056\*2/36] + [0\*0.944\*2/36] + [0\*1.056\*2/36] + [0\*3.056\*2/36] + [1\*-1.944\*1/36] + [1\*0.056\*2/36] + [1\*2.056\*2/36] + [2\*-0.944\*2/36] + [2\*1.056\*2/36] + [3\*-1.944\*1/36] + [3\*0.056\*2/36] + [4\*-0.944\*2/36] + [5\*-1.944\*1/36]

= [9.72 + 77.552 + 5.832 - 0.336 + 3.776 - 4.224 + 1.944 – 0.112 – 4.112 - 1.944 + 0.112 + 4.112 - 3.776 + 4.224 – 5.832 + 0.336 – 7.552 + 9.72] / 36

= 0/ 36 = 0

* 1. **Compute the coefficient of correlation between X and Y.**



|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Y** | y = 0 | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 |
| **fY(y)** | 6/36 | 10/36 | 8/36 | 6/36 | 4/36 | 2/36 |
| **Y - μ** | -1.944 | -0.944 | 0.056 | 1.056 | 2.056 | 3.056 |
| **(y - μ)2** | 3.779 | 0.891 | 0.003 | 1.115 | 4.227 | 9.339 |
| **fY(y) \* (y - μ)2** | 0.104 | 0.247 | 0.001 | 0.185 | 0.469 | 0.518 |

Var(Y) = 2 = ∑ fY(y) \* (y - μ)2 = 0.104 + 0.247 + 0.001 + 0.185 + 0.469 + 0.518

= 1.525

y = Standard deviation =  = square root (1.525) = 1.234

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **X** | x = 2 | x = 3 | x = 4 | x = 5 | x = 6 | x = 7 |
| **f(x)** | 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 |
| **X - μ** | -5 | -4 | -3 | -2 | -1 | 0 |
| **(x - μ)2** | 25 | 16 | 9 | 4 | 1 | 0 |
| **fx(x) \* (x - μ)2** | 25/36 | 32/36 | 27/36 | 16/36 | 5/36 | 0 |
| **X** | x = 8 | x = 9 | X = 10 | x = 11 | x = 12 |  |
| **fx(x)** | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 |  |
| **X - μ** | 1 | 2 | 3 | 4 | 5 |  |
| **(x - μ)2** | 1 | 4 | 9 | 16 | 25 |  |
| **fx(x) \* (x - μ)2** | 5/36 | 16/36 | 27/36 | 32/36 | 25/36 |  |

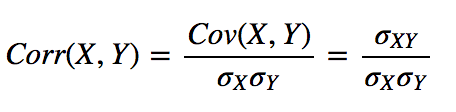
Var(X) = x2 = ∑ fx(x) \* (x - μ)2

= [25 + 32 + 27 + 16 + 5 + 0 + 5 + 16 + 27 + 32 + 25] / 36

= 210/36 = 5.833

x = Standard deviation =  = square root (46.311)

= 6.805



= [0] / [(1.525) \* (5.833)]

= 0

**Interpretation of correlation between X and Y:**

Evidently, ρXY or Correlation(X,Y) = 0, that means Xand Yare completely, un-linearly correlated.

* 1. **Are X and Y independent? Why or why not?**

X and Y are two independent discrete RVs iff



Is P (X = x2, Y = y0) == P (X = x2) \* P (Y = y0)

Is 1/36 == 1/36 \* 6/36

1/36 != 1/216

No! certainly the two sides are not equal. Therefore, X and Y are not independent because we just found out 1/36  1/216

1. **Based on your answers to questions f and g, what conclusion(s) can you draw?**

**Interpretation of correlation between X and Y:**

Evidently, ρXY= 0, that means Xand Yare completely, un-linearly correlated. That is, X and Y may be correlated in some other manner, in a parabolic manner, perhaps, but not in a linear manner. **So, X and Y are uncorrelated**. Also, the two random variables X and Y are not independent.