Estimation : Interesting questions and Back of the envelope calculations

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0.1 Avogadro Number: A Mole.

1. How long one has to roll a Aluminium foil to get 1 mole of Aluminium atoms?
   Answer :
   Step 1. Required information

   Atomic mass of Aluminium = 27. This means 1 mole of Aluminium(Al) atoms weigh 27 grams.
   Density of Al = 2.7 grams/cm^3
   Dimensions of household Al foil:
   Length = 25 meter(2500cm), Width = 30cm, and Thickness = 14 microns(0.0014cm).

   Step 2. Volume of Al foil for above dimensions.

   Volume = Width $\times$ Length $\times$ Thickness
   = (30 $\times$ 0.014 $\times$ 2500) cm^3
   = 105 cm^3

   Step 3. Mass of the Al foil = Volume $\times$ Density
   = 105 $\times$ 2.7
   = 283.5 grams

   Step 4. If 27grams $\rightarrow$ 1mole then 283.5grams $\rightarrow$? moles
   so, $\frac{283.5}{27}$ = 10.5 moles

   Step 5. If 283.5grams $\rightarrow$ 2500cm then 27grams $\rightarrow$? cm
   So, $\frac{(27\times2500)}{283.5}$ = 238.09 cm $\approx$ 2.4 metre

   So, to get 1 mole of Al atoms we have to roll 2.4 meters of Al foil of given dimensions.

2. How much each of 2 SGD, 10 USD, and Rs.50 bills weigh? How many atoms are there in each of the bills?(What fraction of a mole?).

Answer:

Step 1. Required information

1. SGD notes are made of Biaxially oriented polypropylene (PP)$(C_3H_6)_n$. Which is mainly Carbon and Hydrogen atoms.

2. USD and Indian Rupees are made of Paper which is 75% Cotton and 25% Linen. Cotton contains 91% cellulose $(C_6H_{10}O_5)_n$.

3. Dimensions of bills in (width $\times$ length $\times$ thickness)in cm
SGD 2 = (6.3 $\times$ 12.6 $\times$ 0.01)
USD 10 = (6.63 $\times$ 15.6 $\times$ 0.01)
INR 50 = (7.3 $\times$ 14.7 $\times$ 0.01)

4. Density of Polypropylene = 0.93 grams/cm^3.
Density of Cellulose = 1.5grams/cm^3.

5. Molecular weight of Polypropylene(C_3H_6)= 12$\times$3 + 1$\times$6 = 42 grams/mole. Which means 1 mole of PP weighs 42 grams.

Molecular weight of Cellulose $(C_6H_{10}O_5)$= 12$\times$6 + 1$\times$10 + 16$\times$5 = 162 grams/mole. Which means, 1 mole of Cellulose weighs 162 grams.

Step 2. 1. SGD 2

(1.1) Volume = $(6.3 \times 12.6 \times 0.01) \approx 0.8 cm^3$.

(1.2) Mass = Density × Volume
= 0.93 grams/cm^3 × $0.8 cm^3$
= 0.76 grams.

(1.3) If 42 grams of PP → 1 mole of PP molecules then 0.76 grams → ? molecules

So, $\frac{0.76 grams}{42 grams} \times (6 \times 10^{23}) \approx$ **1.1** $\times 10^{21}$ **of PP molecules in 1 SGD 2 note bill.** And to have 1 mole of PP molecules, we should have 55 notes (=SGD 110).

2.USD 10

(2.1) Volume = $(6.63 \times 15.6 \times 0.01) \approx 1 cm^3$.

(2.2) Mass = Density× Volume
= $1.5 grams/cm^3 \times 1 cm^3$
= 1.5 grams (But actual weight of the note is ≈ 1 gram, due to the reduced density of polymer chain after packaging.)

(2.3) If 162 grams of Cellulose → 1 mole of Cellulose molecules then 1 grams → ? molecules

So, $\frac{1 gram}{162 grams} \times 6 \times 10^{23} \approx 3.7 \times 10^{21}$ of Cellulose molecules in 1 USD 10 note bill and to have 1 mole of Cellulose molecules, we should have 162 notes (= USD 1620).

3.INR 50
(3.1) Volume = $(7.3 \times 14.7 \times 0.01) \approx 1.1 cm^3$.
(3.2) Mass = Density × Volume
= 1.5 grams/cm^3 × $1 cm^3$
= 1.7 grams (Similar to the USD, actual weight of the note is ≈ 1 gram due to the reduced density of polymer chain after packaging.)

(3.3) If 162 grams of Cellulose → 1 mole of Cellulose molecules then 1 grams → ? molecules

So, $\frac{1 gram}{162 grams} \times (6 \times 10^{23}) \approx$ **3.7** $\times 10^{21}$ **of Cellulose molecules in 1 INR 50 note bill.** And to have 1 mole of Cellulose molecules, we should have 162 notes (=INR 8100).

3. How many hair are there on the scalp of average adult human? How big a animal has to be to have 1 mole of hair on his body?

Answer:

Step 1: Required information.

1. For humans, average body surface area (BSA) = $\frac{\sqrt{(W \times H)}}{60}$.
Where, W = weight in Kg and H = height in centimetres.
For average adult human with weight = 70Kg and height = 180 cm,
BSA = $\frac{\sqrt{70 \times 180}}{60} \approx 2m^2$.

2. Scalp surface area $\approx 500cm^2$.
No. of hair follicles per$cm^2 \approx 100$. And on an average, each follicle will have 2 hair. So, there are $200hair/cm^2$.

Step 2: 1. If $1cm^2 \rightarrow$ 200 hair, then $500cm^2 \rightarrow$? hair.
So, $\frac{500cm^2 \times 200}{1cm^2}$ = 100,000 hair on human scalp.

2. Considering same density of hair (200 hair/cm^2) over all the body surface area, a human with $2m^2$ of body surface area will have
$\frac{200 \times 2 \times 10^4 cm^2}{1cm^2} = 4 \times 10^6$ hair.

3. Now, if 4 million hair per $2m^2$ of body surface area then 6×10^{23} hair $\rightarrow$? body surface area.
So, $\frac{6 \times 10^{23} \times 2m^2}{4 \times 10^6} \approx 3 \times 10^{19} m^2 = 3 \times 10^{12} km^2$.

4. The Sun is the celestial body with largest surface area in our solar system.
Radius of the Sun = 695,700 Km.$\approx$ 700,000 Km
Area of the Sun = $\pi \times r^2$
$= \pi \times (7 \times 10^5)^2$
$\approx 1.5 \times 10^{12} Km^2$.

So, an animal has to be as big as twice the size of the Sun to have 1 mole of hair.

4. How many sheets of A4 size paper collectively has 1 mole of carbon atoms?

Answer:

Step 1. Required information

1. From given data, A paper of area $1m^2$ ($=10^4 cm^2$) weighs 80 grams.
2. Size of A4 paper = 21cm $\times$ 30cm
3. Molecular weight of Carbon = 12 grams/mole. Area of A4 paper = 21cm $\times$ 30cm = 630 cm^2

Step 2. If $10^4 cm^2 \rightarrow$ 80grams then $630cm^2 \rightarrow$? grams.
So, $\frac{(630 \times 80)}{10^4}$ = 5.04 grams.
Now, If 5 grams $\rightarrow$ 1 A4 paper then 12 grams $\rightarrow$? papers . So, $\frac{12}{5}$ = 2.4 meters.

So, nearly 2 and a half sheets of A4 paper have 1 mole of carbon atoms.

5. How big is 1 cubic volume of 1 mole of Styrofoam?
Answer:
Step 1. Required information
Styrofoam is made of Polystyrene $(C_8H_8)_n$.
Polystyrene density = 0.96-1.04 gram/ cm^3.
Styrofoam weighs = 0.05 gram/cm^3
Molecular weight of Polystyrene(C_8H_8) = $12 \times 8 + 1 \times 8$
= 104 grams/mole.

Step 2. If 0.05 grams $\rightarrow 1cm^3$ then 104 grams $\rightarrow$? cm^3.

So, $\frac{104}{0.05} = 2080cm^3$.

1 mole of Styrofoam has $2080cm^3$ volume. Which is a cube of side 12.76 cm.

6. How big is a diamond of 1 mole of carbon?
Answer:
Step 1. Required information.
Molecular weight of Carbon = 12 grams/mole.
Mass of diamond $\approx$ 12 grams.
Density of diamond = 3.515 grams/cm^3

Step 2. 1. Volume of a diamond of mass 12 grams
$Volume = \frac{mass}{Density}$
$Volume = \frac{12}{3.5} = 3.4cm^3$

2. Volume of a sphere = $\frac{4}{3} \times \Pi \times r^3$
$\implies 3.4\ cm^3 = \frac{4}{3} \times \Pi \times r^3$
$\implies \frac{10.2}{12.56} = r^3$
$\implies 0.82cm^3 = r^3$
$\implies$ r = 0.9 cm

So, A diamond of 1 mole of Carbon will be as big as a sphere of diameter 1.8 cm.

7. How many moles of molecules are there in a dot of white board marker ink?
Answer:
Step 1. Required information.
A white board marker pen ink mainly contains Ethanol(C_2H_6O) or Isopropanol(C_3H_8O).
Density of Ethanol = 0.789 g/cm^3 $\approx$ 0.8 g/cm^3
Density of Isopropanol = 0.786 g/cm^3 $\approx$ 0.8g/cm^3
Considering marker pen tip = 5mm in diameter, a dot on whiteboard is a very tiny disc of thickness 0.1mm
Molecular weight of Ethanol (C_2H_6O) = $12\times2 + 1\times6 + 16\times1$
= $46grams/mole$.

Step 2. 1. Volume of a disk = $\Pi \times r^2 \times thickness$
$= (3.14 \times (0.25)^2 \times 0.001) cm^3$
$\approx 2 \times 10^{-4} cm^3$

2. Mass = Volume × Density
$= 2 \times 10^{-4} \times 0.8$
$= 1.6 \times 10^{-4}$grams

3. So, if 46 grams of Ethanol → 1 mole of Ethanol molecules then 1.6×10^{-4} grams → ? molecules of Ethanol.

So, $\frac{(1.6\times10^{-4})\times(6\times10^{23})}{46} \approx 2 \times 10^{18} molecules.$

So, a $5mm$ diameter dot of marker pen ink has 2×10^{18} molecules.

8. How many moles of electrons does a fully charged phone battery store?
Answer:
Step 1. Required information.

1. Phone battery current rating is in Ampere-Hour(AH). Today's smart phones generally have 3AH of current rating.Which means a fully charged 3AH battery can produce 3Amp current for 1 hour or 1 Amp current for 3 hours.

2. 1 Coulomb = 1 Ampere.Second

3. 1 electron has 1.6×10^{-19} coulombs of charge.
So, 1 Coulomb has $\frac{1}{1.6\times10^{-19}} = 6.25 \times 10^{18}$ electrons.

Step 2. If 1 Amp current → 6.25×10^{18} electrons, then 3 Amp → ? electrons.

So, $\frac{(3\times6.25\times10^{18})}{1} = 1.875 \times 10^{19}$ electrons.

So, a fully charged phone battery stores $\approx 1.9 \times 10^{19}$ electrons.

Step 3. Counting further, if 6.25×10^{18} electrons → $1 Amp$ current, then 6×10^{23}electrons → ? Amp current.

$\frac{6\times10^{23}}{6.25\times10^{18}} \approx 10^5$Amp Current.

So, 1 mole of electrons will produce $10^5 Amp$ currents.

9. Why we weigh lightest in the morning after the sleep? Where does this lost mass go?
Answer:
Step 1. Required information.
1. During sleep human inhales ≈ 500 ml of air per breathe. And take 16 breathes per minute. So, he inhales ≈ 7-8 liters of air per minute.
2. Average sleep duration is 8 hours.

Step 2. 1. Per breathe intake of air is 500 ml. Out of which $\frac{(}{1}5^{th})$ is Oxygen ≈ 100 ml. This inhaled Oxygen combines with carbon and becomes CO_2 which is then exhaled.This exhaled CO_2 is around $\frac{(}{1}5)^{th}$ of the inhaled Oxygen. Which is ≈ 20ml.

Step 3. 1.Exhaled CO_2 carries 0.012 grams of carbon in each breathe.

If per minute 16 breathes, then in 8 hours total 7680 breathes.
So, 0.012 grams× 7680 breathes ≈ 92 grams of mass is lost in the form of carbon during 8 hours of sleep.
Also, some mass is lost in the form of sweat. Hence we weigh lightest in the morning.

10. How many moles of ATP molecules does each cell in a human body synthesize? How many ATP molecules are required to run for 1 hour? And how many ATP molecules are required to run at a speed of 60Km/hr?
Answer:
Step 1: Required information.
Average ATPs required per day by humans ≈ 200-300 moles. Let's consider 240 moles of ATPs a day. Which is ≈ 10 moles of ATPs per hour.

Step 2: Total number of cells in human body ≈ 4×10^{12}

$\Rightarrow 4 \times 10^{12}$ cells synthesize 240 moles of ATPs/day.

If, 4×10^{12} cells $\Longrightarrow$ produce 240 moles of ATPs/day, then 1 cell $\Longrightarrow$? ATPs a day?

$\frac{240 \times 6 \times 10^{23}}{4 \times 10^{1}2} = 36 \times 10^{11}$ ATP molecules.

So, 36×10^{11} **ATP molecules by each cell per day.**

11. How many moles of water molecules does a human consume in a day? And in his life span?
Answer:
Step 1. Required information.

1. 1 mole of water molecules = 18 grams of water.
1 litre water = 1000 grams of water.
1 litre of water contains $\frac{1000}{18} \approx 55$ moles.

Step 2. On an average, daily water intake for humans 2 litres.
So, 2 litres $\times$ 55 moles = 110 moles/day.

Step 3. Number of days per year = 365.
Average life span of humans = 60 years.
So, for 60 *times* 365 = 21900 days, total water consumption is 21900 $\times$ 110 moles $\approx 2.4 \times 10^6$ moles of water. Which is 43362 litres of water.

So, A human consumes 110 moles of water a day and 2.4 million moles in his life span.

12. For a bacteria E. coli, how big a ball of 1 mole of E. coli will be?
Answer:
Step 1: Required information.
E.coli is a cylindrical shape bacteria with a radius of 0.5μm and a length of about 2μm.

Step 2: 1. Volume of single E. coli cell = $\pi \times r^2 \times length$
$= 3.14 \times 0.5^2 \times 2$
$\approx 1.6\mu\ m^3$

2. If 1 E.coli $\rightarrow 1.6\mu\ m^3$ of volume, then 1 mole of E.coli $\rightarrow$? volume.
So, $\frac{(1.6\times(10^{-6})^3\times 6\times 10^{23})}{1} \approx 9.6 \times 10^5 m^3$

3. Volume of a sphere = $\frac{4}{3} \times \pi \times r^3$
$9.6 \times 10^5 m^3 = \frac{4}{3} \times \pi \times r^3$
$\implies r^3 \approx 62$ metres.

1 mole of E.coli will make a ball of diameter 124m.

13. How big a packed ball of E. coli will be, so that there are 1 mole of DNA base pairs?
Answer:
Step 1. Required information.
E.coli genome consists of a single molecule of DNA containing ≈ 4.7 million base pairs.

Step 2. 1. If 4.7×10^6 base pairs $\rightarrow$ in 1 E.coli cell, then 6×10^{23} base pairs $\rightarrow$? E.coli cells.

$\frac{(6\times10^{23})}{(4.7\times10^6)} = 1.27 \times 10^{17}$ E.coli cells.
So, $\approx 1.3 \times 10^{17}$ E.coli cells will have 1 mole of DNA base pairs.

2. Now, if 1 E.coli cell $\rightarrow 1.6\mu\ m^3$ of volume, then 1.3×10^{17} E.coli cells $\rightarrow$?volume.

So, $\frac{1.3\times10^{17}\times1.6\times10^{-18}}{1} \approx 0.2\ m^3$.

3. Volume of a sphere $= \frac{4}{3} \times\pi \times r^3$
$0.2\ m^3 = \frac{4}{3} \times\pi \times r^3$
$\implies$ r ≈ 0.37 m $= 37$cm.

So, a packed ball of E.coli cells containing 1 mole DNA base pairs will be of size 74cm in diameter.

14. How big a ball of E. coli will be to have 1 mole of water?
Answer.
Step 1. Required information
1. E.coli bacteria contains 70% water.
2. 1 mole of E.coli has $9.6 \times 10^5 m^3$ of volume. And 70% of $9.6 \times 10^5 m^3 \approx 6.7 \times 10^5 m^3$.

Step 2. 1.Density of water $= 1000\text{Kg}/m^3$.
If $1m^3 \rightarrow 1000$Kg then $6.7 \times 10^5 m^3 \rightarrow$? Kg.
So, $\frac{6.7\times10^5}{1000} = 6.7 \times 10^8$Kg $= 6.7 \times 10^{11}$grams of water.

2. Now, if 6.7×10^{11}grams $\rightarrow 6.7 \times 10^5 m^3$ volume, then 18 grams $\rightarrow$? volume.

So, $\frac{18\times6.7\times10^5}{6.7\times10^{11}} = 18 \times 10^{-6} m^3 = 18cm^3$.

3. Volume of sphere $= \frac{4}{3}\ \pi \times r^3$
$18cm^3 = \frac{4}{3}\ \pi \times r^3$
$\implies$ r $= 1.65$ cm

So, a ball of E.coli of diameter 3cm will hold 1 mole of water.