

Multilayer neural networks

Can represent non-linear surface

No algorithm for learning in multi-layered networks, and no convergence theorem!

Sigmoid activation function

$$0 = \sigma(w \cdot x), \text{ where } \sigma(z) = (1 / 1 + e^{-z})$$

Forward propagation

Assume two-layer networks (i.e., one hidden layer):

I. For each test example:

1. Present input to the input layer.
2. Forward propagate the activations times the weights to each node in the hidden layer.
3. Forward propagate the activations times weights from the hidden layer to the output layer.
4. Interpret the output layer as a classification.

Momentum

The idea is to keep weight changes moving in the same direction

$$\Delta w^t = (\text{learning rate}) (\delta \text{ error}) (\text{input}) + (\text{Momentum}) (\Delta w^{t-1})$$

SVM

Instances are represented by vector $x \in \mathbb{R}^n$

input = training examples

output = alpha's, support vectors, bias

Margin

Distance from separating hyperplane to nearest positive (or negative) instance.

Length of margin $(1 / ||w||)$

Support Vectors

Elements of the training set that would change the position of the hyperplane if removed.

Kernel function

3. Consider the following three points, x_1 , x_2 , and x_3 , which have been identified as support vectors for a training set. Here, y_i is the class of the point, and α_i is the support vector coefficient.

$$x_1 = (2, 1) \quad y_1 = -1 \quad \alpha_1 = -4$$

$$x_2 = (4, 3) \quad y_2 = -1 \quad \alpha_2 = -4$$

$$x_3 = (2, 3) \quad y_3 = +1 \quad \alpha_3 = 8$$

The bias is $b = 0$.

(a) Using the formula

$$h(x) = \text{sgn} \left(\sum_{i=1}^n \alpha_i (x_i \cdot x) \right) + b$$

give the classification of the new instance $x = (1, 2)$. Show your work.

$$\begin{aligned} h(1, 2) &= \text{sgn} [-4(2, 1) \cdot (1, 2) - 4(4, 3) \cdot (1, 2) + 8(2, 3) \cdot (1, 2)] \\ &= \text{sgn} [-4 \cdot 4 - 4 \cdot 10 + 8 \cdot 8] \\ &= \text{sgn} [-56 + 64] = \text{sgn} [8] = +1 \end{aligned}$$

(b) Use the x_i 's and α_i 's to find the weight vector w associated with the separating hyperplane, where $w = \sum_i \alpha_i x_i$.

$$\begin{aligned} w &= -4(2, 1) - 4(4, 3) + 8(2, 3) \\ &= (-8, -4) + (-16, -12) + (16, 24) \\ &= (-8, 8) \end{aligned}$$

(c) Using the weight vector you obtained in part (b) and the bias $b = 0$, find the equation of the separating hyperplane. Give the equation in the slope-intercept form: $x_2 = (\text{slope} \cdot x_1) + y\text{-intercept}$.

$$-8x_1 + 8x_2 = 0$$

$$x_2 = x_1$$