

Inverse of a Matrix using Minors, Cofactors and Adjugate

We can calculate the Inverse of a Matrix by:

- Step 1: calculating the Matrix of Minors,
- Step 2: then turn that into the Matrix of Cofactors,
- Step 3: then the Adjugate, and
- Step 4: multiply that by 1/Determinant.

But it is best explained by working through an example!

Example: find the Inverse of A :

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

It needs 4 steps. It is all simple arithmetic but there is a lot of it, so try not to make a mistake!

Step 1: Matrix of Minors

The first step is to create a "Matrix of Minors". This step has the most calculations:

For each element of the matrix:

- ignore the values on the current row and column
- calculate the determinant of the remaining values

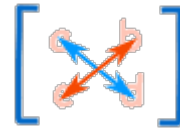
Put those determinants into a matrix (the "Matrix of Minors")

Determinant

For a 2×2 matrix (2 rows and 2 columns) the determinant is easy:
ad-bc

Think of a cross:

- Blue means positive (+ad),
- Red means negative (-bc)



(It gets harder for a 3×3 matrix, etc)

The Calculations

Here are the first two, and last two, calculations of the "**Matrix of Minors**" (notice how I ignore the values in the current row and columns, and calculate the determinant using the remaining values):

$$\begin{array}{c}
 \begin{bmatrix} \bullet & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad 0 \times 1 - (-2) \times 1 = 2 \\
 \begin{bmatrix} 3 & \bullet & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad 2 \times 1 - (-2) \times 0 = 2 \\
 \dots \\
 \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & \bullet & 1 \end{bmatrix} \quad 3 \times -2 - 2 \times 2 = -10 \\
 \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & \bullet \end{bmatrix} \quad 3 \times 0 - 0 \times 2 = 0
 \end{array}$$

And here is the calculation for the whole matrix:

$$\begin{bmatrix} 0 \times 1 - (-2) \times 1 & 2 \times 1 - (-2) \times 0 & 2 \times 1 - 0 \times 0 \\ 0 \times 1 - 2 \times 1 & 3 \times 1 - 2 \times 0 & 3 \times 1 - 0 \times 0 \\ 0 \times (-2) - 2 \times 0 & 3 \times (-2) - 2 \times 2 & 3 \times 0 - 0 \times 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ -2 & 3 & 3 \\ 0 & -10 & 0 \end{bmatrix}$$

Matrix of Minors

Step 2: Matrix of Cofactors

This is easy! Just apply a "checkerboard" of minuses to the "Matrix of Minors". In other words, we need to change the sign of alternate cells, like this:

$$\begin{array}{ccc}
 \begin{bmatrix} 2 & 2 & 2 \\ -2 & 3 & 3 \\ 0 & -10 & 0 \end{bmatrix} & \rightsquigarrow & \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \\
 \text{Matrix of Minors} & & \text{Matrix of CoFactors}
 \end{array}$$

Step 3: Adjugate (also called Adjoint)

Now "Transpose" all elements of the previous matrix... in other words swap their positions over the diagonal (the diagonal stays the same):

$$\begin{bmatrix} 2 & -2 & 0 \\ 2 & 3 & -3 \\ 0 & 10 & 0 \end{bmatrix}$$

Step 4: Multiply by 1/Determinant

Now find the determinant of the original matrix. This isn't too hard, because we already calculated the determinants of the smaller parts when we did "Matrix of Minors".

$$\begin{bmatrix} a \times \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \times \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \times \begin{vmatrix} d & e \\ g & h \end{vmatrix} \end{bmatrix}$$

So: multiply the top row elements by their matching "minor" determinants:

$$\text{Determinant} = 3 \times 2 - 0 \times 2 + 2 \times 2 = \mathbf{10}$$

And now multiply the Adjugate by 1/Determinant:

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.2 & 0 \\ -0.2 & 0.3 & 1 \\ 0.2 & -0.3 & 0 \end{bmatrix}$$

Adjugate Inverse

And we are done!

Compare this answer with the one we got on [Inverse of a Matrix using Elementary Row Operations](#). Is it the same? Which method do you prefer?

Larger Matrices

It is exactly the same steps for larger matrices (such as a 4×4, 5×5, etc), but wow! there is a lot of calculation involved.

For a 4×4 Matrix we have to calculate 16 3×3 determinants. So it is often easier to use computers (such as the [Matrix Calculator](#).)

Conclusion

- For each element, calculate the **determinant of the values not on the row or column**, to make the Matrix of Minors
- Apply a **checkerboard** of minuses to make the Matrix of Cofactors
- **Transpose** to make the Adjugate
- Multiply by **1/Determinant** to make the Inverse

[Question 1](#) [Question 2](#) [Question 3](#) [Question 4](#)

[Question 5](#) [Question 6](#) [Question 7](#) [Question 8](#)
[Question 9](#) [Question 10](#)

[Search](#) :: [Index](#) :: [About](#) :: [Contact](#) :: [Contribute](#) :: [Cite This Page](#) :: [Privacy](#)

Copyright © 2014 MathsIsFun.com