

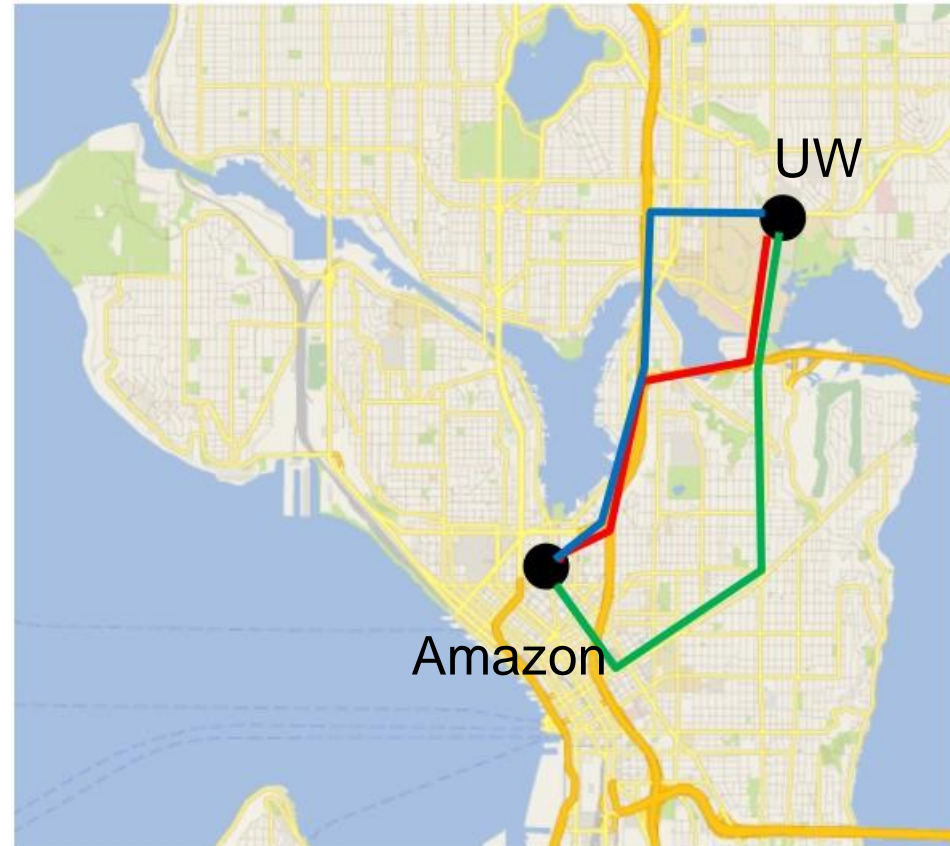
Introduction to Algorithms

Dijkstra's Algorithm

Single Source Shortest Path

Given an (un)directed graph $G=(V,E)$ with **non-negative** edge weights $c_e \geq 0$ and a start vertex s

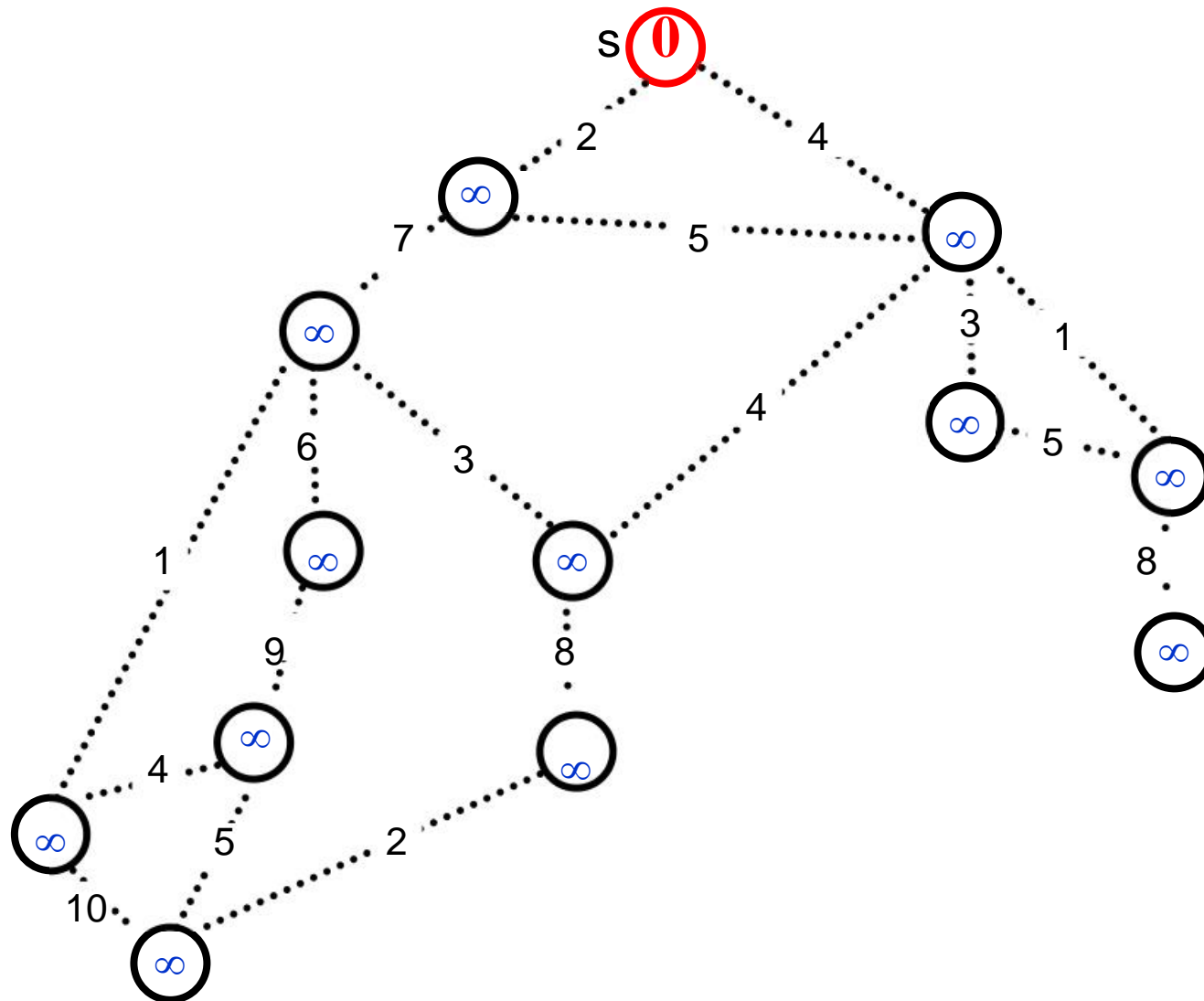
Find length of shortest paths from s to each vertex in G



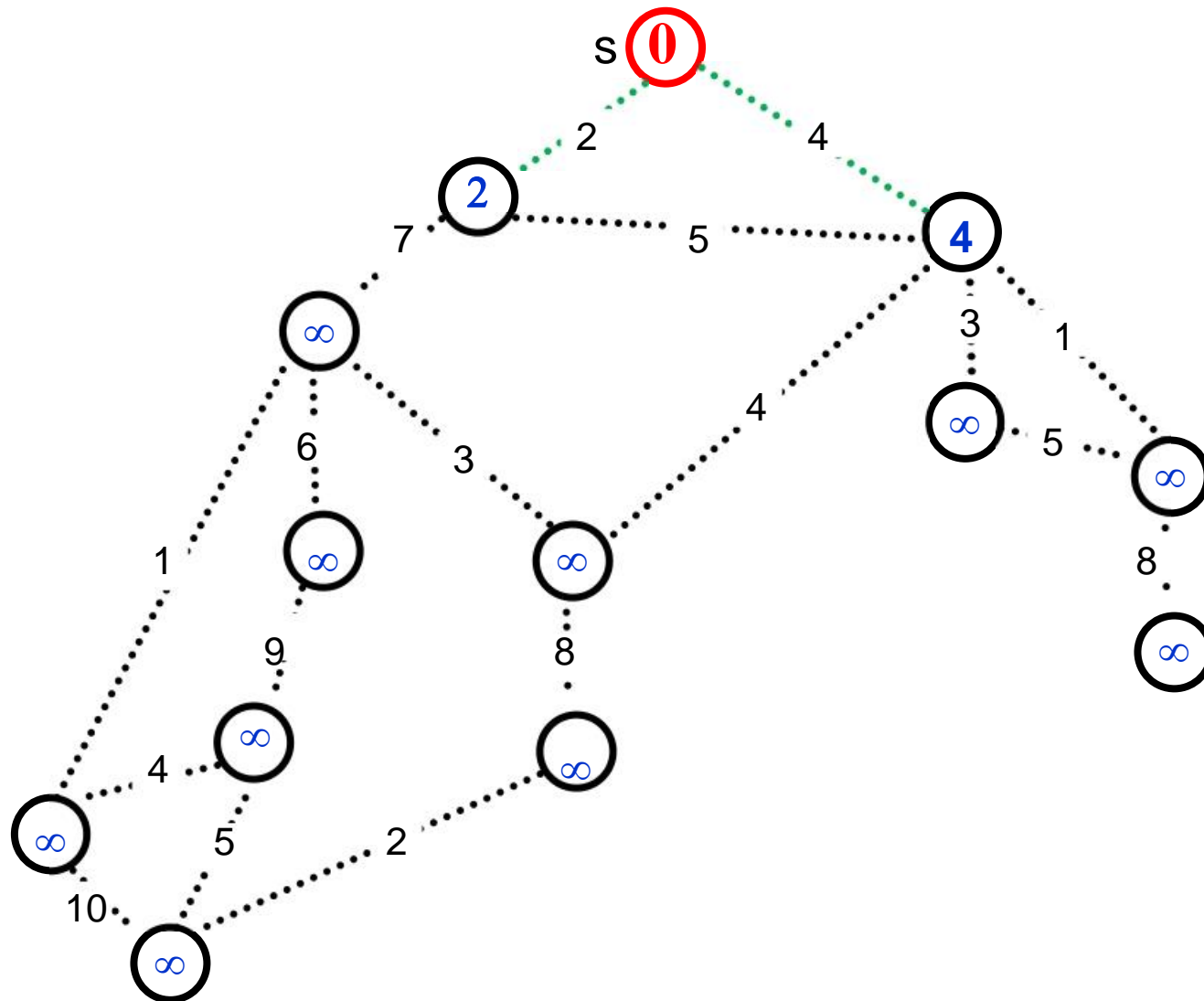
Dijkstra's Algorithm

```
1  function Dijkstra(Graph, source):
2
3      create vertex set Q
4
5      for each vertex v in Graph:
6          dist[v] ← INFINITY
7          prev[v] ← UNDEFINED
8          add v to Q
9
10     dist[source] ← 0
11
12     while Q is not empty:
13         u ← vertex in Q with min dist[u]
14
15         remove u from Q
16
17         for each neighbor v of u:           // only v that are still in Q
18             alt ← dist[u] + length(u, v)
19             if alt < dist[v]:
20                 dist[v] ← alt
21                 prev[v] ← u
22
23     return dist[], prev[]
```

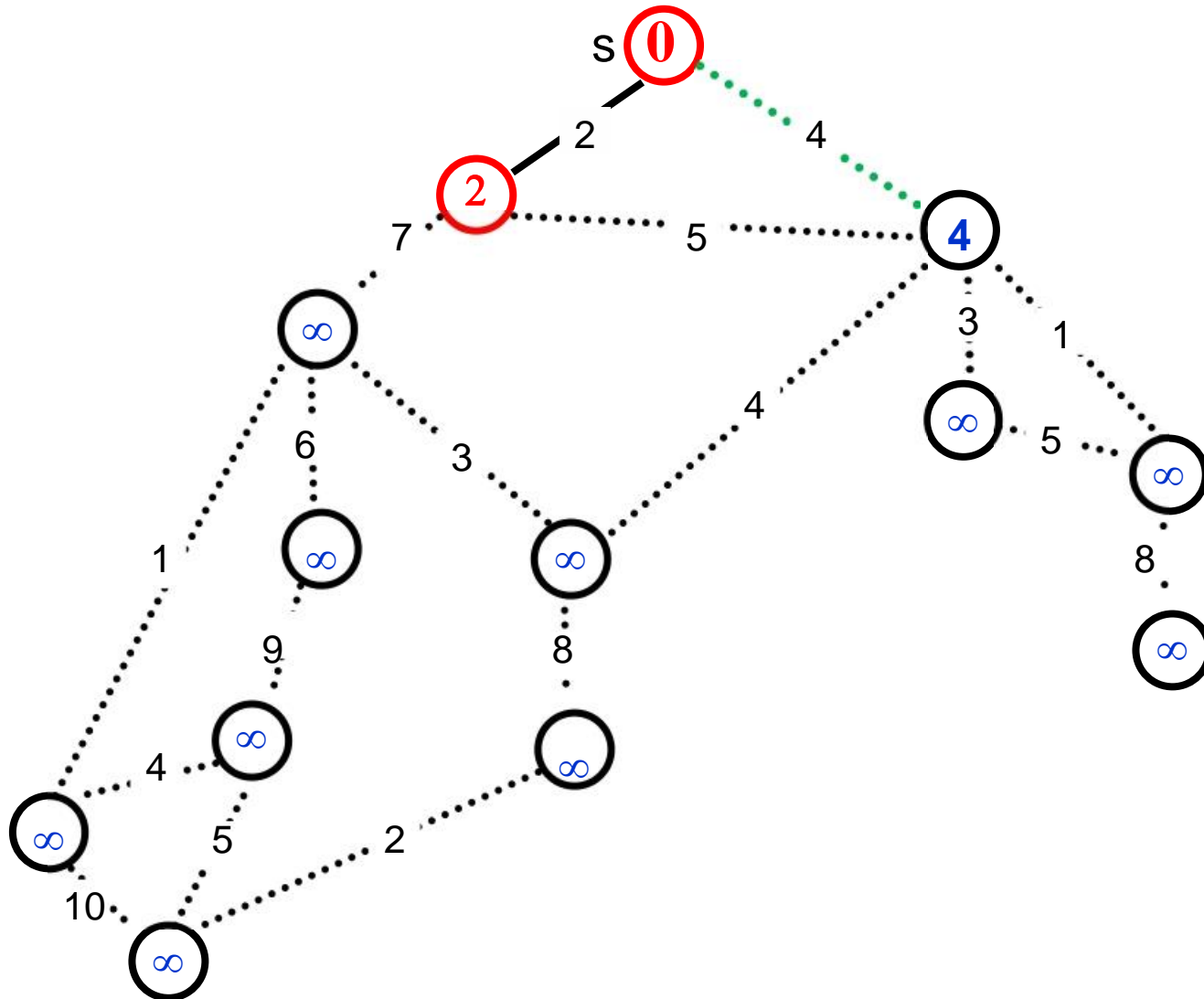
Dijkstra's Algorithm: Example



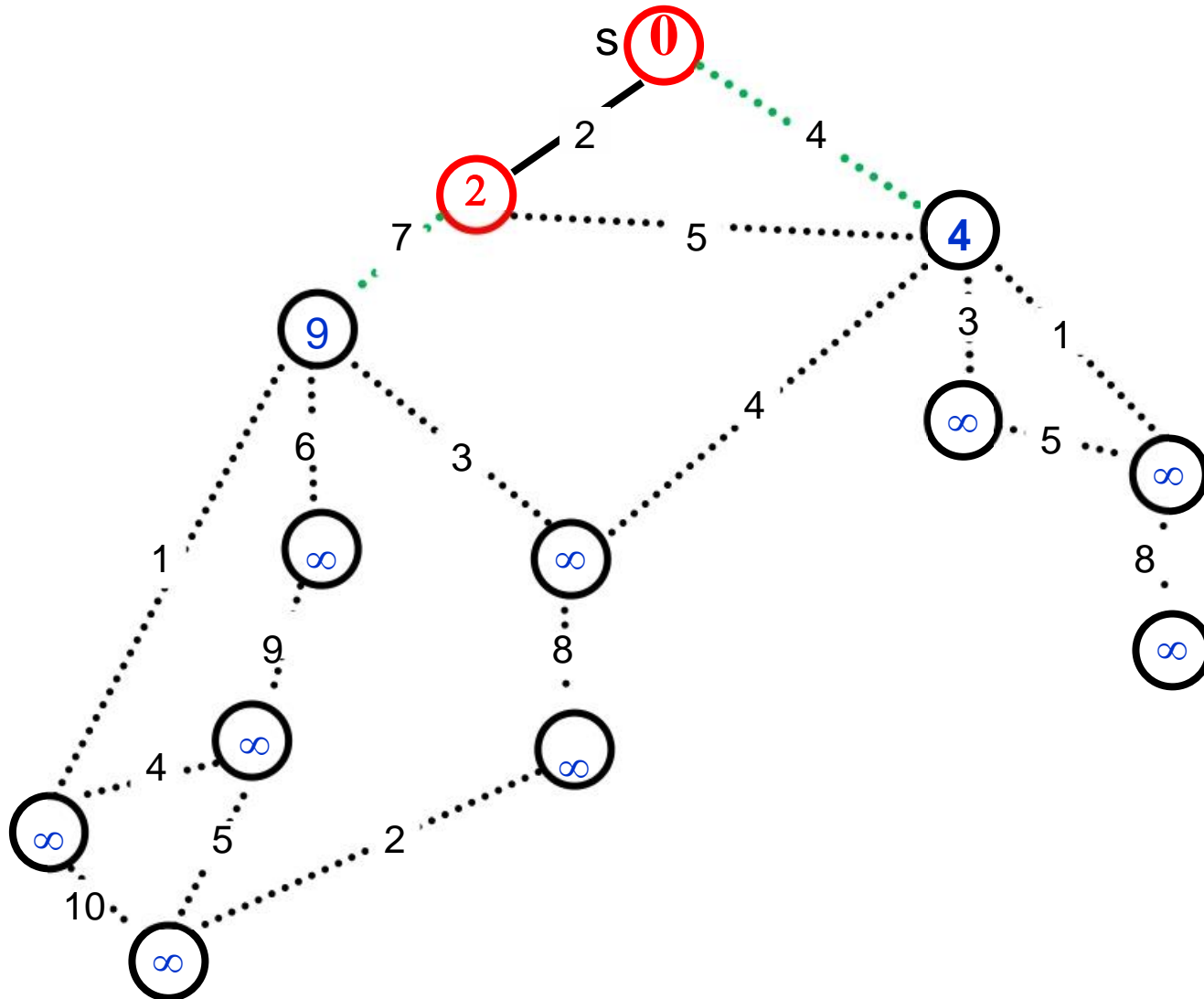
Dijkstra's Algorithm: Example



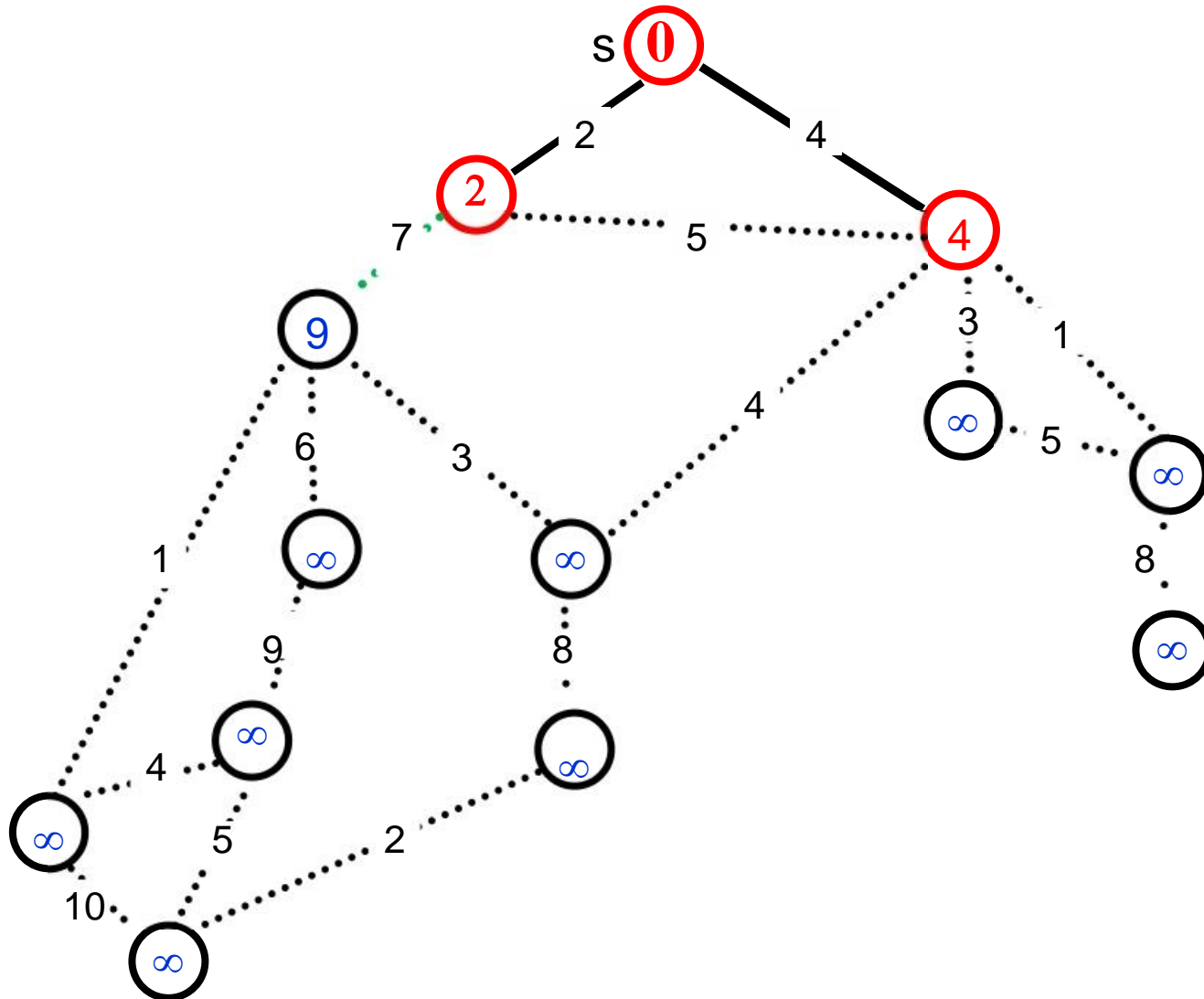
Dijkstra's Algorithm: Example



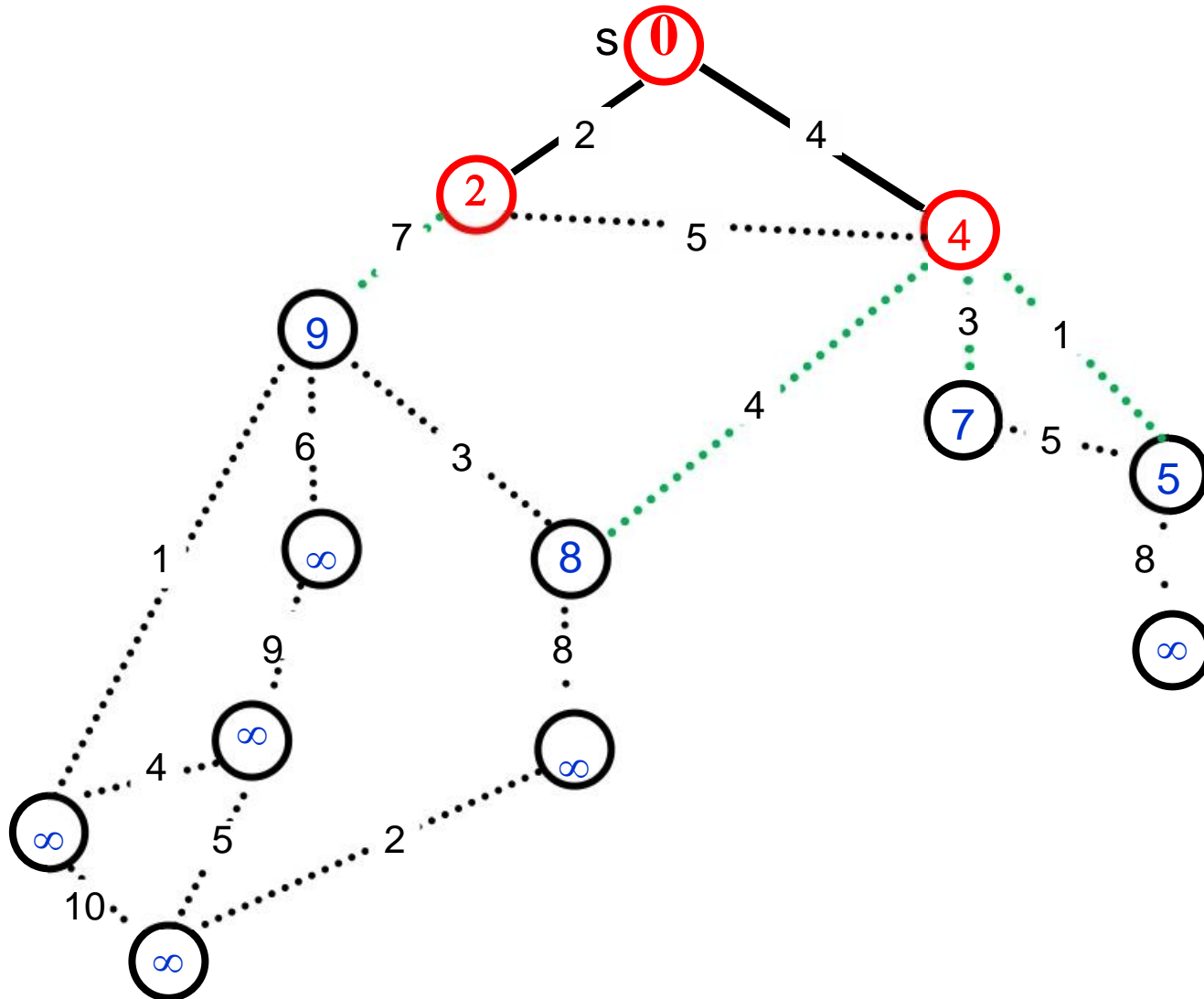
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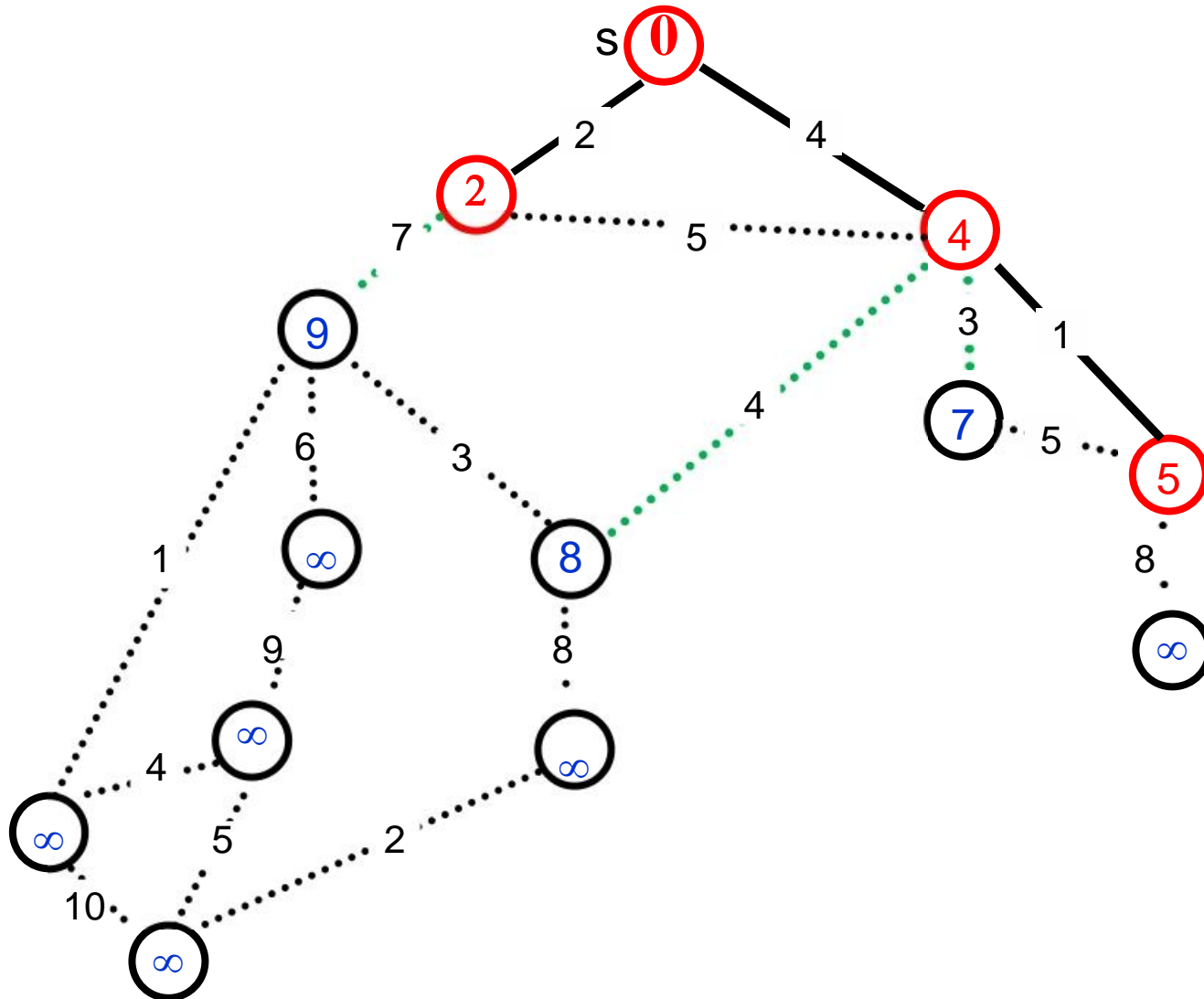
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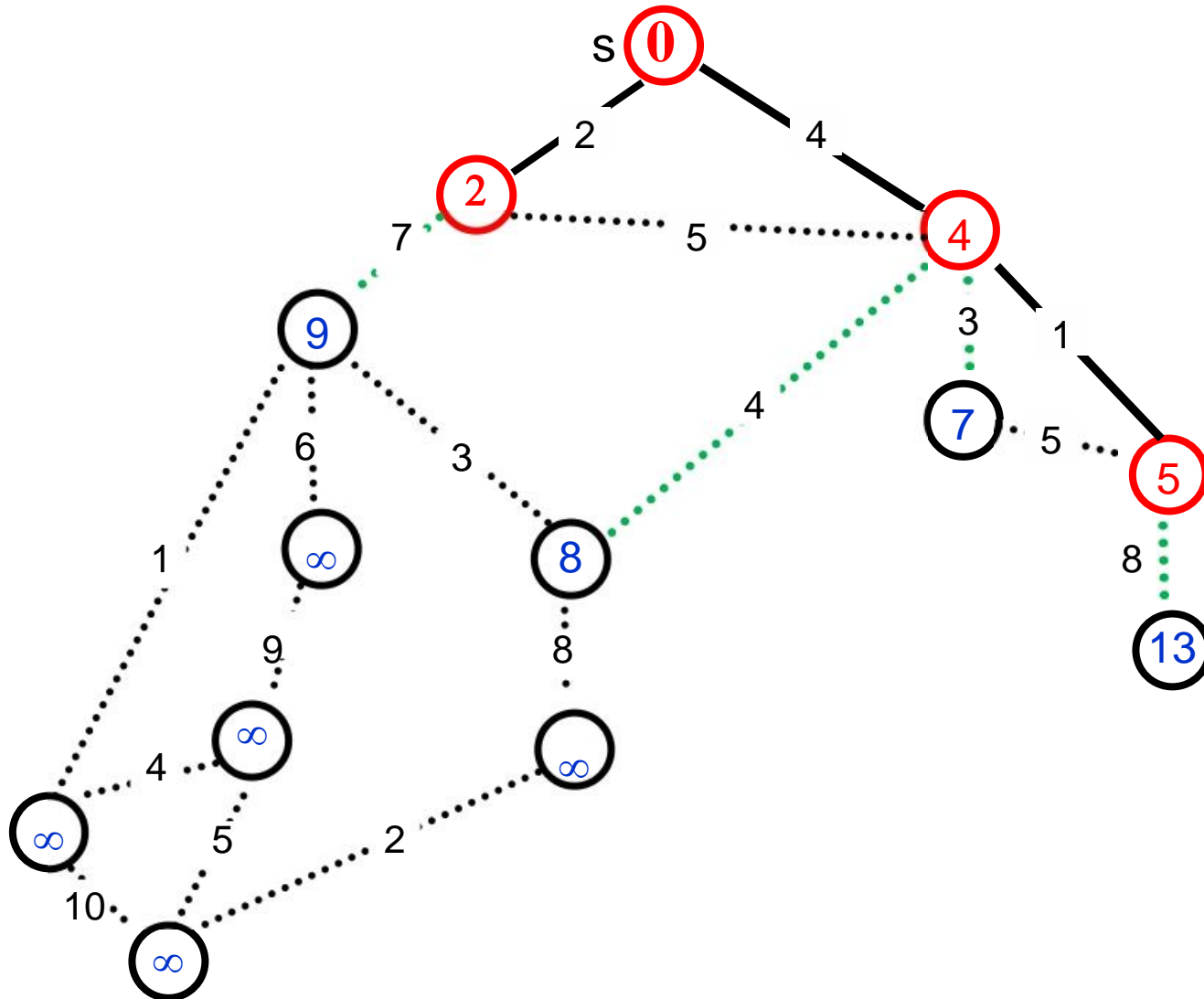
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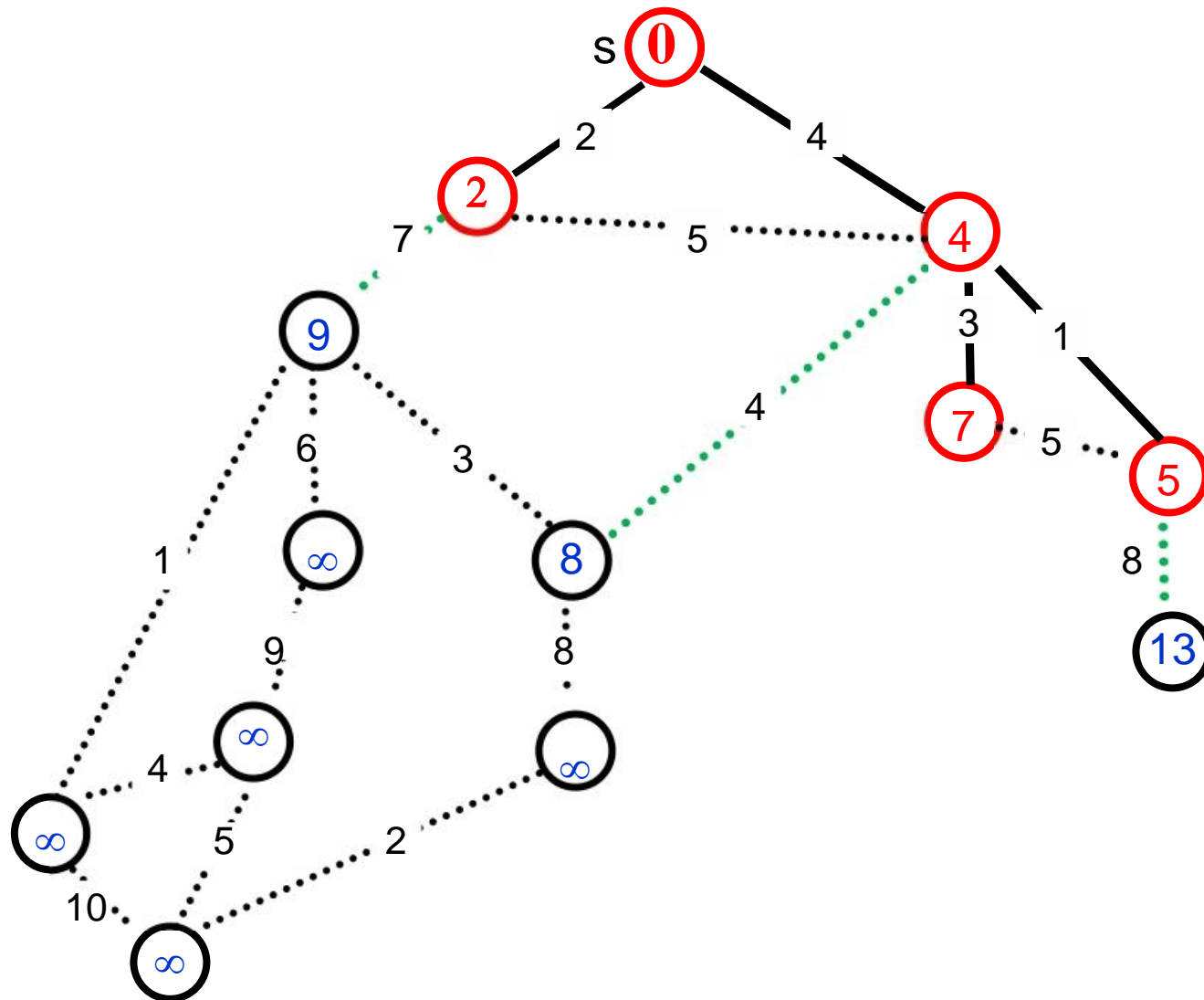
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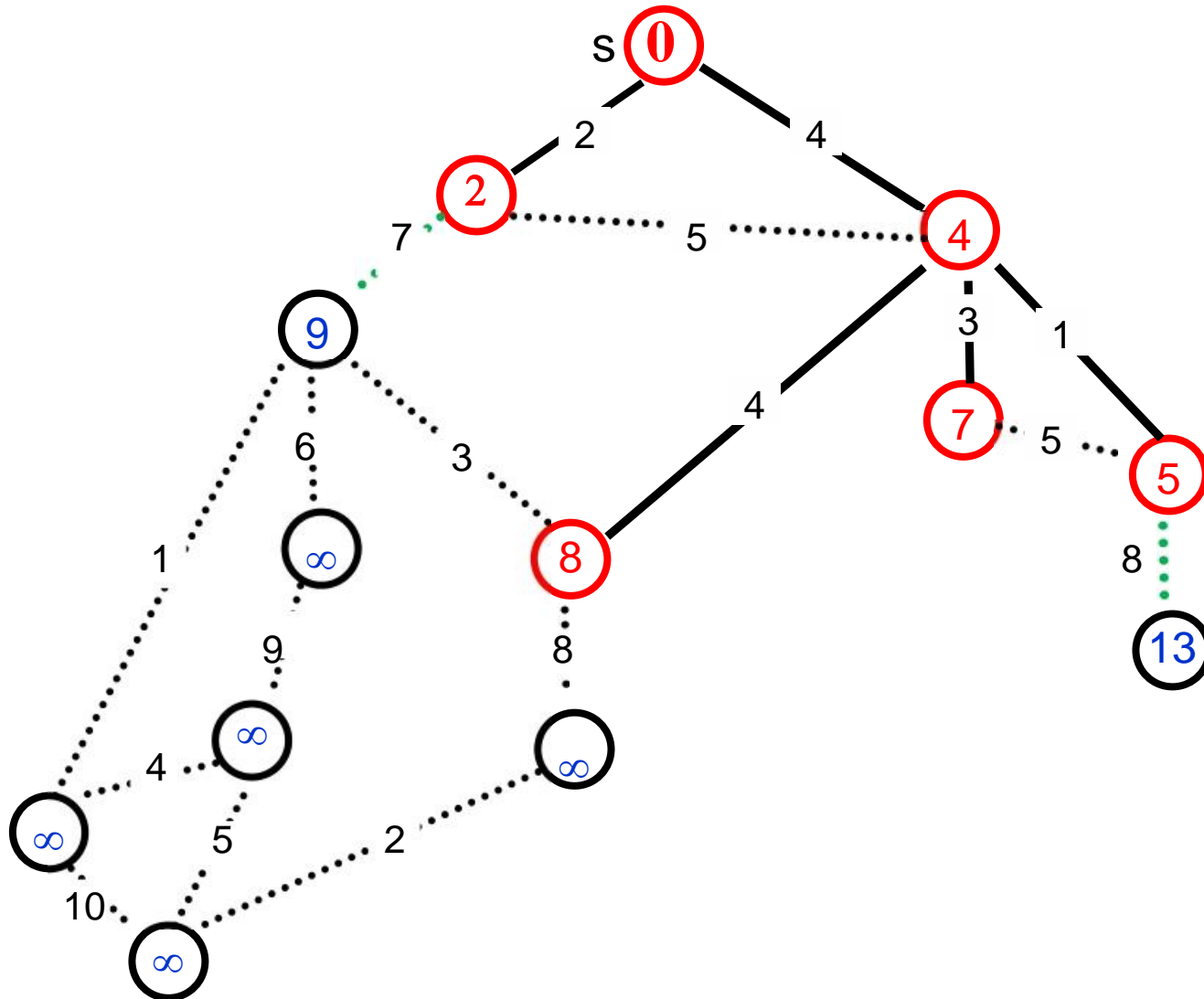
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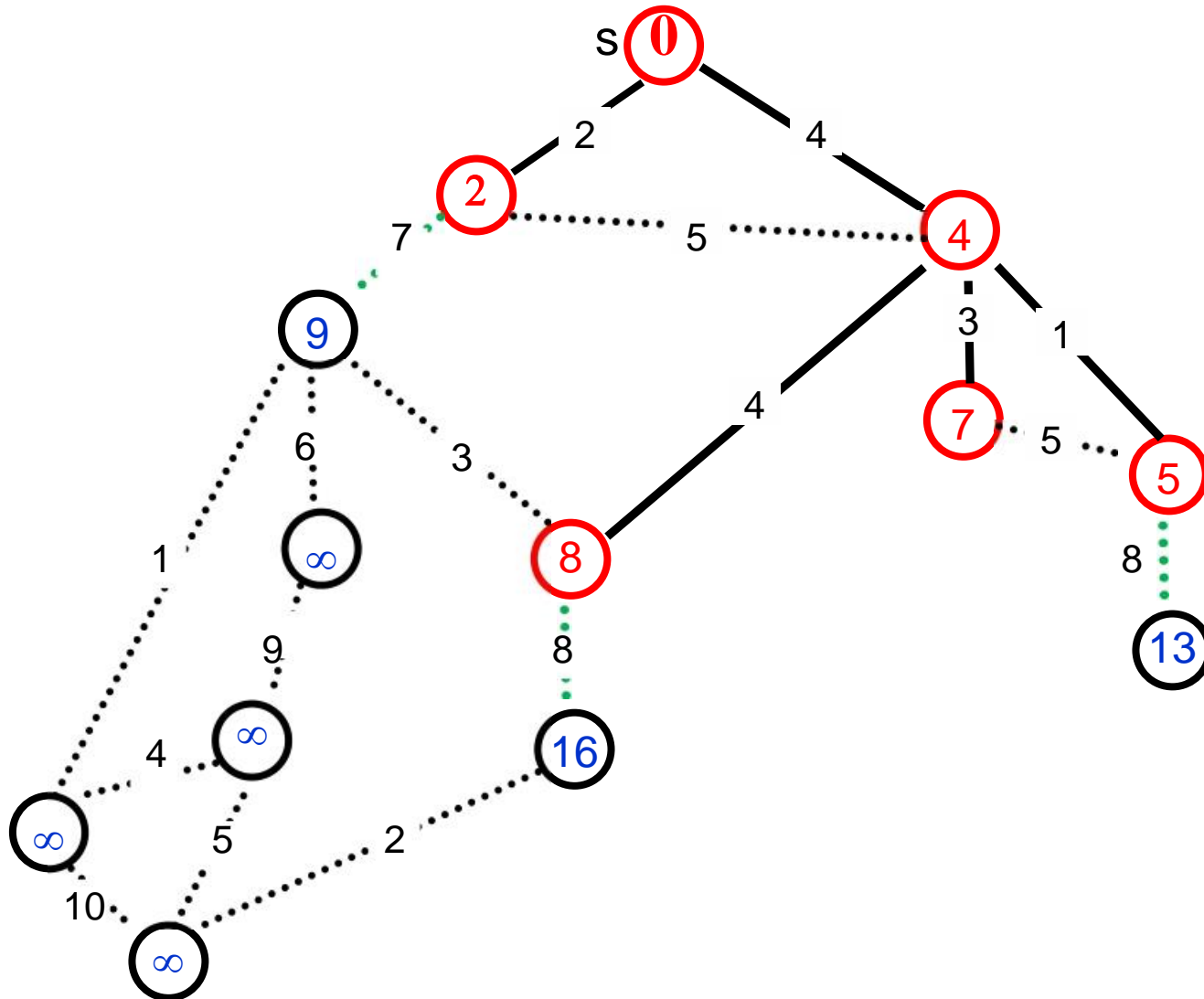
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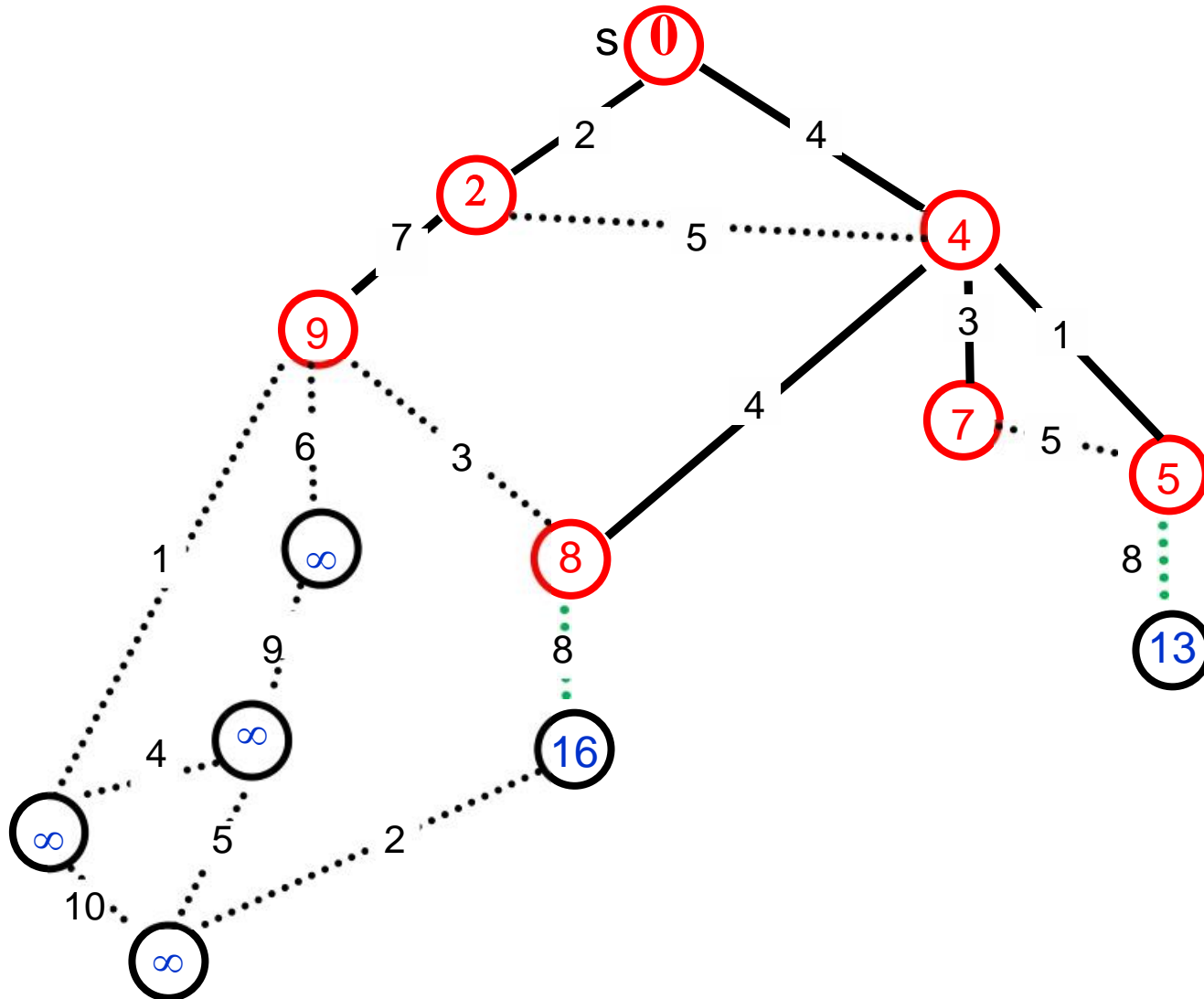
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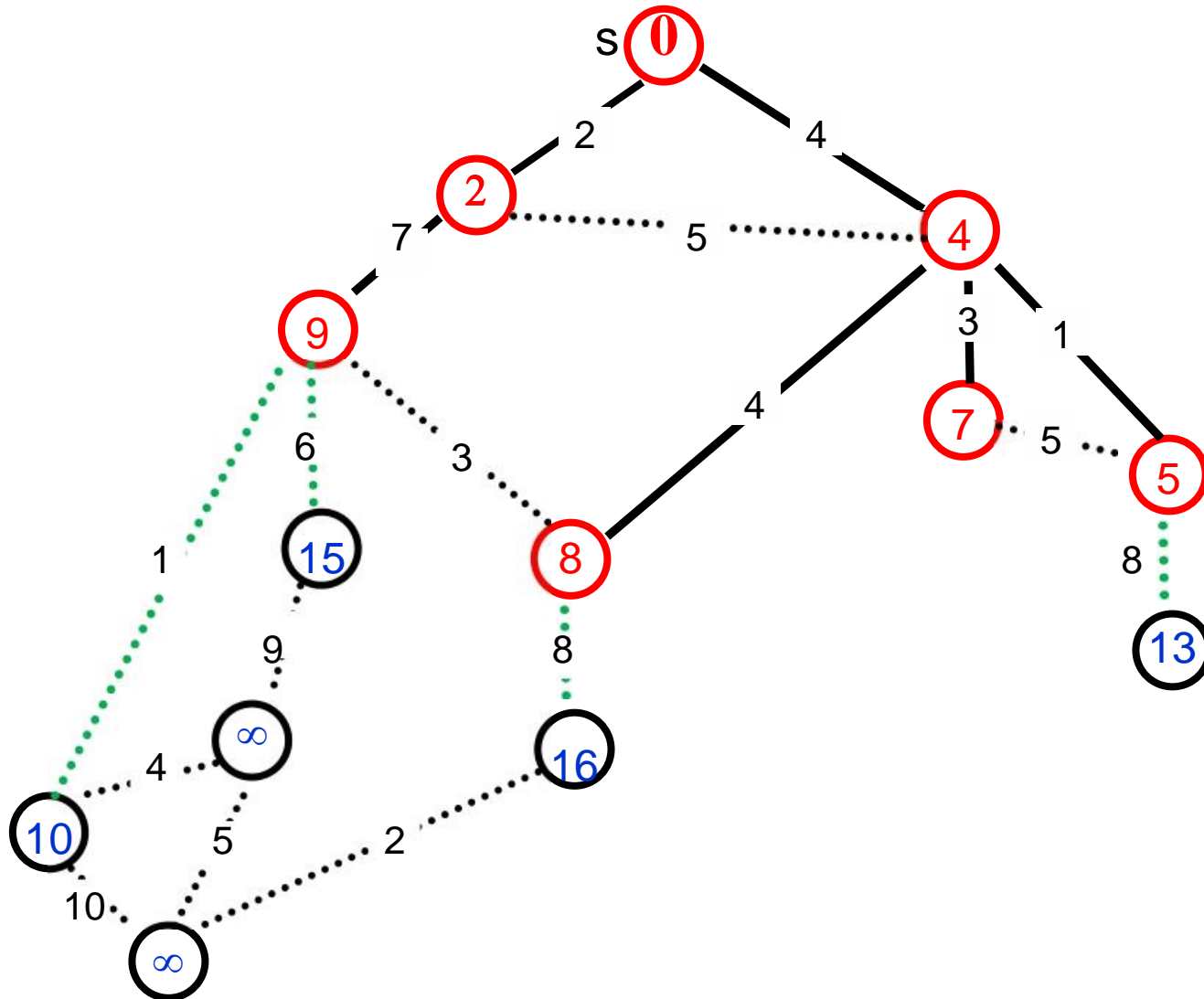
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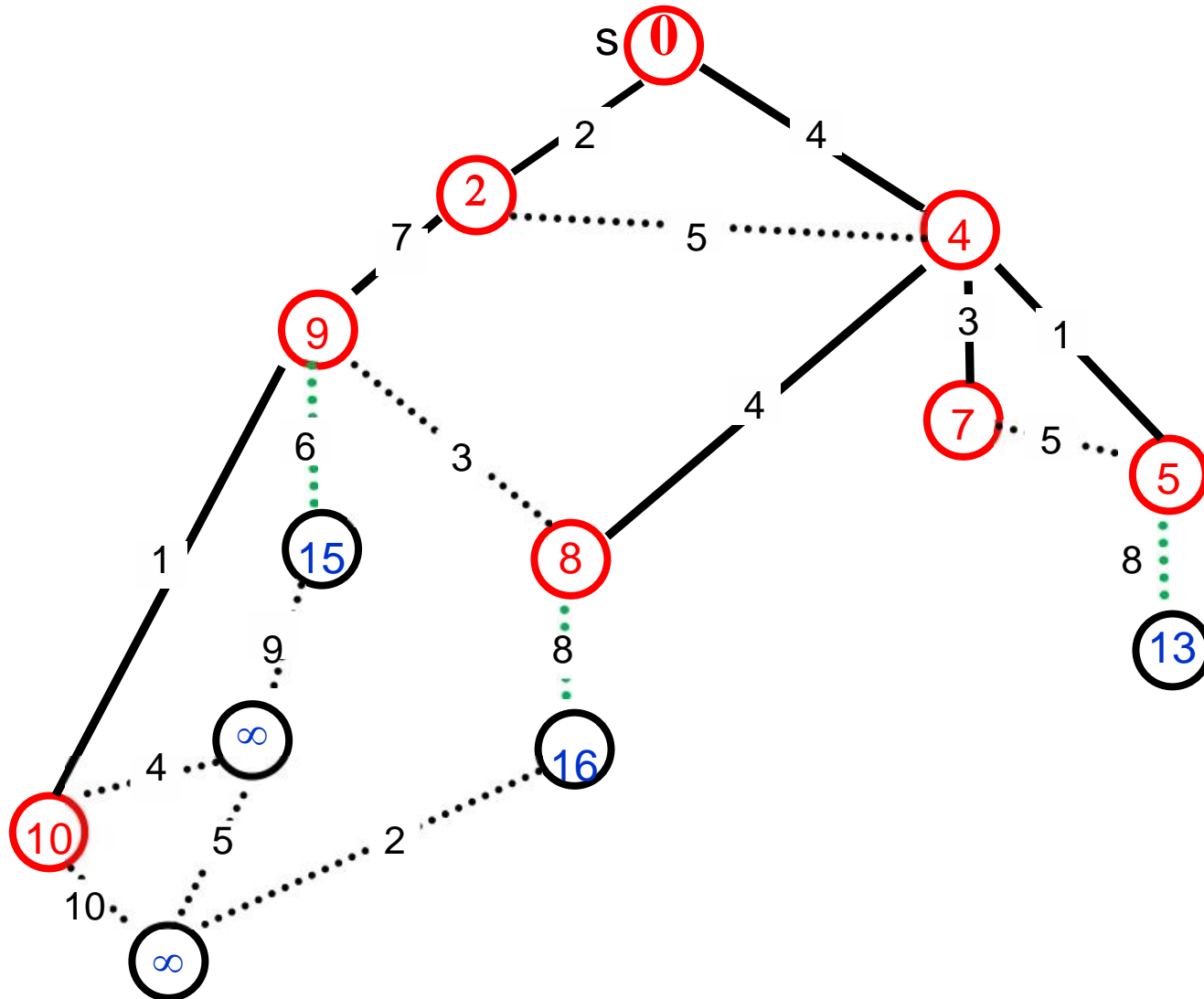
Dijkstra's Algorithm: Example



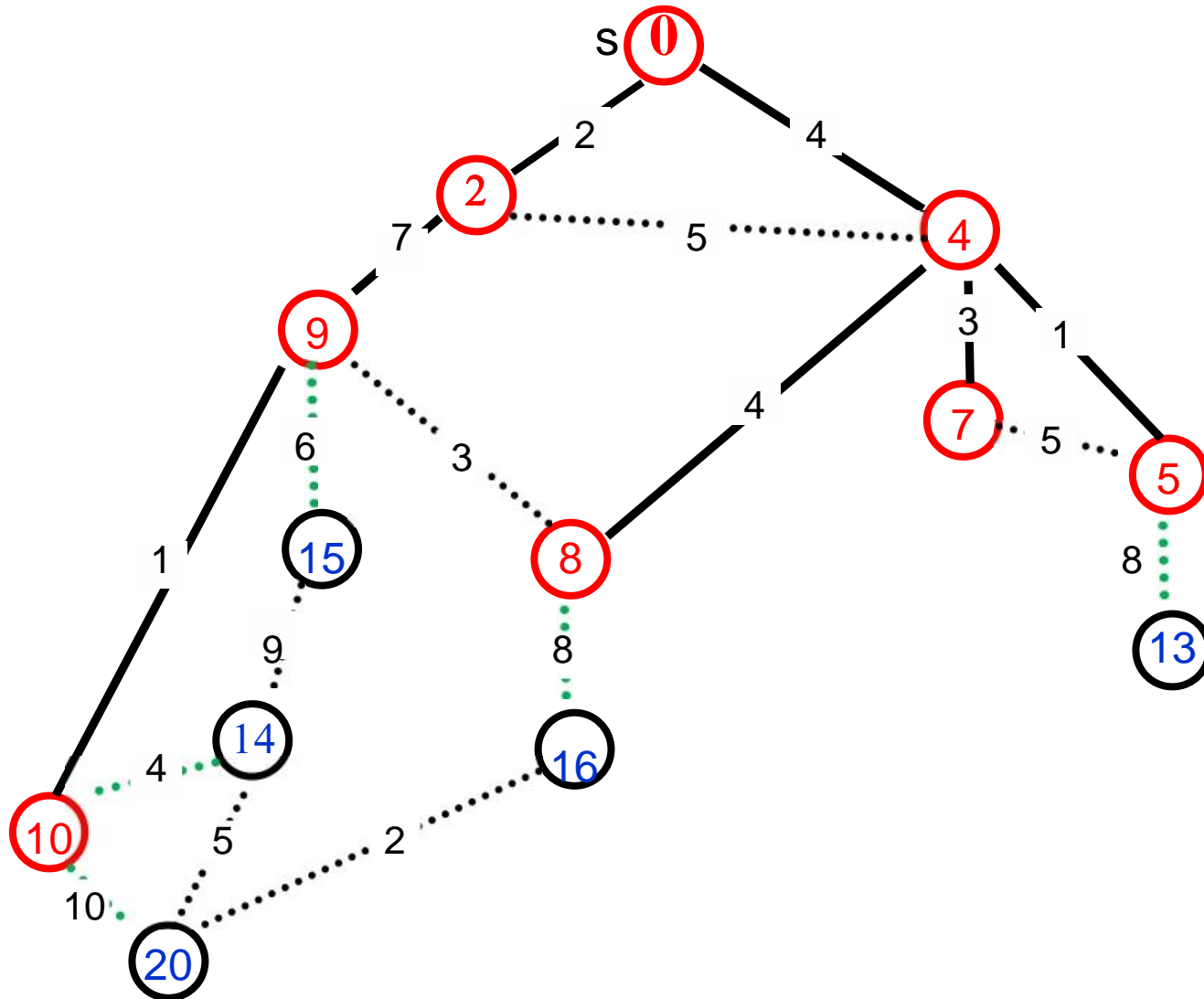
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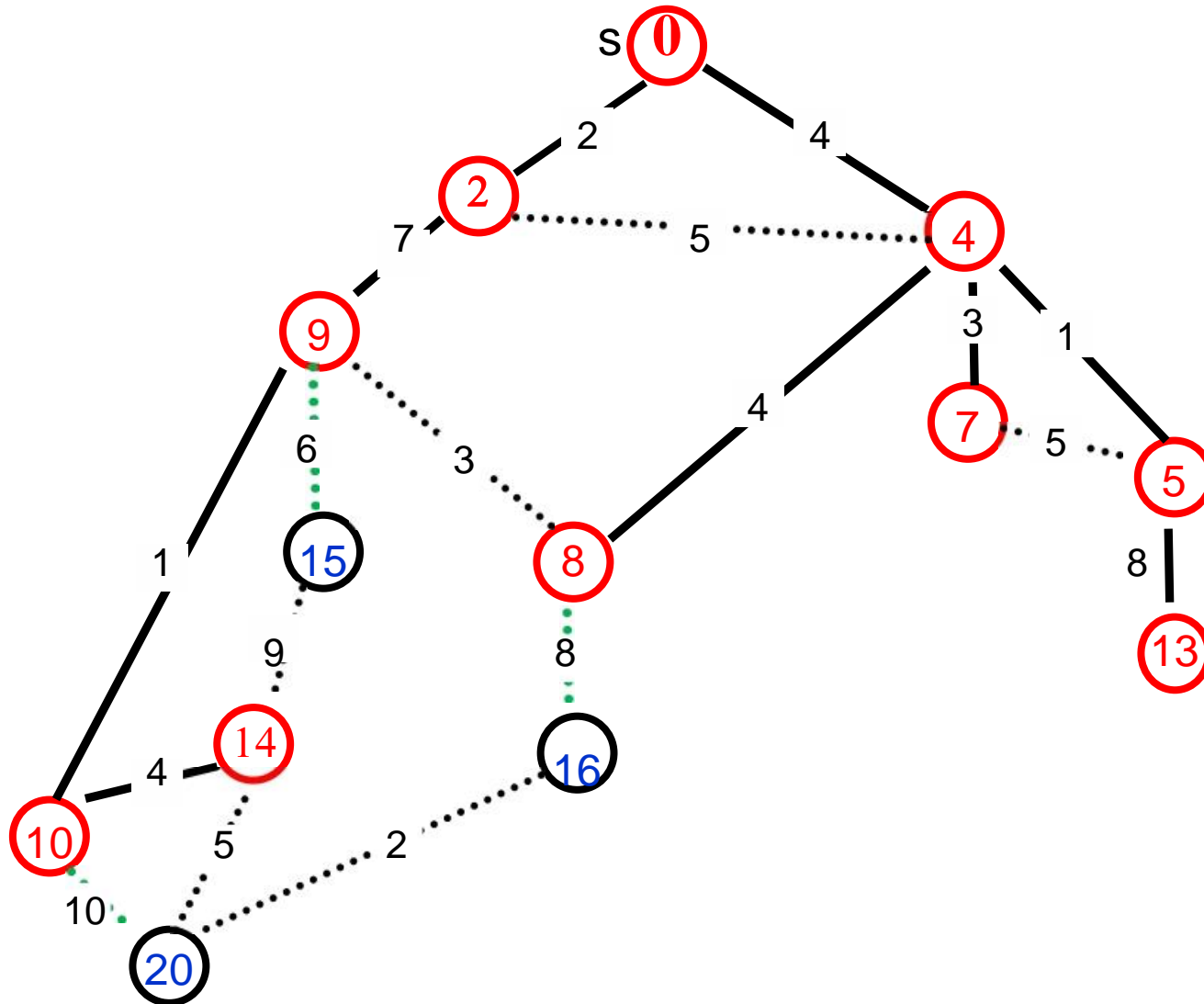
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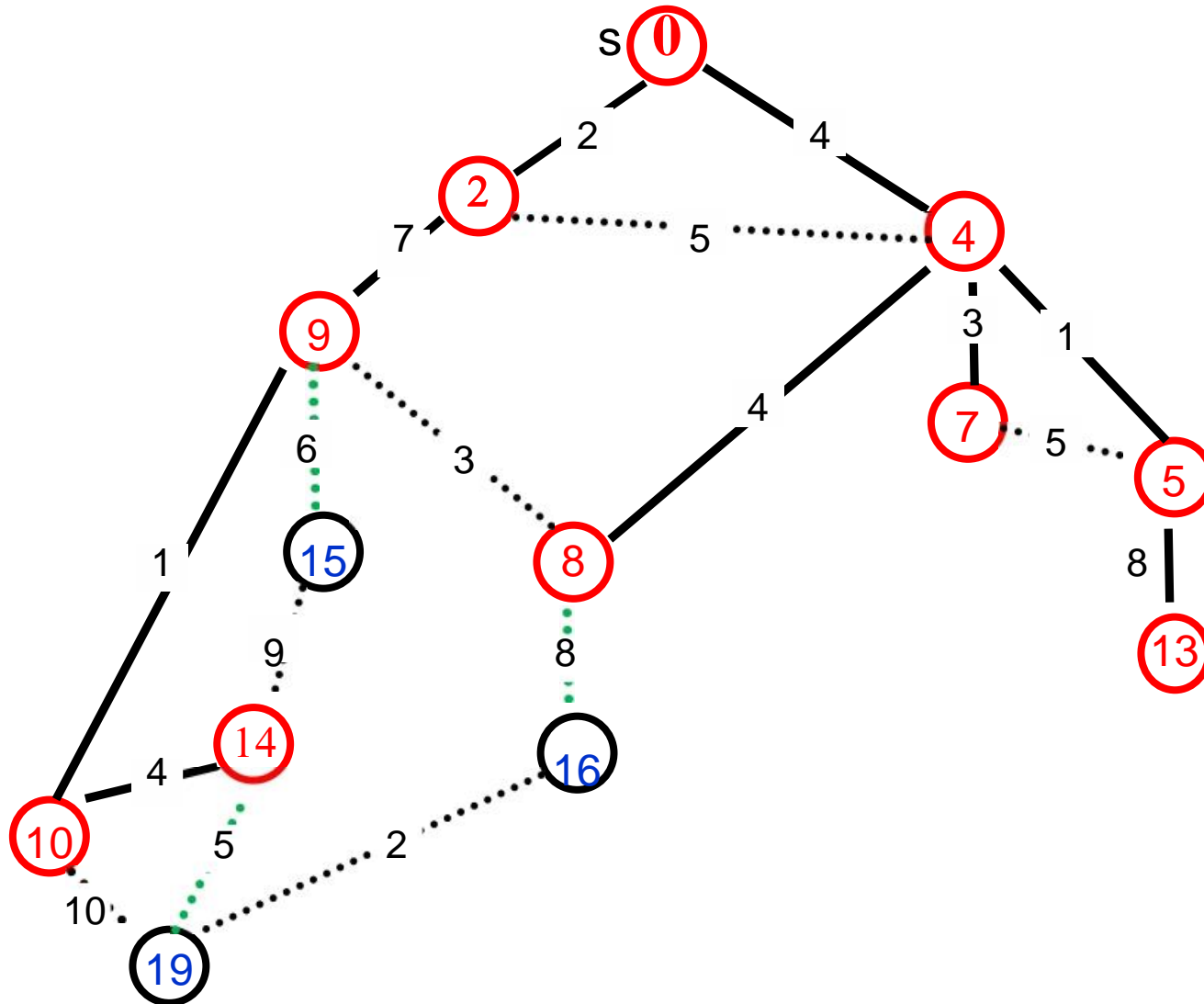
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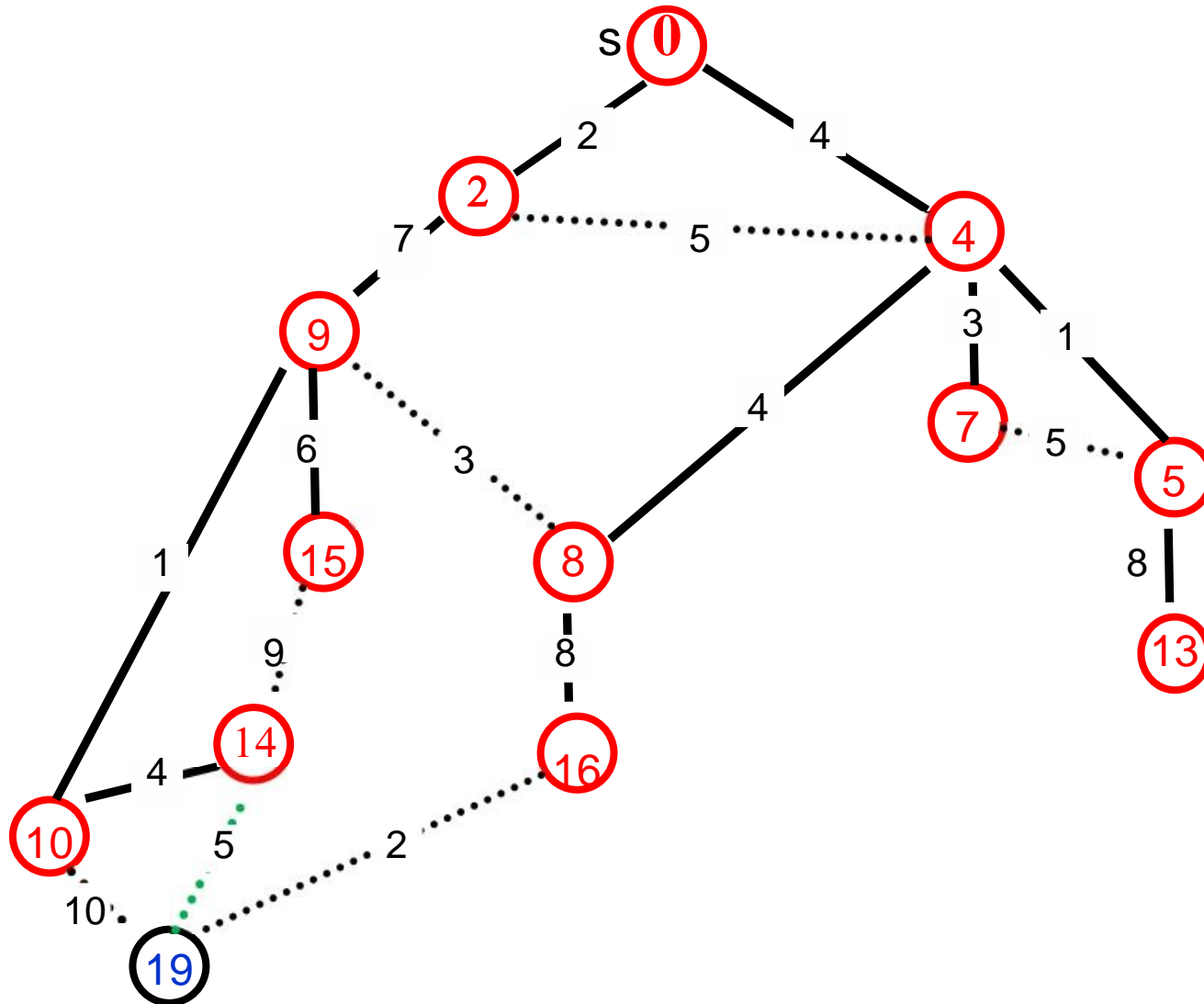
Dijkstra's Algorithm: Example



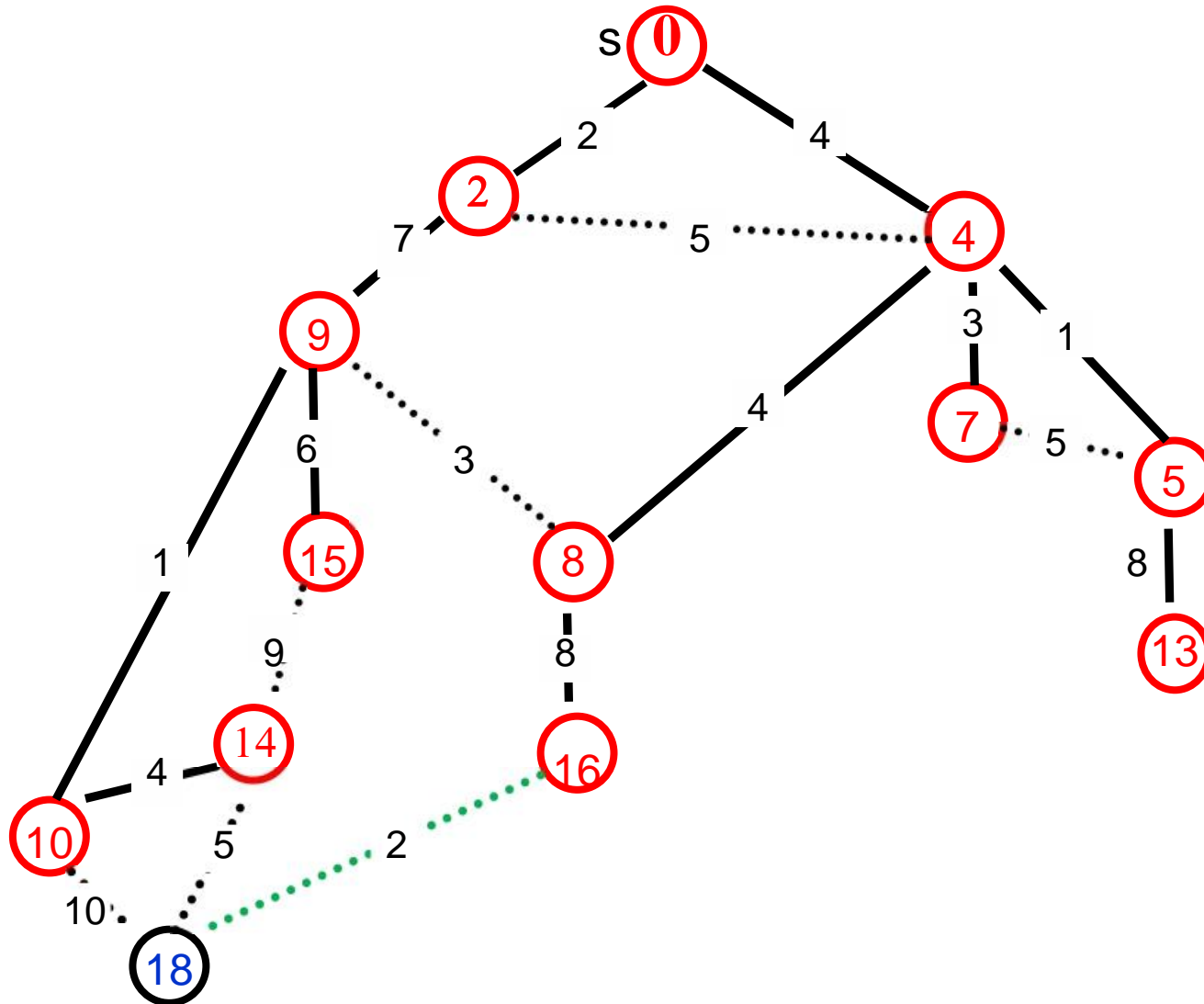
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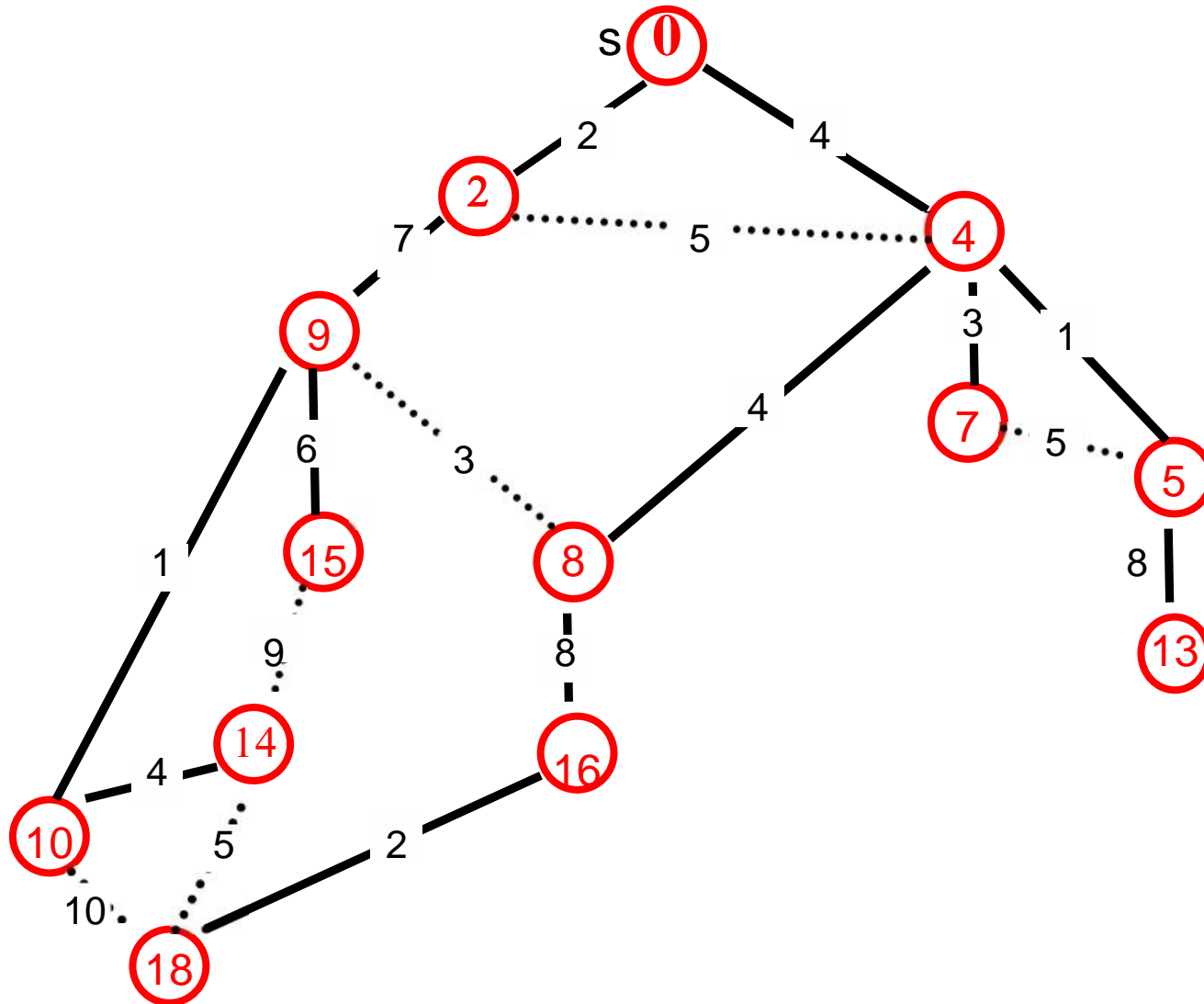
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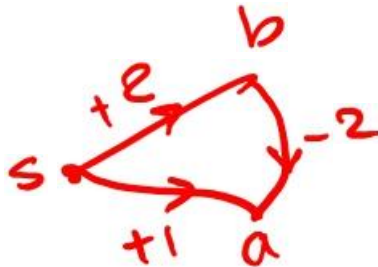
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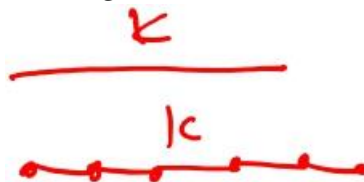
Remarks on Dijkstra's Algorithm

- Algorithm also produces a **tree** of shortest paths to s following Parent links
- Algorithm works on directed graph (with nonnegative weights)
- The algorithm fails with negative edge weights.
 - e.g., some airline tickets

Why does it fail?



- Dijkstra's algorithm is similar to BFS:
 - Substitute every edge with $C_e = k$ with a path of length k , then run BFS.



$\text{dijkstra} = \text{BFS}$.