CHAPTER 17

Theory of Computation

(Solutions to Review Questions and Problems)

Review Questions

Q17-1. The three statements in our Simple Language are the *increment statement*, *decrement statement*, and *loop statement*. The *increment statement* adds 1 to the variable; the *decrement statement* subtracts 1 from the variable; the *loop statement* repeats an action (or a series of actions) while the value of the variable is not zero.

Q17-3. A problem that can be solved by our *Simple Language* can also be solved by the *Turing machine*.

Q17-5. One way to delimit the data on a Turing machine tape is the use of two blanks, one at the beginning of the data and one at the end of the data.

Q17-7. A transition state diagram is a pictorial representation of a program written for the Turing machine.

Q17-9. A Gödel number is an unsigned integer that is assigned to every program that can be written in a specific language. In the halting program, we represent a program as its Godel number when that program is the input to another program.

Problems

P17-1. Table 17.1 shows a solution.

Table 17.1 *Solution to P17-1*

```
Temp ← 0
Y ← 0
while (X)
{
    decr (X)
    incr (Y)
    incr (Temp)
}
while (Temp)
{
    decr (Temp)
```

Table 17.1 *Solution to P17-1*

```
    incr (X)
}
```

P17-3. Table 17.2 shows a solution.

Table 17.2 *Solution to P17-3*

```
Temp ← X        // See solution to P17-1
Z ← 0           // See solution to P17-1
while (Temp)
{
    decr (Temp)
    Z ← Z + Y       // See algorithm 17.7 in the text
}
```

P17-5. Table 17.3 is a solution. We assume that Y > X.

Table 17.3 *Solution to P17-5*

```
while (X)
{
    decr (X)
    decr (Y)
}
```

P17-7. The machine with the single instruction (A, 1, b, R, B) cannot perform any action when it is in the state shown in the text. It crashes.

P17-9. The machine goes through the following states. The last statement is the same as the first statement. The machine goes through an endless loop from statement 1 through statement 9.

	↓					
Start	b	1	1	1	b	State: A
		↓				
1	b	1	1	1	b	State: B
			↓			
2	b	#	1	1	b	State: B
				↓		
3	b	#	#	1	b	State: B
					↓	
4	b	#	#	#	b	State: B
				↓		
5	b	#	#	#	b	State: C
			↓			
6	b	#	#	1	b	State: C
		↓				
7	b	#	1	1	b	State: C
	↓					
8	b	1	1	1	b	State: C
		↓				

9	b 1 1 1 b	State: B

P17-11. The following shows the solution.

	↓	
Start	… b b b b …	State: S_1
	X = 0	
	↓	
(S_1, b, b, R, S_2)	… b b b b …	State: S_2
	X = 0	
	↓	
$(S_2, b, 1, L, S_3)$	… b b 1 b …	State: S_3 (halt)
	X = 1	

P17-13.

- **a.** (S_1, b, b, R, S_2) **S_1 is the *starting* state.**
- **b.** (S_2, b, b, N, S_3) **If X = 0, then go to state S3 (halt).**
- **c.** $(S_2, 1, \#, R, M_R)$ **X is decremented and blank is replaced by #.**
- **d.** $(M_R, 1, 1, R, M_R)$ **M_R is the *move right* state.**
- **e.** (M_R, b, b, N, B_S) **BS is the *start of body loop* state.**
- **f.** (B_H, b, b, L, M_L) **B_H is the *halt of body loop* state.**
- **g.** $(M_L, 1, 1, L, M_L)$ **M_L is the *move left* state.**
- **h.** $(M_L, \#, \#, L, M_L)$
- **i.** (M_L, b, b, N, S_1)

P17-15.

- **a.** (S_1, b, b, R, S_2) **S_1 is the *starting* state.**
- **b.** (S_2, b, b, N, S_3) **S_3 is the *halt* state.**
- **c.** $(S_2, 1, b, R, M_R)$ **S2 is the *decrement X* state.**
- **d.** $(M_R, 1, 1, R, M_R)$ **M_R is the *move right* state.**
- **e.** (M_R, b, b, N, S_4) **S_4 is the *start loop body* state.**
- **f.** (S_4, b, b, R, S_5) **S_5 is the *increment Y* state.**
- **g.** $(S_5, 1, 1, R, S_5)$
- **h.** $(S_5, b, 1, L, S_6)$ **S_6 is the *end loop body* state.**
- **i.** $(S_6, 1, 1, L, S_6)$ **S_6 is the *end loop body* state.**
- **j.** (S_6, b, b, L, M_L) **M_L is the *move left* state.**
- **k.** $(M_L, 1, 1, L, M_L)$
- **l.** (M_L, b, b, N, S_1)

P17-17. Table 17.4 shows the statements for the macro **X** ← **0** and the Gödel number for each statements. The Gödel number for the macro is then $(\text{CF1DBF1E})_{16}$.t

Table 17.4 *Solution to P17-17*

```
while X1                         // Gödel Number: CF1
{                                // Gödel Number: D
    decr X1                      // Gödel Number: BF1
```

Table 17.4 *Solution to P17-17*

}	// Gödel Number: E

P17-19. Table 17.5 shows the statements. The Gödel number for the macro is $(CF3DBF3ECF1DBF1AF3ECF2DBF2AF3E)_{16}$. Notice that this micro does not preserve the value of X_1 or X_2 The Gödel number for the macro will be longer if we want to preserve these two values.

Table 17.5 *Solution to P17-19*

$X_3 \leftarrow 0$	// Gödel Number: CF3DBF3E
while X_1	// Gödel Number: CF1
{	// Gödel Number: D
decr X_1	// Gödel Number: BF1
incr X_3	// Gödel Number: AF3
}	// Gödel Number: E
while X_2	// Gödel Number: CF2
{	// Gödel Number: D
decr X_2	// Gödel Number: BF2
incr X_3	// Gödel Number: AF3
}	// Gödel Number: E