Carnegie Mellon University Relational Model & Algebra

LECTURE #01 >> 15-445/645 FALL 2025 >> PROF. ANDY PAVLO

WAITLIST



We do <u>not</u> control the waitlist. Admins will move students off the waitlist as spots become available.

 \rightarrow See FAQ Q2

To improve your chances of enrolling (though not a guarantee), stay in the class and complete <u>PO</u>.

This class will be offered in Spring 2026!

COURSE OVERVIEW



This course is about the design/implementation of database management systems (DBMSs).

This is not a course about how to use a DBMS to build applications or how to administer a DBMS.

→ See CMU 95-703 (Heinz College)

COURSE LOGISTICS



Course Syllabus + Schedule: Course Web Page

Discussion + Announcements: Piazza

Homework + Projects: <u>Gradescope</u>

Final Grades: Canvas

Non-CMU students can complete assignments using Gradescope (Code: **5R4XPZ**).

- \rightarrow See FAQ Q7
- \rightarrow Do not post your solutions on Github.
- \rightarrow Do not email instructors / TAs for help.
- → Discord Channel: https://discord.gg/YF7dMCg

LECTURE RULES



Do interrupt us for the following reasons:

- \rightarrow I am speaking too fast.
- → You don't understand what I am talking about.
- \rightarrow You have a database-related question.

Do not interrupt for the following reasons:

- \rightarrow Whether you can use the bathroom.
- → Questions about blockchains.

I will <u>not</u> answer questions about the lecture immediately after class.

PROJECTS

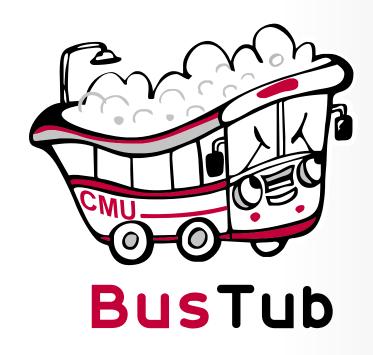


All projects will use the CMU DB Group BusTub academic DBMS.

- \rightarrow Each project builds on the previous one.
- → We will not teach you how to write/debug C++20.
- \rightarrow See the <u>15-445/645 Bootcamp</u>.

Total of four late days the entire semester for projects only.

We will hold an online recitation for each project after it is released.



PROJECT O



Get you started on C++, so you are not surprised later. Get you thinking about algorithms and concurrency.

Project #0 is released today: Count-min Sketch

- \rightarrow Due on Sunday September 7th @ 11:59pm
- \rightarrow No late days are allowed!

Each student must score 100% on this project before the deadline or you will be asked to drop the course.



PLAGIARISM WARNING





The homework and projects must be your own original work. They are not group assignments.

- \rightarrow You may <u>not</u> copy source code from other people or the web.
- \rightarrow You are <u>allowed</u> to use generative AI tools.

Plagiarism is not tolerated. You will get lit up.

→ Please ask instructors (not TAs!) if you are unsure.

See <u>CMU's Policy on Academic Integrity</u> for additional information.

DB FLASH TALKS



Quick 10-minute talks from CMU-DB IAP partners about their DBMSs at end of every Wednesday lecture.

On-campus database recruiting event September 15 + 16.

 \rightarrow Everyone in this class is invited.

Information about internship + fulltime openings will be posted soon.



















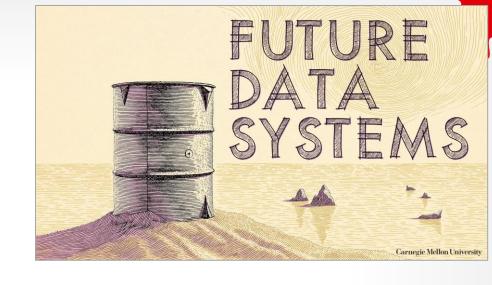




Future Data Systems

Mondays @ 4:30pm starting Sept 22nd Live on Zoom. Videos to YouTube. Open to the public. Optional for class.

https://db.cs.cmu.edu/seminars/fall2025























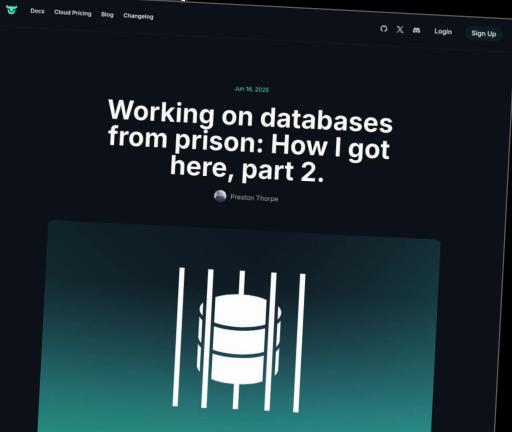


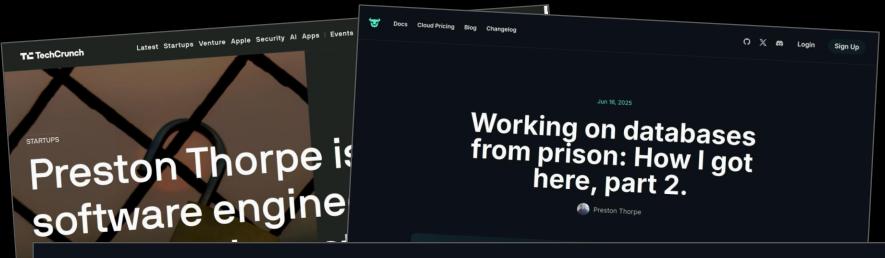


Preston Thorpe is a software engineer at a San Francisco startup he's also serving his 11th year in prison Amanda Silberling

8:48 AM PDT · July 24, 2025

IMAGE CREDITS: ANASTASIJA VUJIC / GETTY IMAGES





Helping build Limbo quickly became my new obsession. I split my time between my job and diving deep into SQLite source code, academic papers on database internals, and Andy Pavlo's CMU lectures. I was active on the <u>Turso Discord</u> but I don't think I considered whether anyone was aware that one of the top contributors was doing so from a prison cell. My story and information are linked on my GitHub, but it's subtle enough where you could miss it if you didn't read the

DATABASE PRISON PROGRAM



Starting in Fall 2025, Intro to Database Systems course available to people locked on the inside at no cost.

If you are in prison or know somebody in prison that wants to learn about databases, please contact: db-prison@cs.cmu.edu



TODAY'S AGENDA

5 14

Database Systems Background

Relational Model

Relational Algebra

Alternative Data Models

DATABASE

Organized collection of inter-related data that models some aspect of the real-world.

Databases are the core component of most computer applications.

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DATABASE EXAMPLE

Create a database that models a digital music store to keep track of artists and albums.

Information we need to keep track of in our store:

- → Information about <u>Artists</u>
- → The <u>Albums</u> those Artists released

FLAT FILE STRAWMAN



Store our database as comma-separated value (CSV) files that we manage ourselves in application code.

- \rightarrow Use a separate file per entity.
- → The application must parse the files each time they want to read/update records.

Artist(name, year, country)

```
"Wu-Tang Clan",1992,"USA"

"Notorious BIG",1992,"USA"

"GZA",1990,"USA"
```

Album(name, artist, year)

```
"<u>Enter the Wu-Tang</u>","Wu-Tang Clan",1993
"<u>St.Ides Mix Tape</u>","Wu-Tang Clan",1994
"<u>Liquid Swords</u>","GZA",1990
```

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FLAT FILE STRAWMAN

Example: Get the year that GZA went solo.

Artist(name, year, country)

```
"Wu-Tang Clan",1992,"USA"

"Notorious BIG",1992,"USA"

"GZA",1990,"USA"
```



```
for line in file.readlines():
    record = parse(line)
    if record[0] == "GZA":
        print(int(record[1]))
```

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FLAT FILES: DATA INTEGRITY

How do we ensure that the artist is the same for each album entry?

What if somebody overwrites the album year with an invalid string?

What if there are multiple artists on an album?

What happens if we delete an artist that has albums?

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FLAT FILES: IMPLEMENTATION

How do you find a particular record?

What if we now want to create a new application that uses the same database? What if that application is running on a different machine?

What if two threads try to write to the same file at the same time?

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FLAT FILES: DURABILITY

What if the machine crashes while our program is updating a record?

What if we want to replicate the database on multiple machines for high availability?

easily corrupt.

What if we want to rep machines for high avail:

SurrealDB is sacrificing data durability to make benchmarks look better

TL;DR: If you are a SurrealDB user running any SurrealDB instance backed by the RocksDB or SurrealKV storage backends you **MUST EXPLICITLY** set SURREAL_SYNC_DATA=true in your environment variables otherwise your instance is NOT crash safe and can very

If you're familiar with Rust or follow what new hype databases are gaining a following, you might have heard of <u>SurrealDB</u>, a jack-of-all-trades database that does it all... So they say.

SurrealDB is the ultimate database for tomorrow's serverless, jamstack, single-page, and traditional applications.

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DATABASE MANAGEMENT SYSTEM

A <u>database management system</u> (DBMS) is software that allows applications to store and analyze information in a database.

A general-purpose DBMS supports the definition, creation, querying, update, and administration of databases in accordance with some data model.

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DATA MODELS

A <u>data model</u> is a collection of concepts for describing the data in a database.

→ Rules that define the types of things that could exist and how they relate.

A <u>schema</u> is a description of a particular collection of data, using a given data model.

- \rightarrow This defines the structure of database for a data model.
- → Otherwise, you have random bits with no meaning.

23 Pal

DATA MODELS

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DATA MODELS

Relational

← Most DBMSs

Key/Value

Graph

Document / JSON / XML / Object

Wide-Column / Column-family

Array (Vector, Matrix, Tensor)

Hierarchical

Network

Semantic

5 24

DATA MODELS

```
Relational
```

Key/Value ← Simple Apps / Caching

Graph

Document / JSON / XML / Object

Wide-Column / Column-family

Array (Vector, Matrix, Tensor)

Hierarchical

Network

Semantic

DATA MODELS

Relational

Key/Value

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DATA MODELS

```
Relational
```

Key/Value

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Document / JSON / XML / Object

Wide-Column / Column-family

Array (Vector, Matrix, Tensor) ← ML / Science

Hierarchical

Network

Semantic

DATA MODELS

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```
Relational
```

Key/Value

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Document / JSON / XML / Object

Wide-Column / Column-family

Array (Vector, Matrix, Tensor)

Hierarchical

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Semantic

Entity-Relationship

← Obsolete / Legacy / Rare

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DATA MODELS

Relational

← This Course

Key/Value

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EARLY DATABASE SYSTEMS

In the late 1960s, early DBMSs required developers to write queries using procedural code.

→ Examples: <u>IDS</u>, <u>IMS</u>, <u>CODASYL</u>

The developer had to choose access paths and execution ordering based on the current database contents.

→ If the database changes, then the developer must rewrite the query code.

1973 ACM Turing

Award Lecture

The Turing Award citation read by Richard G. Canning, chairman of the 1973 Turing Award Committee, at the presentation of this lecture on August 28 at the ACM Annual Conference in Atlanta:

A significant change in the computer field in the last five to cight years has been made in the way we treat and handle data. In the early days of our field, data was intimately field to the application programs that used it. Now we tee that we want to break that tie. We want data that is independent of the application programs that use it—that is, data that is organized and structured to serve many applications and many users. What we seek is the data base.

This movement toward the data base is in its infancy. Even so, it appears that there are now between 1,000 and 2,000 true data base management systems installed worldwide. In ten years very likely, there will be tens of thousands of such systems. Just from the quantities of installed systems, the impact of data bases promises to be huge.

This year's recipient of the A.M. Turing Award is one of the real pioneers of data base technology. No other individual has had the influence that he has had upon this aspect of our field. I

single out three prime examples of what he has done. He was the creator and principal architect of the first commercially available data base management system-the Integrated Data Store-originally developed from 1961 to 1964.1.2.2.4 I-D-S is today one of the three most widely used data base management systems. Also, he was one of the founding members of the CODASYL Data Base Task Group, and served on that task group from 1966 to 1968. The specifications of that task group are being implemented by many suppliers in various parts of the world.^{5,5} Indeed, currently these specifications represent the only proposal of stature for a common architecture for data base management systems. It is to his credit that these specifications, after extended debate and discussion embody much of the original thinking of the Integrated Data Store. Thirdly, he was the creator of a powerful method for displaying data relationships-a tool for data base designers as well as application system designers.2.5

as application system designers.^{1,2}
His contributions have thus represented the union of imagination and practicality. The richness of his work has already had, and will continue to have, a substantial influence upon our field.

I am very pleased to present the 1973 A.M. Turing Award to Classics.

The Programmer as Navigator

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by Charles W. Bachman

This year the whole world celebrates the five-hundredth birthday of Nicolaus Copernicus, the famous Polish astronomer and mathematician. In 1543, Copernicus published his book, Concerning the Revolutions about Celestial Spheres, which described a new theory boot the relative physical movements of the earth, the planets, and the sun. It was in direct contradictions that the earth-centered theories which had been established by Ptolemy 1400 years earlier.

Controllar proposed the heliocentric theory, that faint revolve in a circular orbit around the sun. This theory was subjected to tremendous and persistent criticism. Nearly 100 years later, Galileo was ordered Copyright © 1973. Association for Computing Machinery, the General permission to republish, that not for profit, all or part of this material is granted provided that ACM's copyright notice of noue, and to the fact that registring privileges were granted by permission of the Association for Computing Machinery. Author's addition. Honoyeall Information Systems, Inc., 20

The abstract, key words, etc., are on page 654.

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to appear before the Inquisition in Rome and forced to state that he had given up his belief in the Copernican theory. Even this did not placate his inquisitors, and he was sentenced to an indefinite prison term, while Copernicus's book was placed upon the Index of Prohibited Books, where it remained for another 200 years.

I raise the example of Copernicus today to illustrate a parallel that I believe exists in the computing or, more properly, the information systems world. We have spent the last SO years with almost Ptolemsic information systems. These systems, and most of the thinking about systems, were based on a "computer centered" concept. I choose to speak of 50 years of history tather han 25, for 1 sec today's information systems as dating the property of the property of the stored program compared to the program of the stored fororgam compared to the beginning of the stored fororgam compared to the program of the stored fororgam of the stored

Just as the ancients viewed the earth with the sun revolving around it, so have the ancients of our information systems viewed a tab machine or computer with a sequential file flowing through it. Each was an

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EARLY DATABASE CYCTEMS

In the late 1960s, early DBMSs required developers to write queries using procedural code.

→ Examples: <u>IDS</u>, <u>IMS</u>, <u>CODASYL</u>

The developer had to choose access paths and execution ordering based on the current database contents.

→ If the database changes, then the developer must rewrite the query code.

In order to focus the role of programmer as navigator, let us enumerate his opportunities for record access. These represent the commands that he can give to the database system—singly, multiply or in combination with each other—as he picks his way through the data to resolve an inquiry or to complete an update.

- 1. He can start at the beginning of the database, or at any known record, and sequentially access the "next" record in the database until he reaches a record of interest or reaches the end.
- 2. He can enter the database with a database key that provides direct access to the physical location of a record. (A database key is the permanent virtual memory address assigned to a record at the time that it was created.)
- 3. He can enter the database in accordance with the value of a primary data key. (Either the indexed sequential or randomized access techniques will yield the same result.)
- 4. He can enter the database with a secondary data key value and sequentially access all records having that particular data value for the field.
- 5. He can start from the owner of a set and sequentially access all the member records. (This is equivalent to converting a primary data key into a secondary data key.)
- 6. He can start with any member record of a set and access either the next or prior member of that set.
- 7. He can start from any member of a set and access the owner of the set, thus converting a secondary data key into a primary data key.

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Each of these access methods is interesting in itself, and all are very useful. However, it is the synergistic usage of the entire collection which gives the programmer great and expanded powers to come and go within a large database while accessing only those records of interest in responding to inquiries and updating the database in anticipation of future inquiries.

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Retrieve the names of artists that appear on the DJ Mooshoo Tribute mixtape.

```
PROCEDURE GET_ARTISTS_FOR_ALBUM;
 /* Declare variables */
 DECLARE ARTIST RECORD ARTIST:
 DECLARE APPEARS_RECORD APPEARS;
 DECLARE ALBUM_RECORD ALBUM;
 /* Start navigation */
 FIND ALBUM USING ALBUM NAME = "Mooshoo Tribute"
     ON ERROR DISPLAY "Album not found" AND EXIT:
 /* For each appearance on the album */
 FIND FIRST APPEARS WITHIN APPEARS_ALBUM OF ALBUM_RECORD
     ON ERROR DISPLAY "No artists found for this album" AND EXIT;
 /* Loop through the set of APPEARS */
 REPEAT
      /* Navigate to the corresponding artist */
      FIND OWNER WITHIN ARTIST_APPEARS OF APPEARS_RECORD
          ON ERROR DISPLAY "Error finding artist";
      /* Display artist name */
      DISPLAY ARTIST_RECORD.NAME;
      /* Move to the next APPEARS record in the set */
      FIND NEXT APPEARS WITHIN APPEARS_ALBUM OF ALBUM_RECORD
          ON ERROR EXIT;
 END REPEAT:
END PROCEDURE;
```

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Retrieve the names of artists that appear on the DJ Mooshoo Tribute mixtape.



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Retrieve the names of artists that appear on the DJ Mooshoo Tribute mixtape.

SELECT ARTIST.NAME
FROM ARTIST, APPEARS, ALBUM
WHERE ARTIST.ID=APPEARS.ARTIST_ID
AND APPEARS.ALBUM_ID=ALBUM.ID
AND ALBUM.NAME="Mooshoo Tribute"

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EARLY DATABASE SYSTEMS

The Differences and Similarities Between the Data Base Set and Relational Views of Data.

→ ACM SIGFIDET Workshop on Data

Description, Access, and Control in Ann
Arbor, Michigan, held 1–3 May 1974



Codd



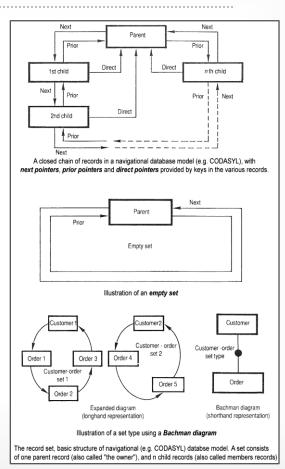
Bachman



Gray



Stonebraker



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EARLY DATABASE SYSTEMS

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→ ACM SIGFIDET Workshop on Data Description, Access, and Control in Ann Arbor, Michigan, held 1–3 May 1974



Codd



Bachman



Gray



Stonebraker

COBOL/CODASYL camp:

- 1. The relational model is too mathematical. No mere mortal programmer will be able to understand your newfangled languages.
- 2. Even if you can get programmers to learn your new languages, you won't be able to build an efficient implementation of them.
- 3. On-line transaction processing applications want to do record-oriented operations.

Relational camp:

- 1. Nothing as complicated as the DBTG proposal can possibly be the right way to do data management.
- 2. Any set-oriented query is too hard to program using the DBTG data manipulation language.
- 3. The CODASYL model has no formal underpinning with which to define the semantics of the complex operations in the model.

RELATIONAL MODEL

Eall

Structure: The definition of the database's relations and their contents independent of their physical representation.

Integrity: Ensure the database's contents satisfy constraints.

Manipulation: Declarative API for accessing and modifying a database's contents via relations (sets).

Information Retrieval

P. BAXENDALE, Editor

A Relational Model of Data for Large Shared Data Banks

E. F. Codd IBM Research Laboratory, San Jose, California

Future users of large data banks must be protected from howing to know hwe data is arganized in the mochine (the internal representation). A prompting service which supplies use hinformation is not a satisfactory solution. Activities of users at terminals and most application programs should remain unaffected when the internal representation of data is danged and even when some supects of the external representation are changes. Alonges in data representation are changed. Changes in data representation will often be needed as a result of changes in query, update, and report raffic and natural growth in the types of stored information.

Existing noninferential, formatted data systems provide users with tree-structured files or slightly more general network models of the data. In Section 1, inadequacies of these models are discussed. A model based on n-ary relations, a normal form for data base relations, and the concept of a universal data sublanguage are introduced. In Section 2, certain operations on relations (other than logical inference) are discussed and applied to the problems of redundancy and consistency in the user's model.

KEY WORDS AND PHRASES: data bank, data base, data structure, data organization, hierarchies of data, networds of data, relations, derivability, redundancy, consistency, composition, join, retrieval language, predicate calculus, scarity, data integrity CR CATECORES 3.70, 3.73, 3.75, 4.20, 4.22, 4.29

1. Relational Model and Normal Form

1.1. Introduction

This paper is concerned with the application of elementary relation theory to systems which provide shared access to large banks of formatted data. Except for a paper by Childs [1], the principal application of relations to data systems has been to deductive question-answering systems. Levein and Maron [2] provide numerous references to work in this area.

In contrast, the problems treated here are those of data independence—the independence of application programs and terminal activities from growth in data types and changes in data representation—and certain kinds of data inconsistency which are expected to become troublesome even in nondeductive systems. The relational view (or model) of data described in Section 1 appears to be superior in several respects to the graph or network model [3, 4] presently in vogue for noninferential systems. It provides a means of describing data with its natural structure only—that is, without superimposing any additional structure for machine representation purposes. Accordingly, it provides a basis for a high level data language which will yield maximal independence between programs on the one hand and machine representation and organization of data on the other.

A further advantage of the relational view is that it forms a sound basis for treating derivability, redundancy, and consistency of relations—these are discussed in Section 2. The network model, on the other hand, has spawned a number of contissions, not the least of which is mistaking the derivation of connections for the derivation of relations (see remarks in Section 2 on the "connection tran").

Finally, the relational view permits a clearer evaluation of the scope and logical limitations of present formatted data systems, and also the relative merits (from a logical standpoint) of competing representations of data within a single system. Examples of this clearer perspective are cited in various parts of this paper. Implementations of systems to support the relational model are not discussed.

1.2. Data Dependencies in Present Systems

The provision of data description tables in recently developed information systems represents a major advance toward the goal of data independence [5, 6, 7]. Such tables callitate changing certain characteristics of the data representation stored in a data bank. However, the variety of data representation characteristics which can be changed without loyically impairing owne application programs is still quite limited. Further, the model of data with which users interest it still cluttered with representational properties, particularly in regard to the representation of collections of data (as opposed to individual items). Three of the principal kinds of data dependencies which still need to be removed are: ordering dependence, in some systems these dependence and access path dependence. In some systems these dependencednence, are not clearly searable from one another.

1.2.1. Ordering Dependence. Elements of data in a data bank may be stored in a variety of ways, some involving no concern for ordering, some permitting each element to participate in one ordering only, others permitting each element to participate in several orderings. Let us consider those existing systems which either require or permit data elements to be stored in at least one total ordering which is closely associated with the hardware-determined ordering of addresses. For example, the records of a file concerning parts might be stored in ascending order by part serial number. Such systems normally permit application programs to assume that the order of presentation of records from such a file is identical to Gr is a subordering of) the

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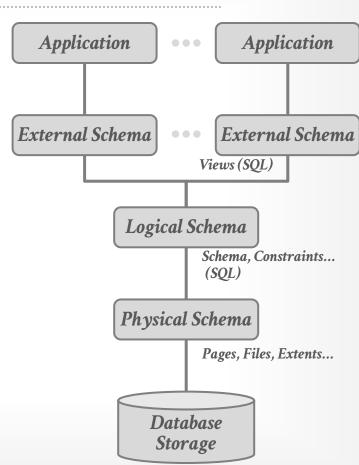
DATA INDEPENDENCE

Isolate the user/application from low-level data representation.

→ The user only worries about high-level application logic.

DBMS optimizes the layout according to operating environment, database contents, and workload.

→ Re-optimize if/when these factors changes.



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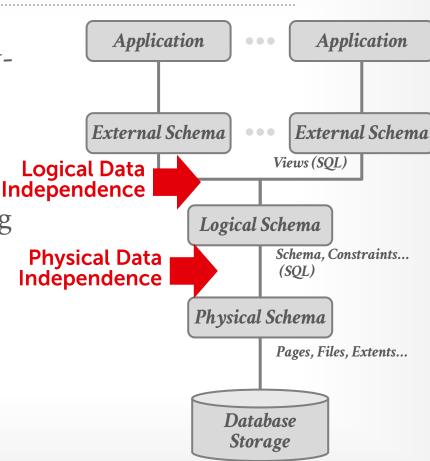
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Isolate the user/application from low-level data representation.

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DBMS optimizes the layout according to operating environment, database contents, and workload.

→ Re-optimize if/when these factors changes.



RELATIONAL MODEL



A <u>relation</u> is an unordered set that contain the relationship of attributes that represent entities.

A <u>tuple</u> is a set of attribute values (aka its **domain**) in the relation.

- → Values are (normally) atomic/scalar.
- → The special value NULL is a member of every domain (if allowed).

Artist(name, year, country)

name	year	country
Wu-Tang Clan	1992	USA
Notorious BIG	1992	USA
GZA	1990	USA

n-ary Relation

Table with *n* columns

RELATIONAL MODEL: PRIMARY KEYS



A relation's **primary key** uniquely identifies a single tuple.

Some DBMSs automatically create an internal primary key if a table does not define one.

DBMS can auto-generation unique primary keys via an <u>identity column</u>:

- → **IDENTITY** (SQL Standard)
- → **SEQUENCE** (PostgreSQL / Oracle)
- → AUTO_INCREMENT (MySQL)

Artist(name, year, country)

name	year	country
Wu-Tang Clan	1992	USA
Notorious BIG	1992	USA
GZA	1990	USA

RELATIONAL MODEL: PRIMARY KEYS



A relation's **primary key** uniquely identifies a single tuple.

Some DBMSs automatically create an internal primary key if a table does not define one.

DBMS can auto-generation unique primary keys via an <u>identity column</u>:

- → **IDENTITY** (SQL Standard)
- → **SEQUENCE** (PostgreSQL / Oracle)
- → AUTO_INCREMENT (MySQL)

Artist(id, name, year, country)

id	name	year	country
101	Wu-Tang Clan	1992	USA
102	Notorious BIG	1992	USA
103	GZA	1990	USA

31

A foreign key specifies that an attribute from one relation maps to a tuple in another relation.



Artist(id, name, year, country)

id	name	year	country
101	Wu-Tang Clan	1992	USA
102	Notorious BIG	1992	USA
103	GZA	1990	USA

Album(id, name, artist, year)

id	name	artist	year
11	Enter the Wu-Tang	101	1993
22	St.Ides Mix Tape	???	1994
33	<u>Liquid Swords</u>	103	1995



ArtistAlbum(artist_id, album_id)

artist_id	album_id
101	11
101	22
103	22
102	22

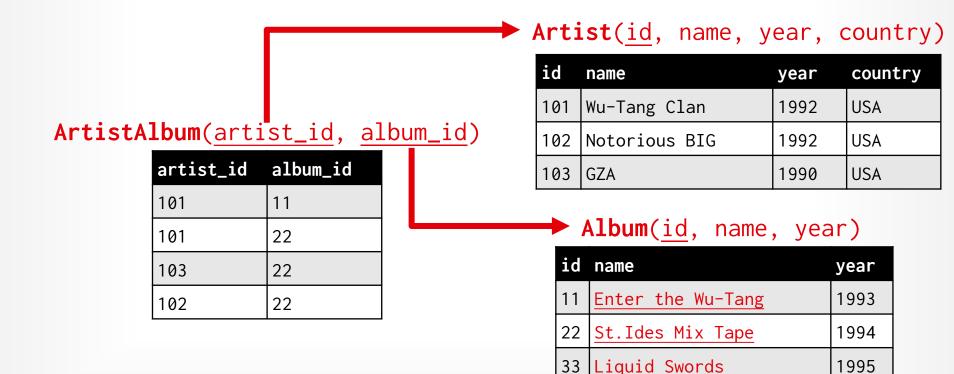
Artist(id, name, year, country)

id	name	year	country
101	Wu-Tang Clan	1992	USA
102	Notorious BIG	1992	USA
103	GZA	1990	USA

Album(id, name, artist, year)

id	name	artist	year
11	Enter the Wu-Tang	101	1993
22	St.Ides Mix Tape	???	1994
33	<u>Liquid Swords</u>	103	1995





RELATIONAL MODEL: CONSTRAINTS



User-defined conditions that must hold for any instance of the database.

- → Can validate data within a single tuple or across entire relation(s).
- → DBMS prevents modifications that violate any constraint.

Unique key and referential (fkey) constraints are the most common.

SQL:92 supports global asserts but these are rarely used (too slow).

Artist(id, name, year, country)

id	name	year	country
101	Wu-Tang Clan	1992	USA
102	Notorious BIG	1992	USA
103	GZA	1990	USA

```
CREATE TABLE Artist (
  name VARCHAR NOT NULL,
  year INT,
  country CHAR(60),
  CHECK (year > 1900)
);
```

RELATIONAL MODEL: CONSTRAINTS



User-defined conditions that must hold for any instance of the database.

- → Can validate data within a single tuple or across entire relation(s).
- → DBMS prevents modifications that violate any constraint.

Unique key and referential (fkey) constraints are the most common.

SQL:92 supports global asserts but these are rarely used (too slow).

Artist(id, name, year, country)

id	name	year	country
101	Wu-Tang Clan	1992	USA
102	Notorious BIG	1992	USA
103	GZA	1990	USA

```
CREATE TABLE Artist (
name VARCHAR NOT NULL,
year INT,
country CHAR(60),
CHECK (year > 1900)

CREATE ASSERTION myAssert
CHECK ( <SQL> );
```

33

DATA MANIPULATION LANGUAGES (DML)

The API that a DBMS exposes to applications to store and retrieve information from a database.

Procedural:

→ The query specifies the (high-level) strategy to find the desired result based on sets / bags.

← Relational Algebra

Non-Procedural (Declarative):

→ The query specifies only what data is wanted and not how to find it.

← Relational Calculus

RELATIONAL ALGEBRA



Fundamental operations to retrieve and manipulate tuples in a relation.

→ Based on set algebra (unordered lists with no duplicates).

Each operator takes one or more relations as its inputs and outputs a new relation.

→ We can "chain" operators together to create more complex operations.

- σ Select
- **π** Projection
- **U** Union
- Intersection
- Difference
- × Product
- Join

RELATIONAL ALGEBRA: SELECT



Choose a subset of the tuples in a relation satisfying selection predicate.

- → Predicate acts as a filter to retain only tuples that fulfill its qualifying requirement.
- → Can combine multiple predicates using conjunctions / disjunctions.

Syntax: $\sigma_{predicate}(R)$

 $R(a_{id},b_{id})$

a_id	b_id
a_1u	D_IU
a1	101
a2	102
a2	103
a3	104

$$\sigma_{a_id='a2'}(R)$$

a_id	b_id
a2	102
a2	103

$$\sigma_{a_id=a_2'} \Lambda_{b_id>102}(R)$$

a_id	b_id
a2	103

RELATIONAL ALGEBRA: SELECT



Choose a subset of the tuples in a relation satisfying selection predicate.

- → Predicate acts as a filter to retain only tuples that fulfill its qualifying requirement.
- → Can combine multiple predicates using conjunctions / disjunctions.

Syntax: $\sigma_{predicate}(R)$

 $R(a_id,b_id)$

<u>/ – </u>
b_id
101
102
103
104

 $\sigma_{a_id='a2'}(R)$ $a_id b_id$

a_id	b_id
a2	102
a2	103

 $\sigma_{a_id='a2'\wedge b_id>102}(R)$

a_id	b_id
a2	103

SELECT * FROM R
WHERE a_id='a2' AND b_id>102

RELATIONAL ALGEBRA: PROJECTION



Generate a relation with tuples that contains only the specified attributes.

- → Rearrange attributes' ordering.
- → Remove unwanted attributes.
- → Manipulate values to create derived attributes.

Syntax: $\Pi_{A1,A2,...,An}(\mathbf{R})$

$R(a_id,b_id)$

	•
a_id	b_id
a1	101
a2	102
a2	103
a3	104
	a1 a2 a2

$$\Pi_{b_id-100,a_id}(\sigma_{a_id-a2}(R))$$

b_id-100	a_id
2	a2
3	a2

RELATIONAL ALGEBRA: UNION



Generate a relation that contains all tuples that appear in either only one or both input relations, eliminating duplicates.

→ Both relations must have the same attributes (based on names).

Syntax: (R U S)

(SELECT	*	FROM R)	
UNION			
(SELECT	*	<pre>FROM S);</pre>	

R(a_id,b_id)

a_id	b_id
a1	101
a2	102
a3	103

S(a_id,b_id)

a_id	b_id
a3	103
a4	104
a5	105

$(R \cup S)$

a_id	b_id
a1	101
a2	102
a3	103
a4	104
a5	105

RELATIONAL ALGEBRA: INTERSECTION



Generate a relation that contains only the tuples that appear in both input relations.

 \rightarrow Both relations must have the same attributes (based on names).

Syntax: $(R \cap S)$

 $R(a_id,b_id)$ $S(a_id,b_id)$

a_id	b_id
a1	101
a2	102
a3	103

a_id	b_id
a3	103
a4	104
a5	105

 $(R \cap S)$

a_id	b_id
a3	103

```
(SELECT * FROM R)
(SELECT * FROM S);
```

RELATIONAL ALGEBRA: DIFFERENCE



Generate a relation that contains only the tuples that appear in the first and not the second of the input relations.

→ Both relations must have the same attributes (based on names).

Syntax: (R - S)

R(a_id,b_id)

a_id	b_id
a1	101
a2	102
a3	103

S(a_id,b_id)

a_id	b_id
a3	103
a4	104
a5	105

RELATIONAL ALGEBRA: DIFFERENCE



Generate a relation that contains only the tuples that appear in the first and not the second of the input relations.

→ Both relations must have the same attributes (based on names).

Syntax: (R - S)

(SELECT	*	FROM R)
EX	CE	PT
(SELECT	*	<pre>FROM S);</pre>

R(a_id,b_id)

a_id	b_id
a1	101
a2	102
a3	103

S(a_id,b_id)

a_id	b_id	
a3	103	
a4	104	
a5	105	

$$(R - S)$$

a_id	b_id
a1	101
a2	102

RELATIONAL ALGEBRA: PRODUCT



Generate a relation that contains all possible combinations of tuples from the input relations.

- → Input relations do not have to have the same attributes.
- → Output includes all the attributes from the input relations.

Syntax: (R × S)

SELECT * FROM R CROSS JOIN S;

SELECT * FROM R, S;

$R(a_id,b_id)$ $S(a_id,b_id)$

a_id	b_id
a1	101
a2	102
a3	103

<u> </u>	/ 	•
a_id	b_id	
a3	103	
a4	104	
a5	105	

$$(R \times S)$$

R.a_id	R.b_id	S.a_id	S.b_id
a1	101	a3	103
a1	101	a4	104
a1	101	a5	105
a2	102	a3	103
a2	102	a4	104
a2	102	a5	105
a3	103	a3	103
a3	103	a4	104
a3	103	a5	105

RELATIONAL ALGEBRA: JOIN



Generate a relation that contains all tuples that are a combination of two tuples (one from each input relation) with a common value(s) for one or more attributes.

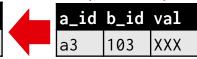
R(a_id,b_id)	S(a_id,	<pre>,b_id,val)</pre>
--------------	---------	-----------------------

a_id	b_id
a1	101
a2	102
a3	103

a_id	b_id	val
a3	103	XXX
a4	104	YYY
a5	105	ZZZ

 $(R \bowtie S)$

R.a_id	${\tt R.b_id}$	$S.a_id$	$S.b_id$	S.val
a3	103	a3	103	XXX



Syntax: (R ⋈ S)

RELATIONAL ALGEBRA: JOIN



Generate a relation that contains all tuples that are a combination of two tuples (one from each input relation) with a common value(s) for one or more attributes.

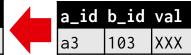
R(a_id,b_id)	S(a_id	l,b_id	,val)
--------------	--------	--------	-------

a_id	b_id
a1	101
a2	102
a3	103

a_id	b_id	val
a3	103	XXX
a4	104	YYY
a5	105	ZZZ

 $(R \bowtie S)$

R.a_id	${\tt R.b_id}$	5	7	_d	5	4	.d	S.val
a3	103	1			1	3		XXX



Syntax: (R ⋈ S)

RELATIONAL ALGEBRA: JOIN



Generate a relation that contains all tuples that are a combination of two tuples (one from each input relation) with a common value(s) for one or more attributes.

$R(a_id,b_id)$	S(a_id	<pre>,b_id,val)</pre>	
----------------	--------	-----------------------	--

a_id	b_id
a1	101
a2	102
a3	103

a_id	b_id	val
a3	103	XXX
a4	104	YYY
a5	105	ZZZ

 $(R \bowtie S)$

a_id	b_id	val
a3	103	XXX

Syntax: $(R \bowtie S)$

SELECT * FROM R **NATURAL JOIN** S;

SELECT * FROM R JOIN S USING (a_id, b_id);

SELECT * FROM R JOIN S
ON R.a_id = S.a_id AND R.b_id = S.b_id;

RELATIONAL ALGEBRA: EXTRA OPERATORS



```
Rename (ρ)
Assignment (R←S)
Duplicate Elimination (δ)
Aggregation (γ)
Sorting (τ)
Division (R÷S)
```

OBSERVATION



Relational algebra defines an ordering of the high-level steps of how to compute a query.

 \rightarrow Example: $\sigma_{b_id=102}(RMS)$ vs. $(RM(\sigma_{b_id=102}(S))$

A better approach is to state the high-level answer that you want the DBMS to compute.

→ Example: Retrieve the joined tuples from R and S where S.b_id equals 102.

RELATIONAL MODEL: QUERIES



The relational model is independent of any query language implementation.

SQL is the de facto standard (many dialects).

```
for line in file.readlines():
   record = parse(line)
   if record[0] == "GZA":
        print(int(record[1]))
```

```
SELECT year FROM artists
WHERE name = 'GZA';
```

DATA MODELS



```
← This Course
Relational
Key/Value
Graph
Document / JSON / XML / Object ← Leading Alternative
Wide-Column / Column-family
Array Vector, Matrix, Tensor) ← New Hotness
Hierarchical
Network
Semantic
Entity-Relationship
```

DATABASE SYSTEMS (FALL 2025)

DOCUMENT DATA MODEL



A collection of record documents containing a hierarchy of named field/value pairs.

- → A field's value can be either a scalar type, an array of values, or another document.
- → Modern implementations use JSON. Older systems use XML or custom object representations.

Avoid object-relational impedance mismatch by tightly coupling objects and database.



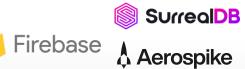
















pouchdb

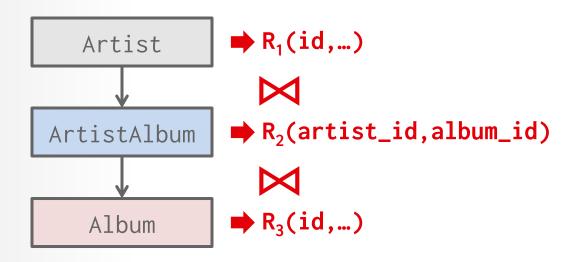






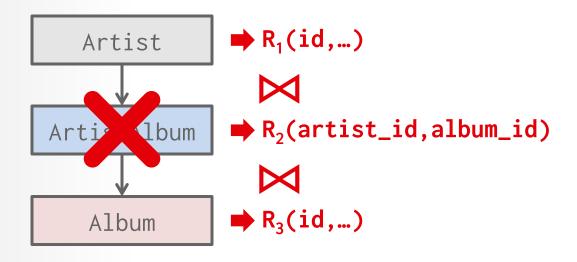
DOCUMENT DATA MODEL





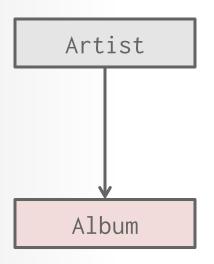
DOCUMENT DATA MODEL





DOCUMENT DATA MODEL





Application Code

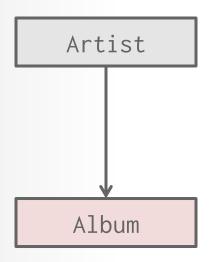
```
class Artist {
   int id;
   String name;
   int year;
   Album albums[];
class Album {
 int id;
  String name;
 int year;
```



```
"name": "GZA",
"year": 1990,
"albums": [
  "name": "Liquid Swords",
  "year": 1995
  },
   "name": "Beneath the Surface",
   "year": 1999
```

DOCUMENT DATA MODEL





Application Code

```
class Artist {
   int id;
   String name;
   int year;
   Album albums[];
class Album {
 int id;
  String name;
 int year;
```



```
"name": "GZA",
"year": 1990,
"albums": [
  "name": "Liquid Swords",
  "year": 1995
   "name": "Beneath the Surface",
   "year": 1999
```



One-dimensional arrays used for nearest-neighbor search (exact or approximate).

- → Used for semantic search on embeddings generated by MLtrained transformer models (think ChatGPT).
- → Native integration with modern ML tools and APIs (e.g., LangChain, OpenAI).

At their core, these systems use specialized indexes to perform NN searches quickly.























One-dimensional arrays used for nearest-neighbor search (exact or approximate).

- → Used for semantic search on embeddings generated by MLtrained transformer models (think ChatGPT).
- → Native integration with modern ML tools and APIs (e.g., LangChain, OpenAI).

At their core, these systems use specialized indexes to perform NN searches quickly.

























Album(id, name, year, lyrics)

id	name	year	lyrics
Id1	Enter the Wu-Tang	1993	<text></text>
Id2	Run the Jewels 2	2015	<text></text>
Id3	<u>Liquid Swords</u>	1995	<text></text>
Id4	We Got It from Here	2016	<text></text>



Album(id, name, year, lyrics)

id	name	year	lyrics
Id1	Enter the Wu-Tang	1993	<text></text>
Id2	Run the Jewels 2	2015	<text></text>
Id3	<u>Liquid Swords</u>	1995	<text></text>
Id4	We Got It from Here	2016	<text></text>



Embeddings

```
Id1 → [0.32, 0.78, 0.30, ...]
Id2 → [0.99, 0.19, 0.81, ...]
Id3 → [0.01, 0.18, 0.85, ...]
Id4 → [0.19, 0.82, 0.24, ...]
```





Album(id, name, year, lyrics)

id	name	year	lyrics
Id1	Enter the Wu-Tang	1993	<text></text>
Id2	Run the Jewels 2	2015	<text></text>
Id3	<u>Liquid Swords</u>	1995	<text></text>
Id4	We Got It from Here	2016	<text></text>



Embeddings

```
Id1 → [0.32, 0.78, 0.30, ...]

Id2 → [0.99, 0.19, 0.81, ...]

Id3 → [0.01, 0.18, 0.85, ...]

Id4 → [0.19, 0.82, 0.24, ...]
```

Query

Find albums with lyrics about running from the police





Album(<u>id</u>, name, year, lyrics)

id	name	year	lyrics
Id1	Enter the Wu-Tang	1993	<text></text>
Id2	Run the Jewels 2	2015	<text></text>
Id3	<u>Liquid Swords</u>	1995	<text></text>
Id4	We Got It from Here	2016	<text></text>

OpenAl Hugging Face Transformer

Embeddings

Id1 → [0.32, 0.78, 0.30, ...]

Id2 → [0.99, 0.19, 0.81, ...]

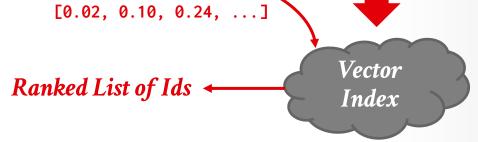
Id3 → [0.01, 0.18, 0.85, ...]

Id4 → [0.19, 0.82, 0.24, ...]

Query

Find albums with lyrics about

running from the police



<u>HNSW</u>, IVFFlat <u>Meta Faiss</u>, <u>Spotify Annoy</u>, <u>Microsoft DiskANN</u>



Album(id, name, year, lyrics)

id	name	year	lyrics
Id1	Enter the Wu-Tang	1993	<text></text>
Id2	Run the Jewels 2	2015	<text></text>
Id3	<u>Liquid Swords</u>	1995	<text></text>
Id4	We Got It from Here	2016	<text></text>



Embeddings

```
Id1 → [0.32, 0.78, 0.30, ...]

Id2 → [0.99, 0.19, 0.81, ...]

Id3 → [0.01, 0.18, 0.85, ...]

Id4 → [0.19, 0.82, 0.24, ...]
```





Album(id, name, year, lyrics)

id	name	year	lyrics
Id1	Enter the Wu-Tang	1993	<text></text>
Id2	Run the Jewels 2	2015	<text></text>
Id3	<u>Liquid Swords</u>	1995	<text></text>
Id4	We Got It from Here	2016	<text></text>



Embeddings

```
Id1 → [0.32, 0.78, 0.30, ...]

Id2 → [0.99, 0.19, 0.81, ...]

Id3 → [0.01, 0.18, 0.85, ...]

Id4 → [0.19, 0.82, 0.24, ...]

:
```

Query

Find albums with lyrics about running from the police and released after 2005

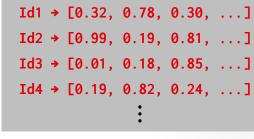


Transformer



Album(id, name, year, lyrics)

id	name	year	lyrics
Id1	Enter the Wu-Tang	1993	<text></text>
Id2	Run the Jewels 2	2015	<text></text>
Id3	<u>Liquid Swords</u>	1995	<text></text>
Id4	We Got It from Here	2016	<text></text>

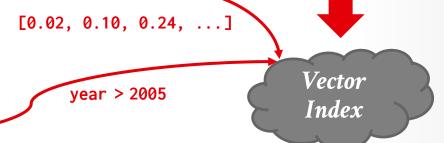


Query

Find albums with lyrics about

running from the police

and released after 2005



Album(id, name, year, lyrics)

id	name	year	lyrics
Id1	Enter the Wu-Tang	1993	<text></text>
Id2	Run the Jewels 2	2015	<text></text>
Id3	<u>Liquid Swords</u>	1995	<text></text>
Id4	We Got It from Here	2016	<text></text>







Embeddings

 $Id1 \rightarrow [0.32, 0.78, 0.30, ...]$ $Id2 \rightarrow [0.99, 0.19, 0.81, ...]$ $Id3 \rightarrow [0.01, 0.18, 0.85, ...]$ $Id4 \rightarrow [0.19, 0.82, 0.24, ...]$

Query

Find albums with lyrics about

running from the police

and released after 2005



[0.02, 0.10, 0.24, ...]



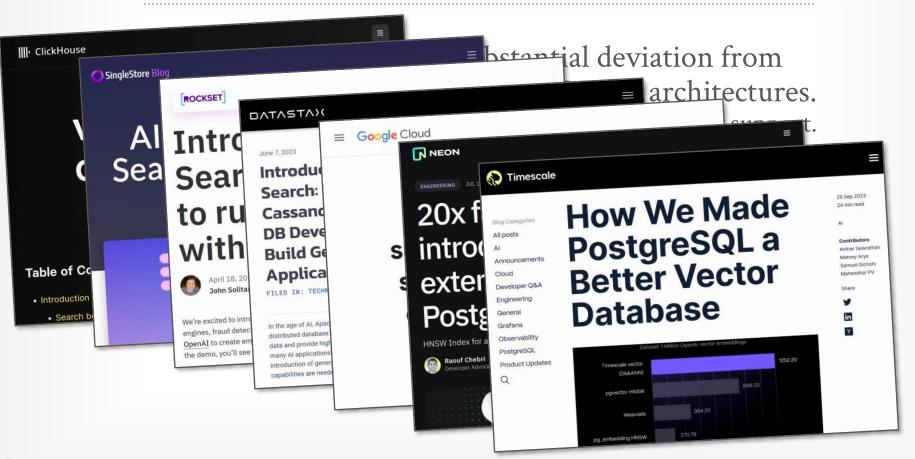


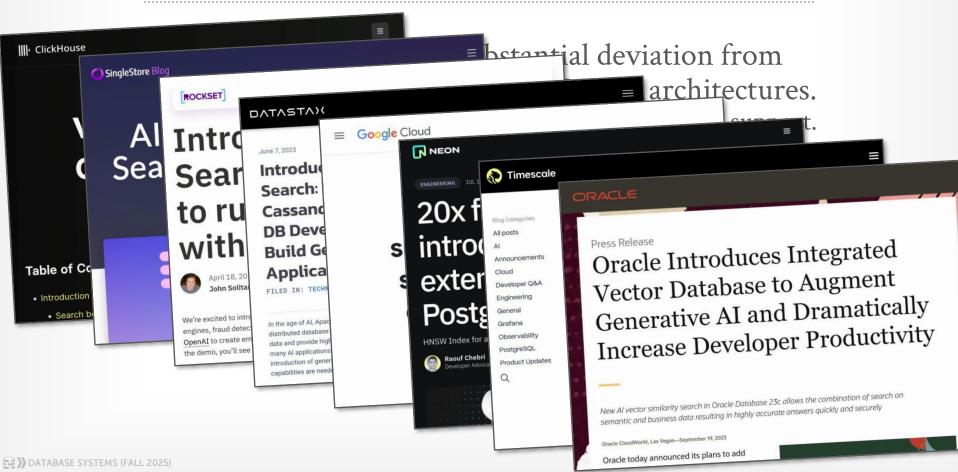
The vector model is <u>not</u> a substantial deviation from existing models that requires new DBMS architectures.

→ Every major DBMS now provides native vector index support.

Vector DBMSs offer better integration with AI tooling ecosystem (e.g., OpenAI, LangChain).







CONCLUSION



Databases are the most important and beautiful software in all of computer science.

Relational algebra defines the primitives for processing queries on a relational database.

We will see relational algebra again when we talk about query optimization + execution.

NEXT CLASS



Modern SQL

→ Make sure you understand basic SQL before the lecture.