

# Temporal Development of Overlapping Communities in Co-Authorship Networks

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**Abstract.** The abstract should summarize the contents of the paper and should contain at least 70 and at most 150 words.

**Keywords:** co-authorship networks, scientific collaboration, overlapping community detection, temporal analysis, clique percolation

## 1 Introduction and Motivation

A social network is a representation of a social structure consisting of actors (e.g. individuals, affiliations) and their social interactions. The network model conceptualizes (e.g. social, economic, political) structure as lasting pattern of interactions between actors. [?]

A co-authorship networks is a (scientific) collaboration network, where actors are scientists and the social interaction is the collaboration work resulting in publishing a paper together. Networks are in mathematical terms graphs, so they consist of nodes (vertex) and links (edges). In a co-authorship graph nodes are individual authors who are considered connected if they have co-authored a paper. [?]

Usually those networks are used to study patterns of scientific collaboration in different fields, their development over time, identify key people regarding different measurement, detection and evolution of communities and many more. [?, ?, ?]

This seminar paper is motivated by a dataset consisting of 2277 publications related to geochemistry about stable metal isotopes (calcium, magnesium, lithium, silicon, etc.). This data was used to build an accumulated co-authorship network with a very simple approach to model collaboration: The dataset was collected using the bibliographic database scopus. It is an almost complete dataset starting from 1997/98 until 2014. The dataset consists of authors and co-authors, title, year, abstract and affiliations. Basic network analysis had already been conducted for example centrality measurement, community detection, degree distribution and average degree.

As the dataset includes temporal information, the question arises how do communities within this type of co-authorship network evolve over time. In geochemistry people often tend to focus on one isotope system, and stick to this topic for a long time. Do researchers in geochemistry really focus on one topic or do they change the field over time?

The goal of this seminar paper therefore is to find and explain an already established and well understood/studied method for analysing the temporal development of communities within co-authorship networks. The main point is to understand the underlying algorithm in detail and to find out implications and challenges when applying this method to the geochemistry dataset.

## 2 Related Work

community detection in complex and large graphs  
 different methods of community detection algorithms  
 communities in collaboration networks (co-authorship)  
 temporal aspects of communities  
 time slicing and dividing the network into snapshots  
 algorithms and implementations  
 focus not on different time slicing methods but more on community detection  
 references to temporal stuff [TODO cite moody]

### 2.1 Scientific collaboration

TODO definition [TODO cite]  
 based on who borad or narrow the definition of collaboration is, a function/measurement for edgeweight can be chosen

### 2.2 Co-Authorship Network

short summary networks, social networks, SNA, co-authorship networks  
 references to who studied those networks [Newmann and Barabasi] just references  
 nodes are authors, links are collaboration, wights for nodes and edges

### 2.3 Communities in Social Networks

Definition of communities, there is no proper definition yet  
 [?]  
<http://www.ams.org/notices/200909/rtx090901082p.pdf>  
 [?]  
 [?]  
[https://en.wikipedia.org/wiki/Community\\_structure](https://en.wikipedia.org/wiki/Community_structure) most real world networks contain parts in which nodes are more highly conected to each other than to the rest of the network, those sets are usually called clusters, communities, cohesive groups or modeuls [TODO cite], they have no wiedely accepted unique definition. the detection algorithms mainly define what a community is.

**Community Detection Algorithms in General** [TODO short summary with further readings] maybe short classification of algorithms from [?]  
 maybe there are so many approaches  
 divisive and agglomerative methods

**Detecting Overlapping Communities** Need for detecting overlapping communities in co-authorship networks [TODO - find citation]

[However, in real graphs vertices are often shared between communities (Section 2), and the issue of detecting overlapping communities has become quite popular in the last few years. We devote this section to the main techniques to detect overlapping communities. [?]]

One popular method for detecting overlapping communities is the *clique percolation method* introduced by Palla et. al in 2005 [?]

Other methods are summarized by Fortunato [?] starting page 131 [TODO summary with references]

### 3 Clique Percolation Method

The clique percolation method (CPM) developed by Palla et. al [?] is used to identify overlapping communities in unweighted and undirected networks. The authors tested and evaluated their method using a co-authorship network, protein-protein interaction network, word associations network and a network constructed from variables within the source code of an open source ftp program. The following section explains the algorithm and summarizes the main findings of the paper regarding co-authorship networks.

The community definition the algorithm implies, relies on the fact that a community consists of fully connected subgraphs, called *cliques*, that share many nodes. *k-cliques* are fully connected subgraphs with  $k$  nodes. In figure 2 example of  $\{3, 4\}$ -cliques are given. A community, in this context, is called a *k-clique-community*, which is defined as a union of all  $k$ -cliques, which can be reached from each other through a number of *adjacent k-cliques*. Two  $k$ -cliques are called adjacent if they share  $k - 1$  nodes. In figure 3 the so called  $k$ -clique template rolling is visualized for a 3-clique. A  $k$ -clique-community is best described by relocating one node until all nodes of the community are discovered.

#### 3.1 Algorithm

The starting point for the algorithm is a undirected and unweighted graph. In section 3.2 we talk about generating the unweighted co-authorship graph. Based on the explained community definition the algorithm consists of the following steps:

- (1) Find all *maximal cliques*.
- (2) Prepare clique-clique overlap matrix (clique graph).
- (3) Threshold the matrix with a certain  $k$ .
- (4) Each connected component in the clique graph form a  $k$ -clique-community.

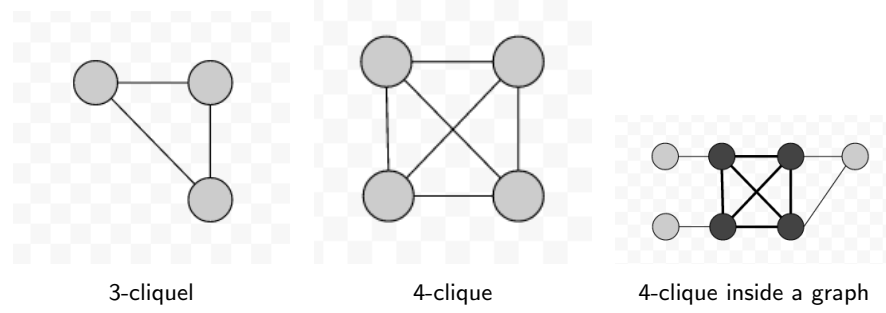


Fig. 2: Example of cliques

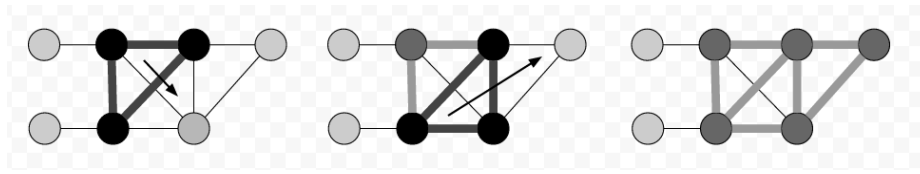


Fig. 3: Template rolling for a 3-clique

**(1) Find all maximal cliques** Instead of finding all  $k$ -cliques (it is a polynomial problem) as a first step. The algorithm searches for all maximal cliques, this can be done in exponential time the authors state. Maximal cliques cannot be subsets of larger cliques. That is why they are detected in decreasing order of their size. The largest possible clique size  $s_{max}$  is determined by the maximal degree  $d_{max}$  found in the network.

- (1) Determine  $s = s_{max}$ .
- (2) Repeatedly choose a node  $v$  from the graph:
  - (2.1) Extract all cliques of size  $s$  containing  $v$ .
  - (2.2) Delete the node and its edges.
- (3) When no nodes are left set  $s = s - 1$  and start with (2) on the original graph.

The set of already found cliques do influence the found cliques in later steps, as the later found cliques are smaller. The detailed algorithm for step (2.1) finding cliques of size  $s$  of  $v$  can be looked up in supplementary material to the paper on section 1.1.2, page 3. The result of this step is a set of all maximal cliques, an example can be seen in figure 1a. This set contains  $n_c$  cliques.

**(2) Prepare clique-clique overlap matrix (clique graph)** The dimension of the overlap matrix is  $n_c \times n_c$ . Each row and column represent a clique. All non-diagonal matrix elements represent the number of nodes those cliques share. The diagonal entries represent the size of the corresponding clique. This matrix is only generated once. In figure 5 the clique-clique overlap matrix of graph in figure 4 and the corresponding clique graph is shown.

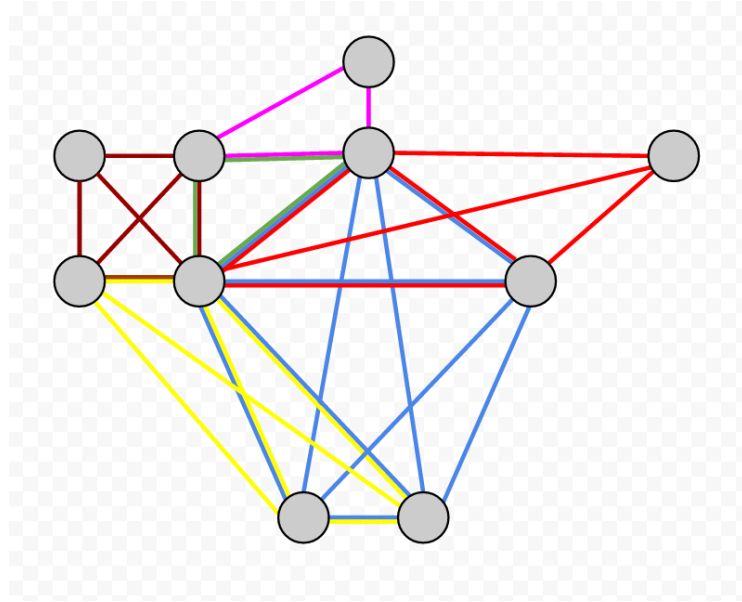


Fig. 4: All maximal cliques in a graph. Each color represents a maximal clique.

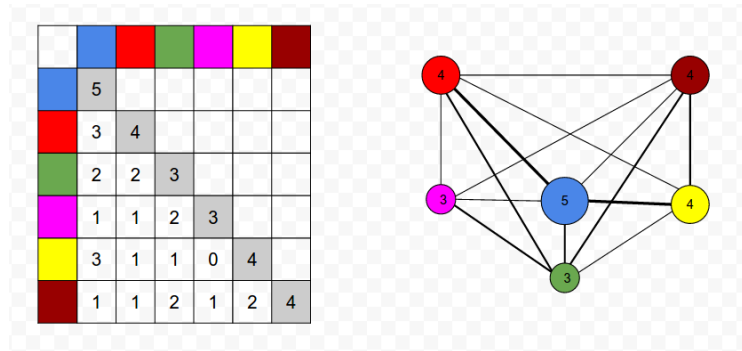


Fig. 5: The clique-clique overlap matrix and the corresponding clique graph. The number inside the node is the number of nodes belonging to that community, the width of the edge represents the shared nodes.

**(3) Threshold the matrix** All off-diagonal entries smaller than  $k - 1$  and diagonal entries smaller than  $k$  are set to 0. The remaining elements are set to 1, resulting in a binary matrix, representing a network of cliques. The thresholding can be done repeatedly without calculating a new matrix. See figure 6.

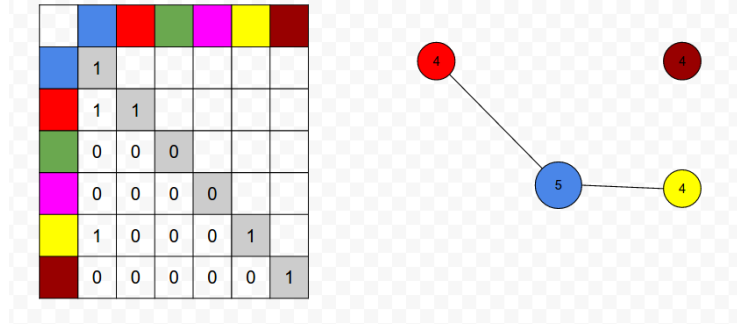


Fig. 6: This is the resulting clique graph after the thresholding with  $k = 4$ . All connected components do represent a  $k$ -clique-community. There are only two communities.

**(4) All connected components represent a community** In the resulting graph we just need to look for connected components, those represent the  $k$ -clique-communities. See figure 7.

### 3.2 Construction of the co-authorship and choosing the right $k$ and threshold.

The edge weights are calculated with  $n/(n - 1)$  with  $n$  as number of author per publication. This results in modeling the collaboration between fewer authors as more intensive than looking at a higher number of authors. To generate an unweighted network all edges need to be removed under a certain threshold  $w^*$ , all others are replaced by weight 1. Increasing  $w^*$  results in fewer edges throughout the network, but stronger links and fewer detected communities.

In addition a good  $k$  needs to be chosen. As  $k$  is increased the detected  $k$ -clique-communities shrink, but at the same time become more cohesive (stronger connected).

So a good balance between  $k$  and  $w^*$  needs to be found resulting in a community structure which is highly structured.

The authors state that  $k$  is usually between 3 and 6. If the number of links in a network are increased above some point a giant component evolves and covers the underlying community structure. So for each  $k$  the threshold  $w^*$  is lowered until the largest component becomes twice as big as the second largest component. For really choosing the best  $k$  some further characteristics describing the community

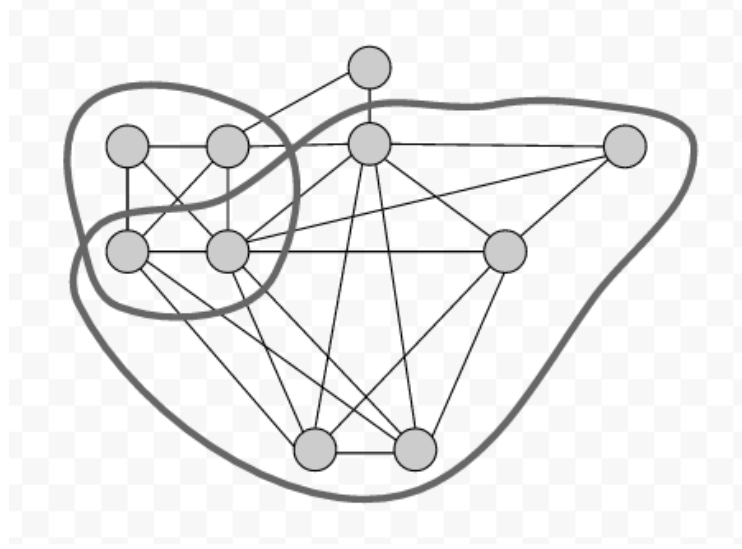


Fig. 7: Here the two resulting  $k$ -clique-communities are shown.

structure need to be investigated, see Supplementary Information of [?] for more details.

### 3.3 Summary of measured statistics

A node  $i$  of a network has the following characteristics:

**membership number**  $m_i$  ... number of communities the node belongs to

**overlap size**  $s_{\alpha,\beta}^{\text{ov}}$  ... between two communities  $\alpha$  and  $\beta$

**community degree**  $d_{\alpha}^{\text{com}}$  ... number of links a community  $\alpha$  has

**community size**  $s_{\alpha}^{\text{com}}$  ... number of nodes belonging to community  $\alpha$

The distributions of all characteristics are measured and analysed.

### 3.4 Main Findings of the paper

Overlaps in networks are significant. The distributions introduced in the paper (community size, community degree, overlap size, membership number) reveal universal features of networks. The network of communities has non-trivial correlations and specific scaling properties. [?]

## 4 Community Evolution based on CPM

Palla et. al [?] developed an algorithm based on clique percolation (see section 3) that allows the investigation of overlapping communities over time. They uncovered basic

relationships characterizing community evolution within a co-authorship network and a phone-call network. The following section describes the main steps of the algorithm. A short summary of their main findings can be found in section 4.4.

#### 4.1 Algorithm

The starting point of the algorithm is a set of undirected and unweighted graphs for each timestep. How to produce those temporal graphs is explained later in section 4.2. In general the algorithm uses the clique percolation method to find communities in each temporal graph, and matches the communities of consecutive timesteps.

- (1) Extract communities with CPM for each graph  $g_t$  at time step  $t$ .
- (2) Match set of communities at consecutive time steps of graph  $g_t, g_{t+1}$ , as follows:
  - (2.1) Construct joint graph  $g_{\cup} = g_t \cup g_{t+1}$ .
  - (2.2) Extract communities  $V$  with CPM in joint graph  $g_{\cup}$ .
  - (2.3) For each extracted community  $V_i$ :
    - Extract communities in  $g_t$  and  $g_{t+1}$  that are contained in  $V_i$ .
    - Calculate relative overlap for each pair.
    - Match communities in descending order.
- (3) Gap filling.

The following describe the steps (2) and (3) in detail. Step (1) was explained in section 3 already.

**Matching Communities** For matching communities, the *relative node overlap*  $C(A, B)$  between two nodes  $A$  and  $B$ , in a simple way, is defined as follows:

$$C(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

As overlapping communities are allowed the matching from consecutive time steps in descending order of their relative node overlap can lead to mismatching. For example when small communities gain a lot of members or vice versa. An example for this problem is given in figure X [TODO-PIC]. As a solution, for each time steps  $t$  and  $t + 1$  a joint graph  $g_{\cup}$  is constructed, containing all links from both networks. Let  $D$  be the set of communities at time step  $t$  and  $E$  the set of communities at time step  $t + 1$ . The set of communities from the joint graph  $g_{\cup}$  are extracted using CPM again and are called  $V$ . For any community  $D_i \in D$  or  $E_j \in E$  exactly one community  $V_k \in V$  can be found. For checking weather  $E_i$  or  $D_j$  is contained in  $V_k$  the links are compared instead of nodes. For each community  $V_i \in V$  the set of communities  $D_i^k \in D$  and  $E_j^k \in E$  contained in  $V_i$  are extracted. Now the relative node overlap between every possible pair can be calculated as follows

$$C_{i,j}^k = \frac{|D_i^k \cap E_j^k|}{|D_i^k \cup E_j^k|}$$



and the pairs can be matched in descending order.

In figures X three examples are given: figXa is a simple matching of a propagating community, figXb showing two merging communities with one community dying and figXc showing the splitting of a community into two communities with one community is new born.

**Gap Filling** In some cases a community which was disintegrated at a certain time step suddenly reappear in a later timestep, due to low publishing rates for example. That means a newborn community includes a formerly dead community. This problem is overcome by just filling the gap with the last step of the almost disintegrated community.

## 4.2 Construction of the temporal co-authorship network

Events in the co-authorship are paper publications. The social connection between people writing a paper together usually starts before the event and last for some time after the event. The higher the frequency the closer the relationship [?].

The edge weight resulting from one paper is  $n/(n-1)$  with  $n$  authors. The *link weight* between nodes  $a$  and  $b$  at a certain time  $t$  is calculated as

$$w_{a,b}(t) = \sum_i w_i e^{\frac{-\lambda|t-t_i|}{w_i}}$$

The summation runs over all collaboration event in which  $a$  and  $b$  are involved. The event  $i$  occurs at time  $t_i$  and the corresponding edge weight at this time is called  $w_i$ . This function which is a decay functions kind of models the strength of collaboration between authors over time considering all event ever occurred in time.

A threshold  $w^*$  is used to only include certain edges to the temporal graph. So for each timestep a graph can be constructed based on the collaboration strength per time step.

The authors of the paper used  $w^* = 1.0$  for the co-authorship network, they said nothing about  $\lambda$  at all. (maybe explain how  $w/k$  was chosen?)

The dataset contains 142 month of publications, but I could find in the paper how the data was aggregated, because in the figures it looks like 50 timesteps in total.

## 4.3 Summary of variables and measured statistics

For evaluating the quality the overall coverage of the community structure (ratio of nodes contained in at least one community) is measured. Also the distribution of community size is measured.

## 4.4 Main Findings of the paper

The paper summarized differences between large and small communities and their development over time. Small communities live longer if the members stay the same

over time. If members in small communities change frequently, they only live for a short time. Large communities live longer if members are changed permanently, if members stay the same they die quickly.

## 5 Implications regarding my dataset

what are problems with this method and what are implications regarding my dataset and network

I need a new function for calculating edge weights

find out what time slices are possible in my data[TODO]

try out CPM (there is an implementation in R) with static network

calculate measurement to see if quality of communities is good, find out right  $k$  and  $w^*$ , maybe use the one from paper but, evaluate if those are good

create snapshots of the paper and try if provided function for calculating weight at timesteps can be used

find an implementation of the community mapping for each time step

say how big is my data

what about time complexity

## 6 Future Work

find out what other methods are out there with implementations, I dont have the time to implement a cutting edge algorithm

compare my network/community structure to null-model(or other) or to other social network to see if everything is nice

maybe some time should be invested for finding a good representation for edge weights calculation, related to collaboration definition

next time choose a less complicated topic

how to visualize the development?