Restrictions Search for Panel Vector Autoregressive Models

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Abstract

The paper introduces the stochastic search variable selection for panel vector autoregressive models (SSVSP). The proposed selection prior allows for a data-based restriction search ensuring the estimation-feasibility. The SSVSP differentiates between domestic and foreign variables, thereby allowing a flexible panel structure and extending Koop and Korobilis's S^4 to a restriction search on single elements. Absent a matrix structure for restrictions, a Monte Carlo simulation shows that SSVSP outperforms S^4 in terms of deviation from the true values. Furthermore, a forecast exercise for G7 countries demonstrates that forecast performance improves for SSVSP focusing on sparsity in form of no dynamic interdependencies.

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1. Introduction

Intensifying international goods and knowledge flows, as well as trade agreements, demonstrate the importance of international interdependencies among economies. With these inter-linkages empirical analyses require taking both the connections and heterogeneities across countries into account. Recent literature stresses the benefits of including a global dimension while forecasting national and international key macroeconomic variables. Studies using factor models with global factors or multi-country models which account for international linkages provide evidence on improved forecast performance.[1] Similar, structural spillover analyses disregarding country specific information and global dependencies could end up with biased results regarding spillover effects and transmission channels.[2]

One tool that is able to consider dynamic and static global interdependencies as well as cross-section heterogeneities is the unrestricted panel vector autoregressive (PVAR) model. A PVAR includes several countries and country-specific variables in one model. Thus, lagged foreign variables can impact domestic variables, meaning that dynamic interdependencies exist. Static interdependencies between two variables of two countries occur if the covariance between the two is unequal to zero. Finally, the PVAR accounts for heterogeneity across countries since the coefficient matrices can vary across economies. This strength of PVARs comes at the cost of a large number of parameters to estimate - usually set against a relatively low number of time series observations for macroeconomic variables. To overcome this problem, the researcher has to set restrictions on the PVAR.

This paper conducts a data-based restriction search for dynamic and static interdependencies and cross-section heterogeneities specifying a selection prior for PVAR models. The prior differentiates between domestic and foreign variables for each country. This extends the selection prior for PVAR models of Koop and Korobilis (2015b), which is called stochastic search specification selection (S^4), from a matrix wide search to single elements, as George et al. (2008) do in their stochastic search variable selection (SSVS) for VAR models. In order to distinguish the algorithm from S^4 and SSVS, the algorithm is called stochastic search variable selection for PVAR models (SSVSP).

This paper adds to the selection prior literature, started by George and McCulloch (1993) who develop a selection prior for multiple regression models. Based on a hierarchical prior variables are selected which are included in the model. George et al. (2008) extend the SSVS to the use for VAR models. Korobilis (2008) and Jochmann et al. (2010) demonstrate that forecast performance is improved when using SSVS for VAR models. Subsequently, Korobilis (2013) extends the selection priors further to nonlinear set-ups. Koop and Korobilis (2015b) develop a selection prior for PVAR models. Their stochastic search specification selection builds closely on George et al. (2008) but adds a restriction search for homogeneity of domestic autoregressive coefficients across countries. Further,

[1]Ciccarelli and Mojon (2010) and Bjørnland et al. (2017) use a factor model for inflation and GDP forecasts, respectively. Koop and Korobilis (2015a) indicate that using a panel VAR, estimated by a factor approach, for forecasting key macroeconomic indicators of Euro zone countries can lead to improvements in forecasts. Pesaran et al. (2009) provide evidence that forecasts based on global VAR (GVAR) models outperform forecasts based on univariate models. Greenwood-Nimmo et al. (2012), Dovern et al. (2016), Huber et al. (2016), and Garratt et al. (2016) provide further evidence that GVAR models improve forecast performance relative to univariate benchmark models.

[2]Compare to Canova and Ciccarelli (2009), Lütkepohl (2014) and Georgiadis (2017).

in contrast to SSVS, they run the restriction search on whole matrices including all variables of one country.

The SSVSP extends the estimation procedure for PVAR models, contributing to the existing literature on PVARs. As yet, to overcome the curse-of-dimensionality problem in PVAR models the literature follows two approaches besides the selection prior of Koop and Korobilis (2015b): setting assumptions of either homogeneity, a lack of dynamic or static interdependencies or using the cross-sectional shrinkage approach proposed by Canova and Ciccarelli (2004) and Canova and Ciccarelli (2009) which reduces the number of coefficients to be estimated by generating lower-dimensional factors.[3]

By implementing their prior on country matrices, Koop and Korobilis (2015b) assume a specific panel structure; namely, all variables of one country are treated in a similar way: either restricted or not. Instead, the SSVSP allows for a less restrictive panel structure and conducts a restriction search for each variable. The underlying panel structure separates domestic and foreign variables, although foreign variables are not separated on a country basis.

This less restrictive panel structure has the advantages that, firstly, the SSVSP can provide evidence supporting the exclusion of a single lag of a variable. A clear ranking of posterior probabilities of which variables to include in the model and which coefficients are homogeneous can be developed. Using the S^4 prior of Koop and Korobilis (2015b) bases the decision of excluding a single variable on the results for a matrix-wide search. Secondly, compared to the commonly used Litterman prior for large Bayesian VAR models, which assumes a specific shrinkage depending on the lag number, the SSVSP takes the panel structure into account. Koop and Korobilis (2015b) as well as Korobilis (2016) provide evidence that a prior for PVAR models which accounts for the inherent panel dimension within the data improves forecast accuracy and reduces mean squared errors. In addition, in a set-up where country grouping for restrictions does not hold, Korobilis (2016) demonstrates that the absolute deviation from the true value is lower for SSVS than it is for S^4. This result contributes to the argument for a restriction search on single elements. Thirdly, the SSVSP prior has a wider range for empirical application than does the more rigid S^4. Applications, including financial and real variables, can especially benefit from a less restrictive form since the SSVSP can incorporate variable specific restrictions.

These advantages are reflected in the results of both a Monte Carlo simulation and a forecasting exercise. Firstly, the results of the Monte Carlo studies show that especially when a more flexible panel structure is present, the posterior estimates of the SSVSP deviate less from the true values than the ones of S^4. Furthermore, the SSVSP is accurate in the selection of the restrictions displayed in the posterior probabilities for no interdependencies and homogeneity. Secondly, the results of the empirical application demonstrate that forecast performance improves for the SSVSP specifications which focus on sparsity in form of no dynamic interdependencies.

The next section describes possible restrictions for PVAR models. Section 3 intro-

[3]Examples setting assumptions include Love and Zicchino (2006) assuming homogeneity and no dynamic interdependencies, and Ciccarelli et al. (2013), restricting for no dynamic interdependencies. Canova et al. (2012) and Ciccarelli et al. (2016) use the factor approach. Billio et al. (2014) build on the the cross-sectional shrinkage approach and expand it to a Markov-switching model. Koop and Korobilis (2015a) broaden it to time-varying parameter PVAR models additionally allowing for time-varying covariance matrices.

duces the stochastic search variable selection for PVARs. Next, the performance of the SSVSP is evaluated based on two Monte Carlo simulations and an empirical application is conducted. Finally, section 6 concludes.

2. PVAR Restrictions

A PVAR model for country i at time t with $i = 1, ..., N$ and $t = 1, ..., T$ is given by

$$y_{it} = A_{i1}Y_{t-1} + A_{i2}Y_{t-2} + ... + A_{iP}Y_{t-P} + u_{it}, \tag{1}$$

where $Y_{t-1} = (y'_{1t-1}, ..., y'_{Nt-1})'$ and y_{it} denotes a vector of dimension $[G \times 1]$ where the number of country-specific variables is defined as G.[4] All matrices A_{ip} have dimension $[G \times NG]$ for lag $p = 1, ..., P$. The index i denotes that the matrices are country specific for country i. The u_{it} are uncorrelated over time and normally distributed with mean zero and covariance matrix Σ_{ii}. The covariance matrix between errors of different countries is defined as $E(u_{it}u'_{jt}) = \Sigma_{ij} \;\; \forall \; i \neq j$ with dimension $[G \times G]$.

The PVAR model for all N countries can then be written as

$$Y_t = A_1Y_{t-1} + A_2Y_{t-2} + ... + A_PY_{t-P} + U_t. \tag{2}$$

The Y_t and U_t are $[NG \times 1]$-vectors. The U_t is normally distributed with mean zero and covariance matrix Σ that is of dimension $[NG \times NG]$. The $[NG \times NG]$-matrix A_p for one particular lag p is defined as

$$A_p = \begin{pmatrix} \alpha^{11}_{p,11} & \cdots & \alpha^{1k}_{p,1j} & \cdots & \alpha^{1G}_{p,1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \alpha^{l1}_{p,i1} & \cdots & \alpha^{lk}_{p,ij} & \cdots & \alpha^{lG}_{p,iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \alpha^{G1}_{p,N1} & \cdots & \alpha^{Gk}_{p,Nj} & \cdots & \alpha^{GG}_{p,NN} \end{pmatrix}.$$

The element $\alpha^{lk}_{p,ij}$ refers to the coefficient of lag p of variable k of country j in the equation of variable l of country i. Thus, it measures the impact of lag p of variable k of country j on variable l of country i.

A structural form of the PVAR model is derived by decomposing the covariance matrix Σ into $\Sigma = \Psi^{-1'}\Psi^{-1}$ where Ψ is a upper triangular matrix. Therefore, the structural identification is based on a recursive order. An element ψ^{lk}_{ij} of the upper triangular matrix Ψ defines the static relation between variable l of country i and variable k of country j.

This structural PVAR model can account for dynamic interdependencies (DI), static interdependencies (SI), and cross-section heterogeneities (CSH).[5] However, the large number of free parameters in the unrestricted PVAR requires some kind of model selection. A straightforward way is setting restrictions using the panel structure inherent in the data. It can be expected that interdependencies and heterogeneities across countries only exist for specific country and variable combinations. Therefore, for some coefficients the following restrictions can hold:

[4] Although this specification does not include a constant, it can be extended to include one.

[5] Canova and Ciccarelli (2013) provide a survey of the PVAR restrictions.

1. **No dynamic interdependencies**: no lagged impact from variable l of country i to variable k of country j for lag p if $\alpha^{lk}_{p,ij} = 0$ for $j \neq i$
2. **No static interdependencies**: no correlation between the error term of equation l of country i, u^l_{it}, with the error term of equation k of country j, u^k_{jt}, if $\psi^{lk}_{ij} = 0$ for $j \neq i$
3. **No cross-section heterogeneities**: homogeneous coefficient across the economies for lag p if $\alpha^{lk}_{p,jj} = \alpha^{lk}_{p,ii}$ for $j \neq i$

In total, $[(NG - G)NG]$ DI, $[(N(N-1)/2)G^2]$ SI, and $[(N(N-1)/2)G^2]$ CSH restrictions can be defined.[6] The essential part is to determine for which country and variable combinations these restrictions hold.

Large Bayesian VAR (BVAR) and global VAR (GVAR) models are other potential tools to account for international spillovers.[7] However, BVARs neglect the existence of a panel dimension in the data. BVARs usually assume identical priors for each country, thus, being especially applicable for analyzing intra-country spillovers, including a large number of variables. GVARs, however, are restrictive by imposing a particular interdependencies structure by the chosen weights for aggregating the foreign component. GVAR models are especially useful for studies focusing on aggregated impacts or on spillovers from one large economy. In contrast to macroeconomic panel regressions, PVAR models allow to focus on effects on each single country and variable in the structural analysis while standard panel regressions deliver averaged or pooled results.

3. Selection Prior for PVAR

The unrestricted PVAR model with one lag can be rewritten as

$$Y_t = Z_{t-1}\alpha + U_t, \tag{3}$$

where α is the vectorized matrix A and $Z_{t-1} = (I_{NG} \otimes Y_{t-1})$. The model is simplified to a model including only one lag. Otherwise, the restriction search for dynamic interdependencies would provide guidance for lag length selection.

Each element of α is drawn from a mixture of two normal distributions centering around the restriction, either with a small or large variance. The coefficient either shrinks to the restriction (small variance case) or is estimated with a looser prior (larger variance case). Thus, the algorithm imposes soft restrictions by allowing for a small variance. In contrast to Koop and Korobilis (2015b), the restriction search is completed for each single element and not on whole matrices that include all variables for a given country.

The SSVSP algorithm has specific priors for the parameters of A_1 and for the covariance matrix building with the DI, SI, and CSH restrictions. The DI restrictions impose limits on the coefficients of the lagged foreign endogenous variables. The DI prior is given by

$$\begin{aligned} \alpha^{lk}_{ij} \mid \gamma^{lk}_{DI,ij} &\sim (1 - \gamma^{lk}_{DI,ij})\mathcal{N}(0, \tau_1^2) + \gamma^{lk}_{DI,ij}\mathcal{N}(0, \tau_2^2) \\ \gamma^{lk}_{DI,ij} &\sim Bernoulli(\pi^{lk}_{DI,ij}). \end{aligned}$$

[6] Note that while SI restrictions are symmetric, this is not (necessarily) the case for DI restrictions.

[7] Detailed descriptions of the two models are in Banbura et al. (2010), Pesaran et al. (2004), and Dees et al. (2007).

The prior distribution of α_{ij}^{lk} is conditional on the hyperparameter $\gamma_{DI,ij}^{lk}$ which is Bernoulli distributed. If $\gamma_{DI,ij}^{lk}$ is equal to zero, α_{ij}^{lk} is drawn from the first part of the normal distribution with mean zero and variance τ_1^2.[8] If $\gamma_{DI,ij}^{lk}$ is equal to one, α_{ij}^{lk} is drawn from the second part of the normal distribution with mean zero and variance τ_2^2. The values of τ_1^2 and τ_2^2 must be chosen such that τ_1^2 is smaller than τ_2^2. Thus, if $\gamma_{DI,ij}^{lk} = 0$, the prior is tight in the sense that the parameter is shrunk to zero and no dynamic interdependency is supported by the data. Whereas the prior is loose for $\gamma_{DI,ij}^{lk} = 1$.

The SI prior is set on the elements of the upper triangular matrix, Ψ. If SI restrictions are found, the structural PVAR is overidentified since additional zero restrictions can be set on top of the recursive ordering. A clear advantage of this decomposition is that it assures that by construction every simulated Σ is positive definite.[9]

The prior for SI restrictions follows the same logic as the DI prior:

$$\begin{aligned}\psi_{ij}^{lk} \mid \gamma_{SI,ij}^{lk} &\sim (1-\gamma_{SI,ij}^{lk})\mathcal{N}(0,\kappa_1^2) + \gamma_{SI,ij}^{lk}\mathcal{N}(0,\kappa_2^2)\\ \gamma_{SI,ij}^{lk} &\sim Bernoulli(\pi_{SI,ij}^{lk}).\end{aligned}$$

The prior is for all $j \neq i$. To assure positive variance elements, the $(\psi_{ii}^{kk})^2$ are gamma distributed, $(\psi_{ii}^{kk})^2 \sim \mathcal{G}(a,b)$. The elements for the same country, ψ_{ii}^{lk} for $l \neq k$, are normally distributed with mean zero and variance κ_2^2. All ψ_{ij}^{lk} elements ($j \neq i$) are drawn from the specified hierarchical prior. The parameter κ_1^2 is smaller than κ_2^2. If $\gamma_{SI,ij}^{lk}$ is equal to zero, the parameter shrinks to zero showing that the data do not support static interdependency. The selection prior can be used to estimate the reduced form of a PVAR model by not applying a Cholesky decomposition and not searching for static interdependency restrictions but by assuming a standard distribution for the PVAR variance, e.g. an inverse Wishart distribution.

Searching for homogeneity across countries is not as straightforward as searching for the zero restrictions for dynamic and static interdependencies. The main contribution of Koop and Korobilis (2015b) is the development of a procedure how to search for CSH restrictions. Possible homogeneity across countries is assessed for the coefficients measuring the impact of domestic variables on variables of the same country. The CSH prior is given by

$$\begin{aligned}\alpha_{jj}^{lk} \mid \gamma_{CSH}^{w} &\sim (1-\gamma_{CSH}^{w})\mathcal{N}(\alpha_{ii}^{lk},\xi_1^2) + \gamma_{CSH}^{w}\mathcal{N}(0,\xi_2^2)\\ \gamma_{CSH}^{w} &\sim Bernoulli(\pi_{CSH}^{w}).\end{aligned}$$

The prior is for all $j \neq i$. There are $(N(N-1)/2)G^2 = K$ combinations of coefficients that are checked for homogeneity. The index $w = 1, ..., K$ refers to a specific combination. Again, ξ_1^2 is smaller than ξ_2^2. The main difference to the DI and SI prior is that instead of shrinking the parameter to zero in the first part of the normal distribution, the mean is equal to the coefficient for which homogeneity is being checked, $\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$ in mean.

To be able to check all possible combinations, the procedure of Koop and Korobilis (2015b) is followed, who define a selection matrix $\Gamma = \prod_{w=1}^{K} \Gamma_w$. The matrix Γ_w is an identity matrix of dimension $[NG \times NG]$ with two exceptions. The diagonal element at

[8] How $\pi_{DI,ij}^{lk}$ is set is described in detail in the appendix. This holds also for the CSH and SI priors.

[9] Compare also to Koop and Korobilis (2015b).

the position α_{jj}^{lk} is set equal to γ_{CSH}^{w} and the off-diagonal element referring to the element α_{ii}^{lk} is set equal to $(1-\gamma_{CSH}^{w})$. If all coefficients are heterogeneous, all Γ_w are identity matrices. To impose the CSH restrictions, the posterior mean of α is multiplied by the selection matrix Γ.

To summarize, consider a simple 3-countries-2-variables example, the coefficients of A, which are checked for dynamic interdependencies, are marked with DI and the coefficients checked for homogeneity are marked with CSH. The elements of the covariance matrix which are checked for static interdependencies are marked with SI:

$$A=\begin{pmatrix} CSH & CSH & DI & DI & DI & DI \\ CSH & CSH & DI & DI & DI & DI \\ DI & DI & CSH & CSH & DI & DI \\ DI & DI & CSH & CSH & DI & DI \\ DI & DI & DI & DI & CSH & CSH \\ DI & DI & DI & DI & CSH & CSH \end{pmatrix}, \Psi=\begin{pmatrix} & & SI & SI & SI & SI \\ & & SI & SI & SI & SI \\ & & & & SI & SI \\ & & & & SI & SI \\ & & & & & \\ & & & & & \end{pmatrix}.$$

The prior specification has the advantage that it allows the usage of the Gibbs sampler to sample from the posterior distributions.[10] The means of the posterior distributions are used as point estimates for the coefficients.

The outcome of the algorithm can be interpreted in two ways: model selection and Bayesian model averaging.[11] The researcher can select one specific restricted PVAR model based on the posterior means of $\gamma_{DI}, \gamma_{SI}, \gamma_{CSH}$. The researcher can set the restrictions successively until the model with the best fit is found. In addition, a threshold value can be used, often set to 0.5 by the selection prior literature. If the posterior mean of γ is below the threshold value, the restriction is set. Alternatively, the outcome of the algorithm can be used as a Bayesian model averaging (BMA) result. Thus, the posterior means averaged over all draws are taken as coefficient estimates. Since each draw leads to a specific restricted model, the BMA results average over all possible restricted models.

4. Monte Carlo Simulation

4.1. Simulation Set-up

In order to evaluate the prior, two Monte Carlo simulations are conducted.[12] For both Monte Carlo simulations data are generated from a panel VAR model which includes three countries, two variables for each country, one lag, and 100 observations. Firstly, it is assumed that both dynamic and static interdependencies as well as cross-sectional heterogeneities exist for specific variable and country combinations. In particular, country 2 has a dynamic impact on country 1 and country 1 on country 3. Country 3 does not impact the other two countries dynamically. Coefficients are homogeneous between countries 2 and

[10] The Gibbs sampler algorithm is described in detail in the appendix.

[11] Compare to the general survey in Koop and Korobilis (2010) or the specific explanation for the S^4 in Koop and Korobilis (2015b).

[12] 100 samples, each with a length of 100 are simulated. The Gibbs sampler is done with 55000 draws, of which 5000 draws are disregarded as draws of the burn-in-phase. The calculation is based on a further development of the MATLAB code provided by Koop and Korobilis (2015b) (https://sites.google.com/site/dimitriskorobilis/matlab/panel_var_restrictions).

3. Static interdependencies exist between country 1 and 2. This example has a clear country grouping structure. A scenario like this is given by the first Monte Carlo simulation where the following parameter values are set:

$$A^{true} = \begin{pmatrix} 0.8 & 0 & 0.2 & 0.2 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0.3 & 0 & 0 \\ 0 & 0 & 0.6 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0.3 & -0.4 & 0 & 0 & 0.6 & 0.5 \\ 0.2 & 0.4 & 0 & 0 & 0 & 0.5 \end{pmatrix}, \Psi^{true} = \begin{pmatrix} 1 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 1 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Secondly, no clear country grouping for interdependencies and homogeneities is assumed. Hence, a less restrictive panel structure exists. The second Monte Carlo simulation incorporates these properties and has the following true parameters:

$$A^{true} = \begin{pmatrix} 0.8 & 0 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0.7 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0 & 0.6 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 \\ 0.3 & -0.4 & 0 & 0 & 0.6 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{pmatrix}, \Psi^{true} = \begin{pmatrix} 1 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The performance of SSVSP is compared to the performance of different prior specifications. A model with no restriction search is used as the benchmark mark model, referred to as unrestricted VAR. This model is estimated using the SSVSP prior but with fixed γ values such that each parameter is drawn from the distribution with the higher variance. Furthermore, the SSVSP is compared to the S^4 and to the SSVS of George et al. (2008).[13] The SSVS prior sets the DI prior on all lagged values and the SI prior on all covariance elements. Thus, no distinction between domestic and foreign variables is made. Additionally, two specifications of the SSVSP are analyzed. Firstly, the SSVSP only searches for DI restrictions (abbreviated with SSVSP_DI), meaning that the γ values for the SI and CSH priors are set to one. Secondly, the restriction search is only done for CSH restrictions (SSVSP_CSH).

The performance of each estimator is checked via the Absolute Percentage Deviation (APD) statistic and Mean Squared Errors (MSE):

$$APD = \frac{1}{(NG)^2} \sum_{i=1}^{(NG)^2} \mid \alpha_i - \alpha_i^{true} \mid \quad \text{and} \quad MSE = \frac{1}{(NG)^2} \sum_{i=1}^{(NG)^2} (\alpha_i - \alpha_i^{true})^2.$$

APD (MSE) measures the absolute (squared) deviation of the estimated coefficient α_i, given by the posterior mean averaged over all simulation draws, from the true value, α_i^{true}.[14] The same statistics are computed for Σ. Furthermore, the accuracy of the SSVSP

[13]The prior hyperparameters used in the Monte Carlo simulations for all different priors are given in the appendix.

[14]Koop and Korobilis (2015b) uses the absolute deviation and Korobilis (2016) use the mean absolute deviation statistics to evaluate the performance of estimators in Monte Carlo simulations.

Table 1: Deviation for estimated coefficient matrix A and Σ from the true values

	Simulation 1				Simulation 2			
	A		Σ		A		Σ	
	APD	MSE	APD	MSE	APD	MSE	APD	MSE
SSVSP	0.043	0.004	0.163	0.042	0.036	0.004	0.085	0.026
SSVSP_DI	**0.026**	**0.001**	0.142	0.030	**0.023**	**0.002**	0.074	0.020
SSVSP_CSH	0.041	0.004	0.182	0.051	0.037	0.004	0.088	0.028
SSSS	0.048	0.010	0.109	0.068	0.056	0.011	0.080	0.022
SSVS	0.067	0.008	**0.060**	**0.007**	0.066	0.009	**0.052**	**0.006**
unrest VAR	0.027	**0.001**	0.109	0.020	0.027	**0.002**	0.079	0.020

Absolute (APD) and squared (MSE) deviation of the estimates from the true value for coefficient matrix A and covariance Σ, average over 100 MC draws and all coefficients. Coefficient estimates are the posterior means averaged over all MC draws. Lowest values for each column are in bold. SSVSP_DI: SSVSP with only DI restrictions. SSVSP_CSH: SSVSP with only CSH restrictions. S^4: prior of Koop and Korobilis (2015b). SSVS: prior of George et al. (2008). Unrest VAR: parameters drawn from unrestricted part of distributions. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

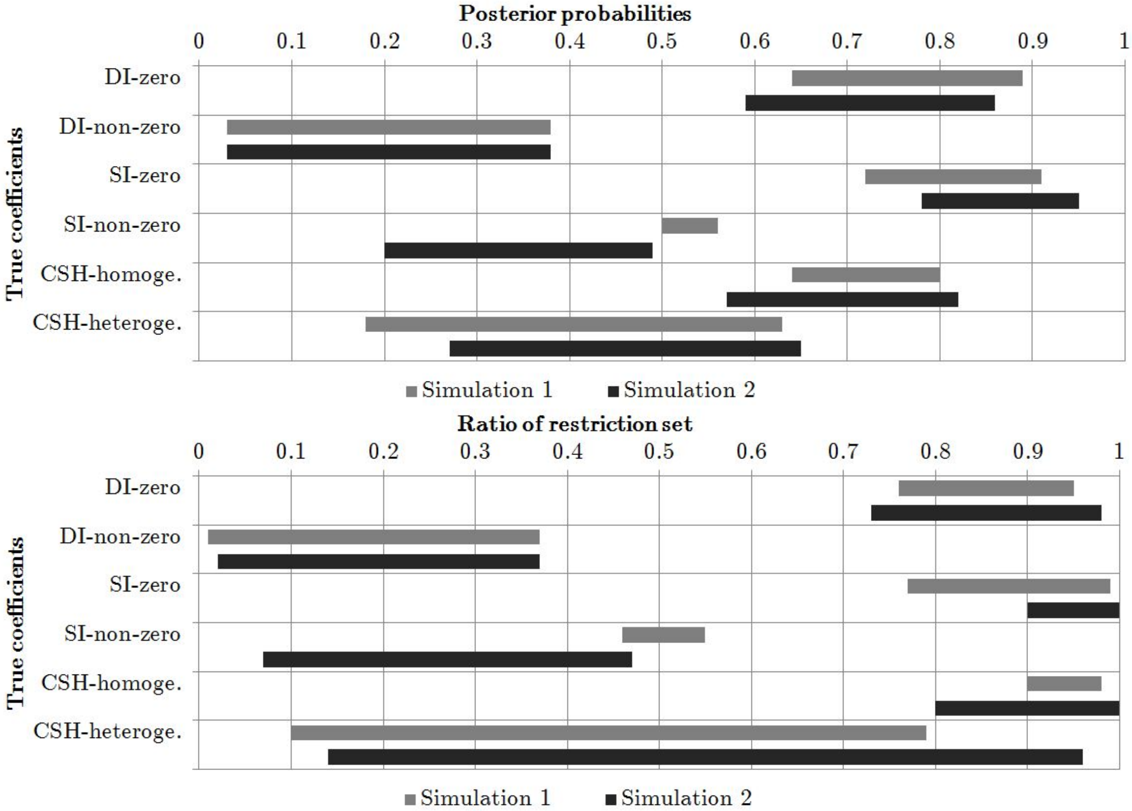
to find the restrictions is evaluated. The posterior probabilities that $\alpha_{ij}^{lk} = 0$, $\psi_{ij}^{lk} = 0$, and $\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$ are compared among themselves and in relation to the true values. These posterior probabilities are calculated as the proportion of $\gamma_{DI,ij}^{lk}$, $\gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. The higher the proportion of γ draws that equal zero is, the higher the probability is that no dynamic and no static interdependencies exist and coefficients are homogeneous.

4.2. Results

The results of the Monte Carlo study demonstrate that, firstly, the SSVSP outperforms the S^4 in terms of closeness to the true coefficients. This especially holds when a less restrictive panel structure is present. Secondly, the SSVSP accurately selects the restrictions for the DGPs of both simulations. This is validated by the higher posterior probabilities for no interdependencies and homogeneity for parameters which are truly zero or homogeneous compared to the probabilities for non-zero and heterogeneous parameters.

As table 1 shows, the estimated coefficients which are the posterior means averaged over all simulation draws from the SSVSP are on average slightly closer to the true values compared to S^4 for both simulations. In particular, the S^4 performs weaker in simulation two, where a less restrictive panel structure is present, since the grouping structure on which the restriction search is done is not present in the data. Furthermore, APD and MSE of A favor SSVSP_DI for both simulations with $APD_{SSVSP\_DI} = 0.026$ and $MSE_{SSVSP\_DI} = 0.001$ for simulation one and $APD_{SSVSP\_DI} = 0.023$ and $MSE_{SSVSP\_DI} = 0.002$ for simulation two. However, the unrestricted VAR outperforms the SSVSP and S^4. Doing the restriction search only for CSH reduces the average deviation from the true values only in simulation one compared to the SSVSP. This indicates that the gain is not explained by the reduced number of restrictions on which the search is done for but rather by searching for no dynamic interdependencies. Thus, the use of a prior which incorpo-

Figure 1: Range of posterior probabilities $\alpha_{ij}^{lk} = 0$, $\psi_{ij}^{lk} = 0$, and $\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$ and ratio of restriction set with threshold value 0.5



Posterior probabilities, $p(\alpha_{ij}^{lk} = 0)$, $p(\psi_{ij}^{lk} = 0)$, and $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are calculated as the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. Ratios are calculated as the number of restrictions set with threshold 0.5 averaged over all Gibbs draws for each simulated draw relative to all simulated draws. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

rates no dynamic interdependencies is beneficial for the DGPs of both simulations. For Σ the lowest values for APD and MSE are obtained with SSVS. In general, deviations from the true values are higher for Σ compared to A. This could be a result of using a symmetric distribution rather than the usually preferred inverse Wishart distribution for covariances.

Furthermore, the SSVSP algorithm is accurate in selecting the restrictions. Posterior probabilities that no interdependencies or heterogeneities exist are higher for true zero or homogeneous values compared to non-zero or heterogeneous values, shown in the first graph of figure 1. The first graph presents the range of posterior probabilities for simulation one and two for true zero or homogeneous coefficients and true non-zero or true heterogeneous coefficients. The second graph demonstrates the range of the ratios of restriction set when doing model selection. The ratios are calculated as the number of restriction set for each MC draw relative to all simulated draws given a threshold value of 0.5.[15] The ratio of restriction set is higher for true zero and homogeneous values than for non-zero and somehow for heterogeneous values. Thus, the first graph demonstrates the accuracy of selecting restrictions in case of BMA while the second graph underlines it for

[15] Results for different threshold values are given in the appendix.

Table 2: Accuracy of selecting DI restrictions: posterior probabilities $p(\alpha_{ij}^{lk} = 0)$

Simulation 1						Simulation 2					
-	-	0.2	0.2	0	0	-	-	0.2	0	0.2	0
		0.26	*0.38*	*0.88*	*0.78*			*0.30*	*0.62*	*0.38*	*0.59*
-	-	0.3	0.3	0	0	-	-	0.2	0	0.2	0
		0.06	*0.18*	*0.83*	*0.76*			*0.31*	*0.70*	*0.30*	*0.70*
0	0	-	-	0	0	0	0	-	-	0	0
0.71	*0.75*			*0.85*	*0.73*	*0.82*	*0.73*			*0.84*	*0.65*
0	0	-	-	0	0	0	0	-	-	0	0
0.74	*0.77*			*0.89*	*0.79*	*0.84*	*0.75*			*0.86*	*0.71*
0.3	-0.4	0	0	-	-	0.3	-0.4	0	0	-	-
0.15	*0.03*	*0.79*	*0.64*			*0.19*	*0.03*	*0.76*	*0.67*		
0.2	0.4	0	0	-	-	0	0	0	0	-	-
0.25	*0.05*	*0.78*	*0.66*			*0.86*	*0.79*	*0.83*	*0.71*		

Posterior probabilities, $p(\alpha_{ij}^{lk} = 0)$ in italic, are calculated as the proportion of $\gamma_{DI,ij}^{lk}$ draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

model selection.

In detail, looking at simulation one and DI restrictions, the probabilities that $\alpha_{ij}^{lk} = 0$ are considerably higher for true zero parameters than for true non-zero values. The first are in a range between 0.64 and 0.89 while the latter one are between 0.03 and 0.38. Table 2 provides the detailed results for the posterior probabilities for $\alpha_{ij}^{lk} = 0$. Turning to simulation two, if no dynamic interdependencies occur in truth, the probabilities that $\alpha_{ij}^{lk} = 0$ are between 0.59 and 0.86. Thus, clearly higher than the probabilities for the parameters which dynamically affect the dependent variables, between 0.03 and 0.38. Findings are in the same range for the ratio of restrictions set.

The SSVPS selects accurately the SI restrictions in both simulations as shown in figure 1 and table 3. This is true since for both simulations the probabilities that $\psi_{ij}^{lk} = 0$ and ratios of restriction set are higher for true zero compared to non-zero parameters. The results for simulation one show that probabilities are in a range of 0.72 and 0.91 for zero values while for the existing static interdependencies probabilities are between 0.50 and 0.56. For simulation two the probabilities for no static interdependencies, between 0.78 and 0.95, are clearly higher for the true zero values compared to the probabilities for non-zero values, 0.20 and 0.49.

Moreover, the SSVSP is mostly accurate in the selection of the cross-section heterogeneity restrictions. The detailed results for $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$ are presented in table 4. For both simulations probabilities that the coefficients are homogeneous are higher for true homogeneous coefficients (with few exceptions for simulation two). However, especially for true values which are close to each other but not equal the probabilities for homogeneity are relatively high with values above 0.5. Again, the same holds for the ratio of restriction set. This slightly weaker performance of the SSVSP to pick the correct CSH restrictions compared to DI and SI was already visible in the APD and MSE results for

Table 3: Accuracy of selecting SI restrictions: posterior probabilities $p(\psi^{lk}_{ij} = 0)$

Simulation 1						Simulation 2					
-	-	0.5	0.5	0	0	-	-	-0.5	0	0	0
		0.56	*0.54*	*0.72*	*0.80*			*0.49*	*0.95*	*0.78*	*0.95*
-	-	-0.5	-0.5	0	0	-	-	0	-0.5	0	0
		0.56	*0.50*	*0.74*	*0.76*			*0.91*	*0.20*	*0.89*	*0.91*
-	-	-	-	0	0	-	-	-	-	0	0
				0.84	*0.86*					*0.90*	*0.93*
-	-	-	-	0	0	-	-	-	-	0	0
				0.91	*0.91*					*0.92*	*0.93*

Posterior probabilities, $p(\psi^{lk}_{ij} = 0)$ in italic, are calculated as the proportion of $\gamma^{lk}_{SI,ij}$ draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

the SSVSP_CSH.

5. Empirical Application

5.1. Data and Procedure

Using an empirical application as an example, the SSVSP is validated based on its forecasting performance, on the restriction posterior probabilities, and on an impulse response analysis. The PVAR(1) includes three key macroeconomic variables for the G7 countries - a growth rate of industrial production (IP), a CPI growth rate (CPI), and a short term interest rate (IR).[16] The data are from the OECD and have monthly frequency from 1990:1 through 2015:2.[17] The model which is considered here serves as an illustration for the performance of SSVSP. In many ways it is not the best model for the DGP as it, for example, takes into account only a fraction of variables which could be of interest for assessing the question of global spillovers or conducting forecasts. Furthermore, the lag order one is set by assumption and is not further validated.

At first, forecasts are provided for 12 horizons for the period beginning from January 2005 through the end of the sample.[18] To obtain the forecasts a predictive distribution is simulated based on the reduced form of the PVAR model with normal distributed error terms. The reduced form model is the model where no SI restriction search is done and the covariance matrix is drawn from an inverse Wishart distribution.[19] The forecasts are evaluated using the mean squared forecast error (MSFE) and the average predictive likelihoods (PL). The MSFE is calculated as the difference between the estimated forecast,

[16]The countries are Canada (CA), Italy (I), United Kingdom (UK), France (F), Japan (J), Germany (D), and United States (US).

[17]The hyperparameters of the prior distributions are set as in the Monte Carlo simulations. Detailed information is provided in the appendix. 110,000 draws are computed for the Gibbs sampler, the first 10,000 are disregarded as burn-in-phase.

[18]The forecasts for the included 21 variables are generated iteratively. Forecasts start conditional on the data from January 1990 to December 2004.

[19]The covariance matrix is drawn from an inverse Wishart distribution with T degrees of freedom and identity matrix plus sum of squared residuals as scaling matrix.

Table 4: Accuracy of selecting CSH restrictions: posterior probabilities $p(\alpha^{lk}_{jj} = \alpha^{lk}_{ii})$

	Simulation 1			Simulation 2		
coefficients	true α^{lk}_{jj}	true α^{lk}_{ii}	$p(\alpha^{lk}_{jj} = \alpha^{lk}_{ii})$	true α^{lk}_{jj}	true α^{lk}_{ii}	$p(\alpha^{lk}_{jj} = \alpha^{lk}_{ii})$
$\alpha^{11}_{11} = \alpha^{11}_{22}$	0.8	0.6	*0.61*	0.8	0.6	*0.65*
$\alpha^{21}_{11} = \alpha^{21}_{22}$	0	0	*0.73*	0	0	*0.79*
$\alpha^{12}_{11} = \alpha^{12}_{22}$	0	0.5	*0.19*	0	0.5	*0.28*
$\alpha^{22}_{11} = \alpha^{22}_{22}$	0.7	0.5	*0.54*	0.7	0.3	*0.30*
$\alpha^{11}_{11} = \alpha^{11}_{33}$	0.8	0.6	*0.63*	0.8	0.6	*0.64*
$\alpha^{21}_{11} = \alpha^{21}_{33}$	0	0	*0.75*	0	0	*0.82*
$\alpha^{12}_{11} = \alpha^{12}_{33}$	0	0.5	*0.18*	0	0.5	*0.27*
$\alpha^{22}_{11} = \alpha^{22}_{33}$	0.7	0.5	*0.57*	0.7	0.5	*0.40*
$\alpha^{11}_{22} = \alpha^{11}_{33}$	0.6	0.6	*0.79*	0.6	0.6	*0.75*
$\alpha^{21}_{22} = \alpha^{21}_{33}$	0	0	*0.80*	0	0	*0.80*
$\alpha^{12}_{22} = \alpha^{12}_{33}$	0.5	0.5	*0.64*	0.5	0.5	*0.57*
$\alpha^{22}_{22} = \alpha^{22}_{33}$	0.5	0.5	*0.65*	0.3	0.5	*0.48*

Posterior probabilities, $p(\alpha^{lk}_{jj} = \alpha^{lk}_{ii})$ in italic, are calculated as the proportion of γ^{w}_{CSH} draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

which is given by the posterior mean of the predictive distribution, and the true value given by the data. The MSFEs for a specific variable and horizon are averaged over all forecasts. The PL is the posterior predictive density evaluated at the true observation y_{t+h}.

The forecast performance is compared to the unrestricted VAR, SSVSP_DI, SSVSP_CSH, S^4, and SSVS. Furthermore, two specifications are added which access the selection property of the SSVSP: SSVSP_setDI_v1 and SSVSP_setDI_v2. SSVSP_setDI_v1 uses the outcome of the SSVSP and sets zeros whenever the posterior probability for a DI restriction is larger than 0.99. SSVSP_setDI_v2 uses 0.5 as a threshold value.

For the structural analysis the variables are ordered in a recursive way. The industrial production growth rate is ordered first, CPI growth rate second, and the short term interest rate third. The monetary policy shock for one country is thus identified by the assumption that the interest rate does not react contemporaneously to unexpected changes in real variables while a monetary policy shock instantaneously impacts the two real variables. The recursive country ordering is based on the openness of a country. Openness is measured based on yearly import and export data for the economies. The higher the trade of a country is, the more open it is. The countries are arranged in ascending order meaning that the most open country, the United States, is ordered last. Thus, US variables can influence all other countries contemporaneously but are not affected by the variables of the remaining G7 countries. The upper triangular matrix Ψ has the following simplified form focusing

Table 5: Mean squared forecast errors and posterior predictive density relative to unrestricted VAR

	number of MSFEs ≤ 1(in %)				
horizon	1	2	4	6	12
SSVSP	38.10	42.86	33.33	28.57	33.33
SSVSP_DI	100.00	85.71	85.71	95.24	90.48
SSVSP_CSH	33.33	38.10	19.05	23.81	52.38
SSVSP_setDI_v1	0.00	0.00	52.38	57.14	66.67
SSVSP_setDI_v2	4.76	4.76	52.38	71.43	80.95
S^4	42.86	33.33	38.10	33.33	47.62
SSVS	19.05	9.52	66.67	95.24	76.19
	number of PLs ≥ 0 (in %)				
horizon	1	2	4	6	12
SSVSP	33.33	28.57	47.62	23.81	38.10
SSVSP_DI	57.14	42.86	42.86	42.86	61.90
SSVSP_CSH	28.57	19.05	38.10	23.81	57.14
SSVSP_setDI_v1	28.57	28.57	47.62	47.62	42.86
SSVSP_setDI_v2	28.57	52.38	52.38	61.90	66.67
S^4	47.62	47.62	33.33	33.33	38.10
SSVS	47.62	38.10	57.14	71.43	61.90

MSFE relative to unrestricted VAR. PL: posterior predictive density evaluated at the true observation y_{t+h}, compared to unrestricted VAR. Forecasts for 12 horizons for Jan 2005 to end of the sample. Unrestricted VAR: parameters drawn from unrestricted part of distributions. S^4: prior of Koop and Korobilis (2015b). SSVS: prior of George et al. (2008). SSVSP_DI: SSVSP with only DI restrictions. SSVSP_CSH: SSVSP with only CSH restrictions. SSVSP_setDI_v1: threshold 0.99 to set zero DI restrictions. SSVSP_setDI_v2: threshold 0.5 to set zero DI restrictions. Σ drawn from inverse Wishart distribution with T degrees of freedom and identity matrix plus sum of squared residuals as scaling matrix. 110,000 Gibbs draws, 10,000 disregarded as burn-in-phase.

on the country order:

$$\begin{array}{c} CA \\ I \\ UK \\ F \\ J \\ D \\ US \end{array} \begin{pmatrix} \times & \times & \times & \times & \times & \times & \times \\ & \times & \times & \times & \times & \times & \times \\ & & \times & \times & \times & \times & \times \\ & \mathbf{0} & & \times & \times & \times & \times \\ & & & & \times & \times & \times \\ & & & & & \times & \times \\ & & & & & & \times \end{pmatrix}.$$

5.2. Results

The results of the empirical application demonstrate three key findings. Firstly, the MSFEs and PLs results favor the SSVSP_DI and the two selection models, SSVSP_setDI_v1 and SSVSP_setDI_v2, indicating that restrictions search is beneficial since sparsity in form of no dynamic interdependencies exist. However, the very large number of restrictions searched for in the SSVSP leads to relatively weak forecast performance. Secondly, the posterior probabilities for the restrictions indicate that domestic interest rates and inflation

Table 6: 10 highest posterior probabilities for the restrictions

DI		SI		CSH		
α_{ij}^{lk}	$p(\alpha_{ij}^{lk}=0)$	ψ	$p(\psi_{ij}^{lk}=0)$	α_{jj}^{lk}	α_{ii}^{lk}	$p(\alpha_{jj}^{lk}=\alpha_{ii}^{lk})$
$\alpha_{UK,D}^{IR,IP}$	1.00	$\psi_{J,US}^{IP,IP}$	0.99	$\alpha_{J,J}^{IR,IP}$	$\alpha_{D,D}^{IR,IP}$	1.00
$\alpha_{J,D}^{IR,IP}$	1.00	$\psi_{J,D}^{IP,IR}$	0.99	$\alpha_{UK,UK}^{IR,IP}$	$\alpha_{J,J}^{IR,IP}$	1.00
$\alpha_{D,J}^{IR,IP}$	1.00	$\psi_{J,US}^{IP,IR}$	0.99	$\alpha_{CA,CA}^{IR,IP}$	$\alpha_{J,J}^{IR,IP}$	1.00
$\alpha_{CA,J}^{IR,IP}$	1.00	$\psi_{J,D}^{IP,CPI}$	0.99	$\alpha_{I,I}^{CPI,IP}$	$\alpha_{F,F}^{CPI,IP}$	1.00
$\alpha_{J,US}^{IP,CPI}$	1.00	$\psi_{J,D}^{IP,CPI}$	0.99	$\alpha_{F,F}^{CPI,IP}$	$\alpha_{J,J}^{CPI,IP}$	1.00
$\alpha_{I,D}^{CPI,IP}$	1.00	$\psi_{J,D}^{IP,IP}$	0.99	$\alpha_{I,I}^{CPI,IP}$	$\alpha_{J,J}^{CPI,IP}$	1.00
$\alpha_{I,F}^{CPI,IP}$	1.00	$\psi_{D,US}^{IP,IR}$	0.99	$\alpha_{UK,UK}^{IR,IP}$	$\alpha_{D,D}^{IR,IP}$	1.00
$\alpha_{UK,J}^{CPI,IP}$	1.00	$\psi_{I,D}^{IP,CPI}$	0.99	$\alpha_{F,F}^{IR,IP}$	$\alpha_{J,J}^{IR,IP}$	1.00
$\alpha_{F,J}^{CPI,IP}$	1.00	$\psi_{D,US}^{IP,IP}$	0.99	$\alpha_{CA,CA}^{IR,IP}$	$\alpha_{D,D}^{IR,IP}$	1.00
$\alpha_{US,J}^{CPI,IP}$	1.00	$\psi_{D,US}^{IP,CPI}$	0.99	$\alpha_{F,F}^{IR,IP}$	$\alpha_{D,D}^{IR,IP}$	1.00

10 highest posterior probabilities are presented for DI, SI, and CSH restrictions. The probabilities, $p(\alpha_{ij}^{lk}=0)$, $p(\psi_{ij}^{lk}=0)$, and $p(\alpha_{jj}^{lk}=\alpha_{ii}^{lk})$, are calculated as one minus the posterior means for $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w}. The probabilities measure the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws meaning that the coefficients are drawn from the restricted part of the distribution.

evolve unaffected by lagged foreign industrial production growth rates, validated by high posterior probabilities for no dynamic interdependencies. The interest rate of a country depends likely statically and dynamically on foreign interest rates. No heterogeneities are in particular found for the effect of domestic industrial production growth on the domestic interest rate and inflation. Thirdly, the impulse response analysis supports the reliability of the results. In the following the key findings are explained in more detail.

The upper part of table 5 shows in percent the number of MSFEs which are smaller or equal to one averaged over all variables for forecast horizon 1, 2, 4, 6, and 12. The MSFEs are given relative to the unrestricted VAR. Thus, a MSFE smaller than one indicates improved forecast performance to the unrestricted VAR. Between 28.57% and 42.86% of the MSFEs of the SSVSP are below or equal to the MSFEs of the unrestricted VAR. Thus, the SSVSP cannot improve the forecasts compared to the unrestricted VAR. This is quite similar to the performance of the S^4. However, the SSVSP_DI performs particularly well. It outperforms the unrestricted model at the best in 100.00% of the cases (horizon one) and at worst in 85.71% of the cases (horizons two and four). Since the number of restrictions which are examined in the SSVSP are high, the information in the data might not be enough for the estimation. Thus, the improved performance of SSVSP_DI could be a result of the reduced number of restrictions. However, only searching for CSH restrictions does not lead to improvements compared to the SSVSP. The SSVSP_DI captures the high probabilities for no dynamic interdependencies which are present in the data. The probability for homogeneity seems to be lower. Excluding dynamic interdependencies based on a specific threshold improves the forecast performance for the higher horizons. The two specifications, SSVSP_setDI_v1 and SSVSP_setDI_v2, can particularly well pick up

Table 7: 10 lowest posterior probabilities for the restrictions

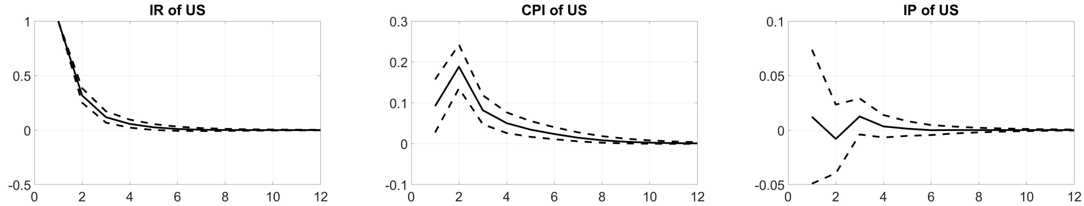
DI		SI		CSH		
α_{ij}^{lk}	$p(\alpha_{ij}^{lk} = 0)$	ψ	$p(\psi_{ij}^{lk} = 0)$	α_{jj}^{lk}	α_{ii}^{lk}	$p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$
$\alpha_{F,D}^{IR,IR}$	0.00	$\psi_{CA,US}^{CPI,CPI}$	0.00	$\alpha_{D,D}^{CPI,CPI}$	$\alpha_{US,US}^{CPI,CPI}$	0.00
$\alpha_{CA,I}^{IR,IR}$	0.00	$\psi_{UK,F}^{CPI,CPI}$	0.00	$\alpha_{D,D}^{IP,IP}$	$\alpha_{US,US}^{IP,IP}$	0.06
$\alpha_{I,D}^{IR,IR}$	0.01	$\psi_{CA,F}^{IR,IR}$	0.00	$\alpha_{F,F}^{IP,IR}$	$\alpha_{J,J}^{IP,IR}$	0.06
$\alpha_{D,UK}^{IP,IR}$	0.01	$\psi_{UK,D}^{IR,IR}$	0.00	$\alpha_{F,F}^{CPI,CPI}$	$\alpha_{US,US}^{CPI,CPI}$	0.07
$\alpha_{UK,US}^{IR,IR}$	0.03	$\psi_{D,US}^{IR,IR}$	0.00	$\alpha_{I,I}^{IP,IR}$	$\alpha_{J,J}^{IP,IR}$	0.07
$\alpha_{F,US}^{CPI,CPI}$	0.04	$\psi_{UK,US}^{IR,IR}$	0.00	$\alpha_{CA,CA}^{IP,IR}$	$\alpha_{J,J}^{CPI,CPI}$	0.07
$\alpha_{CA,US}^{IP,IR}$	0.04	$\psi_{CA,US}^{IR,IR}$	0.00	$\alpha_{UK,UK}^{IP,IR}$	$\alpha_{J,J}^{IP,IR}$	0.09
$\alpha_{F,US}^{IP,IR}$	0.04	$\psi_{CA,F}^{CPI,CPI}$	0.00	$\alpha_{J,J}^{IP,IR}$	$\alpha_{D,D}^{IP,IR}$	0.09
$\alpha_{F,US}^{IP,IP}$	0.05	$\psi_{UK,J}^{CPI,CPI}$	0.00	$\alpha_{J,J}^{IP,IR}$	$\alpha_{US,US}^{IP,IR}$	0.10
$\alpha_{CA,J}^{IR,IR}$	0.05	$\psi_{F,D}^{CPI,CPI}$	0.00	$\alpha_{J,J}^{CPI,CPI}$	$\alpha_{D,D}^{CPI,CPI}$	0.10

10 lowest posterior probabilities are presented for DI, SI, and CSH restrictions. The probabilities, $p(\alpha_{ij}^{lk} = 0)$, $p(\psi_{ij}^{lk} = 0)$, and $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are calculated as one minus the posterior means for $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w}. The probabilities measure the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws meaning that the coefficients are drawn from the restricted part of the distribution.

the sparsity in the data. The model with a lower threshold value, SSVSP_setDI_v2, leads to higher improvements. The performance of the SSVS is volatile, ranging from 9.52% to 95.24% of MSFEs below or equal one. It performs well for the last three reported horizons. The SSVS also searches for dynamic interdependencies, thus, it is similar to the SSVSP_DI specification. With the exception that the SSVSP_DI distinguishes between domestic and foreign variables. The results of the SSVS support the finding that the MSFEs favor priors which capture the possibility of no dynamic interdependencies. The S^4 includes DI but assumes a specific matrix structure which does not seem to be supported by the data.

The lower part of table 5 presents in percent the number of PL, in difference to the unrestricted model, which are higher or equal zero. In general, a higher PL indicates a better performance since the posterior predictive density covers the true observation with a higher probability. The results are generally in line with the findings based on the MSFEs but differ in magnitude and also in horizon. In particular, the PL results favor the SSVSP_DI, SSVSP_setDI_v1, and SSVSP_setDI_v2 as well as the SSVS. In contrast to the extremely volatile MSFE results, the SSVS outperforms the SSVSP at all horizons. However, the SSVSP_DI exceeds the SSVS at two and is equally good at one out of five horizons. Again, the results point to the direction that no dynamic interdependencies are present in the data and a prior which can pick up these characteristics performs well. Compared to the findings based on the MSFEs the prior specifications are less often able to outperform the unrestricted VAR. This could be explained by a higher parameter uncertainty of the selection priors since they are a mixture of two distributions. The higher uncertainty is reflected in the posterior predictive density. The results are in general in

Figure 2: Responses of US variables to a shock to US interest rate



Solid line shows response, dotted lines present 68% Bayesian credible interval.

line with Korobilis (2016) who also shows a high volatility in the performance as well as improved forecasting results for the SSVS compared to the S^4. However, combining the advantages of both priors, the panel dimension of the S^4 and the single restriction search of the SSVS, in the SSVSP does not seem to pay of due to the large number of restrictions to search for. As Korobilis (2016) shows the approach of Canova and Ciccarelli (2009) has no clear advantage, measured by forecasting performance, over the selection priors.

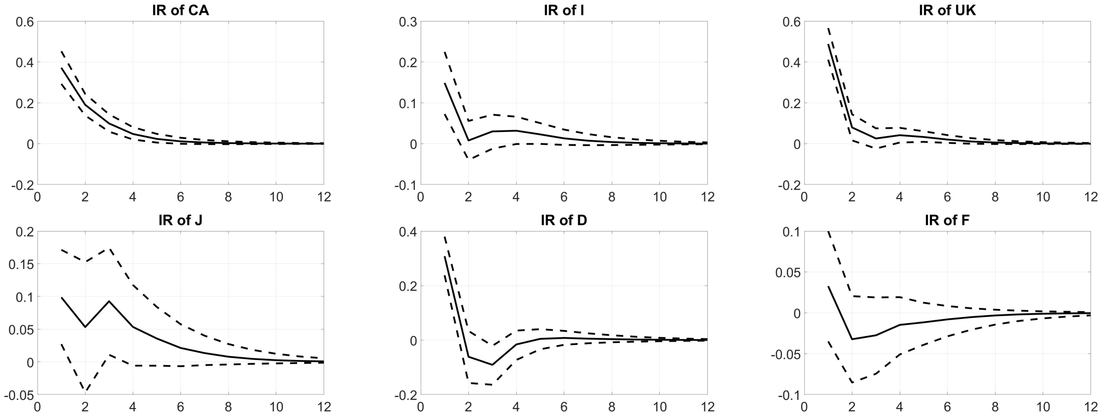
Tables 6 and 7 provide the ten highest and ten lowest posterior probabilities for the restrictions, $p(\alpha_{ij}^{lk} = 0)$, $p(\psi_{ij}^{lk} = 0)$, and $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$. The SSVSP can provide a detailed ranking on how likely a restriction should be set based on the data on a variable basis. The algorithm is able to detect a nuanced structure of the restrictions present in the data. Since the presented PVAR model serves as an illustration, the economic findings should not be over-interpreted.

The posterior probabilities provide evidence that restrictions are especially supported for the industrial production variable while it is vice versa for interest rates. Table 6 provides the ten highest posterior probabilities. Probabilities are high, indicated by ones, that no dynamic impacts of foreign lagged IP on interest rates and CPI growth exist. Additionally, industrial production growth seems to be fairly independent from other variables, shown by the high probabilities for no static interdependencies between IP and other variables. Finally, the probabilities for homogeneity of coefficients are especially high for the industrial production variables in other equations.

Table 7 presents the ten lowest posterior probabilities for restrictions. Lagged foreign interest rates seem to affect the domestic variables. Furthermore, US variables have a dynamic impact on other countries' variables. Both findings are supported by low probabilities for no DI. The lowest probabilities for the SI restrictions are found for combinations of the same variable, in particular for inflation and the interest rate. Heterogeneity is favored - low probability for no CSH - for the effect of inflation on inflation and of the interest rate on industrial production growth.

The impulse response analysis sheds light on the reliability of the findings. Exemplary, the responses to a shock to the US interest rate, presented in figure 2 for US variables and in figure 3 for foreign interest rates, will be discussed. A contractionary US monetary policy, shown by an increase in the US interest rate, leads to a rise in US CPI in this system. The response of industrial production growth is insignificant. The increase of inflation in response to a tightening in the monetary policy is in line with the price puzzle. The price puzzle refers to this result contradicting theoretical models and empirical findings which would claim that a rise in the interest rate leads to a decline in inflation. The puzzle is expected for VAR models which just include industrial production growth, inflation, and a short term interest rate and have a structural identification based on a recursive system.

Figure 3: Responses of foreign interest rates to a shock to US interest rate



Solid line shows response, dotted lines present 68% Bayesian credible interval.

The finding of the price puzzle underlines that the here estimated PVAR model can only serve as an illustration and has its clear limitations.

The foreign interest rates immediately raise in response to a tightening in the US monetary policy. The increases in the interest rates are lower, below 0.5, than the initial raise in the US interest rate, which is normalized to one. The UK interest rate is initially affected most, followed by the Canadian and German interest rate responses. After around two horizons the effect of the US shock is insignificant for the interest rate of the United Kingdom, Germany, and Italy. The responses of the interest rates of Japan and France are lowest. For Japan the response is insignificant after the first horizon while for France the response is insignificant for all horizons. The raise in the Canadian interest rate lasts longest and comes to zero after six horizons. To sum up, the impulse repose functions support that the results based on SSVSP are in line with theoretically expected responses from a recursively identified system with the three included variables. The illustrative model provides evidence that the results obtained using SSVSP are plausible.

6. Conclusions

This paper introduces the SSVSP as an extension of the Bayesian S^4 proposed by Koop and Korobilis (2015b). The SSVSP is an alternative Bayesian estimation procedure for PVARs that is able to fully incorporate dynamic and static interdependencies as well as cross-country heterogeneities. It allows for a flexible panel structure since it only distinguishes between domestic and foreign variables. Using a hierarchical prior, the SSVSP searches for restrictions that are supported by the data.

The results of the Monte Carlo simulations demonstrate that the SSVSP outperforms the S^4 in terms of deviation from the true values in particular when a less restrictive panel structure is present. The average deviation of the estimated parameters from the true values for the simulation with a flexible panel structure is less for the SSVSP, $APD_{SSVSP} = 0.036$, than for S^4, $APD_{S^4} = 0.056$. The SSVSP_DI, where a mixture prior is only set on the parameters measuring dynamic interdependencies, has the smallest deviation from the true values of all models. Furthermore, the accuracy of the SSVSP in selecting the restrictions is proven by the posterior probabilities for no interdependencies and homogeneity.

The results of the empirical application are summarized in three main findings. Firstly, the forecast performance is especially good for the SSVSP_DI and the two selection models, SSVSP_setDI_v1 and SSVSP_setDI_v2. Thus, restrictions search for no dynamic interdependencies is beneficial. However, the performance of the SSVSP is limited by the very large number of restrictions searched for. Secondly, posterior probabilities for DI and SI restrictions show that interest rates likely depend on foreign interest rates while foreign industrial production growth does not impact other domestic variables. Thirdly, responses to a shock in the US interest rate are in line with expected response functions.

The SSVSP prior can be further developed. The SI restriction search, based on data, is an initial way to achieve structural identification, but it is limited by the fact it is built on a recursive system. Furthermore, in this specification, the hyperparameters are fixed for all parameters that are estimated. George et al. (2008) propose a default semi-automatic approach to select the hyperparameters which vary for each coefficient. Trying this approach leads to hyperparameters that tend to be so small that the majority of values are drawn from the loose part of the prior. Koop and Korobilis (2015b) specify distributions for the hyperparameters as also suggested in Giannone et al. (2015). This allows them to have varying and less subjectively chosen hyperparameters.

To sum up, the findings of the Monte Carlo simulations conducted and the exemplary empirical application encourage the use of the SSVSP to estimate PVAR models. However, further research regarding both the recursive structural identification and the specified hyperparameters can be undertaken.

7. Acknowledgements

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Appendix A. Gibbs Sampler Algorithm

The full unrestricted PVAR model with one lag including N countries and for each country G variables can be written as

$$Y_t = Z_{t-1}\alpha + U_t,$$

where α is the vectorized $[NG \times NG]$-coefficient matrix A for lag one. The $Z_{t-1} = (I_{NG} \otimes Y_{t-1})$ where $Y_{t-1} = (y'_{1t-1}, ..., y'_{Nt-1})'$ and y_{it} denotes a vector of dimension $[G \times 1]$. The Y_t and U_t are $[NG \times 1]$-vectors. The U_t is normally distributed with mean zero and covariance matrix Σ that is of dimension $[NG \times NG]$. The element α_{ij}^{lk} refers to the coefficient of variable k of country j in the equation of variable l of country i.

The Gibbs sampler algorithm has the following three steps:

Step 1:
Sample α from a normal posterior conditional on Σ, γ_{DI}, γ_{CSH}.

$$\alpha \mid \Sigma, \gamma_{DI}, \gamma_{CSH} \sim \mathcal{N}(\Gamma\mu_\alpha, V_\alpha),$$

where $V_\alpha = ((D'D)^{-1} + \Sigma^{-1} \otimes X'X)^{-1}$ with $X = Y_{t-1}$ and $\mu_\alpha = V_\alpha((\Sigma^{-1} \otimes X'X)\alpha_{OLS})$. D is a diagonal matrix with $D = diag(h_{11}^{11}, ..., h_{NN}^{GG})$. The value of h depends on γ_{DI} and γ_{CSH}: $h_{ij}^{lk} = \begin{cases} \tau_1, & if\ \gamma_{DI,ij}^{lk} = 0 \\ \tau_2, & if\ \gamma_{DI,ij}^{lk} = 1 \end{cases}$ for the parameters, where DI restriction search is done ($i \neq j$) and $h_{jj}^{lk} = \begin{cases} \xi_1, & if\ \gamma_{CSH}^{w} = 0 \\ \xi_2, & if\ \gamma_{CSH}^{w} = 1 \end{cases}$ for the block diagonal parameters where CSH restriction search is done. α_{OLS} is the OLS estimate of α. The posterior mean is restricted with the selection matrix Γ.

Step 2:
Update γ_{DI} and γ_{CSH} from Bernoulli distribution:

$$\begin{aligned} \gamma_{DI,ij}^{lk} &\sim Bernoulli(\pi_{DI,ij}^{lk}) \\ \pi_{DI,ij}^{lk} &= \frac{u2_{DI,ij}^{lk}}{u1_{DI,ij}^{lk} + u2_{DI,ij}^{lk}} \\ \gamma_{CSH}^{w} &\sim Bernoulli(\pi_{CSH}^{w}) \\ \pi_{CSH}^{w} &= \frac{v2_{CSH}^{w}}{v1_{CSH}^{w} + v2_{CSH}^{w}}. \end{aligned}$$

Hereby, $u1_{DI,ij}^{lk} = f(\alpha_{ij}^{lk} \mid 0, \tau_1^2)prob_{DI}$ and $u2_{DI,ij}^{lk} = f(\alpha_{ij}^{lk} \mid 0, \tau_2^2)(1 - prob_{DI})$. $f()$ denotes the p.d.f. of the normal distribution with mean zero and variance τ_1^2 or τ_2^2 evaluated at α_{ij}^{lk}. The parameter $prob_{DI}$ is set equal to 0.5. This shows that *a priori* the researcher assumes that it is equally likely that a dynamic interdependency between two variables of country i and j are zero or nonzero. $v1_{CSH}^{w} = f(\alpha_{jj}^{lk} \mid \alpha_{ii}^{lk}, \xi_1^2)prob_{CSH}$ and $v2_{CSH}^{w} = f(\alpha_{jj}^{lk} \mid 0, \xi_2^2)(1 - prob_{CSH})$. Again, $prob_{CSH}$ is set equal 0.5. Depending on γ_{CSH}^{w} the elements in Γ_w are updated.

Step 3:
Update $\Sigma = \Psi^{-1\prime}\Psi^{-1}$ and γ_{SI}. The variance elements, ψ_{ii}^{kk}, are drawn from a Gamma distribution:

$$(\psi_{ii}^{kk})^2 \sim \mathcal{G}(a + \frac{T}{2}, B_n),$$

where $n = 1, ..., NG$ and

$$B_n = \begin{cases} b + 0.5SSE_{nn} & n = 1 \\ b + 0.5(SSE_{nn} - s_n'(S_{n-1} + (R'R)^{-1})^{-1}s_n) & n = 2, ..., NG \end{cases}.$$

Note that ψ_{11}^{11} is assigned to B_2, ψ_{11}^{22} to B_2, ..., and ψ_{NN}^{GG} to B_{NG}. T is defined as the length of the time series and SSE as the sum of squared residuals. S_n is the upper-left $n \times n$ submatrix of SSE, and $s_n = (s_{1n}, ..., s_{n-1,n})'$ contains the upper diagonal elements of SSE. R is a diagonal matrix with $R = diag(r_{11}^{11}, ..., r_{NN}^{GG})$. The value of r depends on γ_{SI}: $r_{ij}^{lk} = \begin{cases} \kappa_1, & if\ \gamma_{SI,ij}^{lk} = 0 \\ \kappa_2, & if\ \gamma_{SI,ij}^{lk} = 1 \end{cases}.$

Define the vector $\psi = (\psi_{12}^{11}, ..., \psi_{N-1,N}^{GG})'$. Thus, ψ contains the covariance elements, ψ_{ij}^{lk} for all $i \neq j$ and has the dimension $n_{SI} \times 1$, where $n_{SI} = 1, .., N_{SI}$ and N_{SI} is the length equal to the number of SI restrictions. The elements of ψ are updated from a normal distribution:

$$\psi_{n_{SI}} \mid \alpha, \psi, \gamma_{SI} \sim \mathcal{N}(\mu_{n_{SI}}, V_{n_{SI}}).$$

Hereby, $\mu_{n_{SI}} = -\psi_{ii}^{kk}(S_{n_{SI}-1} + (R'R)^{-1})^{-1}s_{n_{SI}}$ and $V_{n_{SI}} = (S_{n_{SI}-1} + (R'R)^{-1})^{-1}$. The element ψ_{ii}^{kk} is the variance element in the same row of Psi as $\psi_{ij}^{lk} = \psi_{n_{SI}}$ for all $i \neq j$. The off-diagonal elements of the covariance matrix that belong to one country are drawn from a normal distribution with mean zero and variance κ_2.

Appendix B. Hyperparameter

Table B.8: Hyperparameters

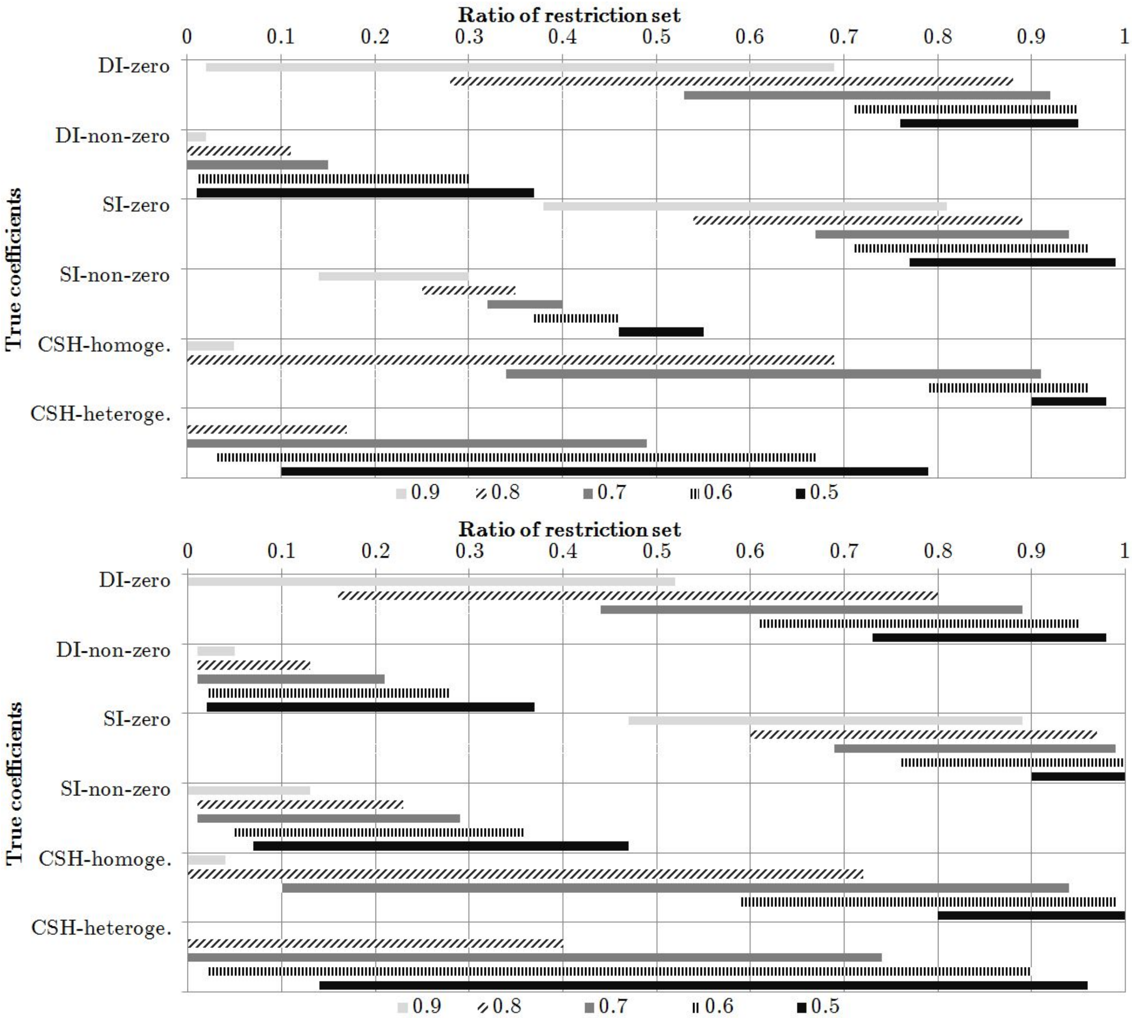
τ_1	τ_2	ξ_1	ξ_2	κ_1	κ_2	a	b
0.2	4	0.2	4	0.3	4	0.01	0.01

A value of $\tau_1 = 0.2$ and $\tau_2 = 4$ means that the variance of the tight prior equals 0.04 and 16 for the loose prior. The criterion that the variance of the first part of the normal distribution is smaller than the second part is clearly fulfilled. Several other specifications are also checked. The accuracy of the algorithm in selecting the restrictions varies with the specification of the hyperparameters. If the τ_1, κ_1, and ξ_1 are chosen too small, the majority of values is drawn from the second part of the normal distribution (γ equals one with a very high probability). Still, γ equals more often one in the cases no restriction is set in the true specification of the Monte Carlo simulation. Values for hyperparameters smaller than or equal to 0.1 prove to be too small, resulting in the difficulties mentioned. George et al. (2008) propose a default semi-automatic approach to selecting the hyperparameters.

The values are not fixed, but varying for each coefficient. For example $\tau_{1,i} = c_1 \sqrt{var(\alpha_i)}$ and $\tau_{2,i} = c_2 \sqrt{var(\alpha_i)}$ whereby $c_1 = 0.1$ and $c_2 = 10$. $var(\alpha_i)$ is a OLS estimated of the variance of the coefficient in an unrestricted model. κ and ξ are set in an equal manner. Trying this approach also leads to hyperparameters smaller than 0.1. The hyperparameters of the other priors are set to the proposed default values of the authors. For S^4 the small variance values are set to 0.1 and the high variance values to square root of 10, for the SSVS small variance values are set to 0.1 and high variance values to 5 as used by Koop and Korobilis (2015b) and George et al. (2008).

Appendix C. Monte Carlo Simulation

Figure C.4: Share of restrictions set according to threshold value - Simulation one and two



First graph shows results for simulation one, second for simulation two. Number of draws of $\gamma^{lk}_{DI,ij}$, $\gamma^{lk}_{SI,ij}$, and γ^{w}_{CSH} that equal zero averaged over all Gibbs sampler draws as a ratio of all simulated samples for a given threshold. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.