Notes on applying super momentum balance law to constraint GW waveform (Update on Daily Progress)

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I. LOCAL SUPER MOMENTUM BALANCE LAW

The local super-momentum balance law for asymptotically flat space-time is given by

$$\mathcal{F}(\theta,\phi)\Big|_{u_1}^{u_2} := -\int_{u_1}^{u_2} du\left[|\dot{\sigma^o}|^2 - \Re\left(\eth^2\dot{\bar{\sigma}}^o\right)\right](u,\theta,\phi) = \Re\left[\Psi_2^o + \bar{\sigma}^o\dot{\sigma^o}\right](u_1,\theta,\phi) - \Re\left[\Psi_2^o + \bar{\sigma}^o\dot{\sigma^o}\right](u_2,\theta,\phi) \tag{1.1}$$

where

$$2\Re\,\Psi_2^o(u,\theta,\phi) = \lim_{r\to+\infty} r^3\, C_{abcd}n^a l^b n^c l^d \tag{1.2}$$

$$\Psi_4^o(u,\theta,\phi) = \lim_{r\to+\infty} r\, C_{abcd}n^a\bar{m}^b n^c\bar{m}^d = \ddot{\bar{\sigma}}^o = r\left(\ddot{h}_+ + i\,\ddot{h}_\times\right) \tag{1.3}$$

We simplify the above equation for the case for perturbed black hole using the results in [add ref]. Equation 2.59 in ref[add ref] the mass aspect

$$\Psi = \psi_2^0 + \eth^2\bar{\sigma}^o + \sigma^o\dot{\bar{\sigma}}^o \tag{1.4}$$

and the spherical harmonic expansion of the above result is

$$\Psi = \Psi^0 + \Psi^i Y^0_{1\,i} + +\Psi^{i\,j}Y^0_{2\,i\,j} \tag{1.5}$$

The interior mass and three-momentum with the $l = 0$ and $l = 1$ harmonic contributions is given by

$$M_B = \frac{-c^2}{2\sqrt{2}\,G}\Psi^0 \tag{1.6}$$

$$P^i = -\frac{c^3}{6\,G}\Psi^i \tag{1.7}$$

Using the above results, at leading order we can relate mass and energy momentum of interior space time to ψ_2 as

$$-2\sqrt{2}\,M_B = \Re\left[\psi_2^0 + \eth^2\bar{\sigma}^o + \sigma^o\dot{\bar{\sigma}}^o\right] \tag{1.8}$$

We can use the above result in the local balance law, to rewrite the local balance law. As a first step just considering change in mass we have

$$-\int_{u_1}^{u_2} du\left[|\dot{\sigma^o}|^2 - \Re\left(\eth^2\dot{\bar{\sigma}}^o\right)\right](u,\theta,\phi) = -\left[\left(2\sqrt{2}M_B + \eth^2\bar{\sigma}^o\right)(u_1,\theta,\phi) - \left(2\sqrt{2}M_B + \eth^2\bar{\sigma}^o\right)(u_2,\theta,\phi)\right] \tag{1.9}$$

$$= \int_{-\infty}^{t} du\left(2\sqrt{2}\,\dot{M}_B + \eth^2\bar{\sigma}^o\right) \tag{1.10}$$

$$\implies -|\dot{\sigma^o}|^2 + \Re\left(\eth^2\bar{\sigma}^o\right) = 2\sqrt{2}\,\dot{M}_B + \eth^2\bar{\sigma}^o \tag{1.11}$$

$$\implies -|\dot{\sigma^o}|^2 = 2\sqrt{2}\,\dot{M}_B \tag{1.12}$$

Higher order corrections can be obtained in a similar way

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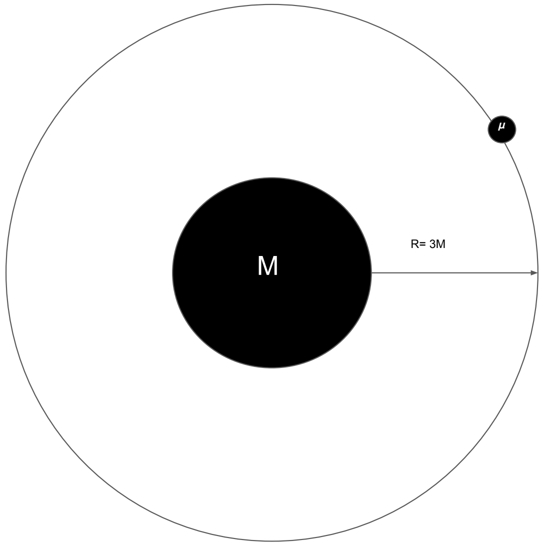

FIG. 1: The figure

II. WEYL SCALAR Ψ_2^o FOR BOB AS PERTURBER CROSS THE LIGHT RING

In Backwards One-Body (BOB)[add ref] description of black hole binary merger, the system is described as a single black hole spacetime with a inspiraling in perturber. At the leading order we can just assume that the mass of the perturber and the source is evolving.

A. Evolution of Ψ_2^o for perturbed Swarzchild black hole

We assume that the perturber is loosing mass at given rate $\dot{\epsilon}$. Given this and looking at Bianchi identity on asymptotically flat space time, we can also deduce that the source mass is evolving as $\dot{\epsilon}^2$. In this model where we assume both perturber and source mass evolving in time, we can relate this to gravitational wave strain. The gravitational wave luminosity is given by,

$$\mathcal{L} = -\frac{dE}{dt} = \lim_{r\to+\infty} \frac{r^2}{16\pi} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \Big| \int_{-\infty}^{\infty} dt' \Psi_4^{lm} \Big|^2$$

for l=2, m=2 mode (2.1)

$$\mathcal{L} = \lim_{r\to+\infty} \frac{r^2}{16\pi} \Big| \int_{-\infty}^{\infty} dt' \Psi_4^{2m} \Big|^2 \tag{2.2}$$

And the News function in related to Ψ_4 by

$$\sigma = \int_{-\infty}^{t} dt' \Psi_4^{22} \tag{2.3}$$

Now let us look at how this luminosity evolves with time.

$$\frac{d\mathcal{L}_{GW}}{dt} = \frac{1}{16\pi} \frac{d}{dt} \Big| \int_{-\infty}^{\infty} dt' \Psi_4^{22} \Big|^2 = \frac{1}{16\pi} \frac{d}{dt} \Big[\int_{-\infty}^{t} dt' \bar{\Psi}_4^{22}(t') \int_{-\infty}^{t} dt'' \Psi_4^{22}(t'') \Big] \tag{2.4}$$

$$= \frac{1}{16\pi} \Big[\bar{\Psi}_4^{22}(t) \int_{-\infty}^{t} dt' \Psi_4^{22}(t') + \Psi_4^{22}(t) \int_{-\infty}^{t} dt' \bar{\Psi}_4^{22}(t') \Big]$$

$$= \frac{1}{16\pi} \Big[\ddot{\bar{\sigma}}(t) \dot{\sigma}(t) + \dot{\bar{\sigma}}(t) \ddot{\sigma}(t) \Big] = \frac{1}{8\pi} \Re \Big[\ddot{\bar{\sigma}}(t) \dot{\sigma}(t) \Big] \tag{2.5}$$

$$\Psi_2 = \Psi_2^{(0)} + \Psi_2^{(1)}$$

$$\Psi_2^{(0)} = M\,\rho^3$$
$$\text{and}\,\Psi_2^{(1)} = \frac{E}{r^3}, \tag{2.6}$$
$$E := -\mu\, u_t = \mu\frac{(1-2M/r_0)}{(1-3M/r_0)^{1/2}} \tag{2.7}$$

The radiation reaction has a negligible effect on dynamics of the perturber inside ISCO, so we model the rate of mass loss to the Ψ_2^o. First we observe the the luminosity of gravitational wave from binary system is given by

$$\mathcal{L} = \frac{1}{r}\mathcal{M}^{5/3}\Omega^{2/3} \;\text{ where } \mathcal{M} = \mu^{3/5}M^{2/5} \tag{2.8}$$

This luminosity is proportional to square of the gravitational wave strain amplitude. Let us consider the perturber which loose an ϵ mass how it is related to the gravitational wave luminosity. In our model we let the half of the perturber's mass fall into the black hole and half as radiated into gravitational waves.

$$\mathcal{L} = (\mu-\epsilon)^2\,(M+\frac{\epsilon}{2})^{4/3} \approx \mu^2\,M^{4/3}\left(1-\frac{2\epsilon}{\mu}\right) \propto |\Omega^2\Psi_4|^2 \tag{2.9}$$

$$\Psi_2 = M + E(r = r_{ISCO}) = \,\mu\frac{(1-2M/6M)}{(1-3M/6M)^{1/2}} = \frac{2\sqrt{2}\mu}{3} \tag{2.10}$$

so for the perturbation loosing mass the Ψ_2 would change as