

Notes on applying super momentum balance law to constraint GW waveform

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I. LOCAL SUPER MOMENTUM BALANCE LAW

$$\mathcal{F}(\theta, \phi) \Big|_{u_1}^{u_2} := - \int_{u_1}^{u_2} du \left[|\dot{\sigma}^o|^2 - \Re \left(\dot{\sigma}^2 \dot{\sigma}^o \right) \right] (u, \theta, \phi) = \Re \left[\Psi_2^o + \bar{\sigma}^o \dot{\sigma}^o \right] (u_1, \theta, \phi) - \Re \left[\Psi_2^o + \bar{\sigma}^o \dot{\sigma}^o \right] (u_2, \theta, \phi) \quad (1.1)$$

where

$$2\Re \Psi_2^o(u, \theta, \phi) = \lim_{r \rightarrow +\infty} r^3 C_{abcd} n^a l^b n^c l^d \quad (1.2)$$

$$\Psi_4^o(u, \theta, \phi) = \lim_{r \rightarrow +\infty} r C_{abcd} n^a \bar{m}^b n^c \bar{m}^d = \ddot{\sigma}^o = r \left(\ddot{h}_+ + i \ddot{h}_\times \right) \quad (1.3)$$

II. WEYL SCALAR Ψ_2^o FOR BOB AS PERTURBER CROSS THE LIGHT RING

In Backwards One-Body (BOB)[add ref] description of black hole binary merger, the system is described as a single black hole spacetime with a inspiraling in perturber. The Ψ_2^o for the unperturbed Kerr space time is given by $\Psi_2^o = M \rho^3$, where $\rho = -(r - i a \cos \theta)^{-1}$. Here we consider the modification to Ψ_2^o for Kerr spacetime due to presence of such perturber is done by MERLIN, ORI, BARACK, POUND, and VAN DE MEENT [add ref] in ingoing radiation gauge. Since we need the asymptotic limit 1.2 in the super momentum balance law, the only use full part in our of their results in given in equation 58 in ref [add ref]

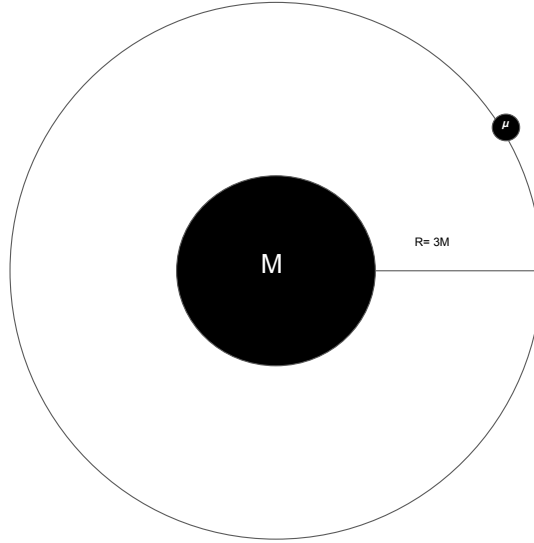


FIG. 1: The figure

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A. Asymptotic axial and asymmetric part of Ψ_2^o for perturbed black hole

$$\begin{aligned}\Psi_2 &= \Psi_2^{(0)} + \Psi_2^{(1)} \\ \Psi_2^{(0)} &= M \rho^3 \\ \text{and } \Psi_2^{(1)} &= \frac{E}{r^3},\end{aligned}\tag{2.1}$$

$$E := -\mu u_t = \mu \frac{(1 - 2M/r_0)}{(1 - 3M/r_0)^{1/2}}\tag{2.2}$$

The radiation reaction has a negligible effect on dynamics of the perturber inside ISCO, so we model the rate of mass loss to the Ψ_2^o . First we observe the the luminosity of gravitational wave from binary system is given by

$$\mathcal{L} = \frac{1}{r} \mathcal{M}^{5/3} \Omega^{2/3} \text{ where } \mathcal{M} = \mu^{3/5} M^{2/5}\tag{2.3}$$

This luminosity is proportional to square of the gravitational wave strain amplitude. Let us consider the perturber which loose an ϵ mass how it is related to the gravitational wave luminosity. In our model we let the half of the perturber's mass fall into the black hole and half as radiated into gravitational waves.

$$\mathcal{L} = (\mu - \epsilon)^2 \left(M + \frac{\epsilon}{2}\right)^{4/3} \approx \mu^2 M^{4/3} \left(1 - \frac{2\epsilon}{\mu}\right) \propto |\Omega^2 \Psi_4|^2\tag{2.4}$$

$$\Psi_2 = M + E(r = r_{ISCO}) = \mu \frac{(1 - 2M/6M)}{(1 - 3M/6M)^{1/2}} = \frac{2\sqrt{2}\mu}{3}\tag{2.5}$$

so for the perturbation loosing mass the Ψ_2 would change as