LEHMAN BROTHERS

The Lehman Brothers Return Attribution Model

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INTRODUCTION

Lehman Brothers, the industry leader in providing domestic indices and benchmarks for the fixed income market, has developed a Return Attribution Model to supplement its index products. The Lehman Brothers Family of Indices has always reported total returns and their components: price, coupon, paydown, and currency returns. This breakdown describes *how* each security contributes to the total return of a particular index or portfolio, but the new model adds insight into *why* those returns have been realized.

The Lehman Brothers Return Attribution Model is designed to help explain returns by ascribing portions of each security's price return to such effects as passage of time, yield curve movement, changes in volatility, and changes in spread. By aggregating these return components, a portfolio manager can better understand a portfolio's performance.

The Lehman Brothers Return Attribution Model was designed to fulfill the two needs of intuitive clarity and computational accuracy. The need for clarity led us to define yield curve movements based on changes in bellwether yields; the need for accuracy led us to use a "full valuation" approach to return attribution. Rather than base the analysis on beginning-of-month sensitivities to various effects, the model calculates each component of return using a full option-adjusted spread (OAS) evaluation.

This report begins by defining the components of price return as they appear in the model. Then we test the model's accuracy and compare it with other methodologies. In the last section, we show how the Return Attribution Model can be used for portfolio management and demonstrate the use of the new model through the Lehman Brothers portfolio analytics platform.

COMPONENTS OF PRICE RETURN

The Lehman Brothers Return Attribution Model itemizes the price return component of total return. The two largest sources of changes in prices for fixed income securities are the passage of time and changes in the Treasury yield curve. This report is concerned primarily with quantifying the effects of changes in the yield curve on price returns. Changes in volatility and spread provide additional sources of price return.

Price returns associated with the passage of time include accretion and roll-down returns. Accretion returns are due to the convergence of a bond's price to par as it approaches maturity. Typically, accretion return will be positive for discounts and negative for premiums, and over the long term accretion is the primary source of return for zero coupon bonds. Accretion return is calculated by holding a bond's yield constant while moving the settlement date forward. For mortgage-backed securities (MBS), the calculation assumes a constant prepayment rate as well. Rolldown return, the other type of price return associated with the passage of time, results from predictable changes in a bond's yield as time elapses, reflecting a change in the bond's placement along the yield curve. In a positively sloping yield curve environment (where longer maturities have higher yields), a bond's yield will fall as its maturity shortens, providing positive rolldown returns. When portions of the yield curve are negatively sloped, bonds may experience negative rolldown returns as their yields rise. For bonds with embedded options, the time return includes the effect of option theta (i.e., erosion of the time value of the option as it nears expiration). For MBS, a constant yield curve assumption does not translate into constant prepayment rates under the Lehman Brothers Prepayment Model. Because the model incorporates a lag of several months between refinancing incentive and prepayment activity, the rolldown return can be strongly influenced by an anticipated increase or decrease in prepayments following significant yield curve movements. As a result, negative rolldown returns are sometimes observed (even on a positively sloped yield curve) as MBS discounts lengthen or premiums shorten.

The largest source of price return for fixed income securities is the movement of the yield curve. The Lehman Brothers Return Attribution Model approximates actual changes in the yield curve using a combination of three curve movements: shift, twist, and butterfly. These curve movements are defined and quantified in the implementation section below.

Components of price returns due to sources other than yield curve changes are volatility and spread returns. Volatility return is due to changes in market volatility and only affects securities with embedded optionality. An increase in volatility, for example, will increase the value of embedded options, giving rise to positive returns on securities that contain an implicit long option position (e.g., putable bonds) and negative returns on securities that contain an implicit short option position (e.g., callable bonds, mortgage pass-throughs). Spread return is due to the widening or tightening of option-adjusted spreads vs. the Treasury yield curve.

MODEL IMPLEMENTATION OF YIELD CURVE CHANGES

In a given month, the Lehman Brothers Return Attribution Model first identifies a set of changes in the marketplace and then divides each security's return into components due to each such change. The model defines the components of yield curve movement as intuitively as possible and strives to attribute accurately the returns realized due to each component.

In an effort to make the model as simple as possible, we based our representation of yield curve movement on changes of bellwether Treasury bond yields, which are directly observable in the marketplace, not on the output of computational procedures such as principal components analysis and linear regression. The Lehman Brothers model decomposes the actual change in the Treasury yield curve in a given month into a combination of three basic shapes (shift, twist and butterfly). Each of these components is a piecewise linear function, with prescribed values at 2-, 5-, 10-, and 30-years,[1] and a single parameter (s for shift, t for twist, and b for butterfly, see Figure 1) that specifies the magnitude of that component in a given month. The values of the parameters s, t, and b are based on observed changes in the yields of Treasury bellwether bonds, selected at the beginning of the month, as follows.

Shift (Figure 1a) returns are due to a parallel yield curve shift, the magnitude s of which we set equal to the average yield change of the 2-, 5-, 10-, and 30-year bellwethers. This type of change produces the same sign for all returns.

$$s = \frac{1}{4}(\Delta y_2 + \Delta y_5 + \Delta y_{10} + \Delta y_{30})$$

Twist (Figure 1b) returns are due to a yield curve flattening or steepening centered on the 5-year. We quantify these returns in a steepening move (t>0) as the 30-year yield moving up by $t/2$ while the 2-year moves down by the same amount; in a flattening move, the opposite occurs. The magnitude of twist t is determined by the change in the spread between 2- and 30-year bellwethers:

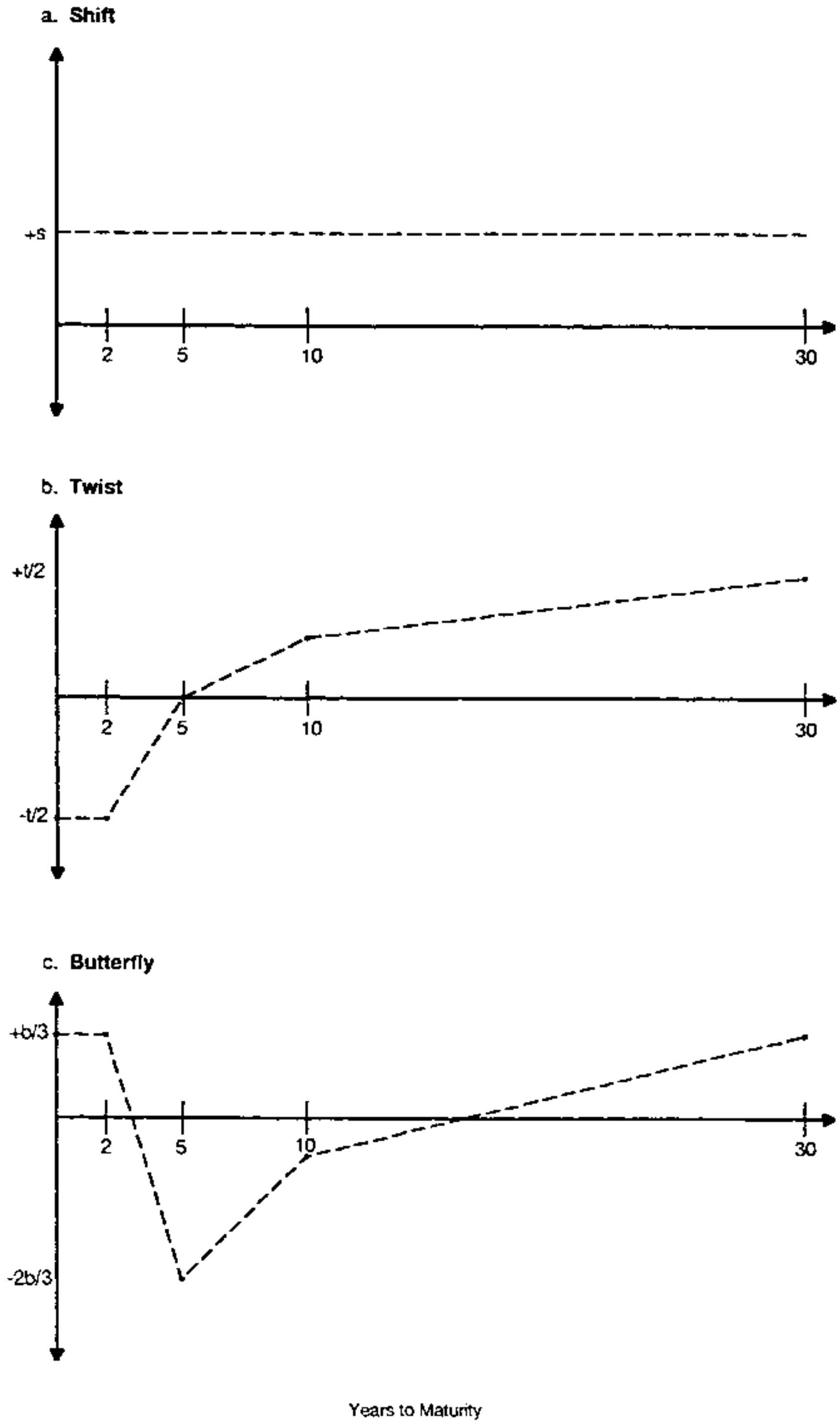
$$t = \Delta y_{30} - \Delta y_2$$

Butterfly (Figure 1c) returns are those produced when the middle of the curve moves in the opposite direction from the wings. We define this as the 2- and 30-year yields moving up by the same amount $b/3$ while the 5-year yield moves down by double the same amount. The magnitude b of the butterfly effect is determined by the extent to which the 5-year bellwether moves differently from the average of the 2- and 30-year yields:

$$b = \frac{1}{2}(\Delta y_2 + \Delta y_{30}) - \Delta y_5$$

[1] The value at 10 years is always set halfway between the values at 5 and 30 years. See the section on Model Development and Validation for a discussion of these assumptions.

Figure 1.
Components of Yield Curve Movement



We define residual return, or shape return, as yield curve returns other than those explained by these three types of yield curve changes. Figure 2 shows the model's treatment of yield change for the month of January 1996.

OAS METHODOLOGY

One strength of the Lehman Brothers model is the method used to attribute each security's return to its various components. The model constructs a sequence of stepwise changes by which the beginning of the month environment is transformed into the end of month environment, and reprices the bond at each of these steps using a full OAS-based valuation. Each component of price return is given by the difference in prices obtained by two successive valuations (see Figure 3).

Figure 2.
Illustration of Breakdown of Yield Curve Movement

a. **Attribution of Changes in the Bellwether Curve between 1/1/96 and 2/1/96,** in bp

Tsy.	Beg.	End	Chg.	Shift	Twist	Bfly.	Shape
2 yr.	5.188%	4.934%	-25.3	-8.1	-16.1	1.7	-2.9
3	5.225	5.032	-19.3	-8.1	-10.7	0.0	-0.5
5	5.404	5.260	-14.4	-8.1	0.0	-3.4	-2.9
10	5.583	5.589	0.6	-8.1	8.0	-0.9	1.5
30	6.005	6.072	6.8	-8.1	16.1	1.7	-2.9

Shift: average of movements of 2-, 5-, 10-, and 30-year = -8.1 bp.
Twist: change in spread between 2- and 30-year = 32.1 bp steepening.
Butterfly: average of movements of 2- and 30-year minus movement of 5-year = 5.1 bp.

b. **Beginning-of-month Bellwethers— Defining Yield Curve Movements**

Tsy.	Coupon	Maturity
2 yr.	5.375%	11/30/1997
3	5.500	11/15/1998
5	5.625	11/30/2000
10	5.875	11/15/2005
30	7.625	2/15/2025

c. **Changes in Term Structure of Volatility**

Tsy.	1/1/96	2/1/96	Change
0 yr.	21.81%	21.87%	0.07%
2	20.85	20.90	0.05
5	19.18	19.19	0.02
7	18.20	18.18	-0.02
10	16.88	16.83	-0.05

Each row of Figure 3 represents a pricing relationship that relates the price of a bond to a particular set of assumptions. The first and last rows correspond to the actual prices of the bond at the beginning and end of the month; the intervening rows show the sequence of pricing calculations that incorporates progressively more of the effects of the month's events. For example, the third row of the table indicates that the bond is to be repriced for settlement at the end of the month, using the beginning of month yield curve, volatility, and OAS. The fourth row indicates that the beginning yield curve should be changed by the parallel shift and the bond repriced using the same OAS over this shifted curve. The difference between the prices calculated by these two operations (as a percentage of beginning price) is reported as the shift return. Each additional row moves one step closer to the end of month environment, and introduces an additional component of return. The right-most column of Figure 3 shows how the components of return are grouped into broader categories.

Figure 3.
Calculation of Return Components by Successive Valuations

Pricing Operation	Settle Date	Yield	PSA	Yield Curve	Volatility	OAS	Return Component[a]	Broader Level Return Component[b]
Beginning Price	Beg	Beg	Beg	Beg	Beg	Beg		
Reprice including Accretion	End	Beg	Beg				Accretion	Time
Reprice including Rolldown	End			Beg	Beg	Beg	Rolldown	
Reprice with shifted yield curve	End			Beg + shift	Beg	Beg	Shift	Yield Curve
Reprice with shifted and twisted yield curve	End			Beg + shift + twist	Beg	Beg	Twist	
Reprice including shift, twist, and butterfly effects	End			Beg + shift + twist + butterfly	Beg	Beg	Butterfly	
Reprice with actual ending yield curve	End			End	Beg	Beg	Shape	
Reprice with ending volatility	End			End	End	Beg	Volatility	OAS Effect
Ending Price	End			End	End	End	Spread	

[a]Each return component in this column is computed as a difference in the current row price from the price in the row immediately above.
[b]Each return component in this column is a sum of two or more related return components from the previous column.

Government and corporate bonds are priced using the Lehman Brothers proprietary implementation of a tree-based lognormal pricing model. The full valuation pricing methodology, in which components of return are calculated by a sequence of OAS-to-price operations based on different yield curves, allows each component of return on a callable bond to be split into bullet and option components, by performing the same sequence of operations on both the callable bond and its bullet-to-maturity equivalent. An example of the return breakdown of a callable corporate bond is shown in Figure 4. The Alabama Power 6.85% of 2002 was priced for settlement on 1/1/96 at 101.712, for a yield to maturity of 6.525%. Repricing at this yield for settlement on 2/1/96, we obtain a price of 101.700, which would give a price return of -1 bp (the change in price is divided by the beginning price + accrued: 101.712 + 2.854 = 104.566). We then reprice once more as of 2/1/96, using the beginning of month OAS of 21.6 bp, to obtain a price of 101.668, and calculate the rolldown return (101.668 - 101.700) / 104.556 = -3 bp. A new lognormal tree is then created by applying a parallel shift of -8.1 bp (see Figure 2) to the beginning-of-month par curve, and the bond is repriced at the same OAS to give a price of 101.919, corresponding to 24 bp of return. This operation is repeated for the twist and butterfly components: a steepening of 32.1 bp causes the price to change to 102.052, for 13 bp of return; the addition of the butterfly movement brings the price to 102.099, giving another 4 bp of return. At the next step, the lognormal tree is constructed using the actual yield curve as of 2/1/96, but still using the term structure of volatility as of 1/1/96, and the bond is repriced at 102.091, for a -1 bp return. Next the new term structure of volatility is put into play, and we reprice the bond using the lognormal tree fully calibrated to the market as of 2/1/96, but still using the beginning OAS. The resulting price of 102.032 gives a -6 bp return due

Figure 4.
Return Attribution for a Callable Corporate Bond, January 1996
Alabama Power 6.850% of 8/1/02

Price Return due to:	
Accretion	-0.01
Rolldown	-0.03
Shift	0.24
Twist	0.13
Butterfly	0.04
Curve Shape	-0.01
Volatility	-0.06
Spread	-0.01
Price Return	0.29
Coupon Return	0.55
Paydown Return	0.00
Currency Return	0.00
Total Return	0.84
Breakdown by Type:	
Time (Accretion + Rolldown)	-0.04
Time + Coupon	0.50
Nonparallel Yield Change	0.16
Overall Yield Curve Change	0.41
OAS (Volatility + Spread)	-0.07

Figure 5.
Return Attribution for a Mortgage Pass-through, January 1996
30-year FNMA 8.0% (1993 origination)

Price Return due to:	
Accretion	0.02
Rolldown	-0.03
Shift	0.11
Twist	0.21
Butterfly	0.01
Curve Shape	0.03
Volatility	0.00
Spread	-0.31
Price Return	0.04
Coupon Return	0.63
Paydown Return	-0.03
Currency Return	0.00
Total Return	0.64
Breakdown by Type:	
Time (Accretion + Rolldown)	-0.01
Time + Coupon	0.62
Nonparallel Yield Change	0.25
Overall Yield Curve Change	0.36
OAS (Volatility + Spread)	-0.31

to the change in volatility. Finally, the actual price of the security as of 2/1/96 is 102.020, which gives an OAS of 22.2 bp. This is interpreted as a -1 bp return due to change in spread.

Mortgage-backed securities are priced using a Monte Carlo approach and the Lehman Brothers Prepayment Model. An example of the return breakdown of a mortgage pass-through is shown in Figure 5. The methodology is identical to the one for the callable corporate, except that as the yield curve is manipulated, we generate new sets of rate paths instead of lognormal trees.

The full valuation methodology employed by the Lehman Brothers model holds a clear advantage over methods that rely on beginning of month sensitivities. For example, shift return could be calculated by multiplying the amount of shift by the beginning of period duration, and twist return by multiplying the measured twist by the beginning twist sensitivity. This approach, however, ignores the fact that changes in the yield curve can cause profound changes in the risk characteristics (e.g., duration, twist sensitivity) of all securities with interest rate-dependent cash flows, and can lead to significant errors in the reporting of return components.

MODEL DEVELOPMENT & VALIDATION

The decomposition of yield curve movement in the Lehman Brothers Return Attribution Model was guided by statistical studies of the historical behavior of Treasury yield changes. Some of these studies were used to help decide on various details of model implementation, while others were used to help measure the accuracy of the completed model.

Figure 6.
Explanatory Power of Different Definitions of Yield Curve Shift
5/90-2/95

Shift Definition	R-squared
3-point shift	85.9%
4-point shift*	88.2
5-point shift	85.1

*Indicates the set of assumptions chosen for the Lehman Brothers model.

1. YIELD CURVE SHIFT

We investigated the definition of yield curve shift based on changes in bellwether bond yields using five years of monthly yield changes for off-the-run coupon Treasuries. Figure 6 summarizes the percentage of yield change explained by each of the three definitions of curve shift:

- 3-point shift (average yield change of 2-, 5-, and 30-year)
- 4-point shift (average yield change of 2-, 5-, 10-, and 30-year)
- 5-point shift (average yield change of 2-, 3-, 5-, 10-, and 30-year)

In measuring fit, we chose to weight the residual yield changes by duration, as a proxy for percentage of explained return. Based on these results, we selected the 4-point shift as superior to both the 3-point shift, which may use too sparse a representation of the curve, and the 5-point shift, which overrepresents the short end of the curve.

2. YIELD CURVE TWIST AND BUTTERFLY

The magnitude of the twist in the yield curve is defined in the model by the change in the 2- to 30-year spread. Several choices were considered for the shape of the term dependence to be assumed for the twist.

- Linear: the twist is applied linearly between 2 and 30 years. This implies a pivot (no change from shifted curve) at 16 years.
- 2-piece linear, pivot at 5: The 5-year point is assumed to be the pivot of the twist (experiences no twist). One wing of the twist is interpolated linearly from 2 to 5 years, and the second (opposite in sign) from 5 to 30.
- 2-piece linear, pivot at 10: same as above, but with pivot at 10 years.
- 3-piece linear, pivot at 5: the 5-year point is considered the pivot of the twist, but the 5-30 year twist is evenly split into 2 linear segments, 5-10 and 10-30, as shown in Figure 1(b).

The definition of the 3-piece linear twist was motivated by a separate historical study, which showed that the yield change of the on-the-run 10-year is close to the average of the changes in the 5-year and the 30-year. (Regression of Δy_{10} against Δy_5 and Δy_{30} indicates coefficients close to 0.5.)

We studied the explanatory power of these various twist shapes, using the same dataset and methodology as in our study of shift. Figure 7 shows the explanatory power added by each of the twist shapes when used in conjunction with the 4-point shift defined above. The two definitions in which the twist has its pivot at 5 years are clearly superior; we have chosen the 3-piece twist based on the improvement in explanatory power and the regression results described above.

The third component usually considered for yield curve decomposition and sometimes observed historically is the butterfly movement. This component results in a modest improvement in the explanatory power of the model, from 97.3% to 98.3%, but can be significant in certain periods.

3. MEASURING YIELD CHANGE

Our definitions of shift, twist, and butterfly are based on the yield changes at specific points (2-, 5-, 10-, and 30-years) along the curve. These yield changes are defined in the model by a "beginning-of-month bellwether" method: we measure the month to month yield change of the beginning-of-month on-the-run Treasury securities of these maturities. We compared the explanatory power of this model with that of two alternative methods: a "rolling bellwether" method in which the yield change may be obtained by subtracting beginning and ending

Figure 7. Explanatory Power of Different Twist Shapes*
5/90-2/95

	Cumulative Fit		
Twist Definition	**Shift**	**Twist**	**Butterfly**
Linear	88.2%	94.2%	94.5%
2-piece linear, pivot at 5	88.2	97.2	98.2
2-piece linear, pivot at 10	88.2	96.5	97.1
3-piece linear, pivot at 5**	88.2	97.3	98.3

*All use Lehman Brothers definitions of shift and butterfly.
**Indicates the set of assumptions chosen for the Lehman Brothers model.

yields of two different bonds in auction months, and a theoretical par curve method in which the beginning and ending yields for the specified maturities are those of hypothetical par coupon Treasuries derived from an off-the-run spline model.

As shown in Figure 8, the beginning-of-month bellwether method used in the Lehman Brothers model exhibits explanatory power roughly equivalent to that based on the par curve, and is more intuitive and accessible to market practitioners. It performs significantly better than the rolling bellwether assumption, which leads to large modeling errors in some auction months.

4. COMPARISON WITH OTHER MODELS

A frequently used method of decomposing yield curve movement is known as principal components analysis. In this method, a detailed analysis of historical yield curve movement is used to find the set of components that can be combined to approximate typical curve behavior. The three most significant components of such analysis tend to have shapes that can be roughly identified as shift, twist, and curvature, but are not explicitly defined as such. We applied this methodology to a monthly time series of changes in the par curve, where the monthly par curves were derived from a spline fitted to off-the-run Treasuries. For each month of data, these three components were then regressed to find the best fit with the set of changes in yields of all noncallable Treasuries. As shown in Figure 9, this analysis explained 97.0% of the changes in yield, while the Lehman Brothers model explained 98.3%.[2]

[2]Theoretically, the regression of this set of components against actual yield changes should have explanatory power greater than that of any ad hoc breakdown. Indeed, when R-squared is computed based on equally weighted (rather than duration weighted) yield residuals, the principal components model slightly outperforms the Lehman Brothers model, 97.4% to 97.1%.

Figure 8.
Effect of Different Techniques for Measuring Yield Change
10/93-12/94

	Cumulative Fit		
Measurement of Yield Change	**Shift**	**Twist**	**Butterfly**
Beginning-of-month Bellwethers*	90.9%	98.1%	99.0%
Rolling Bellwethers	90.5	98.0	98.8
Off-the-run par curves	90.7	98.3	99.1

*Indicates the set of assumptions chosen for the Lehman Brothers model.

Our analysis of historical data demonstrates that the power of the Lehman Brothers model to capture the movement of the yield curve rivals that of more sophisticated approaches. The straightforward treatment of yield curve movement, which makes the model accessible and intuitive, does not incur any significant performance penalty.

APPLICATIONS TO PORTFOLIO MANAGEMENT

A precise attribution of the returns of portfolios and benchmarks can be invaluable in assessing portfolio performance, in both active and passive management modes. In active portfolio management, a manager will typically attempt to enhance portfolio returns by executing a strategy corresponding to a particular view on the market, such as anticipation of a yield curve flattening or a systematic tightening of corporate spreads. To reflect this view, the portfolio will purposely be made structurally different from the benchmark in one or more ways. To assess the contribution of each such strategy to the overall outperformance (especially when multiple strategies are executed simultaneously), it is helpful to break returns into the appropriate components. These uses of return attribution are illustrated in Figure 10.

Figure 9.
Evaluation of Model Accuracy
5/90-2/95

	Cumulative Fit		
Method of Yield Curve Decomposition	**Shift**	**Twist**	**Butterfly**
Lehman Brothers Model*	88.2%	97.3%	98.3%
Principal Components Analysis	91.8	96.6	97.0

*Indicates the set of assumptions chosen for the Lehman Brothers model.

Figure 10.
Application of Return Attribution to Active Portfolio Management

	Sample Outlook	Sample Strategy vs. Benchmark	Measure of Outperformance
1	Unchanged yield curve, volatility and spreads	Buy high current yield.	Excess Coupon & Accretion Returns
2	("no view").	Buy steepest part of curve.	Excess Rolldown Return
3	Upward parallel shift in curve.	Shorten duration.	Excess Shift Return
4	Yield curve to flatten.	Barbell portfolio vs. benchmark.	Excess Twist Return
5	Implied volatility to decline.	Buy callables, derivatives, MBS.	Excess Volatility Return
6	Industrial spreads to tighten.	Overweight industrials.	Excess Spread Return

In passive portfolio management where the goal is to match benchmark performance, any significant deviations between portfolio and benchmark performance can be cause for concern. In such situations, a good understanding of the sources of these return differences can help identify structural differences that have given rise to an observed underperformance and suggest corrective action. Examples of this application of return attribution are given in Figure 11.

We can use the model to compare the return of the actively managed portfolio XYZ to that of its benchmark, the Lehman Brothers Intermediate Corporate Index, for the month of January 1996. The portfolio was duration-matched to the benchmark at the start of the month but was grossly mismatched in both sector composition and allocation along the yield curve. Figure 12 shows the returns achieved by the portfolio and benchmark, as attributed by the model. As would be expected for a duration-matched portfolio, the parallel shift component of the portfolio's return matches that of the benchmark. However, the portfolio underperforms by 22 bp of price return. The model shows that of this 22 bp, 4 bp is attributable to twist return and 14 bp to spread return.

January was characterized by an unusually large steepening of 32.1 bp and a modest parallel shift of -8.1 bp (see Figure 2). For many securities at the short

Figure 11. Application of Return Attribution to Passive Portfolio Management

	Observed Return Deviation vs. Benchmark	Implied Structural Difference
1	Different Coupon/Accretion Returns	Coupon distribution.
2	Different Rolldown Return	Allocation along yield curve.
3	Different Shift Return	Duration mismatch.
4	Different Twist Return	Cash flow distribution along yield curve.
5	Different Volatility Return	Exposure to callables, MBS.
6	Different Spread Return	Sector/quality distribution. Overexposure to individual issuers.

Figure 12. Return Attribution of Portfolio versus Index, January 1996
Portfolio: xyz.prt
Benchmark: icorp.q

	Portfolio	Benchmark	Difference
Price Return due to:			
Accretion	-0.09	-0.08	-0.01
Rolldown	0.02	0.00	0.01
Shift	0.34	0.34	0.00
Twist	-0.07	-0.02	-0.04
Butterfly	0.05	0.07	-0.02
Curve Shape	-0.10	-0.08	-0.02
Volatility	0.00	0.00	0.00
Spread	-0.02	0.12	-0.14
Price Return	0.13	0.35	-0.22
Coupon Return	0.6	0.58	0.02
Paydown Return	0.00	0.00	0.00
Currency Return	0.00	0.00	0.00
Total Return	0.73	0.93	-0.21
Breakdown by Type:			
Time (Accretion + Rolldown)	-0.07	-0.08	0.01
Time + Coupon	0.53	0.50	0.02
Nonparallel Yield Change	-0.12	-0.03	-0.09
Overall Yield Curve Change	0.23	0.32	-0.09
OAS (Volatility + Spread)	-0.02	0.11	-0.14
Implied Spread Change	0.01	-0.03	0.03

and long ends of the yield curve, this steepening caused the twist return to dominate the shift return. (For the FNMA 30-year pass-through shown in Figure 5, with a duration of 1.5 years, the 21 bp twist return is almost double the 11 bp shift return.) The twist returns for this particular portfolio and benchmark are relatively small because their duration of 4.28 years is very near the assumed pivot point of our twist component at 5 years. This effect is illustrated in Figure 13. Both the portfolio and benchmark achieve small twist returns overall by averaging together positive twist returns for short durations with negative twist returns for longer durations. The 4 bp underperformance arises because the portfolio is strongly barbelled versus the index. It is overexposed by 35.31% versus 14.99% in the 0- to 2-year duration cell, underexposed throughout the 2- to 6-year duration cells, and overexposed to durations of 6 years and longer. This barbell position underperformed in the steepening environment; the negative twist returns in the longest cell outweighed the smaller positive twist returns in the shortest.

Figure 13.
Market Structure Comparison of Portfolio vs. Benchmark Twist Return, by Duration
Portfolio: xyz.prt
Benchmark: icorp.q
Pricing Date: 1/31/96

	Modified Adjusted Duration (in years)						
	0 - 2	**2 - 3**	**3 - 4**	**4 - 5**	**5 - 6**	**6 +**	**Total**
Percent	35.31	5.84	6.54	1.61	6.50	44.20	100.00
Percent	14.99	11.01	17.81	14.65	20.84	20.70	100.00
Percent	20.32	-5.17	-11.27	-13.04	-14.34	23.50	0.00
Ret Twist	0.25	0.29	0.14	-0.06	-0.23	-0.37	-0.07
Ret Twist	0.22	0.27	0.15	-0.03	-0.16	-0.38	-0.03
Ret Twist	0.03	0.02	-0.01	-0.03	-0.06	0.00	-0.04

A similar analysis can explain the 14 bp underperformance in spread return. Figure 14 compares the sector and quality allocations of the portfolio and benchmark, along with the resulting spread returns. The portfolio is overweighted in industrials (75.10% vs. 30.98%) and utilities (20.12% vs. 13.01%), has no exposure to the financial or Yankee sectors, and has a 4.78% allocation to the MBS sector, which is not represented in the benchmark. Thus, the portfolio overemphasized the worst performing sector in the benchmark (industrials in the index earned 4 bp of spread return) while missing out on 18 bp of spread return in the finance sector and 12 bp in Yankees. However, this poor showing in sector allocation does not account for the full extent of the underperformance. Had the portfolio matched the benchmark performance within each sector, this allocation to industrials and finance would have given a spread return of about 5 bp (75.10% x 4 + 20.12% x 10), while the portfolio's achieved spread return for these two sectors was only -0.3 bp (75.10% x -1 + 20.12% x 2). This underperformance is attributable to poor security choices in these sectors. A closer look at the performance within each cell reveals especially poor performance in BAA industrials, where the portfolio achieved a spread return of -15 bp versus +2 bp for the index, and in BAA utilities (-14 bp vs. +11 bp). Figure 15 shows a security-level view of portfolio XYZ, with total return separated into three broad components: total time return (coupon, rolldown, and accretion), overall yield curve return, and spread return. Performance in the BAA utility sector was determined by the two bonds representing it in the portfolio—GTE Corp and Gulf States Utilities—which had spread returns of

-12 bp and -18 bp, respectively. The underperformance in BAA industrials was primarily due to the Northrop-Grumman issue, which widened from 81 bp to 141 bp in OAS after being placed on credit watch by Standard & Poor's and provided a huge -373 bp of spread return.

Figure 14. Market Structure Comparison of Portfolio vs. Benchmark Spread Return, by Sector and Quality

Portfolio: xyz.prt
Benchmark: icorp.q
Pricing Date: 1/31/96

1 - Percent
2 - Spread Return

		Sector					
Quality		Indus.	Tele.-Utility	Finan.	Canadian-Supranat'l	MBS	Total
AAA	1	1.98	5.68	0.00	0.00	4.78	12.44
	1	0.67	0.49	0.62	2.47	0.00	4.25
	1	1.31	5.19	-0.62	-2.47	4.78	8.19
	2	-0.04	0.15	0.00	0.00	-0.41	-0.10
	2	0.02	0.02	0.03	0.08	0.00	0.06
	2	-0.06	0.13	-0.03	-0.08	-0.41	-0.15
AA	1	11.78	5.88	0.00	0.00	0.00	17.65
	1	4.62	1.99	5.20	7.83	0.00	19.64
	1	7.16	3.89	-5.20	-7.83	0.00	-1.99
	2	0.05	-0.02	0.00	0.00	0.00	0.03
	2	0.07	0.09	0.11	0.09	0.00	0.09
	2	-0.01	-0.11	-0.11	-0.09	0.00	-0.06
A	1	35.10	3.56	0.00	0.00	0.00	38.66
	1	14.36	5.63	25.32	7.98	0.00	53.28
	1	20.74	-2.06	-25.32	-7.98	0.00	-14.62
	2	0.07	0.13	0.00	0.00	0.00	0.08
	2	0.06	0.10	0.18	0.13	0.00	0.13
	2	0.01	0.03	-0.18	-0.13	0.00	-0.05
BAA	1	26.25	5.00	0.00	0.00	0.00	31.24
	1	11.34	4.90	4.46	2.12	0.00	22.82
	1	14.91	0.10	-4.46	-2.12	0.00	8.42
	2	-0.15	-0.14	0.00	0.00	0.00	-0.15
	2	0.02	0.11	0.28	0.27	0.00	0.11
	2	-0.17	-0.25	-0.28	-0.27	0.00	-0.26
Total	1	75.10	20.12	0.00	0.00	4.78	100.00
	1	30.98	13.01	35.61	20.40	0.00	100.00
	1	44.12	7.11	-35.61	-20.40	4.78	0.00
	2	-0.01	0.02	0.00	0.00	-0.41	-0.02
	2	0.04	0.10	0.18	0.12	0.00	0.11
	2	-0.06	-0.07	-0.18	-0.12	-0.41	-0.14

The return breakdown used in Figure 15 handles zero coupon bonds in a manner consistent with coupon-bearing securities. By grouping the accretion and rolldown components of price return together with coupon return as "total time return," the return of the Archer-Daniels-Midland zero coupon bond is seen on an equal footing with the other bonds in the portfolio. In a more traditional breakdown of total return into price return and coupon return, the entire return of the zero coupon bond is shown in the price return column, offering no additional insight.

Figure 15.
Security Level View of Portfolio XYZ

Issuer	Cpn. Rate (%)	Mty. Date	Par Value	Mod. Adj. Dur. (yrs.)	Quality Ending	OAS-Begin (bp)	OAS (bp)	Returns (%) Total Time	Yld. Crv.	Total Sprd.	Total
AT&T CORP	7.000	5/15/05	1,200	6.77	AA3	39	39	0.53	-0.06	-0.02	0.46
AMERICAN HOME PRODUCTS	7.900	2/15/05	1,200	6.38	A2	46	45	0.53	-0.01	0.07	0.59
ARCHER-DANIELS-MIDLAND	0.000	5/1/02	600	6.07	AA2	38	32	0.52	0.34	0.35	1.21
BP AMERICA INC	8.875	12/1/97	600	1.67	AA3	29	29	0.46	0.43	0.01	0.89
BELLSOUTH TELE	6.500	6/15/05	1,200	6.95	AAA	32	30	0.52	-0.08	0.15	0.59
BLACK + DECKER MFG CO	7.500	4/1/03	300	5.43	BAA3	77	76	0.54	0.25	0.08	0.87
BOEING CO	6.350	6/15/03	300	5.80	A1	38	38	0.52	0.21	0.00	0.72
COCA - COLA ENTERPRISES INC.	6.500	11/15/97	700	1.65	A3	32	31	0.46	0.43	0.04	0.92
CONAGRA INC	9.750	11/1/97	1,000	1.58	BAA1	43	43	0.47	0.41	0.01	0.88
DUPONT E I DE NEMOURS	8.650	12/1/97	500	1.67	AA3	29	29	0.46	0.43	0.01	0.89
DUPONT E I DE NEMOURS	8.125	3/15/04	300	5.90	AA3	40	43	0.52	0.12	-0.18	0.46
EASTMAN CHEMICAL	6.375	1/15/04	700	6.16	A3	69	62	0.54	0.12	0.42	1.07
FORD CAPITAL B.V.	9.125	5/1/98	400	2.00	A1	41	37	0.48	0.48	0.08	1.03
FORD HOLDINGS, INC	9.250	7/15/97	1,200	1.36	A1	34	33	0.44	0.34	0.03	0.82
GTE CORP	8.850	3/1/98	600	1.83	BAA1	74	80	0.50	0.46	-0.12	0.84
GENERAL ELECTRIC	7.875	9/15/98	400	2.30	AAA	23	24	0.47	0.51	-0.04	0.94
GENERAL MOTORS	9.625	12/1/00	400	3.88	A3	62	48	0.51	0.51	0.57	1.59
GULF STATES UTILITIES	8.250	4/1/04	400	5.85	BAA3	122	125	0.59	0.12	-0.18	0.53
INTL BUSINESS MACHINES	6.375	11/1/97	1,000	1.62	A1	28	29	0.45	0.42	0.01	0.88
LOCKHEED CORPORATION	9.375	10/15/99	300	3.08	A3	42	47	0.49	0.53	-0.18	0.84
MCI COMMUNICATIONS	7.500	8/20/04	700	6.18	A2	54	52	0.53	0.05	0.13	0.72
MOBIL OIL CORP ESOP	9.170	2/29/00	300	1.95	AA2	27	21	0.47	0.39	0.12	0.98
NABISCO	8.300	4/15/99	300	2.75	BAA2	78	72	0.52	0.53	0.15	1.19
NORTHROP GRUMMAN	8.625	10/15/04	800	6.07	BAA3	81	141	0.56	0.05	-3.73	-3.12
PENNEY J C	6.375	9/15/00	300	3.88	A1	41	42	0.50	0.52	-0.06	0.96
PHILIP MORRIS COS. INC	9.000	5/15/98	200	2.04	A2	43	39	0.48	0.48	0.08	1.04
RJR NABISCO, INC	8.750	8/15/05	1,100	6.22	BAA3	265	239	0.71	-0.02	1.72	2.41
TCI COMM INC	8.000	8/1/05	1,200	6.70	BAA3	130	124	0.60	-0.05	0.37	0.92
TIME WARNER ENT	9.625	5/1/02	300	4.71	BAA3	100	101	0.56	0.38	-0.07	0.86
USX CORP	6.375	7/15/98	300	2.24	BAA3	73	69	0.50	0.50	0.10	1.10
WMX TECHNOLOGIES	6.375	12/1/03	500	6.06	A1	46	51	0.53	0.14	-0.31	0.36
WAL-MART STORES, INC	6.125	10/1/99	300	3.19	AA1	38	38	0.49	0.55	0.00	1.04
FNMA CONVNTNL LG TSY 30YR	8.000	8/1/22	500	1.47	AAA+	46	55	0.62	0.36	-0.31	0.64
GNMA I SINGLE FAMILY 30YR	9.000	11/1/20	500	1.34	AAA+	67	83	0.70	0.34	-0.50	0.43

The subcomponents of price return offer useful additional information on indices and benchmarks. Figure 16 shows the returns on some of the Lehman Brothers bond indices for the month of January 1996. Shift returns are positive for all indices, while twist returns once again reflect January's drastic steepening. The various intermediate indices, with a maturity range of 1-10 years, have relatively small twist returns since the 5-year pivot of the twist falls in the center of this range. Conversely, the long indices, containing maturities of 10 years and longer, experience large negative twist returns. Also apparent in the figure is the relative performance of the various spread sectors. Outperforming sectors for the month were long finance with a spread return of 45 bp, and long Yankees with a spread return of 33 bp.

Figure 16.
Return Attribution for some Lehman Brothers Indices, January 1996

	Accret	Rlldwn	Shift	Twist	Bfly	Shape	Volat	Spread	Price	Cpn	Paydwn	Total
Aggregate	-0.05	0.00	0.36	-0.17	0.01	-0.01	0.00	-0.03	0.11	0.57	-0.01	0.66
Government/Corporate	-0.08	0.00	0.42	-0.29	0.01	-0.01	0.00	0.02	0.07	0.55	0.00	0.62
Int. Gov/Corp	-0.09	0.01	0.26	0.09	0.04	-0.02	0.00	0.03	0.31	0.55	0.00	0.86
Long Gov/Corp	-0.03	-0.02	0.81	-1.23	-0.06	0.01	-0.01	-0.01	-0.53	0.57	0.00	0.03
Governments	-0.08	0.01	0.40	-0.27	0.00	0.01	0.00	0.00	0.07	0.54	0.00	0.61
Int. Governments	-0.09	0.01	0.24	0.12	0.03	-0.01	0.00	0.00	0.30	0.54	0.00	0.84
Long Governments	-0.04	0.00	0.88	-1.37	-0.07	0.04	0.00	-0.01	-0.58	0.55	0.00	-0.03
1-3 year Govt.	-0.09	0.01	0.13	0.24	-0.02	0.05	0.00	0.01	0.32	0.52	0.00	0.85
Treasuries	-0.09	0.01	0.41	-0.26	0.00	0.00	0.00	0.01	0.09	0.54	0.00	0.63
Int. Treasuries	-0.10	0.02	0.24	0.13	0.03	-0.01	0.00	0.00	0.32	0.54	0.00	0.86
Long Treasuries	-0.05	0.00	0.87	-1.35	-0.07	0.03	0.00	0.01	-0.55	0.56	0.00	0.00
20+ year Treasuries	-0.04	0.00	0.95	-1.57	-0.10	0.11	0.00	-0.01	-0.65	0.53	0.00	-0.12
Agencies	-0.03	-0.02	0.36	-0.30	-0.01	0.02	-0.01	-0.03	-0.03	0.53	0.00	0.50
Int. Agencies	-0.05	-0.02	0.18	0.09	0.02	-0.02	-0.01	0.01	0.21	0.54	0.00	0.75
Long Agencies	0.02	-0.03	0.91	-1.51	-0.11	0.14	-0.02	-0.17	-0.76	0.51	0.00	-0.26
Corporates	-0.06	-0.02	0.47	-0.37	0.04	-0.07	-0.01	0.07	0.06	0.59	0.00	0.65
Int. Corporates	-0.08	0.00	0.34	-0.02	0.07	-0.08	0.00	0.12	0.35	0.58	0.00	0.93
Long Corporates	-0.02	-0.05	0.68	-0.95	-0.03	-0.06	-0.02	0.00	-0.44	0.60	0.00	0.16
Industrials	-0.06	-0.01	0.49	-0.47	0.03	-0.06	0.00	-0.07	-0.16	0.60	0.00	0.45
Int. Industrials	-0.09	0.00	0.34	-0.02	0.07	-0.07	0.00	0.04	0.27	0.60	0.00	0.87
Long Industrials	-0.02	-0.03	0.67	-1.02	-0.03	-0.04	-0.01	-0.20	-0.69	0.61	0.00	-0.08
Utilities	-0.03	-0.04	0.50	-0.45	0.03	-0.07	-0.02	-0.01	-0.09	0.58	0.00	0.49
Int. Utilities	-0.05	0.00	0.35	-0.04	0.08	-0.09	0.00	0.10	0.34	0.56	0.00	0.90
Long Utilities	-0.01	-0.08	0.63	-0.76	-0.01	-0.06	-0.04	-0.10	-0.44	0.61	0.00	0.17
Finance	-0.07	0.00	0.38	-0.12	0.05	-0.08	0.00	0.22	0.38	0.58	0.00	0.96
Int. Finance	-0.08	0.00	0.32	0.01	0.06	-0.06	0.00	0.18	0.44	0.58	0.00	1.02
Long Finance	-0.01	-0.03	0.68	-0.84	0.00	-0.16	0.00	0.45	0.09	0.57	0.00	0.66
Yankees	-0.06	-0.02	0.52	-0.45	0.04	-0.07	0.00	0.20	0.15	0.59	0.00	0.73
Int. Yankees	-0.08	0.01	0.38	-0.08	0.08	-0.10	0.00	0.12	0.34	0.58	0.00	0.91
Long Yankees	-0.02	-0.09	0.77	-1.12	-0.05	-0.02	0.00	0.33	-0.19	0.61	0.00	0.41

DELIVERY THROUGH THE LEHMAN BROTHERS PORTFOLIO ANALYTICS PLATFORM

The Return Attribution Model has been fully integrated into the Lehman Brothers portfolio analytics software. Attribution of returns on all individual securities in the Lehman Brothers Aggregate Index is performed monthly as part of index production and can be incorporated into all reports produced by this software. Examples of these reports are shown in the figures above. In addition, return attribution calculations can be carried out on user-defined securities (on the UNIX platform only) to allow complete return attribution on portfolios.

CONCLUSION

The Lehman Brothers Return Attribution Model offers several benefits. It breaks down portfolio and index returns into a practical set of components, which correspond to strategies commonly used by portfolio managers. It analyzes the movement of the yield curve based on changes in bellwether yields, in an intuitive, easily replicable manner. It uses full OAS-based valuation techniques, thus giving the most accurate treatment possible for securities with curve-dependent cash flows. The model also has been shown to have superior explanatory power in validation testing against historical Treasury yield data. The model will be used to attribute the returns of the domestic Lehman Brothers indices, the most popular domestic benchmarks. Introduction of this model provides to managers of portfolios benchmarked against the Lehman Brothers indices a new and powerful tool for performance measurement and analysis in the fixed income arena.

LEHMAN BROTHERS