Virtual Bidding for Coordination of Power and Natural Gas Markets Under Uncertainty

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This document serves as an electronic companion for the paper "Virtual Bidding for Coordination of Power and Natural Gas Markets Under Uncertainty". It contains 5 sections. Section 1 presents the detailed formulation of all optimization problems from the original manuscript including the equivalent equilibrium problems following Remark 1. The Karush Kuhn Tucker (KKT) conditions of all optimization and equilibrium problems are provided in Section 2. Sections 3 and 4 show the proofs of Propositions 1 and 2, respectively. Section 5 contains the overview over the computational performance.

Nomenclature

Sets

- I Set of dispatchable power production units i.
- Z Subset of non-gas power plants ($Z \subset I$).
- G Subset of natural gas-fired power plants ($G \subset I$).
- S Subset of slow-start power plants ($S \subset I$).
- F Subset of fast-start power plants ($F \subset I$).
- SS Subset of self-scheduling power plants ($SS \subset I$).
- J Set of wind power units j.
- K Set of natural gas supply units k.
- R Set of electricity virtual bidders r.
- Q Set of natural gas virtual bidders q.
- Ω Set of wind power scenarios ω.
- T Set of time periods t.

Note that $Z \cup G = I$, $F \cap S = \emptyset$, $F \cap SS = \emptyset$ and $S \cap SS = \emptyset$.

Variables

- $p^{\mathrm{DA}}_{i,t}, w^{\mathrm{DA}}_{j,t}$ Day-ahead dispatch of units i and j in period t, respectively [MW].
- $p^{\mathrm{RT}}_{i,t,\omega}$ Power production adjustment of unit i in scenario ω, period t [MW].
- $w^{\mathrm{RT}}_{j,t,\omega}$ Wind power production adjustment of unit j in scenario ω, period t [MW].
- $l^{\mathrm{sh,E}}_{t,\omega}, l^{\mathrm{sh,G}}_{t,\omega}$ Electricity and natural gas load shedding under scenario ω, period t [MW, kcf/h].
- $g^{\mathrm{DA}}_{k,t}$ Day-ahead dispatch of unit k in period t [kcf/h].
- $g^{\mathrm{RT}}_{k,t,\omega}$ Natural gas adjustment by unit k in scenario ω, period t [kcf/h].
- $\hat{\lambda}^{\mathrm{E}}_{t}$ Day-ahead electricity price in period t [\$/MWh].
- $\tilde{\lambda}^{\mathrm{E}}_{t,\omega}$ Probability-weighted real-time electricity price in period t, scenario ω [\$/MWh].
- $\hat{\lambda}^{\mathrm{G}}_{t}$ Day-ahead natural gas price in period t [\$/kcf].
- $\tilde{\lambda}^{\mathrm{G}}_{t,\omega}$ Probability-weighted real-time natural gas price in period t, scenario ω [\$/kcf].
- μ, ν Set of dual variables in day-ahead and real-time markets, respectively.
- $c^{\mathrm{DA}}_{i,t}$ Start-up cost of dispatchable unit i in period t [\$].
- $c^{\mathrm{RT}}_{i,t,\omega}$ Start-up cost adjustment of dispatchable fast-start unit i in period t under scenario s [\$].
- $u^{\mathrm{DA}}_{i,t}$ Relaxed unit commitment status of dispatchable unit i in period t.
- $u^{\mathrm{RT}}_{i,t,\omega}$ Relaxed unit commitment adjustment of fast-start unit i in period t, scenario ω.
- $v^{\mathrm{DA,E}}_{r,t}$ Day-ahead trade of electricity virtual bidder r in period t [MW].
- $v^{\mathrm{RT,E}}_{r,t}$ Real-time trade of electricity virtual bidder r in period t [MW].
- $v^{\mathrm{DA,G}}_{q,t}$ Day-ahead trade of natural gas virtual bidder q in period t [kcf/h].
- $v^{\mathrm{RT,G}}_{q,t}$ Real-time trade of natural gas virtual bidder q in period t [kcf/h].

Parameters

D_t^{E} Electricity demand in period t [MW].
D_t^{G} Natural gas demand in period t [kcf/h].
C_i^{E} Production cost of unit i [$/MWh].
$C^{\mathrm{sh,E}}$ Value of electricity lost load [$/MWh].
C_k^{G} Day-ahead offer price of unit k [$/kcf].
$C^{\mathrm{sh,G}}$ Value of natural gas lost load [$/kcf].
$P_i^{\max}$ Capacity of dispatchable unit i [MW].
$P_i^{\min}$ Minimum production level of dispatchable unit i [MW].
ϕ_i Power conversion factor of natural gas unit $i \in G$ [kcf/MWh].
$W_{j,t,\omega}$ Wind power realization of unit j in period t, scenario ω [MW].
$W_{j,t}^{\mathrm{DA}}$ Day-ahead wind power forecast for unit j in period t [MW].
$\overline{W}_j$ Capacity of wind power unit j [MW].
$G_k^{\max}$ Capacity of natural gas unit k [kcf].
G_k^{adj} Adjustment limit of natural gas unit k [kcf/h].
π_ω Probability of scenario ω.
C_i^{SU} Start-up cost of dispatchable unit i [$].
U_i^{ini} Initial commitment status of dispatchable unit i [0/1].
P_i^{ini} Initial dispatch of unit i [MW].
R_i Up/down ramping limit of dispatchable unit i [MW/h].

1. Optimization Problems

1.1. Explicit Electricity Virtual Bidder

The profit maximization problem of each virtual bidder r participating in the electricity market given the day-ahead and expectation of real-time prices $\hat{\lambda}_t^{\mathrm{E}}$ and $\tilde{\lambda}_{t,\omega}^{\mathrm{E}}$, respectively, is given below:

$$\left\{\begin{aligned} &\max_{v_{r,t}^{\mathrm{DA,E}},v_{r,t}^{\mathrm{RT,E}}} \sum_{t\in T}\left(v_{r,t}^{\mathrm{DA,E}}\ \hat{\lambda}_t^{\mathrm{E}} + \sum_{\omega\in\Omega} v_{r,t}^{\mathrm{RT,E}}\ \pi_\omega\ \tilde{\lambda}_{t,\omega}^{\mathrm{E}}\right) && \text{(1a)}\\ &\text{subject to } v_{r,t}^{\mathrm{DA,E}} + v_{r,t}^{\mathrm{RT,E}} = 0 : \rho_{r,t},\ \ \forall t, && \text{(1b)}\end{aligned}\right\},\ \ \forall r \in R,$$

where $\Theta^{\mathrm{VBE}} = \{v_{r,t}^{\mathrm{DA,E}}, v_{r,t}^{\mathrm{RT,E}}, \forall r,t\}$ is the set of primal optimization variables. Objective function (1a) maximizes the expected profit of arbitraging in the day-ahead and real-time electricity markets. Equation (1b) ensures that each virtual bidder sells (buys) the same amount back in the real-time market that was bought (sold) in the day-ahead market. The market operators treat the virtual bidders' dispatch decision as fixed input into the market clearing in the following.

1.2. Day-Ahead Electricity Market

The day-ahead electricity market clears with given day-ahead positions of virtual trade as:

$$\min_{\Theta^{\mathrm{EDA}}} \sum_{t\in T}\Big(\sum_{i\in Z} C_i^{\mathrm{E}}\ p_{i,t}^{\mathrm{DA}} + \sum_{i\in G}\hat{\lambda}_t^{\mathrm{G}}\ \phi_i\ p_{i,t}^{\mathrm{DA}} + \sum_{i\in I} c_{i,t}^{\mathrm{DA}}\Big) \tag{2a}$$

$$\text{subject to } \sum_{i\in I} p_{i,t}^{\mathrm{DA}} + \sum_{j\in J} w_{j,t}^{\mathrm{DA}} - D_t^{\mathrm{E}} + \sum_{r\in R} v_{r,t}^{\mathrm{DA,E}} = 0 : \hat{\lambda}_t^{\mathrm{E}},\ \forall t, \tag{2b}$$

$$0 \le w_{j,t}^{\mathrm{DA}} \le W_{j,t}^{\mathrm{DA}} : \underline{\mu}_{j,t}^{\mathrm{W}}, \overline{\mu}_{j,t}^{\mathrm{W}},\ \ \forall j,t, \tag{2c}$$

$$u_{i,t}^{\mathrm{DA}}\ P_i^{\min} \le p_{i,t}^{\mathrm{DA}} \le u_{i,t}^{\mathrm{DA}}\ P_i^{\max} : \underline{\mu}_{i,t}^{\mathrm{P}}, \overline{\mu}_{i,t}^{\mathrm{P}},\ \ \forall i \in I, t, \tag{2d}$$

$$-u_{i,(t-1)}^{\mathrm{DA}}\ R_i \le \left(p_{i,t}^{\mathrm{DA}} - p_{i,(t-1)}^{\mathrm{DA}}\right) \le u_{i,t}^{\mathrm{DA}}\ R_i : \underline{\mu}_{i,t}^{\mathrm{R}}, \overline{\mu}_{i,t}^{\mathrm{R}},\ \ \forall i \in I, t > 1, \tag{2e}$$

$$-U_i^{\mathrm{ini}}\ R_i \le \left(p_{i,t}^{\mathrm{DA}} - P_i^{\mathrm{ini}}\right) \le u_{i,t}^{\mathrm{DA}}\ R_i : \underline{\mu}_{i,t}^{\mathrm{R}}, \overline{\mu}_{i,t}^{\mathrm{R}},\ \ \forall i \in I, t = 1, \tag{2f}$$

$$0 \leq u^{\mathrm{DA}}_{i,t} \leq 1 : \underline{\mu}^{\mathrm{B}}_{i,t}, \overline{\mu}^{\mathrm{B}}_{i,t}, \quad \forall i \in I, t, \tag{2g}$$

$$C^{\mathrm{SU}}_{i} \left(u^{\mathrm{DA}}_{i,t} - u^{\mathrm{DA}}_{i,(t-1)} \right) \leq c^{\mathrm{DA}}_{i,t} : \overline{\mu}^{\mathrm{SU}}_{i,t}, \quad \forall i \in I, t > 1, \tag{2h}$$

$$C^{\mathrm{SU}}_{i} \left(u^{\mathrm{DA}}_{i,t} - U^{\mathrm{ini}}_{i} \right) \leq c^{\mathrm{DA}}_{i,t} : \overline{\mu}^{\mathrm{SU}}_{i,t}, \quad \forall i \in I, t = 1, \tag{2i}$$

$$0 \leq c^{\mathrm{DA}}_{i,t} : \underline{\mu}^{\mathrm{SU}}_{i,t}, \quad \forall i \in I, t, \tag{2j}$$

where $\Theta^{\mathrm{EDA}} = \{p^{\mathrm{DA}}_{i,t}, c^{\mathrm{DA}}_{i,t}, u^{\mathrm{DA}}_{i,t}, \forall i \in I, t; w^{\mathrm{DA}}_{j,t}, \forall j, t\}$ is the set of primal optimization variables. The objective (2a) of the deterministic day-ahead market-clearing problem is to minimize the day-ahead generation cost. The total cost stems from the cost of non-gas and gas-fired power plants. The price offer for energy sale in the market is assumed equal to the marginal cost of production. For the case of gas-fired units, we assume that the marginal cost of production is described by a linear function of the estimated natural gas price, i.e., $C_i = \hat{\lambda}^{\mathrm{G}}_t \phi_i, \forall i \in G$. Constraint (2b) is the day-ahead power balance with inelastic demand treating the virtual day-ahead positions $\sum_{r \in R} v^{\mathrm{DA,E}}_{r,t}$ as given inputs. Constraints (2c) and (2d) enforce lower and upper bounds on the day-ahead dispatch of wind and conventional generation. Constraints (2e), (2f) ensure the ramping limits of conventional generators and represent in combination with (2g) the tight relaxation of unit commitment. Constraints (2h), (2i), (2j) enforce the start-up cost of each generator.

The optimization problem for day-ahead electricity market clearing can be equivalently formulated as the following equilibrium model with each unit maximizing their profit and a price-setting agent according to Remark 1. Each non gas-fired generator $\mathcal{C}$ maximizes its day-ahead profit with respect to its operational constraints according to

$$\left\{ \begin{aligned} &\max_{p^{\mathrm{DA}}_{i,t}, c^{\mathrm{DA}}_{i,t}, u^{\mathrm{DA}}_{i,t}} \sum_{t \in T} \left[\left(\hat{\lambda}^{\mathrm{E}}_t - C^{\mathrm{E}}_i \right) p^{\mathrm{DA}}_{i,t} - c^{\mathrm{DA}}_{i,t} \right] && \text{(3a)} \\ &\text{subject to } (2\text{d}) - (2\text{j}) \end{aligned} \right\} \forall (i \in \mathcal{C}). \qquad \text{(3b)}$$

Similarly, the gas-fired generators $\mathcal{G}$ decide their day-ahead offers based estimated marginal cost from the natural gas price forecast $\hat{\lambda}^{\mathrm{G}}_t$:

$$\left\{ \begin{aligned} &\max_{p^{\mathrm{DA}}_{i,t}, c^{\mathrm{DA}}_{i,t}, u^{\mathrm{DA}}_{i,t}} \sum_{t \in T} \left[\left(\hat{\lambda}^{\mathrm{E}}_t - \hat{\lambda}^{\mathrm{G}}_t \ \phi_i \right) p^{\mathrm{DA}}_{i,t} - c^{\mathrm{DA}}_{i,t} \right] && \text{(4a)} \\ &\text{subject to } (2\text{d}) - (2\text{j}) \end{aligned} \right\} \forall (i \in \mathcal{G}). \qquad \text{(4b)}$$

Wind farms maximize their profit according to the day-ahead wind forecast $W^{\mathrm{DA}}_{j,t}$ as

$$\left\{ \begin{aligned} &\max_{w^{\mathrm{DA}}_{j,t}} \sum_{t \in T} \hat{\lambda}^{\mathrm{E}}_t \ w^{\mathrm{DA}}_{j,t} && \text{(5a)} \\ &\text{subject to } (2\text{c}) \end{aligned} \right\} \forall j. \qquad \text{(5b)}$$

A price setting agent decides the day-ahead electricity price $\hat{\lambda}^{\mathrm{E}}_t$ according to

$$\min_{\hat{\lambda}^{\mathrm{E}}_t} \sum_{t \in T} \hat{\lambda}^{\mathrm{E}}_t \Big(\sum_{i \in I} p^{\mathrm{DA}}_{i,t} + \sum_{j \in J} w^{\mathrm{DA}}_{j,t} - D^{\mathrm{E}}_t + \sum_{r \in R} v^{\mathrm{DA,E}}_{r,t}. \Big) \tag{6a}$$

The equilibrium problem (3)-(6) is equivalent to the day-ahead market optimization problem (2), since the Karush-Kuhn-Tucker (KKT) conditions are identical, see Section 2.

1.3. Real-Time Electricity Market

In real-time operation, wind power production $W_{j,t,\omega}$ is realized and the real-time markets are cleared to adjust for imbalances. The day-ahead schedule is treated as fixed parameters in the following formulation.

$$\Bigg\{ \min_{\Theta^{\mathrm{ERT}}} \sum_{t\in T} \Big(\sum_{i\in Z} C_i^{\mathrm{E}}\ p_{i,t,\omega}^{\mathrm{RT}} + \sum_{i\in G} \hat{\lambda}_t^{\mathrm{G}}\ \phi_i\ p_{i,t,\omega}^{\mathrm{RT}} + C^{\mathrm{sh,E}}\ l_{t,\omega}^{\mathrm{sh,E}} + \sum_{i\in F} c_{i,t,\omega}^{\mathrm{RT}} \Big) \tag{7a}$$

$$\text{subject to} \sum_{i\in I} p_{i,t,\omega}^{\mathrm{RT}} + l_{t,\omega}^{\mathrm{sh,E}} + \sum_{r\in R} v_{r,t}^{\mathrm{RT,E}} + \sum_{j\in J} w_{j,t,\omega}^{\mathrm{RT}} = 0 : \tilde{\lambda}_{t,\omega}^{\mathrm{E}},\ \forall t, \tag{7b}$$

$$0 \le l_{t,\omega}^{\mathrm{sh,E}} \le D_t^{\mathrm{E}} : \underline{\nu}_{t,\omega}^{\mathrm{DE}}, \overline{\nu}_{t,\omega}^{\mathrm{DE}},\ \ \forall t, \tag{7c}$$

$$0 \le \left(w_{j,t}^{\mathrm{DA}} + w_{j,t,\omega}^{\mathrm{RT}}\right) \le W_{j,t,\omega} : \underline{\nu}_{j,t,\omega}^{\mathrm{W}}, \overline{\nu}_{j,t,\omega}^{\mathrm{W}},\ \ \forall j,t, \tag{7d}$$

$$u_{i,t}^{\mathrm{DA}}\ P_i^{\min} \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}}\right) \le u_{i,t}^{\mathrm{DA}}\ P_i^{\max} : \underline{\nu}_{i,t,\omega}^{\mathrm{P}}, \overline{\nu}_{i,t,\omega}^{\mathrm{P}},\ \ \forall i\in S,t, \tag{7e}$$

$$-\,u_{i,(t-1)}^{\mathrm{DA}}\ R_i \le \Big(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - p_{i,(t-1)}^{\mathrm{DA}} - p_{i,(t-1),\omega}^{\mathrm{RT}}\Big) \le u_{i,t}^{\mathrm{DA}}\ R_i : \underline{\nu}_{i,t,\omega}^{\mathrm{R}}, \overline{\nu}_{i,t,\omega}^{\mathrm{R}},\ \ \forall i\in S, t>1, \tag{7f}$$

$$-\,U_i^{\mathrm{ini}}\ R_i \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - P_i^{\mathrm{ini}}\right) \le u_{i,t}^{\mathrm{DA}}\ R_i : \underline{\nu}_{i,t,\omega}^{\mathrm{R}}, \overline{\nu}_{i,t,\omega}^{\mathrm{R}},\ \ \forall i\in S, t{=}1, \tag{7g}$$

$$\left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}\right) P_i^{\min} \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}}\right) \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}\right) P_i^{\max} : \underline{\nu}_{i,t,\omega}^{\mathrm{P}}, \overline{\nu}_{i,t,\omega}^{\mathrm{P}},\ \ \forall i\in F,t, \tag{7h}$$

$$-\Big(u_{i,(t-1)}^{\mathrm{DA}} + u_{i,(t-1),\omega}^{\mathrm{RT}}\Big) R_i \le \Big(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - p_{i,(t-1)}^{\mathrm{DA}} - p_{i,(t-1),\omega}^{\mathrm{RT}}\Big) \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}\right) R_i$$

$$: \underline{\nu}_{i,t,\omega}^{\mathrm{R}}, \overline{\nu}_{i,t,\omega}^{\mathrm{R}},\ \ \forall i\in F, t>1, \tag{7i}$$

$$-\,U_i^{\mathrm{ini}}\ R_i \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - P_i^{\mathrm{ini}}\right) \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}\right)\ R_i : \underline{\nu}_{i,t,\omega}^{\mathrm{R}}, \overline{\nu}_{i,t,\omega}^{\mathrm{R}},\ \ \forall i\in F, t{=}1, \tag{7j}$$

$$0 \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}\right) \le 1 : \underline{\nu}_{i,t,\omega}^{\mathrm{B}}, \overline{\nu}_{i,t,\omega}^{\mathrm{B}},\ \ \forall i\in F,t, \tag{7k}$$

$$C_i^{\mathrm{SU}}\ \Big(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} - u_{i,(t-1)}^{\mathrm{DA}} - u_{i,(t-1),\omega}^{\mathrm{RT}}\Big) \le \left(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}}\right) : \overline{\nu}_{i,t,\omega}^{\mathrm{SU}},\ \ \forall i\in F,t, \tag{7l}$$

$$C_i^{\mathrm{SU}}\ \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} - U_i^{\mathrm{ini}}\right) \le \left(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}}\right) : \overline{\nu}_{i,t,\omega}^{\mathrm{SU}},\ \ \forall i\in F, t=1, \tag{7m}$$

$$0 \le \left(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}}\right) : \underline{\nu}_{i,t,\omega}^{\mathrm{SU}},\ \ \forall i\in F,t \Bigg\}\ \forall\omega, \tag{7n}$$

where $\Theta^{\mathrm{ERT}} = \{p_{i,t,\omega}^{\mathrm{RT}},\ \forall i\in I,t,\omega; w_{j,t,\omega}^{\mathrm{RT}},\ \forall j,t,\omega; l_{t,\omega}^{\mathrm{sh,E}},\ \forall t,\omega;\ u_{i,t,\omega}^{\mathrm{RT}},\ c_{i,t,\omega}^{\mathrm{RT}}\ \ \forall i\in F,t,\omega\}$ is the set of primal optimization variables. Objective function (7a) describes the real-time cost of power adjustments to cover excess or lack of wind power production. Electricity load shedding cost is also taken into account. Constraint (7b) balances the deviations in real-time from the day-ahead schedule with the position of virtual bidders $\sum_{r\in R} v_{r,t}^{\mathrm{RT,E}}$ as fixed input. Constraints (7c),(7d), (7e), and (7h) enforce lower and upper bounds on the real-time adjustment of load levels, wind generation, and conventional slow- and fast-starting generators, respectively. Constraints (7f),(7g),(7i), (7j) ensure the ramp-rate limits of conventional slow- and fast-starting generators and represent in combination with (7k) the tight relaxation of unit commitment for fast-starting units. Constraints (7l), (7m), (7n) enforce the start-up cost of fast-starting generators.

Also the optimization problem of real-time market clearing (7) can be equivalently formulated as the following equilibrium problem. Slow-starting non gas-fired generators $\mathcal{Z}\cap\mathcal{S}$ maximize their profit in real-time with respect to their day-ahead commitment decisions as

$$\Bigg\{ \max_{p_{i,t,\omega}^{\mathrm{RT}}} \sum_{t\in T} \left(\tilde{\lambda}_{t,\omega}^{\mathrm{E}} - C_i^{\mathrm{E}}\right) p_{i,t,\omega}^{\mathrm{RT}} \tag{8a}$$

$$\text{subject to (7e)} - \text{(7g)} \Bigg\}\ \forall (i\in\mathcal{Z}\cap\mathcal{S}),\omega, \tag{8b}$$

while fast-starting generators $\mathcal{Z} \cap \mathcal{F}$ can update their commitment decisions in real-time

$$\left\{ \max_{p^{\mathrm{RT}}_{i,t,\omega}, c^{\mathrm{RT}}_{i,t,\omega}, u^{\mathrm{RT}}_{i,t,\omega}} \sum_{t\in T} \left[\left(\tilde{\lambda}^{\mathrm{E}}_{t,\omega} - C^{\mathrm{E}}_{i} \right) p^{\mathrm{RT}}_{i,t,\omega} - c^{\mathrm{RT}}_{i,t,\omega} \right] \right. \tag{9a}$$

$$\left. \text{subject to } (7h) - (7n) \right\} \forall (i \in \mathcal{Z} \cap \mathcal{F}), \omega. \tag{9b}$$

Similarly, gas-fired generators optimize their offers in real-time based on real-time natural gas price estimation $\hat{\lambda}^{\mathrm{G}}_{t}$ as slow-starters $\mathcal{G} \cap \mathcal{S}$

$$\left\{ \max_{p^{\mathrm{RT}}_{i,t,\omega}} \sum_{t\in T} \left(\tilde{\lambda}^{\mathrm{E}}_{t,\omega} - \hat{\lambda}^{\mathrm{G}}_{t} \ \phi_i \right) p^{\mathrm{RT}}_{i,t,\omega} \right. \tag{10a}$$

$$\left. \text{subject to } (7e) - (7g) \right\} \forall (i \in \mathcal{G} \cap \mathcal{S}), \omega \tag{10b}$$

and fast-starters $\mathcal{G} \cap \mathcal{F}$, respectively,

$$\left\{ \max_{p^{\mathrm{RT}}_{i,t,\omega}, c^{\mathrm{RT}}_{i,t,\omega}, u^{\mathrm{RT}}_{i,t,\omega}} \sum_{t\in T} \left[\left(\tilde{\lambda}^{\mathrm{E}}_{t,\omega} - \hat{\lambda}^{\mathrm{G}}_{t} \ \phi_i \right) p^{\mathrm{RT}}_{i,t,\omega} - c^{\mathrm{RT}}_{i,t,\omega} \right] \right. \tag{11a}$$

$$\left. \text{subject to } (7h) - (7n) \right\} \forall (i \in \mathcal{G} \cap \mathcal{F}), \omega. \tag{11b}$$

Wind farms adjust their offers in real-time according to the actual wind power realization $W_{j,t,\omega}$:

$$\left\{ \max_{w^{\mathrm{RT}}_{j,t,\omega}} \sum_{t\in T} \tilde{\lambda}^{\mathrm{E}}_{t,\omega} \ w^{\mathrm{RT}}_{j,t,\omega} \right. \tag{12a}$$

$$\left. \text{subject to } (7d) \right\} \forall j, \omega. \tag{12b}$$

Power demand is able to shed load in real-time incurring cost as

$$\left\{ \min_{l^{\mathrm{sh,E}}_{t,\omega}} \sum_{t\in T} \left(C^{\mathrm{sh,E}} - \tilde{\lambda}^{\mathrm{E}}_{t,\omega} \right) l^{\mathrm{sh,E}}_{t,\omega} \right) \tag{13a}$$

$$\left. \text{subject to } (7c) \right\} \forall \omega. \tag{13b}$$

For each scenario, the real-time electricity price $\tilde{\lambda}^{\mathrm{E}}_{t,\omega}$ is set according to

$$\left\{ \min_{\tilde{\lambda}^{\mathrm{E}}_{t,\omega}} \sum_{t\in T} \tilde{\lambda}^{\mathrm{E}}_{t,\omega} \left(\sum_{i\in I} p^{\mathrm{RT}}_{i,t,\omega} + l^{\mathrm{sh,E}}_{t,\omega} + \sum_{j\in J} w^{\mathrm{RT}}_{j,t,\omega} + \sum_{r\in R} v^{\mathrm{RT,E}}_{r,t} \right) \right\} \forall \omega. \tag{14a}$$

The equilibria problems (8)-(14) are equivalent to the real-time market optimization problems (7) for each scenario ω.

1.4. Explicit Natural Gas Virtual Bidder

We also introduce virtual bidding in the natural gas markets. Similarly to electricity virtual bidding, the profit maximization problem of each virtual bidder q participating in the natural gas market is given below for day-ahead and real-time distribution of natural gas spot price $\hat{\lambda}^{\mathrm{G}}_{t}, \tilde{\lambda}^{\mathrm{G}}_{t,\omega}$:

$$\left\{ \max_{\Theta^{\mathrm{VBG}}} \sum_{t\in T} \left(v^{\mathrm{DA,G}}_{q,t} \ \hat{\lambda}^{\mathrm{G}}_{t} + \sum_{\omega\in\Omega} v^{\mathrm{RT,G}}_{q,t} \ \pi_\omega \ \tilde{\lambda}^{\mathrm{G}}_{t,\omega} \right) \right. \tag{15a}$$

$$\left.\text{subject to } v_{q,t}^{\mathrm{DA,G}} + v_{q,t}^{\mathrm{RT,G}} = 0 : \psi_{q,t}, \ \forall t, \right\}, \ \forall q \in Q, \tag{15b}$$

where $\Theta^{\mathrm{VBG}} = \{v_{q,t}^{\mathrm{DA,G}}, v_{q,t}^{\mathrm{RT,G}}, \forall q, t\}$ is the set of primal optimization variables. Objective function (15a) maximizes the expected profit of virtual bidder participating in the day-ahead and real-time natural gas markets and equation (15b) balances the virtual bidders day-ahead and real-time trade.

1.5. Day-Ahead Natural Gas Market

Both the day-ahead dispatch of virtual traders and gas-fired units are inputs into the natural gas day-ahead market clearing problem. The gas-fired units' dispatch is translated into a time-varying demand for natural gas by $\sum_{i\in G} \phi_i \, p_{i,t}^{\mathrm{DA}}, \forall t$.

$$\min_{\Theta^{\mathrm{GD}}} \sum_{t\in T} \Big(\sum_{k\in K} C_k^{\mathrm{G}} \, g_{k,t}^{\mathrm{DA}} \Big) \tag{16a}$$

$$\text{subject to } \sum_{k\in K} g_{k,t}^{\mathrm{DA}} - \sum_{i\in G} \phi_i \, p_{i,t}^{\mathrm{DA}} - D_t^{\mathrm{G}} + \sum_{q\in Q} v_{q,t}^{\mathrm{DA,G}} = 0 : \hat{\lambda}_t^{\mathrm{G}}, \ \forall t, \tag{16b}$$

$$0 \le g_{k,t}^{\mathrm{DA}} \le G_k^{\max} : \underline{\mu}_{k,t}^{\mathrm{G}}, \overline{\mu}_{k,t}^{\mathrm{G}}, \ \forall k, t, \tag{16c}$$

where $\Theta^{\mathrm{GD}} = \{g_{k,t}^{\mathrm{DA}}, \forall k, t\}$ is the set of primal optimization variables. The aim is to minimize the operating cost of the natural gas system that is represented in the objective function (16a) as the cost of natural gas supply. Equation (16b) represents the day-ahead gas supply balance with inelastic demand including given gas demand for power production $\sum_{i\in G} \phi_i \, p_{i,t}^{\mathrm{DA}}$ and virtual trade $v_{q,t}^{\mathrm{DA,G}}$. Constraint (16c) enforces lower and upper bounds on the gas supply.

The optimization problem for day-ahead gas market clearing can be equivalently formulated as the following equilibrium model with each supplier maximizing their profit and a price-setting agent. Each natural gas supplier or producer maximizes its day-ahead profit with respect to its operational constraints according to

$$\left\{ \max_{\Theta^{\mathrm{GD}}} \sum_{t\in T} \Big(\hat{\lambda}_t^{\mathrm{G}} - C_k^{\mathrm{G}} \Big) g_{k,t}^{\mathrm{DA}} \right. \tag{17a}$$

$$\left. \text{subject to } 0 \le g_{k,t}^{\mathrm{DA}} \le G_k^{\max} : \underline{\mu}_{k,t}^{\mathrm{G}}, \overline{\mu}_{k,t}^{\mathrm{G}}, \ t, \right\} \ \forall k, \tag{17b}$$

with the day-ahead price for natural gas $\hat{\lambda}_t^{\mathrm{G}}$ set by

$$\min_{\hat{\lambda}_t^{\mathrm{G}}} \sum_{t\in T} \hat{\lambda}_t^{\mathrm{G}} \Big(\sum_{k\in K} g_{k,t}^{\mathrm{DA}} - \sum_{i\in G} \phi_i \, p_{i,t}^{\mathrm{DA}} - D_t^{\mathrm{G}} + \sum_{q\in Q} v_{q,t}^{\mathrm{DA,G}} \Big) \tag{18a}$$

The equilibrium problem's (17)-(18) KKTs are equivalent to the optimization problem's (16).

1.6. Real-Time Natural Gas Market

The real-time natural gas market is cleared independently for adjusted fuel consumption by gas-fired units converted to a time-varying demand deviation via $\sum_{i\in G} \phi_i \, p_{i,t,\omega}^{\mathrm{RT}}, \forall t, \omega$. The day-ahead schedule of electricity and natural gas systems as well as the real-time electricity adjustments and dispatch decisions by virtual traders are treated as fixed parameters in the following formulation.

$$\left\{ \min_{\Theta^{\mathrm{GR}}} \sum_{t\in T} \Big(\sum_{k\in K} C_k^{\mathrm{G}} \, g_{k,t,\omega}^{\mathrm{RT}} + C^{\mathrm{sh,G}} \, l_{t,\omega}^{\mathrm{sh,G}} \Big) \right. \tag{19a}$$

$$\text{subject to } \sum_{k\in K} g_{k,t,\omega}^{\mathrm{RT}} - \sum_{i\in G} \phi_i \, p_{i,t,\omega}^{\mathrm{RT}} + l_{t,\omega}^{\mathrm{sh,G}} + \sum_{q\in Q} v_{q,t}^{\mathrm{RT,G}} = 0 : \tilde{\lambda}_{t,\omega}^{\mathrm{G}}, \ \forall t, \tag{19b}$$

$$0 \le \big(g_{k,t}^{\mathrm{DA}} + g_{k,t,\omega}^{\mathrm{RT}} \big) \le G_k^{\max} : \underline{\nu}_{k,t,\omega}^{\mathrm{G}}, \overline{\nu}_{k,t,\omega}^{\mathrm{G}}, \ \forall k, t, \tag{19c}$$

$$\left.\begin{aligned} & -G_k^{\text{adj}} \leq g_{k,t,\omega}^{\text{RT}} \leq G_k^{\text{adj}} : \underline{\nu}_{t,\omega}^{\text{GR}}, \overline{\nu}_{t,\omega}^{\text{GR}}, \ \ \forall k,t, && (19\text{d}) \\ & 0 \leq l_{t,\omega}^{\text{sh,G}} \leq D_t^{\text{G}} : \underline{\nu}_{t,\omega}^{\text{DG}}, \overline{\nu}_{t,\omega}^{\text{DG}}, \ \ \forall t, \end{aligned}\right\} \forall \omega \qquad (19\text{e})$$

where $\Theta^{\text{GR}} = \{g_{k,t,\omega}^{\text{RT}}, \forall k,t,\omega; l_{t,\omega}^{\text{sh,G}}, \forall t,\omega\}$ is the set of primal optimization variables. The real-time cost of the natural gas system is given in objective function (19a). The cost of natural gas adjustments along with natural gas load shedding comprise (19a). Constraint (19b) represents the balance of gas supply adjustments in real-time including fixed virtual trade $\sum_{q\in Q} v_{q,t}^{\text{RT,G}}$. Constraints (19c), (19d), and (19e) enforce lower and upper bounds on gas supply, gas adjustments and gas load shedding, respectively.

Market-clearing problem (19) is equivalent to the following equilibrium problem (20)-(22). Each gas supplier updates its offers in real-time as

$$\left\{\begin{aligned} & \max_{\Theta^{\text{GR}}} \sum_{t\in T} \left(\tilde{\lambda}_{t,\omega}^{\text{G}} - C_k^{\text{G}}\right) g_{k,t,\omega}^{\text{RT}} && (20\text{a}) \\ & \text{subject to } 0 \leq \left(g_{k,t}^{\text{DA}} + g_{k,t,\omega}^{\text{RT}}\right) \leq G_k^{\max} : \underline{\nu}_{k,t,\omega}^{\text{G}}, \overline{\nu}_{k,t,\omega}^{\text{G}}, \ \ \forall t, && (20\text{b}) \\ & -G_k^{\text{adj}} \leq g_{k,t,\omega}^{\text{RT}} \leq G_k^{\text{adj}} : \underline{\nu}_{t,\omega}^{\text{GR}}, \overline{\nu}_{t,\omega}^{\text{GR}}, \ \ \forall t, \end{aligned}\right\} \forall k,\omega, \qquad (20\text{c})$$

and gas demand can be curtailed incurring the following cost problem

$$\left\{\begin{aligned} & \min_{\Theta^{\text{GR}}} \sum_{t\in T} \left(C^{\text{sh,G}} - \tilde{\lambda}_{t,\omega}^{\text{G}}\right) l_{t,\omega}^{\text{sh,G}} && (21\text{a}) \\ & \text{subject to } 0 \leq l_{t,\omega}^{\text{sh,G}} \leq D_t^{\text{G}} : \underline{\nu}_{t,\omega}^{\text{DG}}, \overline{\nu}_{t,\omega}^{\text{DG}}, \ \ \forall t, \end{aligned}\right\} \forall \omega. \qquad (21\text{b})$$

The real-time natural gas price is decided for each scenario ω as

$$\left\{ \min_{\tilde{\lambda}_{t,\omega}^{\text{G}}} \sum_{t\in T} \tilde{\lambda}_{t,\omega}^{\text{G}} \left(\sum_{k\in K} g_{k,t,\omega}^{\text{RT}} - \sum_{i\in G} \phi_i \ p_{i,t,\omega}^{\text{RT}} + l_{t,\omega}^{\text{sh,G}} + \sum_{q\in Q} v_{q,t}^{\text{RT,G}} \right) \right\} \forall \omega. \qquad (22\text{a})$$

1.7. Self-Scheduling Gas-Fired Generators

For improving the inter-market coordination, we allow natural gas-fired units to self-schedule outside the markets for optimally allocating their flexibility in the power and natural gas markets. Each gas-fired unit maximizes its expected profit given a perfect anticipation of the distribution of both electricity and natural gas real-time market prices across all scenarios.

1.7.1. *Self-scheduling slow-starting gas-fired unit*

The profit maximization problem of each self-scheduling gas-fired unit $\mathcal{G} \cap \mathcal{S}$ participating in the electricity and natural gas market is given below:

$$\left\{\begin{aligned} & \max_{\Theta^{\text{SSS}}} \sum_{t\in T} \left[p_{i,t}^{\text{DA}} \left(\hat{\lambda}_t^{\text{E}} - \phi_i \ \hat{\lambda}_t^{\text{G}} \right) - c_{i,t}^{\text{DA}} + \sum_{\omega\in\Omega} \pi_\omega \ p_{i,t,\omega}^{\text{RT}} \left(\tilde{\lambda}_{t,\omega}^{\text{E}} - \phi_i \ \tilde{\lambda}_{t,\omega}^{\text{G}} \right) \right] && (23\text{a}) \\ & \text{subject to} (2\text{d}) - (2\text{j}) && (23\text{b}) \\ & (7\text{e}) - (7\text{g}) \end{aligned}\right\} \forall i \in (G\cap S), \qquad (23\text{c})$$

where $\Theta^{\text{SSS}} = \{p_{i,t}^{\text{DA}}, u_{i,t}^{\text{DA}}, c_{i,t}^{\text{DA}}, \forall i \in (G\cap S), t; p_{i,t,\omega}^{\text{RT}}, \forall i \in (G\cap S), t, \omega\}$ is the set of primal optimization variables. Objective function (23a) maximizes the expected profit of self-scheduling gas-fired generators and simultaneously considering the day-ahead (2d)-(2j) and real-time (7e)-(7g) constraints for all scenarios

$\omega \in \Omega$. Note that the self-scheduler's dispatch decisions $p^{\text{DA}}_{i,t}$, $u^{\text{DA}}_{i,t}$, $p^{\text{RT}}_{i,t,\omega}$ are fixed input in the market-clearing problems (2), (7), (16), and (19).

1.7.2. *Self-scheduling fast-starting gas-fired unit*

The profit maximization problem of each fast-start self-scheduling gas-fired unit $\mathcal{G} \cap \mathcal{F}$ participating in the electricity and natural gas market is given below:

$$\left\{ \begin{aligned} &\max_{\Theta^{\text{SSF}}} \sum_{t \in T} \Big[p^{\text{DA}}_{i,t} \left(\hat{\lambda}^{\text{E}}_t - \phi_i \, \hat{\lambda}^{\text{G}}_t \right) - c^{\text{DA}}_{i,t} + \sum_{\omega \in \Omega} \pi_\omega \; p^{\text{RT}}_{i,t,\omega} \left(\tilde{\lambda}^{\text{E}}_{t,\omega} - \phi_i \, \tilde{\lambda}^{\text{G}}_{t,\omega} \right) - c^{\text{RT}}_{i,t,\omega} \Big] && \text{(24a)} \\ &\text{subject to } (2\text{d}) - (2\text{j}) && \text{(24b)} \\ &(7\text{h}) - (7\text{n}) \end{aligned} \right\} \forall i \in (G \cap F), \quad \text{(24c)}$$

where $\Theta^{\text{SSF}} = \{p^{\text{DA}}_{i,t}, u^{\text{DA}}_{i,t}, c^{\text{DA}}_{i,t}, \forall i \in (G \cap F), t; p^{\text{RT}}_{i,t,\omega}, u^{\text{RT}}_{i,t,\omega}, c^{\text{RT}}_{i,t,\omega}, \forall i \in (G \cap F), t, \omega\}$ is the set of primal optimization variables. Objective function (24a) maximizes the expected profit of self-scheduling gas-fired generators and simultaneously considering the day-ahead (2d)-(2j) and real-time (7h)-(7n) constraints for all scenarios $\omega \in \Omega$.

1.8. **Ideal Benchmark: Stochastic Integrated Electricity and Natural Gas Market**

The stochastic and fully-coupled dispatch model simulates the integrated electric power and natural systems by jointly modeling the day-ahead and real-time stages. The problem is formulated as a two-stage stochastic program aiming to minimize the total expected cost and writes as follows,

$$\begin{aligned} &\min_{\Theta^{\text{SC}}} \sum_{t \in T} \Big[\sum_{i \in Z} (C^{\text{E}}_i \; p^{\text{DA}}_{i,t}) + \sum_{i \in I} c^{\text{DA}}_{i,t} + \sum_{k \in K} C^{\text{G}}_k \; g^{\text{DA}}_{k,t} + \sum_{\omega \in \Omega} \pi_\omega \\ &\Big(\sum_{i \in Z} C^{\text{E}}_i \; p^{\text{RT}}_{i,t,\omega} + \sum_{i \in F} c^{\text{RT}}_{i,t} + \sum_{k \in K} C^{\text{G}}_k \; g^{\text{RT}}_{k,t,\omega} + C^{\text{sh,E}} \; l^{\text{sh,E}}_{t,\omega} + C^{\text{sh,G}} \; l^{\text{sh,G}}_{t,\omega} \Big) \Big] \end{aligned} \quad \text{(25a)}$$

subject to

$$(2\text{d}) - (2\text{g}), \quad \forall i, \quad (16\text{c}), (16\text{b}), \quad \text{(25b)}$$
$$(7\text{e}) - (7\text{k}), \quad \forall i, \omega, \quad (19\text{c}) - (19\text{b}), \quad \forall \omega, \quad \text{(25c)}$$

where $\Theta^{\text{SC}} = \{\Theta^{\text{ED}}, \Theta^{\text{GD}}, \Theta^{\text{ER}}, \Theta^{\text{GR}}\}$ is the set of primal optimization variables. In this model, the temporal coordination of the two trading floors is taken into account by anticipating the real-time constraints (25c) for all scenarios $\omega \in \Omega$.

2. Karush-Kuhn-Tucker Conditions

2.1. **Explicit Electricity Virtual Bidder**

$$\frac{\partial L}{\partial v^{\text{DA,E}}_{r,t}} = \hat{\lambda}^{\text{E}}_t - \rho_{r,t} = 0, \quad \forall r, t, \quad \text{(26a)}$$
$$\frac{\partial L}{\partial v^{\text{RT,E}}_{r,t}} = \sum_{\omega \in \Omega} \left(\pi_\omega \; \tilde{\lambda}^{\text{E}}_{t,\omega} \right) - \rho_{r,t} = 0, \quad \forall r, t, \quad \text{(26b)}$$
$$v^{\text{DA,E}}_{r,t} + v^{\text{RT,E}}_{r,t} = 0 : \rho_{r,t}, \quad \forall r, t. \quad \text{(26c)}$$

2.2. **Day-Ahead Electricity Market**

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = C^{\mathrm{E}}_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0,\ \ \forall i \in Z \setminus SS, t < T, \tag{27a}$$

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = C^{\mathrm{E}}_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{R}}_{i,t} - \underline{\mu}^{\mathrm{R}}_{i,t} = 0,\ \ \forall i \in Z \setminus SS, t = T, \tag{27b}$$

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = \hat{\lambda}^{\mathrm{G}}_t \phi_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0,\ \ \forall i \in G \setminus SS, t < T, \tag{27c}$$

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = \hat{\lambda}^{\mathrm{G}}_t \phi_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{R}}_{i,t} - \underline{\mu}^{\mathrm{R}}_{i,t} = 0,\ \ \forall i \in G \setminus SS, t = T, \tag{27d}$$

$$\begin{aligned}\frac{\partial L}{\partial u^{\mathrm{DA}}_{i,t}} &= -P^{\max}_i \overline{\mu}^{\mathrm{P}}_{i,t} + P^{\min}_i \underline{\mu}^{\mathrm{P}}_{i,t} - R_i \overline{\mu}^{\mathrm{R}}_{i,t} - R_i \underline{\mu}^{\mathrm{R}}_{i,(t+1)} + C^{\mathrm{SU}}_i (\overline{\mu}^{\mathrm{SU}}_{i,t} - \overline{\mu}^{\mathrm{SU}}_{i,(t+1)}) + \overline{\mu}^{\mathrm{B}}_{i,t} - \underline{\mu}^{\mathrm{B}}_{i,t} = 0,\\ &\forall i \in I \setminus SS, t < T,\end{aligned} \tag{27e}$$

$$\frac{\partial L}{\partial u^{\mathrm{DA}}_{i,t}} = -P^{\max}_i \overline{\mu}^{\mathrm{P}}_{i,t} + P^{\min}_i \underline{\mu}^{\mathrm{P}}_{i,t} - R_i \overline{\mu}^{\mathrm{R}}_{i,t} + C^{\mathrm{SU}}_i \overline{\mu}^{\mathrm{SU}}_{i,t} + \overline{\mu}^{\mathrm{B}}_{i,t} - \underline{\mu}^{\mathrm{B}}_{i,t} = 0,\ \ \forall i \in I \setminus SS, t = T, \tag{27f}$$

$$\frac{\partial L}{\partial w^{\mathrm{DA}}_{j,t}} = \overline{\mu}^{\mathrm{W}}_{j,t} - \underline{\mu}^{\mathrm{W}}_{j,t} - \hat{\lambda}^{\mathrm{E}}_t = 0,\ \ \forall j, t \tag{27g}$$

$$\frac{\partial L}{\partial c^{\mathrm{DA}}_{i,t}} = 1 - \overline{\mu}^{\mathrm{SU}}_{i,t} - \underline{\mu}^{\mathrm{SU}}_{i,t} = 0,\ \ \forall i \in I \setminus SS, t \tag{27h}$$

$$0 \le (p^{\mathrm{DA}}_{i,t} - u^{\mathrm{DA}}_{i,t} P^{\min}_i) \perp \underline{\mu}^{\mathrm{P}}_{i,t} \ge 0\ \ \forall i \in I \setminus SS, t, \tag{27i}$$

$$0 \le (u^{\mathrm{DA}}_{i,t} P^{\max}_i - p^{\mathrm{DA}}_{i,t}) \perp \overline{\mu}^{\mathrm{P}}_{i,t} \ge 0\ \ \forall i \in I \setminus SS, t, \tag{27j}$$

$$0 \le w^{\mathrm{DA}}_{j,t} \perp \underline{\mu}^{\mathrm{W}}_{j,t} \ge 0,\ \ \forall j, t \tag{27k}$$

$$0 \le (W^{\mathrm{DA}}_{j,t} - w^{\mathrm{DA}}_{j,t}) \perp \overline{\mu}^{\mathrm{W}}_{j,t} \ge 0,\ \ \forall j, t, \tag{27l}$$

$$0 \le \Big[(p^{\mathrm{DA}}_{i,t} - p^{\mathrm{DA}}_{i,(t-1)}) + u^{\mathrm{DA}}_{i,(t-1)} R_i\Big] \perp \underline{\mu}^{\mathrm{R}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t > 1, \tag{27m}$$

$$0 \le \Big[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} - p^{\mathrm{DA}}_{i,(t-1)})\Big] \perp \overline{\mu}^{\mathrm{R}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t > 1, \tag{27n}$$

$$0 \le \big[(p^{\mathrm{DA}}_{i,t} - P^{\mathrm{ini}}_i) + U^{\mathrm{ini}}_i R_i\big] \perp \underline{\mu}^{\mathrm{R}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t = 1, \tag{27o}$$

$$0 \le \big[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} - P^{\mathrm{ini}}_i)\big] \perp \overline{\mu}^{\mathrm{R}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t = 1, \tag{27p}$$

$$0 \le \Big[c^{\mathrm{DA}}_{i,t} - C^{\mathrm{SU}}_i (u^{\mathrm{DA}}_{i,t} - u^{\mathrm{DA}}_{i,(t-1)})\Big] \perp \overline{\mu}^{\mathrm{SU}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t > 1, \tag{27q}$$

$$0 \le \big[c^{\mathrm{DA}}_{i,t} - C^{\mathrm{SU}}_i (u^{\mathrm{DA}}_{i,t} - U^{\mathrm{ini}}_i)\big] \perp \overline{\mu}^{\mathrm{SU}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t = 1, \tag{27r}$$

$$0 \le c^{\mathrm{DA}}_{i,t} \perp \underline{\mu}^{\mathrm{SU}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t, \tag{27s}$$

$$0 \le u^{\mathrm{DA}}_{i,t} \perp \underline{\mu}^{\mathrm{B}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t, \tag{27t}$$

$$0 \le (1 - u^{\mathrm{DA}}_{i,t}) \perp \overline{\mu}^{\mathrm{B}}_{i,t} \ge 0,\ \ \forall i \in I \setminus SS, t, \tag{27u}$$

$$\sum_{i \in I} p^{\mathrm{DA}}_{i,t} + \sum_{j \in J} w^{\mathrm{DA}}_{j,t} - D^{\mathrm{E}}_t + \sum_{r \in R} v^{\mathrm{DA,E}}_{r,t} = 0 : \hat{\lambda}^{\mathrm{E}}_t,\ \forall t. \tag{27v}$$

$$\tag{27w}$$

2.3. **Real-Time Electricity Market**

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = \pi_\omega C^{\mathrm{E}}_i + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} - \tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0,$$

$$\forall i \in Z \setminus SS, t < T, \omega, \tag{28a}$$

$$\frac{\partial L}{\partial p_{i,t,\omega}^{\mathrm{RT}}} = \pi_\omega C_i^{\mathrm{E}} + \overline{\nu}_{i,t,\omega}^{\mathrm{P}} - \underline{\nu}_{i,t,\omega}^{\mathrm{P}} - \tilde{\lambda}_{t,\omega}^{\mathrm{E}} + \overline{\nu}_{i,t,\omega}^{\mathrm{R}} - \underline{\nu}_{i,t,\omega}^{\mathrm{R}} = 0, \;\; \forall i \in Z \setminus SS, t = T, \omega, \tag{28b}$$

$$\frac{\partial L}{\partial p_{i,t,\omega}^{\mathrm{RT}}} = \pi_\omega \tilde{\lambda}_t^{\mathrm{G}} \phi_i + \overline{\nu}_{i,t,\omega}^{\mathrm{P}} - \underline{\nu}_{i,t,\omega}^{\mathrm{P}} - \tilde{\lambda}_{t,\omega}^{\mathrm{E}} + \overline{\nu}_{i,t,\omega}^{\mathrm{R}} - \overline{\nu}_{i,(t+1),\omega}^{\mathrm{R}} - \underline{\nu}_{i,t,\omega}^{\mathrm{R}} + \underline{\nu}_{i,(t+1),\omega}^{\mathrm{R}} = 0,$$
$$\forall i \in G \setminus SS, t < T, \omega, \tag{28c}$$

$$\frac{\partial L}{\partial p_{i,t,\omega}^{\mathrm{RT}}} = \pi_\omega \tilde{\lambda}_t^{\mathrm{G}} \phi_i + \overline{\nu}_{i,t,\omega}^{\mathrm{P}} - \underline{\nu}_{i,t,\omega}^{\mathrm{P}} - \tilde{\lambda}_{t,\omega}^{\mathrm{E}} + \overline{\nu}_{i,t,\omega}^{\mathrm{R}} - \underline{\nu}_{i,t,\omega}^{\mathrm{R}} = 0, \;\; \forall i \in G \setminus SS, t = T, \omega, \tag{28d}$$

$$\frac{\partial L}{\partial w_{j,t,\omega}^{\mathrm{RT}}} = \overline{\nu}_{j,t,\omega}^{\mathrm{W}} - \underline{\nu}_{j,t,\omega}^{\mathrm{W}} - \tilde{\lambda}_{t,\omega}^{\mathrm{E}} = 0, \;\; \forall j, t, \omega, \tag{28e}$$

$$\frac{\partial L}{\partial l_{t,\omega}^{\mathrm{sh,E}}} = \pi_\omega \; C^{\mathrm{sh,E}} + \overline{\nu}_{t,\omega}^{\mathrm{DE}} - \underline{\nu}_{t,\omega}^{\mathrm{DE}} - \tilde{\lambda}_{t,\omega}^{\mathrm{E}} = 0, \;\; \forall t, \omega, \tag{28f}$$

$$\frac{\partial L}{\partial u_{i,t,\omega}^{\mathrm{RT}}} = -P_i^{\max} \overline{\nu}_{i,t,\omega}^{\mathrm{P}} + P_i^{\min} \underline{\nu}_{i,t,\omega}^{\mathrm{P}} - R_i \overline{\nu}_{i,t,\omega}^{\mathrm{R}} - R_i \underline{\nu}_{i,(t+1),\omega}^{\mathrm{R}} + C_i^{\mathrm{SU}} (\overline{\nu}_{i,t,\omega}^{\mathrm{SU}} - \overline{\nu}_{i,(t+1),\omega}^{\mathrm{SU}})$$
$$+ \overline{\nu}_{i,t,\omega}^{\mathrm{B}} - \underline{\nu}_{i,t,\omega}^{\mathrm{B}} = 0, \;\; \forall i \in F, t < T, \omega, \tag{28g}$$

$$\frac{\partial L}{\partial u_{i,t,\omega}^{\mathrm{RT}}} = -P_i^{\max} \overline{\nu}_{i,t,\omega}^{\mathrm{P}} + P_i^{\min} \underline{\nu}_{i,t,\omega}^{\mathrm{P}} - R_i \overline{\nu}_{i,t,\omega}^{\mathrm{R}} + C_i^{\mathrm{SU}} \overline{\nu}_{i,t,\omega}^{\mathrm{SU}} + \overline{\nu}_{i,t,\omega}^{\mathrm{B}} - \underline{\nu}_{i,t,\omega}^{\mathrm{B}} = 0, \;\; \forall i \in F, t = T, \omega, \tag{28h}$$

$$\frac{\partial L}{\partial c_{i,t,\omega}^{\mathrm{RT}}} = \pi_\omega - \overline{\nu}_{i,t,\omega}^{\mathrm{SU}} - \underline{\nu}_{i,t,\omega}^{\mathrm{SU}} = 0, \;\; \forall i \in F, t, \omega, \tag{28i}$$

$$0 \le \left[(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}}) - u_{i,t}^{\mathrm{DA}} P_i^{\min} \right] \perp \underline{\nu}_{i,t,\omega}^{\mathrm{P}} \ge 0, \;\; \forall i \in S, t, \omega, \tag{28j}$$

$$0 \le \left[u_{i,t}^{\mathrm{DA}} P_i^{\max} - (p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}}) \right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{P}} \ge 0, \;\; \forall i \in S, t, \omega, \tag{28k}$$

$$0 \le \left[(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}}) - (u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}) P_i^{\min} \right] \perp \underline{\nu}_{i,t,\omega}^{\mathrm{P}} \ge 0, \forall i \in F, t, \omega, \tag{28l}$$

$$0 \le \left[(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}) P_i^{\max} - (p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}}) \right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{P}} \ge 0, \forall i \in F, t, \omega, \tag{28m}$$

$$0 \le (w_{j,t}^{\mathrm{DA}} + w_{j,t,\omega}^{\mathrm{RT}}) \perp \underline{\nu}_{j,t,\omega}^{\mathrm{W}} \ge 0, \;\; \forall j, t, \omega, \tag{28n}$$

$$0 \le \left[W_{j,t,\omega} - (w_{j,t}^{\mathrm{DA}} + w_{j,t,\omega}^{\mathrm{RT}}) \right] \perp \overline{\nu}_{j,t,\omega}^{\mathrm{W}} \ge 0, \;\; \forall j, t, \omega, \tag{28o}$$

$$0 \le l_{t,\omega}^{\mathrm{sh,E}} \perp \underline{\nu}_{t,\omega}^{\mathrm{DE}} \ge 0, \;\; \forall t, \omega, \tag{28p}$$

$$0 \le D_t^{\mathrm{E}} - l_{t,\omega}^{\mathrm{sh,E}} \perp \overline{\nu}_{t,\omega}^{\mathrm{DE}} \ge 0, \;\; \forall t, \omega, \tag{28q}$$

$$0 \le \left[(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - p_{i,(t-1)}^{\mathrm{DA}} - p_{i,(t-1),\omega}^{\mathrm{RT}}) + u_{i,(t-1)}^{\mathrm{DA}} R_i \right] \perp \underline{\nu}_{i,t,\omega}^{\mathrm{R}} \ge 0, \;\; \forall i \in S, t > 1, \omega, \tag{28r}$$

$$0 \le \left[u_{i,t}^{\mathrm{DA}} R_i - (p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - p_{i,(t-1)}^{\mathrm{DA}} - p_{i,(t-1),\omega}^{\mathrm{RT}}) \right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{R}} \ge 0, \forall i \in S, t > 1, \omega, \tag{28s}$$

$$0 \le \left[(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - P_i^{\mathrm{ini}}) + U_i^{\mathrm{ini}} R_i \right] \perp \underline{\nu}_{i,t,\omega}^{\mathrm{R}} \ge 0, \forall i \in S, t = 1, \omega, \tag{28t}$$

$$0 \le \left[u_{i,t}^{\mathrm{DA}} R_i - (p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - P_i^{\mathrm{ini}}) \right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{R}} \ge 0, \forall i \in S, t = 1, \omega, \tag{28u}$$

$$0 \le \left[(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - p_{i,(t-1)}^{\mathrm{DA}} - p_{i,(t-1),\omega}^{\mathrm{RT}}) + (u_{i,(t-1)}^{\mathrm{DA}} + u_{i,(t-1)}^{\mathrm{RT}}) R_i \right] \perp \underline{\nu}_{i,t,\omega}^{\mathrm{R}} \ge 0, \;\; \forall i \in F, t > 1, \omega, \tag{28v}$$

$$0 \le \left[(u_{i,t}^{\mathrm{DA}} + u_{i,t}^{\mathrm{RT}}) R_i - (p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - p_{i,(t-1)}^{\mathrm{DA}} - p_{i,(t-1),\omega}^{\mathrm{RT}}) \right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{R}} \ge 0, \;\; \forall i \in F, t > 1, \omega, \tag{28w}$$

$$0 \le \left[p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - P_i^{\mathrm{ini}} + U_i^{\mathrm{ini}} R_i \right] \perp \underline{\nu}_{i,t,\omega}^{\mathrm{R}} \ge 0, \forall i \in F, t = 1, \omega, \tag{28x}$$

$$0 \le \left[(u_{i,t}^{\mathrm{DA}} + u_{i,t}^{\mathrm{RT}}) R_i - (p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - P_i^{\mathrm{ini}}) \right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{R}} \ge 0, \forall i \in F, t = 1, \omega, \tag{28y}$$

$$0 \le \left[(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}}) - C_i^{\mathrm{SU}} (u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} - u_{i,(t-1)}^{\mathrm{DA}} - u_{i,(t-1),\omega}^{\mathrm{RT}}) \right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{SU}} \ge 0, \;\; \forall i \in F, t > 1, \omega, \tag{28z}$$

$$0 \le \left[(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}}) - C_i^{\mathrm{SU}} (u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} - U_i^{\mathrm{ini}}) \right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{SU}} \ge 0, \;\; \forall i \in F, t = 1, \omega, \tag{28aa}$$

$$0 \le \left(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}} \right) \perp \underline{\nu}_{i,t,\omega}^{\mathrm{SU}} \ge 0, \;\; \forall i \in F, t, \omega, \tag{28ab}$$

$$0 \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}\right) \perp \underline{\nu}_{i,t,\omega}^{\mathrm{B}} \ge 0, \ \ \forall i \in F, t, \omega, \tag{28ac}$$

$$0 \le \left[1 - (u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}})\right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{B}} \ge 0, \ \ \forall i \in F, t, \omega, \tag{28ad}$$

$$\sum_{i \in I} p_{i,t,\omega}^{\mathrm{RT}} + l_{t,\omega}^{\mathrm{sh,E}} + \sum_{r \in R} v_{r,t}^{\mathrm{RT,E}} + \sum_{j \in J} w_{j,t,\omega}^{\mathrm{RT}} = 0 \colon \tilde{\lambda}_{t,\omega}^{\mathrm{E}}, \ \forall t. \tag{28ae}$$

2.4. **Explicit Natural Gas Virtual Bidder**

$$\frac{\partial L}{\partial v_{q,t}^{\mathrm{DA,G}}} = \hat{\lambda}_t^{\mathrm{G}} - \psi_{q,t} = 0, \ \ \forall q, t, \tag{29a}$$

$$\frac{\partial L}{\partial v_{q,t}^{\mathrm{RT,G}}} = \sum_{\omega \in \Omega} \left(\pi_\omega \ \tilde{\lambda}_{t,\omega}^{\mathrm{G}}\right) - \psi_{q,t} = 0, \ \ \forall q, t, \tag{29b}$$

$$v_{q,t}^{\mathrm{DA,G}} + v_{q,t}^{\mathrm{RT,G}} = 0 : \psi_{q,t}, \ \ \forall q, t. \tag{29c}$$

2.5. **Day-Ahead Natural Gas Market**

$$\frac{\partial L}{\partial g_{k,t}^{\mathrm{DA}}} = C_k^{\mathrm{G}} + \overline{\mu}_{k,t}^{\mathrm{G}} - \underline{\mu}_{k,t}^{\mathrm{G}} - \hat{\lambda}_t^{\mathrm{G}} = 0 \ \ \forall k, t, \tag{30a}$$

$$0 \le g_{k,t}^{\mathrm{DA}} \perp \underline{\mu}_{k,t}^{\mathrm{G}} \ge 0 \ \ \forall k, t, \tag{30b}$$

$$0 \le (G_k^{\max} - g_{k,t}^{\mathrm{DA}}) \perp \overline{\mu}_{k,t}^{\mathrm{G}} \ge 0 \ \ \forall k, t, \tag{30c}$$

$$\sum_{k \in K} g_{k,t}^{\mathrm{DA}} - \sum_{i \in G} \phi_i \ p_{i,t}^{\mathrm{DA}} - D_t^{\mathrm{G}} + \sum_{q \in Q} v_{q,t}^{\mathrm{DA,G}} = 0 : \hat{\lambda}_t^{\mathrm{G}}, \ \ \forall t. \tag{30d}$$

2.6. **Real-Time Natural Gas Market**

$$\frac{\partial L}{\partial g_{k,t,\omega}^{\mathrm{RT}}} = \pi_\omega C_k^{\mathrm{G}} + \overline{\nu}_{k,t,\omega}^{\mathrm{G}} - \underline{\nu}_{k,t,\omega}^{\mathrm{G}} - \tilde{\lambda}_{t,\omega}^{\mathrm{G}} = 0, \ \ \forall k, t, \omega, \tag{31a}$$

$$\frac{\partial L}{\partial l_{t,\omega}^{\mathrm{sh,G}}} = \pi_\omega C^{\mathrm{sh,G}} + \overline{\nu}_{t,\omega}^{\mathrm{DG}} - \underline{\nu}_{t,\omega}^{\mathrm{DG}} - \tilde{\lambda}_{t,\omega}^{\mathrm{G}} = 0, \ \ \forall t, \omega, \tag{31b}$$

$$0 \le (g_{k,t}^{\mathrm{DA}} + g_{k,t,\omega}^{\mathrm{RT}}) \perp \underline{\nu}_{k,t,\omega}^{\mathrm{G}} \ge 0, \ \ \forall k, t, \omega, \tag{31c}$$

$$0 \le \left[G_k^{\max} - (g_{k,t}^{\mathrm{DA}} + g_{k,t,\omega}^{\mathrm{RT}})\right] \perp \overline{\nu}_{k,t,\omega}^{\mathrm{G}} \ge 0, \ \ \forall k, t, \omega, \tag{31d}$$

$$0 \le (g_{k,t,\omega}^{\mathrm{RT}} + G_k^{\mathrm{adj}}) \perp \underline{\nu}_{t,\omega}^{\mathrm{GR}} \ge 0, \ \ \forall k, t, \omega, \tag{31e}$$

$$0 \le (G_k^{\mathrm{adj}} - g_{k,t,\omega}^{\mathrm{RT}}) \perp \overline{\nu}_{t,\omega}^{\mathrm{GR}} \ge 0, \ \ \forall k, t, \omega, \tag{31f}$$

$$0 \le l_{t,\omega}^{\mathrm{sh,G}} \perp \underline{\nu}_{t,\omega}^{\mathrm{DG}} \ge 0, \ \ \forall t, \omega, \tag{31g}$$

$$0 \le (D_t^{\mathrm{G}} - l_{t,\omega}^{\mathrm{sh,G}}) \perp \overline{\nu}_{t,\omega}^{\mathrm{DG}} \ge 0, \ \ \forall t, \omega, \tag{31h}$$

$$\sum_{k \in K} g_{k,t,\omega}^{\mathrm{RT}} - \sum_{i \in G} \phi_i \ p_{i,t,\omega}^{\mathrm{RT}} + l_{t,\omega}^{\mathrm{sh,G}} + \sum_{q \in Q} v_{q,t}^{\mathrm{RT,G}} = 0 : \tilde{\lambda}_{t,\omega}^{\mathrm{G}}, \ \forall t, \omega. \tag{31i}$$

2.7. **Self-Scheduling Slow-Starting Gas-Fired Generator**

$$\left\{ \frac{\partial L}{\partial p_{i,t}^{\mathrm{DA}}} = -\hat{\lambda}_t^{\mathrm{E}} + \hat{\lambda}_t^{\mathrm{G}} \phi_i + \overline{\mu}_{i,t}^{\mathrm{P}} - \underline{\mu}_{i,t}^{\mathrm{P}} + \overline{\mu}_{i,t}^{\mathrm{R}} - \overline{\mu}_{i,(t+1)}^{\mathrm{R}} - \underline{\mu}_{i,t}^{\mathrm{R}} + \underline{\mu}_{i,(t+1)}^{\mathrm{R}} \right.$$

$$+\sum_{\omega\in\Omega}\Big[\overline{\nu}^{\mathrm{P}}_{i,t,\omega}-\underline{\nu}^{\mathrm{P}}_{i,t,\omega}+\overline{\nu}^{\mathrm{R}}_{i,t,\omega}-\overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega}-\underline{\nu}^{\mathrm{R}}_{i,t,\omega}+\underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega}\Big]=0,\ \ \forall t<T, \tag{32a}$$

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}}=-\hat{\lambda}^{\mathrm{E}}_t+\hat{\lambda}^{\mathrm{G}}_t\phi_i+\overline{\mu}^{\mathrm{P}}_{i,t}-\underline{\mu}^{\mathrm{P}}_{i,t}+\overline{\mu}^{\mathrm{R}}_{i,t}-\underline{\mu}^{\mathrm{R}}_{i,t}+\sum_{\omega\in\Omega}\Big[\overline{\nu}^{\mathrm{P}}_{i,t,\omega}-\underline{\nu}^{\mathrm{P}}_{i,t,\omega}+\overline{\nu}^{\mathrm{R}}_{i,t,\omega}-\underline{\nu}^{\mathrm{R}}_{i,t,\omega}\Big]=0, t=T, \tag{32b}$$

$$\frac{\partial L}{\partial u^{\mathrm{DA}}_{i,t}}=-P^{\max}_i\overline{\mu}^{\mathrm{P}}_{i,t}+P^{\min}_i\underline{\mu}^{\mathrm{P}}_{i,t}-R_i\overline{\mu}^{\mathrm{R}}_{i,t}-R_i\underline{\mu}^{\mathrm{R}}_{i,(t+1)}+C^{\mathrm{SU}}_i(\overline{\mu}^{\mathrm{SU}}_{i,t}-\overline{\mu}^{\mathrm{SU}}_{i,(t+1)})+\overline{\mu}^{\mathrm{B}}_{i,t}-\underline{\mu}^{\mathrm{B}}_{i,t}$$
$$+\sum_{\omega\in\Omega}\Big(-P^{\max}_i\overline{\nu}^{\mathrm{P}}_{i,t,\omega}+P^{\min}_i\underline{\nu}^{\mathrm{P}}_{i,t,\omega}-R_i\overline{\nu}^{\mathrm{R}}_{i,t,\omega}-R_i\underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega}\Big)=0,\ \ \forall t<T, \tag{32c}$$

$$\frac{\partial L}{\partial u^{\mathrm{DA}}_{i,t}}=-P^{\max}_i\overline{\mu}^{\mathrm{P}}_{i,t}+P^{\min}_i\underline{\mu}^{\mathrm{P}}_{i,t}-R_i\overline{\mu}^{\mathrm{R}}_{i,t}+C^{\mathrm{SU}}_i\overline{\mu}^{\mathrm{SU}}_{i,t}+\overline{\mu}^{\mathrm{B}}_{i,t}-\underline{\mu}^{\mathrm{B}}_{i,t}$$
$$+\sum_{\omega\in\Omega}\big(-P^{\max}_i\overline{\nu}^{\mathrm{P}}_{i,t,\omega}+P^{\min}_i\underline{\nu}^{\mathrm{P}}_{i,t,\omega}-R_i\overline{\nu}^{\mathrm{R}}_{i,t,\omega}\big)=0,\ \ t=T, \tag{32d}$$

$$\frac{\partial L}{\partial c^{\mathrm{DA}}_{i,t}}=1-\overline{\mu}^{\mathrm{SU}}_{i,t}-\underline{\mu}^{\mathrm{SU}}_{i,t}=0,\ \ \forall t, \tag{32e}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}}=-\pi_\omega\big(\frac{\tilde{\lambda}^{\mathrm{E}}_{t,\omega}}{\pi_\omega}-\phi_i\frac{\tilde{\lambda}^{\mathrm{G}}_{t,\omega}}{\pi_\omega}\big)+\overline{\nu}^{\mathrm{P}}_{i,t,\omega}-\underline{\nu}^{\mathrm{P}}_{i,t,\omega}+\overline{\nu}^{\mathrm{R}}_{i,t,\omega}-\overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega}-\underline{\nu}^{\mathrm{R}}_{i,t,\omega}+\underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega}=0,$$
$$\forall t<T,\omega, \tag{32f}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}}=-\pi_\omega\big(\frac{\tilde{\lambda}^{\mathrm{E}}_{t,\omega}}{\pi_\omega}-\phi_i\frac{\tilde{\lambda}^{\mathrm{G}}_{t,\omega}}{\pi_\omega}\big)+\overline{\nu}^{\mathrm{P}}_{i,t,\omega}-\underline{\nu}^{\mathrm{P}}_{i,t,\omega}+\overline{\nu}^{\mathrm{R}}_{i,t,\omega}-\underline{\nu}^{\mathrm{R}}_{i,t,\omega}=0,\ \ t=T,\omega, \tag{32g}$$

$$0\le(p^{\mathrm{DA}}_{i,t}-u^{\mathrm{DA}}_{i,t}P^{\min}_i)\perp\underline{\mu}^{\mathrm{P}}_{i,t}\ge 0\ \ \forall t, \tag{32h}$$

$$0\le(u^{\mathrm{DA}}_{i,t}P^{\max}_i-p^{\mathrm{DA}}_{i,t})\perp\overline{\mu}^{\mathrm{P}}_{i,t}\ge 0\ \ \forall t, \tag{32i}$$

$$0\le\Big[(p^{\mathrm{DA}}_{i,t}-p^{\mathrm{DA}}_{i,(t-1)})+u^{\mathrm{DA}}_{i,(t-1)}R_i\Big]\perp\underline{\mu}^{\mathrm{R}}_{i,t}\ge 0,\ \ \forall t>1, \tag{32j}$$

$$0\le\Big[u^{\mathrm{DA}}_{i,t}R_i-(p^{\mathrm{DA}}_{i,t}-p^{\mathrm{DA}}_{i,(t-1)})\Big]\perp\overline{\mu}^{\mathrm{R}}_{i,t}\ge 0,\ \ \forall t>1, \tag{32k}$$

$$0\le\big[(p^{\mathrm{DA}}_{i,t}-P^{\mathrm{ini}}_i)+U^{\mathrm{ini}}_iR_i\big]\perp\underline{\mu}^{\mathrm{R}}_{i,t}\ge 0,\ \ \forall t=1, \tag{32l}$$

$$0\le\big[u^{\mathrm{DA}}_{i,t}R_i-(p^{\mathrm{DA}}_{i,t}-P^{\mathrm{ini}}_i)\big]\perp\overline{\mu}^{\mathrm{R}}_{i,t}\ge 0,\ \ \forall t=1, \tag{32m}$$

$$0\le\Big[c^{\mathrm{DA}}_{i,t}-C^{\mathrm{SU}}_i(u^{\mathrm{DA}}_{i,t}-u^{\mathrm{DA}}_{i,(t-1)})\Big]\perp\overline{\mu}^{\mathrm{SU}}_{i,t}\ge 0,\ \ \forall t>1, \tag{32n}$$

$$0\le\big[c^{\mathrm{DA}}_{i,t}-C^{\mathrm{SU}}_i(u^{\mathrm{DA}}_{i,t}-U^{\mathrm{ini}}_i)\big]\perp\overline{\mu}^{\mathrm{SU}}_{i,t}\ge 0,\ \ \forall t=1, \tag{32o}$$

$$0\le c^{\mathrm{DA}}_{i,t}\perp\underline{\mu}^{\mathrm{SU}}_{i,t}\ge 0,\ \ \forall t, \tag{32p}$$

$$0\le u^{\mathrm{DA}}_{i,t}\perp\underline{\mu}^{\mathrm{B}}_{i,t}\ge 0,\ \ \forall t, \tag{32q}$$

$$0\le(1-u^{\mathrm{DA}}_{i,t})\perp\overline{\mu}^{\mathrm{B}}_{i,t}\ge 0,\ \ \forall t, \tag{32r}$$

$$0\le\big[(p^{\mathrm{DA}}_{i,t}+p^{\mathrm{RT}}_{i,t,\omega})-u^{\mathrm{DA}}_{i,t}P^{\min}_i\big]\perp\underline{\nu}^{\mathrm{P}}_{i,t,\omega}\ge 0,\ \ \forall t,\omega, \tag{32s}$$

$$0\le\big[u^{\mathrm{DA}}_{i,t}P^{\max}_i-(p^{\mathrm{DA}}_{i,t}+p^{\mathrm{RT}}_{i,t,\omega})\big]\perp\overline{\nu}^{\mathrm{P}}_{i,t,\omega}\ge 0,\ \ \forall t,\omega, \tag{32t}$$

$$0\le\Big[(p^{\mathrm{DA}}_{i,t}+p^{\mathrm{RT}}_{i,t,\omega}-p^{\mathrm{DA}}_{i,(t-1)}-p^{\mathrm{RT}}_{i,(t-1),\omega})+u^{\mathrm{DA}}_{i,(t-1)}R_i\Big]\perp\underline{\nu}^{\mathrm{R}}_{i,t,\omega}\ge 0,\ \ \forall t>1,\omega, \tag{32u}$$

$$0\le\Big[u^{\mathrm{DA}}_{i,t}R_i-(p^{\mathrm{DA}}_{i,t}+p^{\mathrm{RT}}_{i,t,\omega}-p^{\mathrm{DA}}_{i,(t-1)}-p^{\mathrm{RT}}_{i,(t-1),\omega})\Big]\perp\overline{\nu}^{\mathrm{R}}_{i,t,\omega}\ge 0,\forall t>1,\omega, \tag{32v}$$

$$0\le\big[(p^{\mathrm{DA}}_{i,t}+p^{\mathrm{RT}}_{i,t,\omega}-P^{\mathrm{ini}}_i)+U^{\mathrm{ini}}_iR_i\big]\perp\underline{\nu}^{\mathrm{R}}_{i,t,\omega}\ge 0,\forall t\!=\!1,\omega, \tag{32w}$$

$$\left.0\le\big[u^{\mathrm{DA}}_{i,t}R_i-(p^{\mathrm{DA}}_{i,t}+p^{\mathrm{RT}}_{i,t,\omega}-P^{\mathrm{ini}}_i)\big]\perp\overline{\nu}^{\mathrm{R}}_{i,t,\omega}\ge 0,\forall t\!=\!1,\omega\right\}\ \forall i\in(G\cap SS) \tag{32x}$$

3. Proof of Proposition 1

Each self-scheduler's KKT optimality conditions if its DA dispatch is restricted by operational bounds enforce

$$\begin{cases} \dfrac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = -\hat{\lambda}^{\mathrm{E}}_t + \hat{\lambda}^{\mathrm{G}}_t \phi_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} \\ \qquad + \sum_{\omega\in\Omega} \Big[\overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega}\Big] = 0, \ \ \forall t < T, \end{cases} \tag{33a}$$

and

$$\left.\begin{aligned} &\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = -\pi_\omega\big(\frac{\tilde{\lambda}^{\mathrm{E}}_{t,\omega}}{\pi_\omega} - \phi_i \frac{\tilde{\lambda}^{\mathrm{G}}_{t,\omega}}{\pi_\omega}\big) + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0, \\ &\qquad \forall t < T, \omega, \end{aligned}\right\} \ \forall i \in (G \cap SS). \tag{33b}$$

The summation of condition (33b) over all scenarios, i.e., $\sum_\omega$(33b), shows that when VB in electricity and natural gas markets enforce $\hat{\lambda}^{\mathrm{E}}_t = \sum_\omega \tilde{\lambda}^{\mathrm{E}}_{t,\omega}$ and $\hat{\lambda}^{\mathrm{G}}_t = \sum_\omega \tilde{\lambda}^{\mathrm{G}}_{t,\omega}$, the problem is feasible only if $\overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0, \ \forall i, t$.

4. Proof of Proposition 2

The KKT optimality conditions of the stochastic two-stage optimization problem (25) and GNE problem (1), (2), (7), (15), (16), (19), (23), (24) when all gas-fired units are implicit virtual bidders are identical under the conditions that the DA operational bounds on $p^{\mathrm{DA}}_{i,t}$, $w^{\mathrm{DA}}_{j,t}$, $g^{\mathrm{DA}}_{k,t}$, $c^{\mathrm{DA}}_{i,t}$, $u^{\mathrm{DA}}_{i,t}$ are non-binding (e.g., $\overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0, \ \forall i, t$) so that the DA and the RT prices converge in expectation (i.e., $\hat{\lambda}^{\mathrm{E}}_t = \sum_\omega \tilde{\lambda}^{\mathrm{E}}_{t,\omega}$ and $\hat{\lambda}^{\mathrm{G}}_t = \sum_\omega \tilde{\lambda}^{\mathrm{G}}_{t,\omega}$), see (26)-(32) .

Deterministic generator:

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = -\hat{\lambda}^{\mathrm{E}}_t + C^{\mathrm{E}}_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0, \tag{34a}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = -\tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \pi_\omega C^{\mathrm{E}}_i + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0, \tag{34b}$$

Stochastic generator:

$$\begin{aligned} \frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} &= -\hat{\lambda}^{\mathrm{E}}_t + C^{\mathrm{E}}_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} \\ &\quad + \sum_{\omega\in\Omega} \Big[\overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega}\Big] = 0, \end{aligned} \tag{34c}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = -\pi_\omega\big(\frac{\tilde{\lambda}^{\mathrm{E}}_{t,\omega}}{\pi_\omega} - C^{\mathrm{E}}_i\big) + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0, \tag{34d}$$

5. Computational Performance

Model	**Postsolved residual**	**Computational time[s]**
Seq	-	0.125
Seq+eVB	2.46E-9 & 2.46E-7	12.64 + 0.34
Seq+SS	2.22	526.42
Seq+VB	1.08E-08	29.84
Ideal	-	0.16

TABLE 1. Computational performance