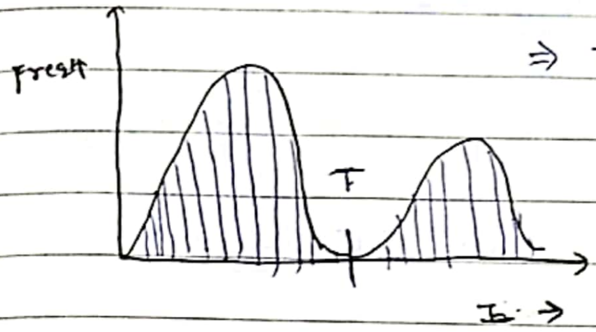
Otsu's Thresholding

→ Ideology behind this thresholing technique.



⇒ Threshold T is the optimal threshold if the segments produced have distinct intensity patterns or it gives best separation w.r.t intensities of two thresholds.

⇒ Maximizes the between class variance:- (Variance diff b/w both classes is very high) or both the classes are very distinct [very different features from each other]

⇒ Also computationally inexpensive. Uses a 1D array of histogram of intensities. Global threshold required many partitions of original picture intensities again and again. (ie go through pixels one by one again and again)

#1

Variance b/w classes:- It's the variance b/w mean of class. Let $C_1, C_2, \ldots C_K$ are K classes with mean $\mu_1, \mu_2 \ldots \mu_K$. B/w class variance is variance of $\mu_1, \mu_2 \ldots \mu_K$.

$\{0, 1, \ldots L-1\}$:- Set of L intensity levels.

for grayscale

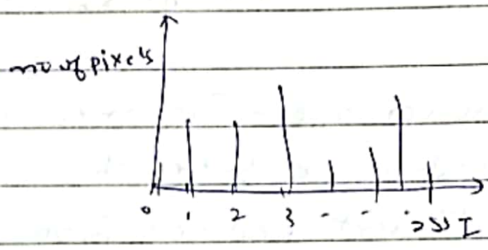
$L = 256$

$\{0, 1 \ldots 255\}$ intensities.

MN :- total pixels in the image (shape of image).

n_i :- no of pixels with intensity i.

$i \in \{0, 1 \ldots L-1\}$



$\therefore MN = \#\text{zero} + \#\text{one} + \cdots \#L-1$

$MN = n_0 + n_1 + \cdots n_{L-1}$

P_i :- given image, what is the prob of finding pixel with i intensity (prob of choosing a pixel and it's i intensity)

We will use frequentist probability formula.

\#

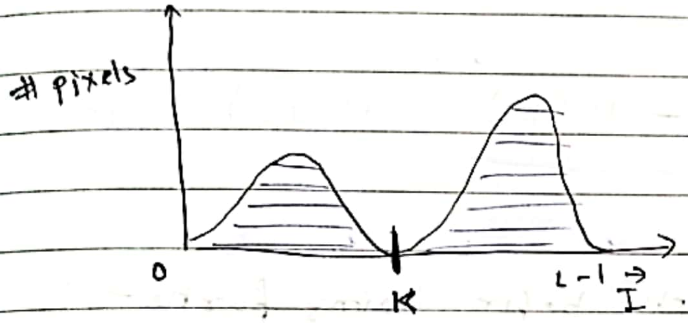
$$\boxed{P_i = \frac{n_i}{MN}}$$

$\therefore$ Since all these events are mutualy exclusive and exhaustive

$\Rightarrow \cancel{\sum_{i=0}^{L-1} p_i \leq 0}$

$$\Rightarrow \boxed{\sum_{i=0}^{L-1} p_i = 1 \quad , \; p_i \geq 0}$$

$\therefore$ Suppose we now threshold the image.



$0 < K < L-1$

Now there a two classes of pixels C_1 and C_2

C_1 contains pixels with intensities $\in [0, K]$

C_2 contains pixels with intensities $\in [K+1, L-1]$

$\therefore \boxed{P_1(K)}$:- Probability that a random pixel chosen belongs to class C_1.

$$P_1(K) = \frac{n_0 + n_1 + \dots n_K}{MN}$$

$$= \frac{n_0}{MN} + \frac{n_1}{MN} + \dots \frac{n_K}{MN}$$

$$P_1(K) = P_0 + P_1 + \dots P_K$$

$$\boxed{P_1(K) = \sum_{i=0}^{K} P_i} = P(C_1)$$

$P_2(K)$:- Probability that a pixel chosen at random belongs to class C_2

$$P_2(K) = \frac{n_{K+1} + n_{K+2} + \dots n_{L-1}}{MN}$$

$$P_2(K) = P_{K+1} + P_{K+2} + \dots P_{L-1}$$

$$\boxed{P_2(K) = \sum_{i=K+1}^{L-1} P_i}$$

$$\boxed{P_2(K) = 1 - P_1(K)} = P(C_2) \quad \dots (a)$$

Probability Concepts before Going further :-

Expected value :-

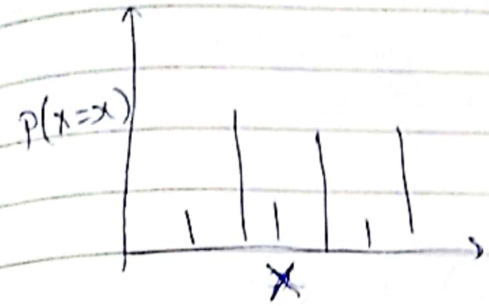
Expected value of a random variable is a long-run average value of the random variable. It is long run mean in a sense that if more and more values are collected for a variable, the sample mean will eventually reach the expected value.

$E(X)$

$$\boxed{E(X) = \sum x \cdot P(X = x)}$$ (for discrete random var).

x :- possible values of X

Let's say sample size is N_s.



$$\therefore E(X) = P(X=x_1)\cdot x_1 + \cdots P(X=x_m)\cdot x_m$$

$$E(X) = x_1 \cdot \frac{N_{x_1}}{N_s} + x_2 \cdot \frac{N_{x_2}}{N_s} + \cdots x_m \frac{N_{x_m}}{N_s}$$

$$E(X) = \frac{x_1 \cdot N_{x_1} + x_2 \cdot N_{x_2} + \cdots x_m N_{x_m}}{N_s}$$

$N_{x_1}, N_{x_2} \cdots N_{x_m}$ = weights,

so expected value is also called probability weighted average.

Derivation continue :-

Given class C1 (not assuming all intensities)

~~Probabili~~

$P(i|C_1)$:- Probability that given class C1, pixels taken from that class has an intensity i.

These are dependent events because, ~~probability~~ intensity class depend upon which class the pixel is in.

$\therefore$ Expected value of mean intensity (m) of pixels in class C_1 is.

$$m_1(K) = \sum_{i=0}^{K} i\, P(i/C_1)$$

$$m_1(K) = \sum i \times P(i \cap C_1) / P(C_1)$$

$$m_1(K) = \sum i \times \frac{P(C_1|i) \cdot P(i)}{P(C_1)}$$

$$P(C_1|i) = 1 \quad \text{why?}$$

Ans:- Because all pixels of intensity i are in class C_i (we are summation over pixels in class C_1)
So, the chance of finding i in $C_1 = 1$.

$$\cancel{\boxed{m_1(K) = \sum_i \frac{i\, P(i)}{P_1(K)}}}$$

$$\cancel{m(K) = \sum_{i=0} P(i)}$$

$$\therefore \boxed{m_1(K) = \frac{\sum_{i=0}^{K} i \cdot P(i)}{P_1(K)}} \cdot (i)$$

7

Similarily for class C_2.

$$\boxed{m_2(K) = \frac{\sum_{i=K+1}^{L-1} i \cdot P_i}{P_2(K)}} \quad \cdot \text{(ii)}$$

∴ Considering entire image.

Expected mean intensity upto K is given by.

$$\boxed{m(K) = \sum_{i=0}^{K} i \cdot P(i)} \quad \cdot \text{(iii)}$$

Global mean intensity is.

$$\boxed{m_G = \sum_{i=0}^{L-1} i \cdot P(i)} \quad \cdot \text{(iv)} .$$

∴ Now,

$$P_1 . m_1 + P_2 \cdot m_2 = m_G \cdot\cdot \text{(v)} .$$

Also, $P_1 + P_2 = 1$ [from a].

Now, Global variance σ_G^2

$$\boxed{\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 \cdot P(i)} \cdot\cdot \text{(vi)} \quad \text{[weighted variance]}$$

Now σ_B^2 = between class variance.

$$\sigma^2_B = P_1(m_1 - m_\sigma)^2 + P_2(m_2 - m_\sigma)^2$$

$$\sigma^2_B = P_1 m_1^2 + P_1 m_\sigma^2 - 2P_1 m_1 m_\sigma$$
$$+ P_2 m_2^2 + P_2 m_\sigma^2 - 2P_2 m_2 m_\sigma.$$

$$\sigma^2_B = m_\sigma^2(P_1 + P_2) + P_1 m_1^2 + P_2 m_2^2$$
$$- 2P_1 m_1 m_\sigma - 2P_2 m_2 m_\sigma$$

$$\sigma^2_B = m_\sigma^2 + P_1 m_1^2 + P_2 m_2^2$$
$$- 2P_1 m_1 m_\sigma - 2P_2 m_2 m_\sigma.$$

$$\sigma^2_B = m_\sigma^2 + P_1 m_1^2 + P_2 m_2^2$$
$$- 2m_\sigma(P_1 m_1 + P_2 m_2).$$

$$\sigma^2_B = m_\sigma^2 + P_1 m_1^2 + P_2 m_2^2 - 2m_\sigma^2$$

$$\sigma^2_B = P_1 m_1^2 + P_2 m_2^2 - m_\sigma^2$$

$$\sigma^2_B = P_1\left(\frac{m}{P_1}\right)^2 + P_2\left(\frac{m}{P_2}\right)^2 - m_\sigma^2.$$

$$\sigma^2_B = \frac{m^2}{P_1} + \frac{m^2}{P_2} - m_\sigma^2$$

Now,

$$\sum_{i=0}^{k} i P_i + \sum_{i=k+1}^{L-1} i P_i = m_G$$

$$m + \sum_{i=k+1}^{L-1} P_i = m_G$$

$$\sum_{i=k+1}^{L-1} P_i = m_G - m.$$

$$\therefore m_2 \cdot P_2 = m_G - m$$

$$\boxed{m_2 = \frac{m_G - m}{P_2}}$$

$$\therefore \sigma_B^2 = P_1\left(\frac{m}{P_1}\right)^2 + P_2\left(\frac{m_G - m}{P_2}\right)^2 - m_G^2$$

$$\sigma_B^2 = \frac{m^2}{P_1} + \frac{(m_G - m)^2}{P_2} - m_G^2$$

$$\sigma_B^2 = \frac{m^2}{P_1} + \frac{m_G^2 + m^2 - 2m_G \cdot m}{P_2} - m_G^2$$

$$\sigma_B^2 = \frac{P_2 \cdot m^2 + P_1 m_G^2 + P_1 m^2 - 2P_1 m_G \cdot m}{P_1 \cdot P_2} - m_G^2$$

$$\sigma_B^2 = \frac{(1-P_1)m^2 + P_1 m_G^2 + P_1 m^2 - 2P_1 m_G \cdot m - m_G^2 (P_1)(1-P_1)}{P_1(1-P_1)}$$

$$= \frac{m^2 - \cancel{P_1 m^2} + \cancel{P_1 m_G^2} + \cancel{P_1 m^2} - 2P_1 m_G \cdot m - \cancel{P_1 m_G^2} + P_1^2 m_G^2}{P_1(1-P_1)}$$

$$= \frac{P_1^2 m_G^2 - 2P_1 m_G \cdot m + m^2}{P_1(1-P_1)}$$

$$\therefore \boxed{\sigma_B^2 = \frac{(P_1 m_G - m)^2}{P_1(1-P_1)}}$$

Also,

$$\sigma^2_B = P_1 P_2 (m_1 - m_2)^2$$

$\Downarrow$

Farther both means are, more will be between-class variance.

So, we want to maximize it.

$$\therefore \eta = \frac{\sigma^2_B}{\sigma^2_G} \Rightarrow \text{Constant}$$

$\eta \propto \sigma^2_B$. ($\because$ η is also a parameter).

<u>Objective</u>:- ~~Ma~~ Find a value of K, such that σ^2_B or η is maximized.

$$\eta(K) = \frac{\sigma^2_B(K)}{\sigma^2_G}$$

$$\boxed{\sigma^2_B(K) = \frac{(m_G P_1(K) - m(K))^2}{[P_1(K)(1 - P_1(K))]}}$$

$\therefore$ optimal threshold K^*

$$\boxed{\sigma^2_B(K^*) = \max_{0 \le K \le L-1} \sigma^2_B(K)}$$

To find K^*, we iterate over all values of K. (such that $0 < P_1(K) < 1$) and select value of K which gives max $\sigma^2_B(K)$

If ~~avera~~ $\sigma^2_B(K)$ is same (max) for many K's, we need to average for all values of K for which $\sigma^2_B(K)$ is maximum.

<u>Segmentation</u> :-

$$g(x,y) = \begin{cases} 1, & \text{if } f(x,y) > K^* \\ 0, & \text{if } f(x,y) \le K^* \end{cases}$$

<u>Additional</u> :-

Formula for b/w class variance is.

~~$\sigma^2(B) = P_1(m_1 - m_G)^2$~~

$$SS(B) = \sum n(\bar{x} - \bar{X}_{GM})^2$$

SS(B) :- sum of squares b/w group.

$\bar{x}$:- sample mean ~~for each group~~

$\bar{X}_{GM}$:- Group mean,

$$(\bar{x} - \bar{X}_{GM})^2 = (X_{GM} - \bar{x})^2$$

Scanned with CamScanner

classmate

Date

Page

$\therefore$ Groups in our case are below K and above it.

$\therefore \sigma_B^2 = \sum P_i (m_i - m_G)^2$

$$\boxed{\sigma_B^2 = P_1 (m_1 - m_G)^2 + P_2 (m_2 - m_G)^2}$$