L-19: Determinant Formulas and Co-factor Matrix

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1 Extended Formula

Method: Take each row of matrix at a time and split row into no.of column pieces.(property 3b)

example

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & 0 \\ c & 0 \end{vmatrix} + \begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ 0 & d \end{vmatrix} + \begin{vmatrix} 0 & b \\ c & 0 \end{vmatrix}$$

Only second and last determinants are useful as the rest have 0 columns.(singular matrix)

Consider a 3*3 matrix with elements:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

For a 3*3 matrix 1st row will be divided into 3 pieces. For each of these pieces, 2nd row will give three pieces, therefore equating to 9 pieces. Similarily, considering third row, we get a total of 27 pieces. Explanation is in supplementary notes 1. However, out of these 27 pieces only some are useful. The useful determinants give the formula(6 total):

$\mid A \mid = a_{11}(a_{22}a_{33} - a_{32}a_{23}) + a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{13}(a_{33}a_{21} - a_{31}a_{22})$

1.1 General Formula

For 2*2 = 2 useful
For 3*3 = 6 useful
For n*n = n! useful

$\det A = \sum_{r=1}^{n!} a_{1\alpha}a_{2\beta}a_{3\gamma}....a_{n\omega}$

Where r=number of permutations and $[\alpha, \beta, \gamma, \delta$ etc. are permutations of 1 to nth column]

2 Co-Factors

$| A |= a_{11}(a_{22}a_{33} - a_{32}a_{23}) + a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{13}(a_{33}a_{21} - a_{31}a_{22})$

Here, the elements in parenthesis are co-factors.

1 **General formula for co-factor**

Cofactor of $a_{ij} = C_{ij} = \pm \det (n - 1 \textit{ dimensional matrix with row } i \textit{ and column } j \textit{ erased})$

$$Where\ sign\ is, \begin{cases} + & if\ i+j = even \\ - & if\ i+j = odd \end{cases}$$

2.0.1 Cofactor Formula for Determinants

Determinant formula along ith row is given by(we can take along column too as transpose is equal in case of determinants.)

$\det A = a_{i1}C_{i1} + a_{i2}C_{i2} + ... + a_{in}C_{in}$

2 **Extra material**

Extra material to develop more intuition can be found in book and supplementary notes 1.