L-20: Determinants and their Applications

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1 Inverse Matrix

For an n dimensional matrix, the formula for inverse can also be given by:

$$A^{-1} = \frac{1}{\det(A)} * C^T$$

Where, C is the matrix of co-factors.

For a 2*2 matrix, the formula will be:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

1.1 Why does this work?

$A.A^T = I$

$A.C^T = det(A).I$

$$\begin{bmatrix} a_{11} & a_{12} & ... & a_{2n} \\ a_{21} & a_{22} & ... & a_{2n} \\ . & . & . & . \\ a_{n1} & a_{n2} & ... & a_{nn} \end{bmatrix} . \begin{bmatrix} c_{11} & c_{21} & ... & c_{n1} \\ c_{12} & c_{22} & ... & c_{n2} \\ . & . & . & . \\ c_{1n} & c_{2n} & ... & c_{nn} \end{bmatrix} = \begin{bmatrix} detA & ... & 0 \\ 0 & detA & ... \\ 0 & ... & detA \end{bmatrix}$$

The first element of res matrix

$detA_{11} = a_{11}c_{11} + a_{12}c_{12} + ... + a_{1n}c_{1n}$ [This is the formula for determinant.(Cofactor formula)]

Detailed proof can be seen from the book. Some information is mentioned in supplementary notes 2.

2 Cramers rule

$Ax = b$
$x = A_{-1}b$

$x = \frac{1}{\det A}.C^T b$

According to cramer's rule, the individual components of x can be written as:

$x_1 = \frac{\det B_1}{\det A}$

$x_2 = \frac{\det B_2}{\det A}$

$x_i = \frac{\det B_i}{\det A}$

Where, B_i is A with column i replaced by b.

1 **Note**

Use elimination wherever possible as it is computationally less expensive than these formulas.

3 Area and volumes of shapes

These formulas and their proofs can be found in supplementary material 2 and the book.