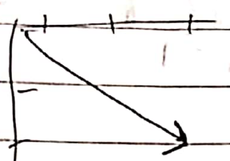
Lin. Algebra practice book :

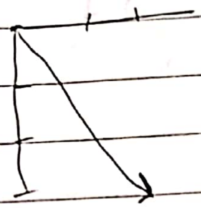
Example 1.2 :

Homework 1.2.1.1 :

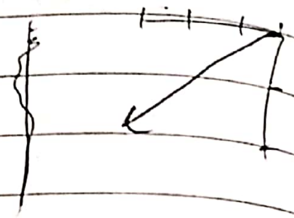
(b)



(c)



(d) $x = (-3 \ \ -2)$



Homework 1.2.1.2 :

(a) $a = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ (b) $b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (c) $c = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

(d) $d = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$ (e) $e = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (f) $f = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(g) $g = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

Homework 1.2.1.3 :

(a) $(-1,2)$ to $(0,0)$ $\rightarrow$ $v = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(b) $(0,0)$ to $(-1,2)$ $\rightarrow$ $v = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(c) $(-1,2,4)$ to $(0,0,1)$ $\rightarrow$ $v = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$

(d) $(1,0,0)$ to $(4,2,-1)$

$v = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

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Homework 1.2.2.1 :-

Answer :- (c)

Homework 1.3.1.1 :-

VQ

v_1 is from $(-1,2)$ to $(0,0)$, $\Rightarrow v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

v_2 is from $(1,-2)$ to $(2,-1)$ $\Rightarrow v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

False, they are not equal.

v_1 is from $(1,-1,2)$ to $(0,0,0)$ $\Rightarrow v_1 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

v_2 is from $(1,1,-2)$ to $(0,2,-4)$ $\Rightarrow v_2 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$

True, both are equal,

Homework 1.3.2.1 :-

$\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$ Ans

Homework 1.3.2.2 :-

$\begin{pmatrix} -3 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$ Ans

Homework 1.3.2.3 :

For x, y in $\in \mathbb{R}^m$

$$x + y = y + x$$

$$x = \begin{bmatrix} \chi_0 \\ \vdots \\ \chi_{n-1} \end{bmatrix}, \quad y = \begin{bmatrix} \psi_0 \\ \vdots \\ \psi_{n-1} \end{bmatrix}$$

$$x + y = \begin{bmatrix} \chi_0 + \psi_0 \\ \vdots \\ \chi_{n-1} + \psi_{n-1} \end{bmatrix}, \quad y + x = \begin{bmatrix} \psi_0 + \chi_0 \\ \vdots \\ \psi_{n-1} + \chi_{n-1} \end{bmatrix}$$

True Q.E.D (Always)

Homework 1.3.2.4 :

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \left(\begin{pmatrix} -3 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right)$$

$$= \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Homework 1.3.2.5 :

$$\left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \end{pmatrix} \right) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

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Homework 1.3.2.6 :

L.H.S

$$(x+y)+z = \left[\begin{bmatrix} x_0 \\ \vdots \\ x_{n-1} \end{bmatrix} + \begin{bmatrix} y_0 \\ \vdots \\ y_{n-1} \end{bmatrix}\right] + \begin{bmatrix} z_0 \\ \vdots \\ z_{n-1} \end{bmatrix}$$

$$= \begin{bmatrix} x_0 + y_0 + z_0 \\ \vdots \\ x_{n-1} + y_{n-1} + z_{n-1} \end{bmatrix}$$

$\therefore (x+y)+z) = x+(y+z)$ Always

Homework 1.3.2.7 -

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Homework 1.3.2.8 -

$x + 0 = x$ Always.

No linear addition on components.

Homework 1.3.3.1 -

$$\left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}\right) + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$$

Homework 1.3.3.2 -

$$3\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$$

Homework 1.3.3.3

$a = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

(a) $2a = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$

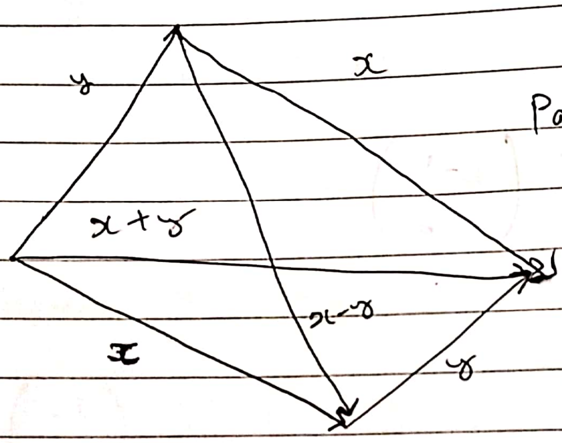
(c) $-\left(\frac{1}{2}\right)a = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

$c = \frac{-1}{2} a$

Vector $d = 2a$

(b) $\left(\frac{1}{2}\right)a = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Vector $b = \frac{1}{2} a$



Parallelogram law.

Homework 1.3.4.1 :

$\forall x \in \mathbb{R}^n, x - x = 0.$

Algebraically

$x = \begin{bmatrix} x_0 \\ \vdots \\ x_{n-1} \end{bmatrix}$ $-x = \begin{bmatrix} -x_0 \\ \vdots \\ -x_{n-1} \end{bmatrix}$

$x + (-x) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ $\therefore$ Q.E.D

Geometrically

No vector formed when we connect head of second vector to tail of first vec. So, no vector or zero vector in this case.

Always

Homework 1.3.4.2 :-

$\forall\ x, y \in \mathbb{R}^n \quad x - y = y - x$

$$x = \begin{bmatrix} x_0 \\ \vdots \\ x_{n-1} \end{bmatrix}, \quad y = \begin{bmatrix} y_0 \\ \vdots \\ y_{n-1} \end{bmatrix}$$

$$x - y = \begin{bmatrix} x_0 - y_0 \\ \vdots \\ x_{n-1} - y_{n-1} \end{bmatrix}, \quad y - x = \begin{bmatrix} y_0 - x_0 \\ \vdots \\ y_{n-1} - x_{n-1} \end{bmatrix}$$

If $x = y$ only then equal. Otherwise signs interchanged depending upon which component is greater.

So sometimes is correct.

Homework 1.4.1.1 :- $\boxed{n+n+n+n}$

(a) memops:- $3n + 1$

(b) flops :- $2n$

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Homework 1.4.2.1 -

$$3\begin{bmatrix}2\\4\\-1\\0\end{bmatrix} + 2\begin{bmatrix}1\\0\\1\\0\end{bmatrix} = \begin{bmatrix}8\\12\\-1\\0\end{bmatrix}$$

Homework 1.4.2.2 -

$$-3\begin{bmatrix}1\\0\\0\end{bmatrix} + 2\begin{bmatrix}0\\1\\0\end{bmatrix} + 4\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-3\\2\\4\end{bmatrix}$$

Homework 1.4.2.3 -

$$\alpha\begin{bmatrix}1\\0\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\\0\end{bmatrix} + \gamma\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}2\\-1\\3\end{bmatrix}$$

$\alpha = 2, \ \beta = -1, \ \gamma = 3$

H/W 1.4.3.2 -

$$\begin{bmatrix}2 & 5 & -6 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$= 2 + 5 - 6 + 1 = 2$

H/W 1.4.3.3 -

$$\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}\begin{bmatrix}2\\5\\-6\\1\end{bmatrix} = 2$$

$w = 0$

$\downarrow$

m [memops]

$v_j \in m$ memops

writes $\rightarrow m$ memops.

$(n \times m + m)$

$m + n \times (m + m)$

$m + 2mn$ $\quad m +$

$m(2n + 1)$ $\quad (n \times m$

$2mn$ $\quad = m(2n$

$\cancel{(m + m)}$ $\quad m(n^2 + 2$

$\cancel{m(2(n+1))}$

H/W 1.4.3.4 :

$\forall\ x, y \in \mathbb{R}^n \quad x^T y = y^T x$.

$x = \begin{bmatrix} x_0 \\ \vdots \\ x_{n-1} \end{bmatrix}, x^T = [x_0 \ \cdots \ x_{n-1}], \ y = \begin{bmatrix} y_0 \\ \vdots \\ y_{n-1} \end{bmatrix}, y^T = [y_0 \ \cdots \ y_{n-1}]$

$x^T y = x_0 y_0 + x_1 y_1 + \cdots x_{n-1} \cdot y_{n-1}$

$y^T x = y_0 x_0 + y_1 x_1 + \cdots y_{n-1} \cdot x_{n-1}$

Now, addition of vectors ~~in~~ is comm and multiplication of $x_i y_i \in R$ is also commutative.

$\therefore \ x^T y = y^T x$ Always

H/W 1.4.3.5 :

$[1 \ 1 \ 1 \ 1] \left[\begin{pmatrix} 2 \\ 5 \\ -6 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \right]$

$= [1 \ 1 \ 1 \ 1] \begin{bmatrix} 3 \\ 7 \\ -3 \\ 5 \end{bmatrix}$

$= 12$ Ans

H/w 1.4.3.6

$$[1 \ \ 1 \ \ 1 \ \ 1]\begin{bmatrix} 2 \\ 5 \\ -6 \\ 1 \end{bmatrix} + [1 \ \ 1 \ \ 1 \ \ 1]\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$= 2 + 10 = 12$ Ans

H/w 1.4.3.8

$\forall \ x, y, z \in \mathbb{R}^m, \ x^T(y+z) = x^T y + x^T z$

LHS

$$[x_0 \cdots x_m]\begin{bmatrix} y_0 + z_0 \\ \vdots \\ y_{n-1} + z_{n-1} \end{bmatrix} = x_0(y_0 + z_0) + \cdots \ x_{n-1}(y_{n-1} + z_{n-1})$$

& R.H.S

$$x^T y + x^T z = \begin{bmatrix} x_0 y_0 + \cdots \\ x_{n-1} + y_{n-1} \end{bmatrix} + \begin{bmatrix} x_0 z_0 + \cdots \\ x_{n-1} z_{n-1} \end{bmatrix}$$

$= x_0(y_0 + z_0) + x_1(y_1 + z_1) + \cdots x_{n-1}(y_{n-1} +$

$\therefore$ LHS = RHS Always

H/w 1.4.3.9 :

$\forall \ x, y, z \in \mathbb{R}^m, \ (x+y)^T z = x^T z + y^T z$

L.h.s :

$$[x_0 + y_0 \ \cdots \ x_{n-1} + y_{n-1}]\begin{bmatrix} z_0 \\ \vdots \\ z_{n-1} \end{bmatrix} = z_0(x_0 + y_0) + \cdots \ z_{n-1}(x_{n-1} + y_{n-1})$$

R.H.S

$x^Tz + y^Tz = [x_0 z_0 + \cdots + x_{n-1} z_{n-1}]$
$+ [y_0 z_0 + \cdots + y_{n-1} z_{n-1}]$

$= z_0(x_0 + y_0) + \cdots + z_{n-1}(x_{n-1} + y_{n-1})$

$\therefore$ LHS = RHS Always

H/W 1.4.3.10+

$\forall\, x, y \in \mathbb{R}^n,\ (x+y)^T(x+y) = x^Tx + 2x^Ty + y^Ty$

L.H.S

$$[x_0 + y_0 \ \cdots\ x_{n-1} + y_{n-1}] \begin{bmatrix} x_0 + y_0 \\ \vdots \\ x_{n-1} + y_{n-1} \end{bmatrix} = (x_0 + y_0)^2 + \cdots + (x_{n-1} + y_{n-1})^2$$

R.H.S :-

$$x^Tx + 2x^Ty + y^Ty = [x_0^2 + \cdots x_{n-1}^2] + 2[x_0 y_0 + \cdots x_{n-1} y_{n-1}] + [y_0^2 + \cdots y_{n-1}^2]$$

$$= (x_0^2 + 2x_0 y_0 + y_0^2) + \cdots + (x_{n-1}^2 + 2x_{n-1} y_{n-1} + y_{n-1}^2)$$

$$= (x_0 + y_0)^2 + \cdots + (x_{n-1} + y_{n-1})^2$$

$\therefore$ L.H.S = R.H.S Always

H/w 1.4.3.11 :-

$x, y \in \mathbb{R}^n$. $x^T y = 0$, x or y is a zero vector

$\therefore x_0 y_0 + x_1 y_1 + \cdots (x_{n-1} \cdot y_{n-1}) = 0$

but

$\Rightarrow (x_{n-1} \cdot y_{n-1}) = -[x_0 y_0 + \cdots x_{n-2} \cdot y_{n-2}]$

$\therefore$ Not necessarily zero vector

example :

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix} = 3 + 6 - 9 = 0.$$

Sometimes

H/w 1.4.3.12 -

$x \in \mathbb{R}^n$, $e_i^T x = x^T e_i = x_i$ where x_i is ith component of x

Now, $x^T y = y^T x$

$\Rightarrow e_i^T x = x^T e_i$

example

$$\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = 2 = x_2 .$$ It's true.

ith component is taken and rest ignored.

$\therefore$ It's true all times.

$1 + m($ $\alpha : \to 1$

$m(1+1)$ $2m$ $2m+2$

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H/w 1.4.3.13

1) Mem cost = ~~2cR+1~~ $2(n+1)$

2) flops :- $n(2) = 2n$

H/w 1.4.4.1 -

(a) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\|a\|_2 = \sqrt{0+0+0} = 0$

(b) $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$ $\|b\|_2 = \sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}$
$= 1$

(c) $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $\|c\|_2 = \sqrt{1^2+4+4}$
$= 3$

(f) $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \sqrt{1}$
$= 1$ Ans

Homework 1.4.4.2 -

$x \in \mathbb{R}^n$

$\|x\|_2 < 0$

$\sqrt{(x_1)^2 + \dots (x_m)^2} \to$ Positive always ~~unless~~ for $x \in \mathbb{R}^n$ (not complex space)

Never Ans

Homework 1.4.4.3 -

False $x = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

$\Downarrow$

Is a unit vector but not a unit basis as no 0

~~False $x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, y = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$~~

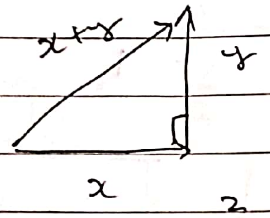
~~x is a unit vector but not a unit basis vector for 3-D space~~ 2-D space.

H/w 1.4.4.4 :-

$x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ It's a unit basis vector and also a unit vector.

H/w 1.4.4.5 :-

$x \perp y$ $\therefore$ To prove:- $x^T y = 0$.



$$\|x\|_2^2 + \|y\|_2^2 = \|x+y\|_2^2$$

$$\cancel{(x_0)^2} + \cancel{(x_1)^2} + \dots \cancel{(x_{n-1})^2} + \cancel{y_0^2} + \cancel{y_1^2} + \dots \cancel{y_{n-1}^2} =$$

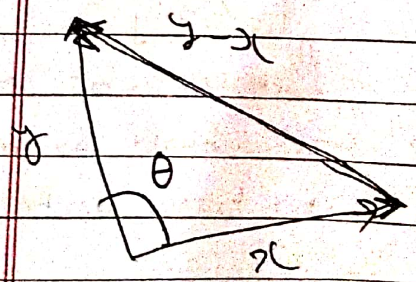
$$\cancel{x_0^2} + \cancel{y_0^2} + 2x_0y_0 + \dots \cancel{x_{n-1}^2} + \cancel{y_{n-1}^2} + 2x_{n-1}y_{n-1}$$

$$2\left[x_0y_0 + \dots + x_{n-1}y_{n-1}\right] = 0$$

$\therefore \boxed{x^T y = 0}$ TRUE

H/w 1.4.4.6 :-

$x, y \in \mathbb{R}^n$ $\therefore$ To prove:- $\cos\theta = \dfrac{x^T y}{\|x\|_2 \|y\|_2}$



$\cancel{(y-x)^2 = \|y\|_2^2 + \|x\|^2 - 2x^T y \cos\theta}$.

$\|y-x\|_2^2 = \|y\|_2^2 + \|x\|_2^2 - 2\|x\|_2\|y\|_2 \cos\theta$.

$\cancel{y_0^2} + \cancel{x_0^2} - 2y_0x_0 + \cdots \cancel{y_{n-1}^2} + \cancel{x_{n-1}^2} - 2x_{n-1}y_{n-1}$

$= \cancel{y_0^2} + \cdots + \cancel{y_{n-1}^2} + \cancel{x_0^2} + \cdots + \cancel{x_{n-1}^2}$
$- 2\|x\|_2\|y\|_2 \cos\theta$.

$\cancel{2}[x_0y_0 + \cdots + x_{n-1}y_{n-1}] = \cancel{2}\|x\|_2\|y\|_2 \cos\theta$.

$x^Ty = \|x\|_2\|y\|_2 \cos\theta$

$\Rightarrow \boxed{\cos\theta = \dfrac{x^Ty}{\|x\|_2\|y\|_2}}$ Q.E.D..

TRUE

H/W 1.4.4.7

$x^Ty = 0$ only if x and y are $\perp$.

$\cos\theta = \dfrac{x^Ty}{\|x\|_2\|y\|_2 \ (\neq 0)}$

~~cos~~ Suppose $x^Ty = 0$

$\cos\theta = 0$
$\theta = \cos^{-1}(0)$

$\boxed{\theta = (2m+1)\pi/2}$ ∴ Yes ~~angle~~ the vectors are $\perp$'r.

H/W 1.4.5.1

(a) $f\left(\alpha, \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}\right) = \begin{pmatrix} x_0 + \alpha \\ x_1 + \alpha \\ x_2 + \alpha \end{pmatrix}$

$f\left(1, \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{pmatrix} 7 \\ 3 \\ 4 \end{pmatrix}$

(b) $f\left(\alpha, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} \alpha \\ \alpha \\ \alpha \end{bmatrix}$

(c) $f\left(0, \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$

(d) $f\left(\beta, \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_0 + \beta \\ x_1 + \beta \\ x_2 + \beta \end{bmatrix}$

(e) $\alpha f\left(\beta, \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}\right) = \alpha \begin{bmatrix} x_0 + \beta \\ x_1 + \beta \\ x_2 + \beta \end{bmatrix}$

(f) $f\left(\alpha, \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{bmatrix}\right) = \begin{bmatrix} \alpha(x_0 + \psi_0) \\ \alpha(x_1 + \psi_1) \\ \alpha(x_2 + \psi_2) \end{bmatrix}$

(g) $f\left(\alpha, \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}\right) + f\left(\alpha, \begin{bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{bmatrix}\right)$

$= \begin{bmatrix} \alpha(x_0 + \psi_0) \\ \alpha(x_1 + \psi_1) \\ \alpha(x_2 + \psi_2) \end{bmatrix}$

H/W 1.4.6.1 -

$$f\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_0+1 \\ x_1+2 \\ x_2+3 \end{pmatrix}$$

(a) $f\begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 6 \end{pmatrix}$ (b) $f\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

(c) $f\left(2\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}\right) = 2\begin{pmatrix} x_0+1 \\ x_1+2 \\ x_2+3 \end{pmatrix}$ (d) $f\left(\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{bmatrix}\right)$

$$= \begin{bmatrix} x_0+\psi_0+1 \\ x_1+\psi_1+2 \\ x_2+\psi_2+3 \end{bmatrix}$$

(e) $f\left(\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}\right) + f\left(\begin{bmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{bmatrix}\right) = \begin{bmatrix} x_0+1 \\ x_1+2 \\ x_2+3 \end{bmatrix} + \begin{bmatrix} \psi_0+1 \\ \psi_1+2 \\ \psi_2+3 \end{bmatrix}$

$$= \begin{bmatrix} x_0+\psi_0+2 \\ x_1+\psi_1+4 \\ x_2+\psi_2+6 \end{bmatrix}$$

H/W 1.4.6.2 -

$$f\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} x_0 \\ x_0+x_1 \\ x_0+x_1+x_2 \end{bmatrix}$$

$$f\begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} = \begin{bmatrix} 6 \\ 8 \\ 11 \end{bmatrix}$$

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H/W 1.4.6.3

If $f: \mathbb{R}^n \to \mathbb{R}^m$, then $f(0) = 0$.

Suppose

$f: \mathbb{R}^n \to \mathbb{R}^m$ be a function such that

$$f\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha+\beta \\ \beta+\gamma \\ \alpha+\gamma \\ 1 \end{bmatrix}$$

This means

$$f\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

So, it is true sometimes.

H/W 1.4.6.4 :-

$f: \mathbb{R}^n \to \mathbb{R}^m$, $\lambda \in \mathbb{R}$ and $x \in \mathbb{R}^n$, then

$$f(\lambda x) = \lambda f(x)$$

Suppose $f\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} 1 \\ x_0 \\ \vdots \\ x_{n-1} \end{bmatrix}$

$\therefore f(\lambda x) = \begin{bmatrix} 1 \\ \lambda x_0 \\ \vdots \\ \lambda x_{n-1} \end{bmatrix}$ but $\lambda f(x) = \begin{bmatrix} \lambda \\ \lambda x_0 \\ \vdots \\ \lambda x_{n-1} \end{bmatrix}$

So, it is true but sometimes.

H/W 1.4.6.5 :-

$f: \mathbb{R}^n \to \mathbb{R}^m$ and $x, y \in \mathbb{R}^n$, then

$$f(x+y) = f(x) + f(y).$$

Suppose :- $f\begin{bmatrix} x_0 \\ \vdots \\ x_{n-1} \end{bmatrix} = f\begin{bmatrix} 1 \\ x_0 \\ \vdots \\ x_{n-1} \end{bmatrix}$

$$f\begin{bmatrix} x_0 + y_0 \\ x_1 + y_1 \\ \vdots \\ x_{n-1} + y_{n-1} \end{bmatrix} = \begin{bmatrix} 1 \\ x_0 + y_0 \\ \vdots \\ x_{n-1} + y_{n-1} \end{bmatrix}$$

$$f(x) + f(y) = \begin{bmatrix} 1 \\ x_0 \\ \vdots \\ x_{n-1} \end{bmatrix} + \begin{bmatrix} 1 \\ y_0 \\ \vdots \\ y_{n-1} \end{bmatrix} = \begin{bmatrix} 2 \\ x_0 + y_0 \\ \vdots \\ x_{n-1} + y_{n-1} \end{bmatrix}$$

So, sometimes equal.

Overflow fix norm 2 :-

$$\|x\|_2 = \sqrt{\sum_{i=0}^{N-1} |x_i|^2} = \sqrt{\alpha^2 \frac{\sum_{i=0}^{N-1} |x_i|^2}{\alpha^2}}.$$

$$= \alpha \sqrt{\sum_{i=0}^{N-1} \left(\frac{x_i}{\alpha}\right)^2} = \alpha \sqrt{\left(\frac{1}{\alpha}x\right)^T \left(\frac{1}{\alpha}x\right)}$$

$$= \alpha \left(\cancel{\frac{x}{\alpha}}\right)$$

where

$$\alpha = \max_{i=0}^{N-1} |x_i|$$

Now x is scaled b/w 0 and 1

No overflow in between

but

$\alpha \times \sqrt{n}$ might cause it.

Underflow might happen for very small values.

H/W 1.8.1.1 :

$x = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $y = \begin{pmatrix} \alpha \\ \beta - \alpha \end{pmatrix}$ and $x = y$.

$\alpha = 2$ ✓

$\beta - \alpha = -1$

$\beta - 2 = -1$

$\boxed{\beta = 1}$ ✓

(a) $\alpha = 2$ ✓ (b) $\beta = 1$ ✓ (c) $\beta - \alpha = -1$ ✓ (d) $\beta - 2 = -1$ ✓

(e) $x = 2e_0 - e_1$

$2e_0 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $e_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$2e_0 - e_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ ✓

All are true.

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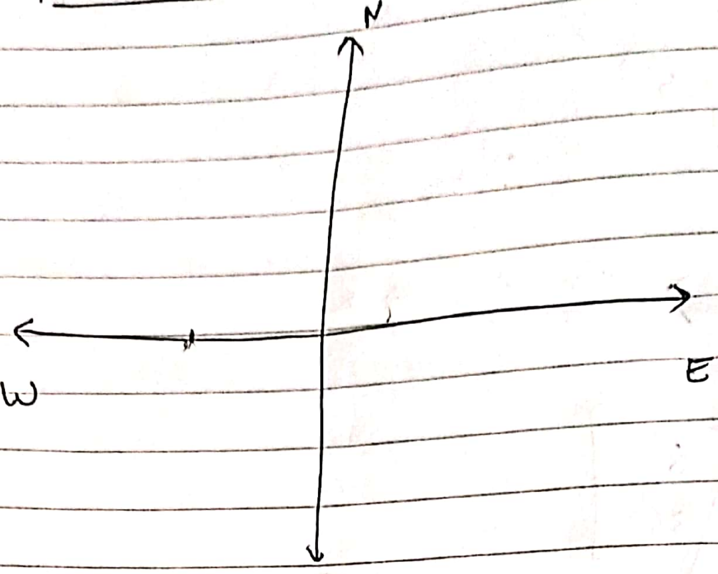
H/w 1.8.1.2:



300
- 39
261

5°
30° - 39
26'1

(a) $S^1 = \begin{pmatrix} 102^\circ & 06' \\ 20^\circ & 36' \end{pmatrix}$

261

$66\overline{)261}4$
240
21

$A = \begin{pmatrix} -97^\circ & 45' \\ 30^\circ & 15' \end{pmatrix}$

$x - A = \begin{pmatrix} 102^\circ & 06' \\ 20^\circ & 36' \end{pmatrix}$

$x = \begin{pmatrix} 5^\circ & -39'E \\ 50^\circ & 51'N \end{pmatrix}$

$x = \begin{pmatrix} 4^\circ & 21'E \\ 50^\circ & 51'N \end{pmatrix}$

$\boxed{x = \text{Brussels}}$

(b) $S^2 = \begin{pmatrix} 04^\circ & 18' \\ -00^\circ & 59' \end{pmatrix}$

$$B = \begin{pmatrix} 4^\circ & 21'E \\ 50^\circ & 51'N \end{pmatrix}$$

$$x - B = \begin{pmatrix} 04^\circ & 18' \\ -00^\circ & 59' \end{pmatrix}$$

$$x = \begin{pmatrix} 8^\circ & 39'E \\ 50^\circ & -8'N \end{pmatrix}$$

$$\boxed{x = \begin{pmatrix} 8^\circ & 39'E \\ 49^\circ & 52'N \end{pmatrix}}$$

$\boxed{x = \text{Darmstadt}}$

(c) $$S_3 = \begin{pmatrix} -00^\circ & 06' \\ -02^\circ & 30' \end{pmatrix}$$

$$x - D = \begin{pmatrix} -00^\circ & 06' \\ -02^\circ & 30' \end{pmatrix}$$

$$\boxed{x = \begin{pmatrix} 8^\circ & 33'E \\ 47^\circ & 22'N \end{pmatrix}}$$

$\boxed{x = \text{Zürich}}$

(d)

$S_4 = \begin{pmatrix} 01^\circ & 48' \\ -03^\circ & 39' \end{pmatrix}$

$x - z = \begin{pmatrix} 01^\circ & 48' \\ -03^\circ & 39' \end{pmatrix}$

$x = \begin{pmatrix} 9^\circ & 81'E \\ 44^\circ & -17'N \end{pmatrix}$

$= \begin{pmatrix} 9^\circ & 81'E \\ 43^\circ & 43'N \end{pmatrix}$

$= \begin{pmatrix} 10^\circ & 21'E \\ 43^\circ & 43'N \end{pmatrix}$

$\boxed{x = \text{Pisa}}$

(e) $S_5 = \begin{pmatrix} 09^\circ & 35' \\ 06^\circ & 21' \end{pmatrix}$

$x = \begin{pmatrix} 19^\circ & 56'E \\ 49^\circ & 64'N \end{pmatrix}$

$x = \begin{pmatrix} 19^\circ & 56'E \\ 50^\circ & 4'N \end{pmatrix}$

$\boxed{x = \text{Krapow}}$

(f) $S_6 = \begin{pmatrix} -20^\circ & 04' \\ 01^\circ & 26' \end{pmatrix}$

$x = \begin{pmatrix} -1^\circ & 52'E \\ 51^\circ & 30'N \end{pmatrix}$

$\boxed{x = \begin{pmatrix} 0^\circ & -8'E \\ 51^\circ & 30'N \end{pmatrix}}$

$\boxed{x = \text{London}}$

(g) $S_7 = \begin{pmatrix} 00^\circ & 31'E \\ -12^\circ & 02'N \end{pmatrix}$

$x = \begin{pmatrix} 1^\circ & -19'E \\ 39^\circ & 28'N \end{pmatrix}$

$x' = \boxed{x = \begin{pmatrix} 0 & 23'E \\ 39^\circ & 28'N \end{pmatrix}}$

$\boxed{x = \text{Valencia}}$

(9) $S_8 = \begin{pmatrix} -98^\circ & 08' \\ -09^\circ & 13' \end{pmatrix}$

$x = \begin{pmatrix} -98^\circ & 15'E \\ 30^\circ & 15'N \end{pmatrix}$

$\boxed{x = \begin{pmatrix} -97^\circ & 45'E \\ 30^\circ & 15'N \end{pmatrix}}$

$\boxed{x = \text{Austin}}$

Path:-

Brussels → Darmstadt → Zürich

Pisa → Krapow → London → Valencia

Austin.

H/W 1.8.1.3 :

Power vector

$$x = \begin{pmatrix} 90 \\ 30 \\ 10 \\ 15 \\ 5 \end{pmatrix} \xrightarrow{KW} \frac{1}{1000}\begin{pmatrix} 90 \\ 30 \\ 10 \\ 15 \\ 5 \end{pmatrix} \quad y = 2\begin{pmatrix} 0.7 \\ 0.6 \\ 0.3 \\ 0.1 \\ 1.0 \end{pmatrix}$$

$$x^T y = \frac{2}{1000}\left(90\times0.7 + 30\times0.6 + 10\times0.3 + 15\times0.1 + 5\times1.0\right)$$

$$= \frac{2}{1000}\left(63 + 18 + 3 + 1.5 + 5\right)$$

$$= \frac{2}{1000}(90.5) = \frac{181}{1000} = 0.181 \text{ KWH}$$

H/W 1.8.1.4 :

$x_i = 0, 1, 2, 3, \ldots n-1$

$$x = \begin{pmatrix} x_0 \\ \vdots \\ x_{n-1} \end{pmatrix}, \quad \vec{I} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

(i)

(a) $x^T x \Rightarrow$ sum of squares x

(b) $\vec{I}^T x = (1 \cdots 1)\begin{pmatrix} x_0 \\ \vdots \\ x_{n-1} \end{pmatrix} = x_0 + x_1 + \cdots x_{n-1}$ ✓

(c) $x^T \vec{I} = (x_0 \cdots x_n)\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = x_0 + x_1 + \cdots x_{n-1}$ ✓

(ii) $\bar{x} = \frac{1}{n} \sum_{i=0}^{n-1} x_i$

(a) $\frac{1}{n} x^T x$ ✗ (b) $\frac{1}{n} \vec{I}^T x = \frac{x_0 + \cdots x_{n-1}}{n}$ ✓

(c) $(\vec{I}^T \vec{I})^{-1} (x^T \vec{I})$

$\vec{I}^T I = 1+1+\cdots n \text{ times} = n$

$(n)^{-1} = \frac{1}{n}$

$\frac{1}{n}(x^T \vec{I}) = \frac{x_0 + \cdots x_{n-1}}{n}$ ✓

(iii)

(a) $x^T x = x_0^2 + x_1^2 + \cdots x_{n-1}^2$ ✓ (b) $\vec{I}^T x$ ✗

(c) $x^T \vec{I}$ ✗

✗ — — — — —