$\therefore$ det (Singular) $= 0$

$\det(A-\lambda I) = 0$ {No x, we can solve λ}
{called characteristic eq or λ equation}

Find λ first. (n ~~[illegible]~~ λ's for $n\times n$)

Example :

$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ {Symmetric matrix}

$\det(A-\lambda I) = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix}$

$= (3-\lambda)^2 - 1$

$(3-\lambda)^2 - 1 = 0$

$9+\lambda^2-6\lambda-1=0$

$\lambda^2-6\lambda+8=0.$

$\lambda^2-6\lambda+8=0$

$\lambda^2-4\lambda-2\lambda+8=0$

$\lambda(\lambda-4)-2(\lambda-4)=0$

$\boxed{\lambda=4,2}$

$\boxed{\text{Trace} = 6 = 4+2}$

Q.E.D

2×2 case

$\boxed{\lambda^2 - \text{trace}\times\lambda + \det(A) = 0}$

Eigen vector for 4

$$A-4I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{array}{cc} P & F \\ \end{array}$$
$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$x_2 = 1$

$x_1 - 1 = 0$

$\boxed{x_1 = 1}$

$$\boxed{N(A) = c\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

Eigen vector for 2 :

$$A-2I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{array}{cc} P & F \\ \end{array}$$
$$(\text{rref}) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$x_2 = 1$

$x_1 + 1 = 0$

$\boxed{x_1 = -1}$

$$\boxed{N(A) = c\begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$

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$\therefore$ Two whole lines of eigen vectors.
but $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$ are basis to those spaces.

$A = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, $\lambda_1 = 1.\begin{bmatrix}1\\1\end{bmatrix}$, $\lambda_2 = -1\begin{bmatrix}-1\\1\end{bmatrix}$

$B = \begin{bmatrix}3 & 1\\1 & 3\end{bmatrix} = A + 3I$, $\lambda_1 = 4\begin{bmatrix}1\\1\end{bmatrix}$, $\lambda_2 = 2\begin{bmatrix}-1\\1\end{bmatrix}$

If we add $3I$ to A, λ increases by 3
Eigen vectors remain the same.

Proof :

If $Ax = \lambda x$

Then $(A+3I)x = \lambda x + 3x = (\lambda+3)x$.

Q.E.D

NOT SO GREAT :

If $Ax = \lambda x$, ~~Bx=~~ B has eigenvalues α

$Bx = \lambda x$

$(A+B)x = (\lambda+\alpha)x$ ✗ [That's false].

It can only work when x is an eigenvector of B.
We have no reason to believe B has x as an eigenvector.

$\boxed{\text{Also for } A \cdot B}$

Example 3:- (Rotation Matrix) {Q}.

Imp examples of orthogonal matrix.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

90° rotation $Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

trace $= 0 + 0 = \lambda_1 + \lambda_2$

$\det(Q) = 1 = \lambda_1 \times \lambda_2$

Something is wrong? What vector comes ||'el to itself after rotation.

$\det(Q - \lambda I) = 0$

$$\begin{vmatrix} -1\lambda & -1 \\ 1 & -1\lambda \end{vmatrix} = +\lambda^2 + 1 = 0$$

$\lambda^2 = -1$ {Only if we consider imaginary complex ~~imaginary~~ numbers}

$$\boxed{\begin{aligned} \lambda_1 &= i \\ \lambda_2 &= -i \end{aligned}}$$

These are complex conjugates.
(switch imaginary signs).

If matrices are symmetric, real eigen values are present.

Anti-symmetric have perfectly imaginary numbers.

Example 4 :

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

$\det(Q - \lambda I) = 0$

$$\begin{vmatrix} 3-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix}$$

$$= (3-\lambda)^2 - 0 = 0$$

$$\cancel{\lambda = 3\,(\lambda - 3)^2 = 0}$$

$$\cancel{\lambda = \pm 3}$$

$$\cancel{\lambda = 3}$$

$$(3-\lambda)(3-\lambda) = 0$$

$$\boxed{\lambda = 3, 3}$$

$$\boxed{\lambda_1 = 3, \quad \lambda_2 = 3}$$

E.V's :

$(A - \lambda I)x = 0$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

~~Basis zero vector for this~~ null space

$$X_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Now, for X_2, there isn't any independent vector. ~~So, this is a degenerate~~ matrix.