# The Art of Forecasting

Gianluca Campanella

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Hello!

My name is **Gianluca** [dʒanˈluːka]

### What I do nowadays

I'm a Data Scientist at



in Algorithms and Data Science

### What I do nowadays

I also run my own company



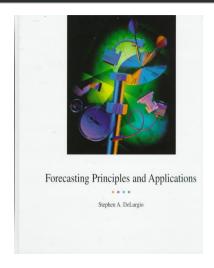
that provides

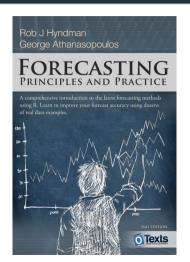
Data Science training and mentoring

## Today's slides

```
https://github.com/gcampanella/
ndr-2018
```

#### References





#### **Contents**

Motivation

Modelling

Results and recommendations

#### What's a time series?

# Any data that change over time

- Typically continuous (including counts)
- Time gives natural ordering

### What's forecasting?

#### Regression

- Value of y given values for the predictors X
- Does not depend on time (or temporal effect is negligible)

## What's forecasting?

#### Regression

- Value of y given values for the predictors X
- Does not depend on time (or temporal effect is negligible)

#### **Forecasting**

- Value of y given previous values of y
- Some models can also incorporate exogenous predictors

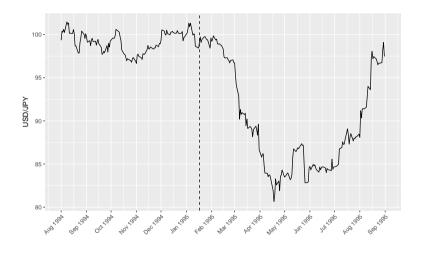
### **Predictability**

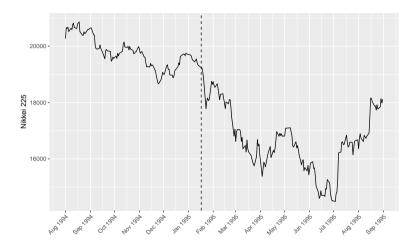
Can we forecast in changing environments?

### **Predictability**

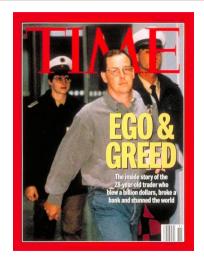
#### Predictability depends on...

- Availability of data
- Our understanding of contributing factors
- Whether our forecasts affect the process we're trying to forecast





What happened?







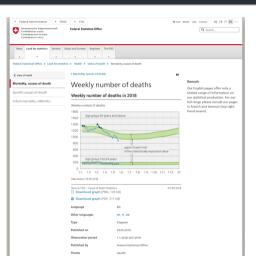


http://demo.istat.it/

 $\downarrow$ 

https://github.com/gcampanella/ istat-demographics





#### **Data**

### Original data

- Births, deaths, and net migration
- Monthly resolution from January 2004 till November 2017
- At municipality (comune) level
- Stratified by sex

#### **Data**

### **Aggregated data**

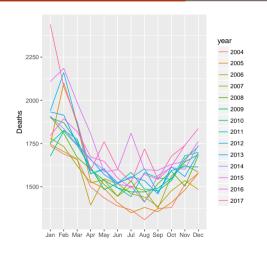
- Deaths only
- Monthly resolution from January 2004 till November 2017
- At region level (N = 20)
- Stratified by sex

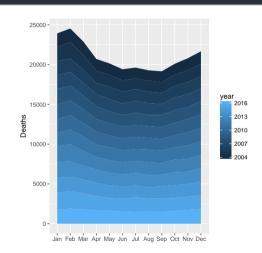
### **Pre-processing**

# Data are unnormalised monthly counts

- Boundary changes
- Population size (pre-census vs post-census)
- Calendar adjustment

### **Exploratory data analysis**





# Analysis

Family	Method	Package
Baseline	Naïve (RW)	forecast
	Seasonal naïve	forecast
	Naïve with drift	forecast
	Average	forecast
Univariate	ETS	forecast
	ARIMA	forecast
	BSTS	bsts
	Prophet	prophet
Hierarchical	HTS	hts

Modelling

### Naïve and average methods

For all 
$$h = 1, 2, ...,$$

Naïve (RW)	$\hat{y}_{T+h T} = y_T$
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Seasonal naïve with period 
$$m$$
  $\hat{y}_{T+h|T} = y_{T+h-m(\lfloor (h-1)/m \rfloor+1)}$ 

Naïve with drift 
$$\hat{y}_{T+h|T} = y_T + h(y_t - y_1)/(T-1)$$

Average 
$$\hat{y}_{T+h|T} = \sum_{t=1}^{T} y_t / T$$

**Modelling** 

**Exponential smoothing** 

### Simple exponential smoothing (SES)

Given a smoothing parameter  $0 \le \alpha \le 1$ ,

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$$

$$\hat{y}_{t+h|t} = \ell_t$$
 (forecast)  $\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$  (smoothing)

#### Holt's linear trend method

Given a smoothing parameter  $0 \le \beta \le 1$ ,

$$egin{aligned} \hat{y}_{t+h|t} &= \ell_t + h b_t \ \ell_t &= lpha y_t + (1-lpha)(\ell_{t-1} + b_{t-1}) \ b_t &= eta(\ell_t - \ell_{t-1}) + (1-eta)b_{t-1} \end{aligned}$$
 (forecast) (level)

### Gardner and McKenzie's damped trend method

Given a damping parameter  $0 < \varphi < 1$ ,

$$\hat{y}_{t+h|t} = \ell_t + (\mathbf{\phi} + \mathbf{\phi}^2 + \ldots + \mathbf{\phi}^h)b_t$$
 (forecast)  
 $\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + \mathbf{\phi}b_{t-1})$  (level)  
 $b_t = \beta(\ell_t - \ell_{t-1}) + (1 - \beta)\mathbf{\phi}b_{t-1}$  (trend)

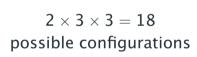
### Holt-Winters' seasonal (additive) method

Given a smoothing parameter  $0 \le \gamma \le 1$  and a frequency  $m \in \mathbb{N}$ ,

$$\begin{split} \hat{y}_{t+h|t} &= \ell_t + hb_t + \frac{s_{t+h-m(\lfloor (h-1)/m \rfloor+1)}}{\ell_t = \alpha(y_t - s_{t-m} + (1-\alpha)(\ell_{t-1} + b_{t-1})} & \text{(forecast)} \\ b_t &= \beta(\ell_t - \ell_{t-1}) + (1-\beta)b_{t-1} & \text{(trend)} \\ s_t &= \gamma(y_t - \ell_t) + (1-\gamma)s_{t-m} & \text{(seasonality)} \end{split}$$

### **ETS** methods

- Error
  - Additive
  - Multiplicative
- Trend
  - None
  - Additive
  - Additive damped
- Seasonality
  - None
  - Additive
  - Multiplicative



 $\sim \rightarrow$ 

# Modelling

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**ARIMA** models

# Backshift operator $\mathcal{B}$

Let's introduce the backshift operator  $\mathcal{B}$ ,

$$\mathcal{B}y_{t} = y_{t-1}$$

$$\mathcal{B}^{2}y_{t} = y_{t-2}$$

$$\vdots$$

$$\mathcal{B}^{m}y_{t} = y_{t-m}$$

# Backshift operator $\mathcal{B}$

We can rewrite first-order differences in terms of B,

$$y_t - y_{t-1} = y_t - \mathcal{B}y_t$$
$$= (1 - \mathcal{B})y_t$$

In general, B follows algebraic rules,

$$(1-\mathcal{B})(1-\mathcal{B}^m)y_t = (1-\mathcal{B}^m-\mathcal{B}+\mathcal{B}^{m+1})y_t \ = y_t - y_{t-m} - y_{t-1} + y_{t-m-1} \ = (y_t - y_{t-m}) - (y_{t-1} - y_{(t-1)-m})$$

# Autoregressive and moving average models

#### Autoregressive AR(p) model of order p

$$y_t = \beta_0 + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \epsilon_t$$

# Moving average MA(q) model of order q

$$y_t = \gamma_0 + \gamma_1 \epsilon_{t-1} + \ldots + \gamma_q \epsilon_{t-q} + \epsilon_t$$

### **ARIMA** models

#### Non-seasonal ARIMA(p, d, q) model

$$(1-eta_1\mathcal{B}-\ldots-eta_p\mathcal{B}^p)(1-\mathcal{B})^dy_t=lpha+(1+\gamma_1\mathcal{B}+\ldots+\gamma_q\mathcal{B}^q)\epsilon_t$$

### ARIMA models

# Non-seasonal ARIMA(p, d, q) model

$$(1 - \beta_1 \mathcal{B} - \ldots - \beta_p \mathcal{B}^p)(1 - \mathcal{B})^d y_t = \alpha + (1 + \gamma_1 \mathcal{B} + \ldots + \gamma_q \mathcal{B}^q) \epsilon_t$$

# Seasonal ARIMA $(p, d, q)(P, D, Q)_m$ model

$$(1 - \beta_1 \mathcal{B} - \dots - \beta_p \mathcal{B}^p)(1 - B_1 \mathcal{B}^m - \dots - B_p \mathcal{B}^{pm})(1 - \mathcal{B})^d (1 - \mathcal{B}^D) y_t$$
  
=  $\alpha + (1 + \gamma_1 \mathcal{B} + \dots + \gamma_q \mathcal{B}^q)(1 + \Gamma_1 \mathcal{B}^m + \dots + \Gamma_Q \mathcal{B}^{Qm}) \epsilon_t$ 

# Modelling

Other methods

# Bayesian Structural Time Series (BSTS) models

- Introduced by S. L. Scott and H. Varian (Google)
- Ensemble method
- Structural time series model + regression component

#### Model evaluated

- Local linear trend
- Seasonal model with m = 12

# **Prophet**

- Introduced by S. J. Taylor and B. Letham (Facebook)
- Curve fitting (similarly to GAMs)
- Decomposition into trend, seasonality, and holidays

#### Model evaluated

- Default settings
- $\,
  ightarrow\,$  No daily or weekly seasonality

# Hierarchical time series models

- Introduced by R. J. Hyndman et al. (Monash University)
- Independent forecasts + aggregation at different levels
- Many different aggregation methods

#### Models evaluated

- Forecasting methods: ARIMA, ETS, RW
- ullet 5 aggregation methods imes 4 weighting schemes

# Modelling

**Measures** 

# **Scale-dependent measures**

Given the prediction errors  $e_{T+h} = y_{T+h} - \hat{y}_{T+h}$ , ...

Measure	
Mean absolute error	$mean( e_t )$
Root-mean-square error	$\sqrt{\operatorname{mean}(e_t^2)}$

# Percentage errors

Given the percentage errors  $p_t = 100e_t/y_t$ , ...

#### Measure

Mean absolute percentage error  $mean(|p_t|)$ 

**Symmetric MAPE**  $mean(200|y_t - \hat{y}_t|/(y_t + \hat{y}_t))$ 

#### **Scaled errors**

Given the scaled errors...

$$q_t = \frac{e_t}{\frac{1}{T-1} \sum_{t'=2}^{T} |y_{t'} - y_{t'-1}|}$$
 or  $q_t = \frac{e_t}{\frac{1}{T-m} \sum_{t'=m+1}^{T} |y_{t'} - y_{t'-m}|}$ ,

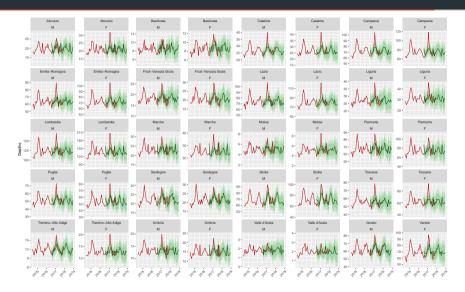
the **mean absolute scaled error** is simply  $mean(|q_t|)$ 

#### Interpretation

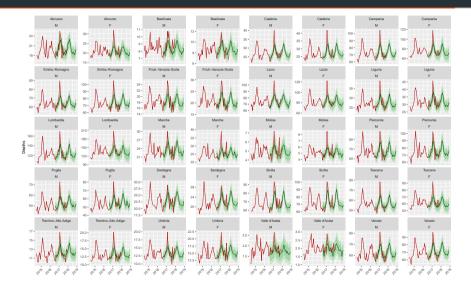
For  $q_t < 1$ , the forecast is better than the average (seasonal) naïve forecast (computed on the training data)

Results and recommendations

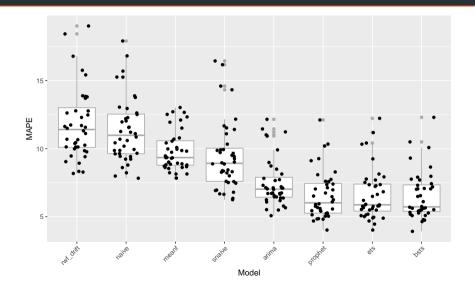
# Seasonal naïve forecasts



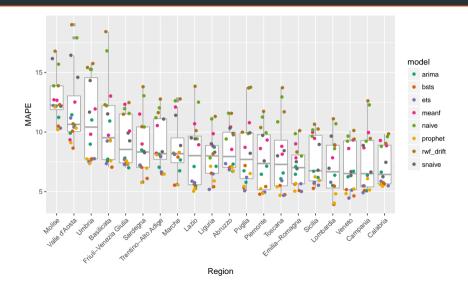
# **ETS** forecasts



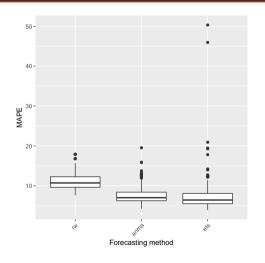
# Univariate models

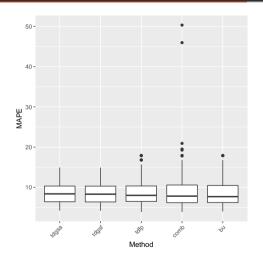


# Univariate models

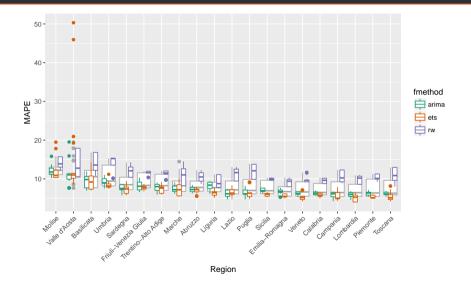


# **HTS models**





### **HTS models**



# And the winner is...

Method	MAPE
BSTS	6.52%
Prophet	6.58%
ETS	6.62%
HTS (bottom-up ETS)	6.62%
ARIMA	7.49%
Seasonal naïve	9.44%
Average	9.83%
Naïve (RW)	11.4%
Naïve with drift	11.8%

# Lessons learned

#### Time series are messy!

- Temporal resolution and spacing
- Calendar adjustment
- Model evaluation and cross-validation
- Hierarchical structure

# Lessons learned

#### Time series are fun!

- Data visualisation
- Models (often) interpretable
- Anomaly detection

#### **Future work**

- Compare even more models (including neural networks)
- Include exogenous covariates such as temperature
- Build a user interface