The Art of Forecasting

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Hello!

My name is **Gianluca** [dʒanˈluːka]

What I do nowadays

I'm a Data Scientist at



in Algorithms and Data Science

What I do nowadays

I also run my own company



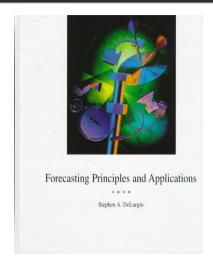
that provides

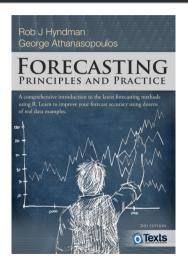
Data Science training and mentoring

Today's slides

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https://github.com/gcampanella/
ndr-2018
```

References





Contents

Motivation

Modelling

Results and recommendations

What's a time series?

Any data that change over time

- Typically continuous (including counts)
- Time gives natural ordering

What's forecasting?

Regression

- Value of y given values for the predictors X
- Does not depend on time (or temporal effect is negligible)

What's forecasting?

Regression

- Value of y given values for the predictors X
- Does not depend on time (or temporal effect is negligible)

Forecasting

- Value of y given previous values of y
- Some models can also incorporate exogenous predictors

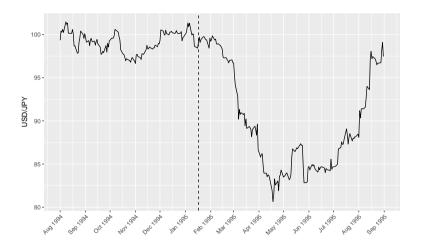
Predictability

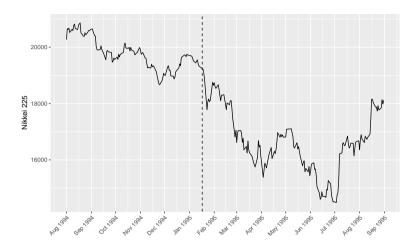
Can we forecast in changing environments?

Predictability

Predictability depends on...

- Availability of data
- Our understanding of contributing factors
- Whether our forecasts affect the process we're trying to forecast





What happened?







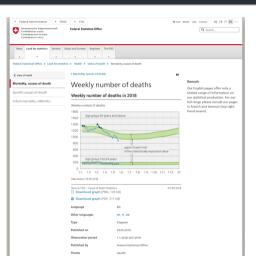


http://demo.istat.it/

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https://github.com/gcampanella/
 istat-demographics





Data

Original data

- Births, deaths, and net migration
- Monthly resolution from January 2004 till November 2017
- At municipality (comune) level
- Stratified by sex

Data

Aggregated data

- Deaths only
- Monthly resolution from January 2004 till November 2017
- At region level (N = 20)
- Stratified by sex

Data

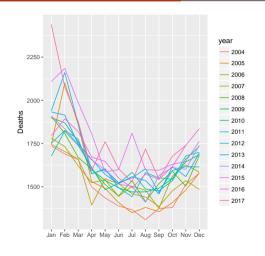
| | Start | End | Length |
|----------|--------------|---------------|------------|
| Training | January 2004 | June 2016 | 12.5 years |
| Test | July 2016 | November 2017 | 17 months |

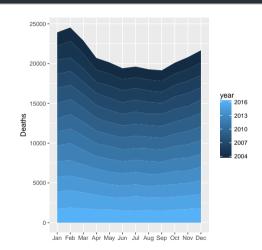
Pre-processing

Data are unnormalised monthly counts

- Boundary changes
- Population size (pre-census vs post-census)
- Calendar adjustment

Exploratory data analysis





Analysis

| Family | Method | Package |
|--------------|------------------|----------|
| | Naïve (RW) | forecast |
| Baseline | Seasonal naïve | forecast |
| baseime | Naïve with drift | forecast |
| | Average | forecast |
| | ETS | forecast |
| Univariate | ARIMA | forecast |
| Ullivariate | BSTS | bsts |
| | Prophet | prophet |
| Hierarchical | HTS | hts |

Modelling

Naïve and average methods

For all
$$h = 1, 2, ...,$$

| Naïve (RW) $\hat{y}_{\bar{1}}$ | $r_{+h T} = y_T$ |
|--------------------------------|------------------|
|--------------------------------|------------------|

Seasonal naïve with period
$$m$$
 $\hat{y}_{T+h|T} = y_{T+h-m(\lfloor (h-1)/m \rfloor+1)}$

Naïve with drift
$$\hat{y}_{T+h|T} = y_T + h(y_t - y_1)/(T-1)$$

Average
$$\hat{y}_{T+h|T} = \sum_{t=1}^{T} y_t/T$$

Time series decomposition

Common components

- Trend-cycle T_t
- Seasonal St
- Remainder R_t

Additive model

$$y_t = T_t + S_t + R_t$$

Multiplicative model

$$y_t = T_t \times S_t \times R_t$$

Modelling

Exponential smoothing

Simple exponential smoothing (SES)

Given a smoothing parameter $0 \le \alpha \le 1$,

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$$

$$\hat{y}_{t+h|t} = \ell_t$$
 (forecast) $\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$ (smoothing)

Holt's linear trend method

Given a smoothing parameter $0 \le \beta \le 1$,

Gardner and McKenzie's damped trend method

Given a damping parameter $0 < \varphi < 1$,

$$\hat{y}_{t+h|t} = \ell_t + (\mathbf{\phi} + \mathbf{\phi}^2 + \ldots + \mathbf{\phi}^h)b_t$$
 (forecast)
 $\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + \mathbf{\phi}b_{t-1})$ (level)
 $b_t = \beta(\ell_t - \ell_{t-1}) + (1 - \beta)\mathbf{\phi}b_{t-1}$ (trend)

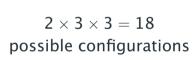
Holt-Winters' seasonal (additive) method

Given a smoothing parameter $0 \le \gamma \le 1$ and a frequency $m \in \mathbb{N}$,

$$\begin{split} \hat{y}_{t+h|t} &= \ell_t + hb_t + \frac{s_{t+h-m(\lfloor (h-1)/m \rfloor+1)}}{\ell_t = \alpha(y_t - s_{t-m} + (1-\alpha)(\ell_{t-1} + b_{t-1})} & \text{(forecast)} \\ b_t &= \beta(\ell_t - \ell_{t-1}) + (1-\beta)b_{t-1} & \text{(trend)} \\ s_t &= \gamma(y_t - \ell_t) + (1-\gamma)s_{t-m} & \text{(seasonality)} \end{split}$$

ETS methods

- Error
 - Additive
 - Multiplicative
- Trend
 - None
 - Additive
 - Additive damped
- Seasonality
 - None
 - Additive
 - Multiplicative



 $\sim \rightarrow$

Modelling

ARIMA models

Backshift operator \mathcal{B}

Let's introduce the backshift operator \mathcal{B} ,

$$\mathcal{B}y_{t} = y_{t-1}$$

$$\mathcal{B}^{2}y_{t} = y_{t-2}$$

$$\vdots$$

$$\mathcal{B}^{m}y_{t} = y_{t-m}$$

Backshift operator \mathcal{B}

We can rewrite first-order differences in terms of B,

$$y_t - y_{t-1} = y_t - \mathcal{B}y_t$$
$$= (1 - \mathcal{B})y_t$$

In general, B follows algebraic rules,

$$(1-\mathcal{B})(1-\mathcal{B}^m)y_t = (1-\mathcal{B}^m-\mathcal{B}+\mathcal{B}^{m+1})y_t$$

= $y_t - y_{t-m} - y_{t-1} + y_{t-m-1}$
= $(y_t - y_{t-m}) - (y_{t-1} - y_{(t-1)-m})$

Autoregressive and moving average models

Autoregressive AR(p) model of order p

$$y_t = \beta_0 + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \epsilon_t$$

Moving average MA(q) model of order q

$$y_t = \gamma_0 + \gamma_1 \epsilon_{t-1} + \ldots + \gamma_q \epsilon_{t-q} + \epsilon_t$$

ARIMA models

Non-seasonal ARIMA(p, d, q) model

$$(1-eta_1\mathcal{B}-\ldots-eta_p\mathcal{B}^p)(1-\mathcal{B})^dy_t=lpha+(1+\gamma_1\mathcal{B}+\ldots+\gamma_q\mathcal{B}^q)\epsilon_t$$

ARIMA models

Non-seasonal ARIMA(p, d, q) model

$$(1 - \beta_1 \mathcal{B} - \ldots - \beta_p \mathcal{B}^p)(1 - \mathcal{B})^d y_t = \alpha + (1 + \gamma_1 \mathcal{B} + \ldots + \gamma_q \mathcal{B}^q) \epsilon_t$$

Seasonal ARIMA $(p, d, q)(P, D, Q)_m$ model

$$(1 - \beta_1 \mathcal{B} - \dots - \beta_p \mathcal{B}^p)(1 - B_1 \mathcal{B}^m - \dots - B_p \mathcal{B}^{pm})(1 - \mathcal{B})^d (1 - \mathcal{B}^D) y_t$$

= $\alpha + (1 + \gamma_1 \mathcal{B} + \dots + \gamma_q \mathcal{B}^q)(1 + \Gamma_1 \mathcal{B}^m + \dots + \Gamma_Q \mathcal{B}^{Qm}) \epsilon_t$

Modelling

Other methods

Bayesian Structural Time Series (BSTS) models

- Introduced by S. L. Scott and H. Varian (Google)
- Ensemble method
- Structural time series model + regression component

Model evaluated

- Local linear trend
- Seasonal model with m = 12

Prophet

- Introduced by S. J. Taylor and B. Letham (Facebook)
- Curve fitting (similarly to GAMs)
- Decomposition into trend, seasonality, and holidays

Model evaluated

- Default settings
- $\,
 ightarrow\,$ No daily or weekly seasonality

Hierarchical time series models

- Introduced by R. J. Hyndman et al. (Monash University)
- Independent forecasts + aggregation at different levels
- Many different aggregation methods

Models evaluated

- Forecasting methods: ARIMA, ETS, RW
- ullet 5 aggregation methods imes 4 weighting schemes

Modelling

Measures

Scale-dependent measures

Given the prediction errors $e_{T+h} = y_{T+h} - \hat{y}_{T+h}$, ...

| Measure | |
|------------------------|----------------------------|
| Mean absolute error | $mean(e_t)$ |
| Root-mean-square error | $\sqrt{mean(\pmb{e}_t^2)}$ |

Percentage errors

Given the percentage errors $p_t = 100e_t/y_t$, ...

Measure

Mean absolute percentage error $mean(|p_t|)$

Symmetric MAPE $mean(200|y_t - \hat{y}_t|/(y_t + \hat{y}_t))$

Scaled errors

Given the scaled errors...

$$q_t = \frac{e_t}{\frac{1}{T-1} \sum_{t'=2}^{T} |y_{t'} - y_{t'-1}|}$$
 or $q_t = \frac{e_t}{\frac{1}{T-m} \sum_{t'=m+1}^{T} |y_{t'} - y_{t'-m}|}$,

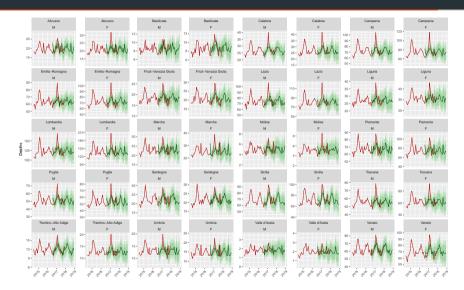
the **mean absolute scaled error** is simply mean($|q_t|$)

Interpretation

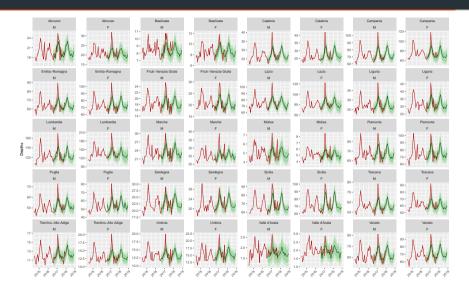
For $q_t < 1$, the forecast is better than the average (seasonal) naïve forecast (computed on the training data)

Results and recommendations

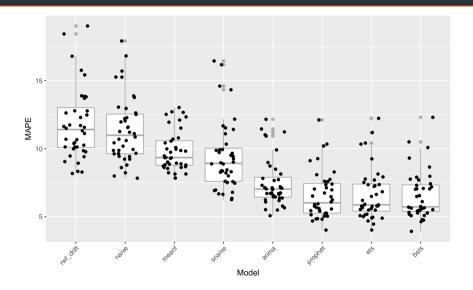
Seasonal naïve forecasts



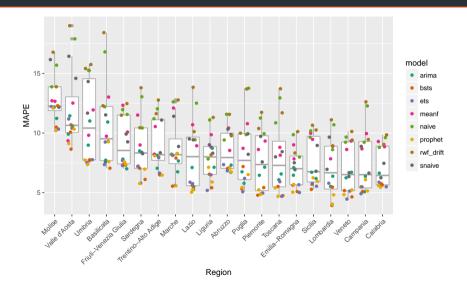
ETS forecasts



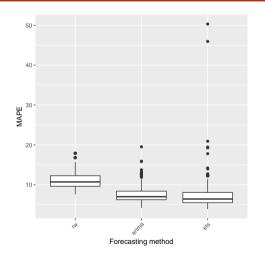
Univariate models

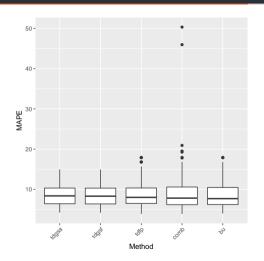


Univariate models

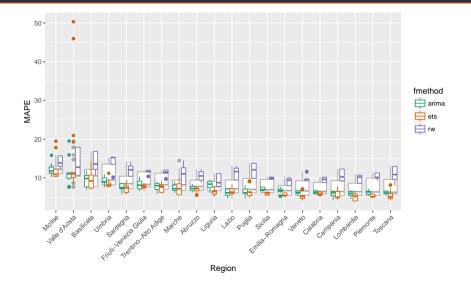


HTS models





HTS models



And the winner is...

| Method | MAPE |
|---------------------|-------|
| BSTS | 6.52% |
| Prophet | 6.58% |
| ETS | 6.62% |
| HTS (bottom-up ETS) | 6.62% |
| ARIMA | 7.49% |
| Seasonal naïve | 9.44% |
| Average | 9.83% |
| Naïve (RW) | 11.4% |
| Naïve with drift | 11.8% |

Lessons learned

Time series are messy!

- Temporal resolution and spacing
- Calendar adjustment
- Model evaluation and cross-validation
- Hierarchical structure

Lessons learned

Time series are fun!

- Data visualisation
- Models (often) interpretable
- Anomaly detection

Future work

- Compare even more models (including neural networks)
- Include exogenous covariates such as temperature
- Build a user interface