# The Art of Forecasting

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Hello!

My name is **Gianluca** [dʒanˈluːka]

# What I do nowadays

I'm a Data Scientist at



in Algorithms and Data Science

## What I do nowadays

I also run my own company



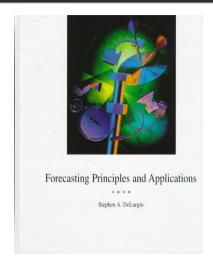
that provides

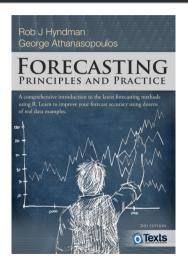
Data Science training and mentoring

# Today's slides

```
https://github.com/gcampanella/
ndr-2018
```

#### References





### **Contents**

Motivation

Modelling

Results and recommendations

#### What's a time series?

# Any data that change over time

- Typically continuous (including counts)
- Time gives natural ordering

# What's forecasting?

#### Regression

- Value of y given values for the predictors X
- Does not depend on time (or temporal effect is negligible)

# What's forecasting?

#### Regression

- Value of y given values for the predictors X
- Does not depend on time (or temporal effect is negligible)

#### **Forecasting**

- Value of y given previous values of y
- Some models can also incorporate exogenous predictors

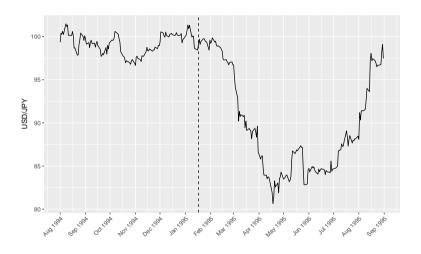
# **Predictability**

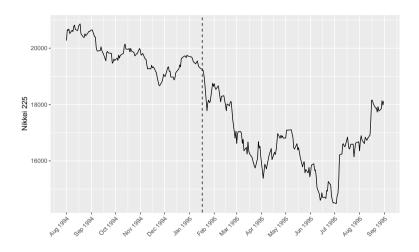
Can we forecast in changing environments?

## Predictability

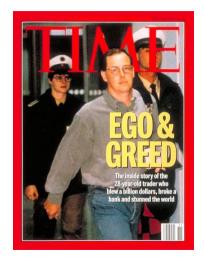
Predictability depends on...

- Availability of data
- Our understanding of contributing factors
- Whether our forecasts affect the process we're trying to forecast





What happened?







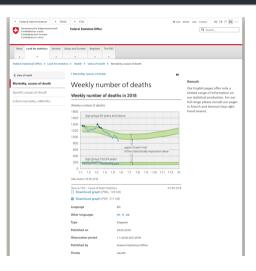


http://demo.istat.it/

 $\downarrow$ 

https://github.com/gcampanella/ istat-demographics





#### **Data**

#### Original data

- Births, deaths, and net migration
- Monthly resolution from January 2004 till November 2017
- At municipality (comune) level
- Stratified by sex

#### **Data**

#### Aggregated data

- Deaths only
- Monthly resolution from January 2004 till November 2017
- At region level (N = 20)
- Stratified by sex

## Data

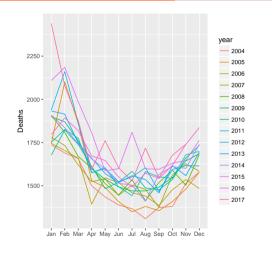
	Start	End	Length
Training	January 2004	June 2016	12.5 years
Test	July 2016	November 2017	17 months

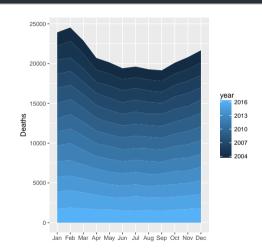
# **Pre-processing**

# Data are unnormalised monthly counts

- Boundary changes
- Population size (pre-census vs post-census)
- Calendar adjustment

# **Exploratory data analysis**





# Analysis

Family	Method	Package
	Naïve (RW)	forecast
Baseline	Seasonal naïve	forecast
baseime	Naïve with drift	forecast
	Average	forecast
	ETS	forecast
Univariate	ARIMA	forecast
Ullivariate	BSTS	bsts
	Prophet	prophet
Hierarchical	HTS	hts

Modelling

# Naïve and average methods

For all 
$$h = 1, 2, ...,$$

Naïve (RW) $\hat{y}_{\bar{1}}$	$r_{+h T} = y_T$
--------------------------------	------------------

Seasonal naïve with period 
$$m$$
  $\hat{y}_{T+h|T} = y_{T+h-m(\lfloor (h-1)/m \rfloor+1)}$ 

Naïve with drift 
$$\hat{y}_{T+h|T} = y_T + h(y_t - y_1)/(T-1)$$

Average 
$$\hat{y}_{T+h|T} = \sum_{t=1}^{T} y_t / T$$

# Time series decomposition

#### **Common components**

- Trend-cycle  $T_t$
- Seasonal St
- Remainder  $R_t$

#### Additive model

$$y_t = T_t + S_t + R_t$$

#### Multiplicative model

$$y_t = T_t \times S_t \times R_t$$

# Modelling

**Exponential smoothing** 

# Simple exponential smoothing (SES)

Given a smoothing parameter  $0 \le \alpha \le 1$ ,

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1}$$

$$\hat{y}_{t+h|t} = \ell_t$$
 (forecast)  $\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$  (smoothing)

#### Holt's linear trend method

Given a smoothing parameter  $0 \le \beta \le 1$ ,

# Gardner and McKenzie's damped trend method

Given a damping parameter  $0 < \varphi < 1$ ,

$$\hat{y}_{t+h|t} = \ell_t + (\mathbf{\phi} + \mathbf{\phi}^2 + \dots + \mathbf{\phi}^h)b_t$$
 (forecast)  
 $\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + \mathbf{\phi}b_{t-1})$  (level)  
 $b_t = \beta(\ell_t - \ell_{t-1}) + (1 - \beta)\mathbf{\phi}b_{t-1}$  (trend)

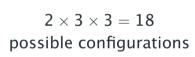
## Holt-Winters' seasonal (additive) method

Given a smoothing parameter  $0 \le \gamma \le 1$  and a frequency  $m \in \mathbb{N}$ ,

$$\begin{split} \hat{y}_{t+h|t} &= \ell_t + hb_t + \frac{s_{t+h-m(\lfloor (h-1)/m \rfloor+1)}}{\ell_t = \alpha(y_t - s_{t-m} + (1-\alpha)(\ell_{t-1} + b_{t-1})} & \text{(forecast)} \\ b_t &= \beta(\ell_t - \ell_{t-1}) + (1-\beta)b_{t-1} & \text{(trend)} \\ s_t &= \gamma(y_t - \ell_t) + (1-\gamma)s_{t-m} & \text{(seasonality)} \end{split}$$

#### **ETS** methods

- Error
  - Additive
  - Multiplicative
- Trend
  - None
  - Additive
  - Additive damped
- **S**easonality
  - None
  - Additive
  - Multiplicative



 $\sim \rightarrow$ 

# Modelling

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**ARIMA** models

# Backshift operator $\mathcal{B}$

Let's introduce the backshift operator  $\mathcal{B}$ ,

$$\mathcal{B}y_{t} = y_{t-1}$$

$$\mathcal{B}^{2}y_{t} = y_{t-2}$$

$$\vdots$$

$$\mathcal{B}^{m}y_{t} = y_{t-m}$$

# Backshift operator $\mathcal{B}$

We can rewrite first-order differences in terms of B,

$$y_t - y_{t-1} = y_t - \mathcal{B}y_t$$
$$= (1 - \mathcal{B})y_t$$

In general, B follows algebraic rules,

$$(1-\mathcal{B})(1-\mathcal{B}^m)y_t = (1-\mathcal{B}^m-\mathcal{B}+\mathcal{B}^{m+1})y_t$$
  
=  $y_t - y_{t-m} - y_{t-1} + y_{t-m-1}$   
=  $(y_t - y_{t-m}) - (y_{t-1} - y_{(t-1)-m})$ 

# Autoregressive and moving average models

#### Autoregressive AR(p) model of order p

$$y_t = \beta_0 + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \epsilon_t$$

# Moving average MA(q) model of order q

$$y_t = \gamma_0 + \gamma_1 \epsilon_{t-1} + \ldots + \gamma_q \epsilon_{t-q} + \epsilon_t$$

#### **ARIMA** models

#### Non-seasonal ARIMA(p, d, q) model

$$(1-eta_1\mathcal{B}-\ldots-eta_p\mathcal{B}^p)(1-\mathcal{B})^dy_t=lpha+(1+\gamma_1\mathcal{B}+\ldots+\gamma_q\mathcal{B}^q)\epsilon_t$$

#### **ARIMA** models

#### Non-seasonal ARIMA(p, d, q) model

$$(1 - \beta_1 \mathcal{B} - \ldots - \beta_p \mathcal{B}^p)(1 - \mathcal{B})^d y_t = \alpha + (1 + \gamma_1 \mathcal{B} + \ldots + \gamma_q \mathcal{B}^q) \epsilon_t$$

## Seasonal ARIMA $(p, d, q)(P, D, Q)_m$ model

$$(1 - \beta_1 \mathcal{B} - \dots - \beta_p \mathcal{B}^p)(1 - B_1 \mathcal{B}^m - \dots - B_p \mathcal{B}^{pm})(1 - \mathcal{B})^d (1 - \mathcal{B}^D) y_t$$
  
=  $\alpha + (1 + \gamma_1 \mathcal{B} + \dots + \gamma_q \mathcal{B}^q)(1 + \Gamma_1 \mathcal{B}^m + \dots + \Gamma_Q \mathcal{B}^{Qm}) \epsilon_t$ 

# Modelling

Other methods

# Bayesian Structural Time Series (BSTS) models

- Introduced by S. L. Scott and H. Varian (Google)
- Ensemble method
- Structural time series model + regression component

#### Model evaluated

- Local linear trend
- Seasonal model with m = 12

# **Prophet**

- Introduced by S. J. Taylor and B. Letham (Facebook)
- Curve fitting (similarly to GAMs)
- Decomposition into trend, seasonality, and holidays

#### Model evaluated

- Default settings
- $\,
  ightarrow\,$  No daily or weekly seasonality

#### Hierarchical time series models

- Introduced by R. J. Hyndman et al. (Monash University)
- Independent forecasts + aggregation at different levels
- Many different aggregation methods

#### Models evaluated

- Forecasting methods: ARIMA, ETS, RW
- ullet 5 aggregation methods imes 4 weighting schemes

# Modelling

**Measures** 

# **Scale-dependent measures**

Given the prediction errors  $e_{T+h} = y_{T+h} - \hat{y}_{T+h}$ , ...

Measure	
Mean absolute error	$mean( e_t )$
Root-mean-square error	$\sqrt{mean(e_t^2)}$

# Percentage errors

Given the percentage errors  $p_t = 100e_t/y_t$ , ...

#### Measure

Mean absolute percentage error  $mean(|p_t|)$ 

**Symmetric MAPE**  $mean(200|y_t - \hat{y}_t|/(y_t + \hat{y}_t))$ 

#### **Scaled errors**

Given the scaled errors...

$$q_t = \frac{e_t}{\frac{1}{T-1} \sum_{t'=2}^{T} |y_{t'} - y_{t'-1}|}$$
 or  $q_t = \frac{e_t}{\frac{1}{T-m} \sum_{t'=m+1}^{T} |y_{t'} - y_{t'-m}|}$ ,

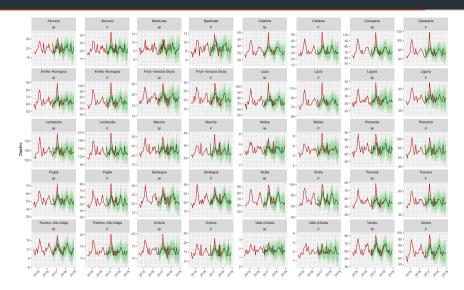
the **mean absolute scaled error** is simply  $mean(|q_t|)$ 

#### Interpretation

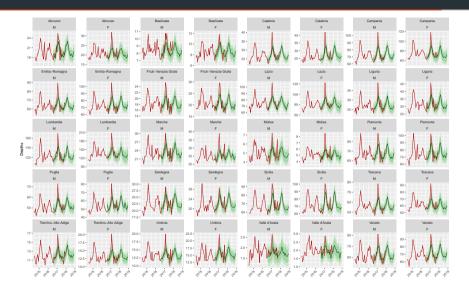
For  $q_t < 1$ , the forecast is better than the average (seasonal) naïve forecast (computed on the training data)

# Results and recommendations

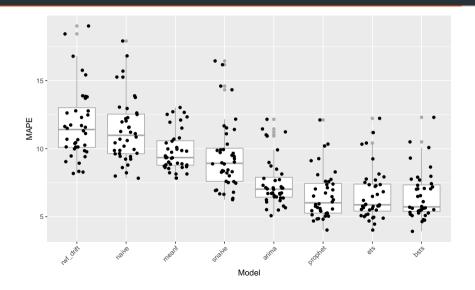
# Seasonal naïve forecasts



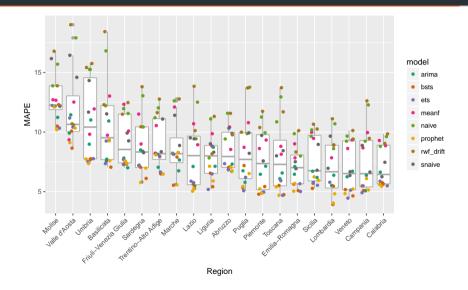
#### **ETS** forecasts



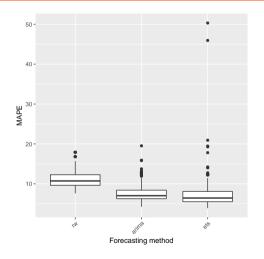
# Univariate models

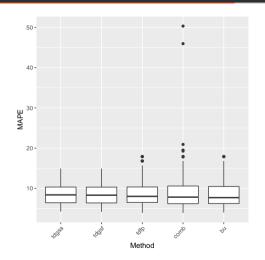


# Univariate models

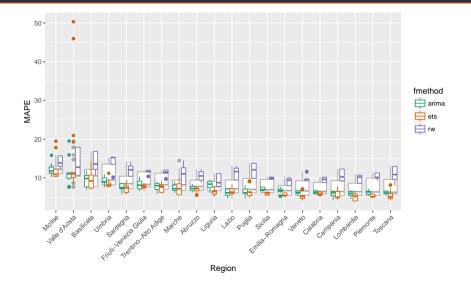


# **HTS models**





#### **HTS models**



# And the winner is...

Method	MAPE
BSTS	6.52%
Prophet	6.58%
ETS	6.62%
HTS (bottom-up ETS)	6.62%
ARIMA	7.49%
Seasonal naïve	9.44%
Average	9.83%
Naïve (RW)	11.4%
Naïve with drift	11.8%

#### Lessons learned

#### Time series are messy!

- Temporal resolution and spacing
- Calendar adjustment
- Model evaluation and cross-validation
- Hierarchical structure

#### Lessons learned

#### Time series are fun!

- Data visualisation
- Models (often) interpretable
- Anomaly detection

1. Visualise — Trend? Seasonality? 'Spikes'?

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- 2. Decide on a measure

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- 4. Plot the ACF of residuals

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- 2. Decide on a measure
- 3. Start with a simple model
- 4. Plot the ACF of residuals
- 5. Iterate

#### **Future work**

- Compare even more models (including neural networks)
- Include exogenous covariates such as temperature
- Build a user interface