Introduction

Every year in the college football season, every team and associated fanbase is out to not only perform well on the field, but to perform as the best on the field. This is majorly due to the College Football Playoffs' top 4 ranking system. This ranking system is made via the CFP selection committee, which is composed of a group of individuals with strong understanding and knowledge of college football, including current athletic directors, former coaches, players, and other experts. Factors that are taken into consideration include win-loss records, strength of schedule, and head-to-head results.

However, college football teams are not easy to rank when it comes to the top 4 criteria. Just this year, media and other sports sources spoke of FSU not being included in the top 4 despite being in the only 3 teams that were undefeated. This came from the upset victory of Alabama v Georgia, where Alabama got 4th place despite having a worse overall record. Given such a close scenario in which either FSU or Alabama enter the top 4, it then becomes important to consider quantifiable metrics to understand team strength in comparison to one another. With a quantifiable metric, it can aid in the decision-making process of not only the CFP selection committee but also for professional sports leagues at large.

Here, we employ a Bradley-Terry Model to extract "team strength" coefficients for each team in College Football as a quantitative metric to compare teams against each other. This model will then be used to compare to the committee's real-time rankings up until week 13. This offers additional backbone to team strength analysis, and therefore, more robust decision making in an extremely competitive league consisting of 11 different conferences.

Methods

In order to have a quantitative metric associated with each team's strength, we use a A win-loss matrix describing all 2023 games among Division 1A teams up through week 13, shown here:

	home_win <int>	Clemson <int>	Northwestern <int>	Oregon.State <int>	LSU <int>	Arkansas.State <int>
1	1	-1	0	0	0	0
2	1	0	-1	0	0	0
3	0	0	0	-1	0	0
4	1	0	0	0	-1	0
5	1	0	0	0	0	-1
6	1	0	0	0	0	0

Figure 1: Data-Matrix on with "home_win": 1 if if the home team won, and 0 if the away team won.

Home win has either 1s or 0s, indicating if the home team won. The other columns represent the other 133 schools in College Football. For example, in line 3, it indicates that Oregon State was an away team and Home win was 0, showing that Oregon State won against the home team. The following shows how the Bradley Terry model is used to predict the probability that one team is preferred over the other:

$$\frac{\Pr(i \text{ beats } j)}{\Pr(j \text{ beats } i)} = \frac{\alpha_i}{\alpha_j},$$

Figure 2: Bradley-Terry Model: Showing how team strength coefficient is derived

The Bradley-Terry model ultimately rates k teams that play n games, each game representing two participants with a distinct winner and loser. It then compares the chance of team i beating team j and team j beating team i, leading to a ratio defined by specific parameters for each team. These parameters represent strength coefficients and can be used to rank highest to lowest.

Throughout this dataset, we will be using four kinds of Bradely-Terry Models to quantify team strength: penalized model taking into account home-field advantage, penalized model not taking into account home-field advantage, non penalized model taking into account home-field advantage, and non penalized model not taking into account home-field advantage.

For the penalized models, we use a bias reduction in binary-response generalized linear model, while the unpenalized model uses a simple generalized linear model. Then, to take into account home-field advantage, we include the intercept in our model and to not take it into account, we employ a -1 (removing the intercept).

First, we compare the non-penalized models and assess their performance through AIC. The non-penalized model that does take home field advantage into account has a lower AIC, at 784.94, against the one that doesn't, at 793.53. Thus far, it seems that keeping the intercept yields a better goodness of fit.

We then compare these four models using cross-validation to estimate respective generalization errors, particularly using RMSE. Then, using one of the penalized models and one of the unpenalized models, we extract out the teams with the largest coefficients to have a Top 25 ranking, which will then be compared with the committee's real time rankings through Nov 28th. We should expect to see similar results with our BT model and the committee's ranking.

To further validate the robustness of our Bradley-Terry (BT) model, we introduce a comparative analysis involving three additional Division 1A teams—USC, Georgia, and Texas—aiming to discern their relative strengths in comparison to Michigan, our selected team. Utilizing the BT model coefficients, we visualize the estimated strengths of the selected teams through a bar chart, providing a qualitative assessment of their standings. To supplement this visual analysis, we conduct t-tests for each team against Michigan, examining the statistical

significance of differences in their BT coefficients. Regarding the issue of multiple comparisons, we apply a Bonferroni multiplicity correction to adjust the significance threshold. The p-values from these tests will offer insights into whether USC, Georgia, or Texas exhibit statistically significant differences in team strength compared to Michigan. This approach enhances the clarity of our model's capacity to differentiate team strength within the competitive landscape of college football.

Results

To use the BT-model, we must also first select a reference team to which these coefficients will be essentially compared to. For the purposes of this study, we will be using Clemson, which is chosen at random. The reference level serves as a baseline against which the other coefficients are compared. Further, the choice of the reference level does not impact the model performance or predictive accuracy. The model will produce the same predictions regardless of the chosen reference level.

Then, when comparing RMSE across all 4 BT-models, we see all of them perform pretty similarly. The highest RMSE is at 1.06 while the lowest is at 1.05, which represents the penalized model with the intercept and the non-penalized model with the intercept. Due to the close RMSE values, we prefer the penalized model with intercept taking home-field advantage into account. Due to the stringent nature of committee rankings, having a penalty for large coefficients can help distinguish ranking choice and order. Below is an image of the top 10 teams according to our BT-Model along with the CFB committee's ranking for a side-by-side comparison:

Team <chr>	Coefficient <dbl>
Michigan	3.8644796
Washington	3.3804628
Georgia	3.0007424
Ohio.State	2.8909324
Florida.State	2.5416345
Oregon	2.4159640
Alabama	2.1070026
Penn.State	1.9509141
Ole.Miss	1.9401012
Texas	1.7480099
LSU	1.5490396
Missouri	1.1897500
Notre.Dame	0.6404760
Oklahoma	0.6297162
Iowa	0.5801919
Utah	0.5575310
Arizona	0.5540584
Louisville	0.4752742
Oregon.State	0.4742144
(Intercept)	0.2919154
Liberty	0.2829503
Tulane	0.2805216
James.Madison	0.2026062
Tennessee	0.1505728
USC	0.1224316

Team <chr>
Georgia
Michigan
Washington
Florida State
Oregon
Ohio State
Texas
Alabama
Missouri
Penn State
Mississippi
Oklahoma
LSU
Louisville
Arizona
Iowa
Notre Dame
Oklahoma State
NC State
Oregon State
Tennessee
Tulane
Clemson
Liberty
Kansas State

Figure 3, Side-by-Side Comparison of Top 25 Rankings, left-side table is the non-penalized model while the right-side table is the committee's ranking

For the penalized model with intercept (taking home-field advantage into account), most rankings are not that different. One of the largest notable differences is just the difference of 4 spots in Louisville's ranking from the college football's top 25. For example, Oregon ranks #6 in the BT model and #5 in the committee's ranking while Louisville ranks #18 in the BT model and #14 in the committee ranking. However, we see several schools in the top 25 for the BT model with intercept b but not in the CFP. Ole Miss, Utah, and USC, for example, are unranked in the CFP but show up in the top 25 from our model.

Now, to compare across teams in the Big 10, we will be using Penn State, Iowa, and Illinois via t-tests for each team against Michigan, along with a Bonferroni multiplicity correction. If the unadjusted p-value is less than 0.05, it shows evidence for the fact that Michigan is significantly different in strength than the other team. On the other hand, if the unadjusted p-value is > 0.05, it shows evidence that Michigan is not significantly different in strength than the other team. The following are the results:

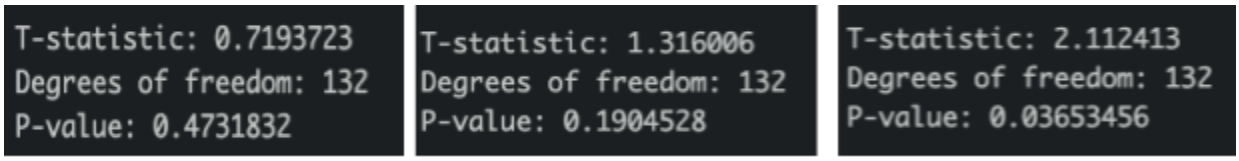

Figure 4: The following p-values, T-statistic and degrees of freedom for a two-sided t-test with Penn State, Iowa, and Illinois, respectively

Here, we see that Michigan is not significantly different from any of these teams except Illinois, with an unadjusted p-value of 0.03. This, of course, makes sense, given the real-time record of Illinois (5-7) and its negative BT coefficient of -1.2. However, it is interesting to see how Iowa, a team out of the top 25, does not reach a p-value of less than 0.05. Now, when we apply a Bonferroni correction, we actually find that no team's t-tests yielded a significant p value of 0.0167 or lower. The following figure shows the Bonferroni correction to these p-values:

	Penn State.Michigan	*Illinois.Michigan*	*Iowa.Michigan*
Adjusted p-values:	1.000	.1096037	0.3937603

Figure 5: Bonferroni-adjusted p values for Penn State, Illinois, and Iowa, respectively

Figure 5 supports the claim that none of the teams are significantly different from Michigan in terms of team strength (team record), including Illinois. Finally, the following is a visualization of the BT coefficients for each team, showing a gaping difference between Illinois and three more competitive teams in the Big 10:

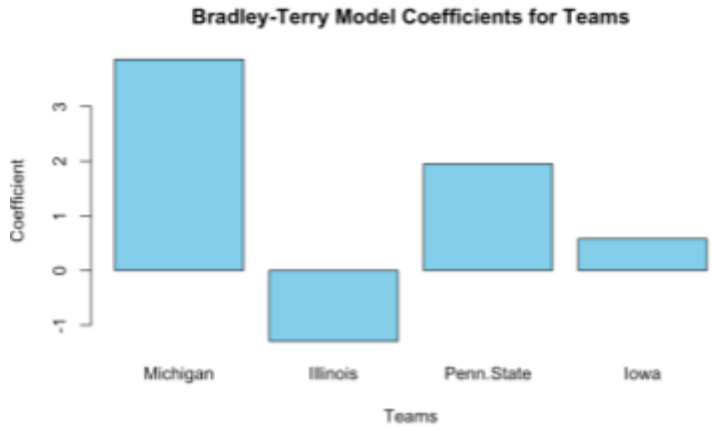

Figure 6: BT coefficient bar graph for Michigan, Iowa, Penn State, and Illinois

Again, we see an extremely low, -1.29 coefficient for Illinois, 0.58 for Iowa, 1.95 for Penn State, and 3.9 for Michigan. This shows the utter dominance of Michigan in this group of 4, which has approximately double the BT coefficient of the 2nd best team in this list.

Discussion

In fitting a BT-model of four different flavors (penalized v unpenalized, home advantage v no home advantage), we have identified that the lowest RMSE lies with the penalized model taking into account the intercept, i.e. home field advantage. This makes intuitive sense, as home field advantage poses a formidable qualitative struggle for the away team to handle. When using this BT model, we see that most of the rankings in comparison to the committee do not differ significantly. However, it is notable that several schools did make it into the top 25 for the model but not in the committee's.

Then, after doing a more individual-to-individual team comparison to compare to Michigan, we used Penn State, Iowa, and Illinois. With an unadjusted p-value, we see that the only major difference between teams compared to Michigan is Illinois. When adjusted for multiplicity via Bonferroni correction, we see that none of these 3 teams are that much better or worse than Michigan. The Bonferroni-adjusted p-value of 1 for Penn State also suggests that,

even after correcting for multiple comparisons, it is not statistically significant at the chosen familywise error rate. This matches up well with the current College Football Playoffs, where Penn State is ranked #11.

However, there is a limitation to our model at large, considering teams like USC did not end up ranking in the Top 50 of College Football rankings but found itself in the Top 25 of the BT model. This is due to the fact that this model only takes into account data from a win loss matrix. It does not count points or strength-of-schedule.

For future research, we can implore a BT model with more variables such as point differential and opponent strength. This is a metric heavily used by the CFB committee, but not used in the BT model. When looking at USC's losses this season, there were games against highly-ranked teams such as Washington and Oregon that determined the downfall of USC's credibility as a high ranking team.

<u>Citations</u>:

College Football Playoff. (2023, November 28). *College football playoff selection committee reveals penultimate rankings of 2023*.

https://collegefootballplayoff.com/news/2023/11/28/cfp-rankings-2023-1128.aspx

Unit 3,Appendix 1, Stats S485
2023-12-05
Outline: The script takes in the win-loss matrix of all teams in College Football through week 13. This matrix has a row for each contest, a home_win column indicating whether the winner was the home (1) or away team (0), and a column for each Division 1A team. Entries in a team's column show as 0 for games that team didn't play in, 1 for the team's home games and -1 for the team's away games. Update: The win-loss matrix through week 14 is here:
Blocks 1 and 2 load in necessary libraries and the data. Blocks 3 and 4 use a penalized and non-penalized models using the Bradley Terry model, while Block 5 calculates RMSE for each. Block 6 ranks the top 25 for a penalized and un-penalized model for comparison to real-time rankings made by the committee. Blocks 7-10 use t-tests based on comparing the estimated difference in teams' BT coefficients to an associated standard error, and incorporating a multiplicity correction such as Bonferroni.
Block 11 visualizes these BT coefficients. Block 12 shows the committee's top 25 rankings.

```
library(BradleyTerry2)
library(tidyverse)
```

```
## ── Attaching core tidyverse packages ──────────────────────── tidyverse 2.0.0 ──
## ✔ dplyr     1.1.0     ✔ readr     2.1.4
## ✔ forcats   1.0.0     ✔ stringr   1.5.0
## ✔ ggplot2   3.4.3     ✔ tibble    3.2.1
## ✔ lubridate 1.9.2     ✔ tidyr     1.3.0
## ✔ purrr     1.0.1
## ── Conflicts ────────────────────────────────────────── tidyverse_conflicts() ──
## ✖ dplyr::filter() masks stats::filter()
## ✖ dplyr::lag()    masks stats::lag()
## ℹ Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(multcomp)
```

```
## Loading required package: mvtnorm
## Loading required package: survival
## Loading required package: TH.data
## Loading required package: MASS
##
## Attaching package: 'MASS'
##
## The following object is masked from 'package:dplyr':
##
```

```
##     select
##
## The following object is masked from 'package:BradleyTerry2':
##
##     glmmPQL
##
##
## Attaching package: 'TH.data'
##
## The following object is masked from 'package:MASS':
##
##     geyser
library(ggplot2)
library(brglm)
## Loading required package: profileModel
## 'brglm' will gradually be superseded by the 'brglm2' R package
(https://cran.r-project.org/package=brglm2), which provides utilities for mean and median bias
reduction for all GLMs.
##  Methods for the detection of separation and infinite estimates in binomial-response models
are provided by the 'detectseparation' R package
(https://cran.r-project.org/package=detectseparation).
library(glmnet)
## Loading required package: Matrix
##
## Attaching package: 'Matrix'
##
## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack
##
## Loaded glmnet 4.1-8
library(dplyr)
getwd()
## [1] "/Users/alonzocortez/Downloads"
df <- read.csv("cfb_wl-2023-11-26.csv")
head(df)
##   home_win Clemson Northwestern Oregon.State LSU Arkansas.State Ball.State
## 1        1      -1            0            0   0             0          0
## 2        1       0           -1            0   0             0          0
## 3        0       0            0           -1   0             0          0
```

```
## 4       1      0          0          0  -1           0         0
## 5       1      0          0          0   0          -1         0
## 6       1      0          0          0   0           0        -1
##   Bowling.Green Colorado East.Carolina Fresno.State Louisiana.Tech N..Illinois
## 1             0       0             0           0             0           0
## 2             0       0             0           0             0           0
## 3             0       0             0           0             0           0
## 4             0       0             0           0             0           0
## 5             0       0             0           0             0           0
## 6             0       0             0           0             0           0
##   Utah.State Virginia Akron Boise.State Buffalo Ohio.State Rice South.Florida
## 1          0       0     0           0       0          0    0             0
## 2          0       0     0           0       0          0    0             0
## 3          0       0     0           0       0          0    0             0
## 4          0       0     0           0       0          0    0             0
## 5          0       0     0           0       0          0    0             0
## 6          0       0     0           0       0          0    0             0
##   UMass California Nevada Army New.Mexico UTSA Texas.State Washington.State
## 1     0          0      0    0          0    0           0                 0
## 2     0          0      0    0          0    0           0                 0
## 3     0          0      0    0          0    0           0                 0
## 4     0          0      0    0          0    0           0                 0
## 5     0          0      0    0          0    0           0                 0
## 6     0          0      0    0          0    0           0                 0
##   Middle.Tenn. Toledo North.Carolina West.Virginia Old.Dominion South.Alabama
## 1            0      0             0             0             0             0
## 2            0      0             0             0             0             0
## 3            0      0             0             0             0             0
## 4            0      0             0             0             0             0
## 5            0      0             0             0             0             0
## 6            0      0             0             0             0             0
##   Texas.Tech Sam.Houston C..Carolina C..Michigan MiamiOH Louisville Stanford
## 1          0           0           0           0       0          0        0
## 2          0           0           0           0       0          0        0
## 3          0           0           0           0       0          0        0
## 4          0           0           0           0       0          0        0
## 5          0           0           0           0       0          0        0
## 6          0           0           0           0       0          0        0
##   Kent.State NC.State Florida Nebraska Navy UTEP Ohio San.Jose.State FIU Hawaii
## 1          0       0       0        0    0    0    0              0   0      0
```

```
## 2        0      0      0      0    0    0    0          0    0      0
## 3        0      0      0      0    0    0    0          1    0      0
## 4        0      0      0      0    0    0    0          0    0      0
## 5        0      0      0      0    0    0    0          0    0      0
## 6        0      0      0      0    0    0    0          0    0      0
##   Vanderbilt James.Madison Notre.Dame Purdue Troy Utah Iowa Ole.Miss UNLV
## 1          0            0         0      0    0    0    0        0    0
## 2          0            0         0      0    0    0    0        0    0
## 3          0            0         0      0    0    0    0        0    0
## 4          0            0         0      0    0    0    0        0    0
## 5          0            0         0      0    0    0    0        0    0
## 6          0            0         0      0    0    0    0        0    0
##   W..Michigan Texas.A.M Marshall Tulsa App..State New.Mexico.State SMU UAB
## 1           0       0       0    0    0          0             0   0   0
## 2           0       0       0    0    0          0             0   0   0
## 3           0       0       0    0    0          0             0   0   0
## 4           0       0       0    0    0          0             0   0   0
## 5           0       0       0    0    0          0             0   0   0
## 6           0       0       0    0    0          0             0   0   0
##   Louisiana Cincinnati North.Texas Houston Jacksonville.State Memphis Oregon
## 1         0         0          0       0                  0       0      0
## 2         0         0          0       0                  0       0      0
## 3         0         0          0       0                  0       0      0
## 4         0         0          0       0                  0       0      0
## 5         0         0          0       0                  0       0      0
## 6         0         0          0       0                  0       0      0
##   Texas UCF UConn Arizona Charlotte E..Michigan Temple UCLA Wisconsin Air.Force
## 1     0   0     0       0         0          0      0    0         0         0
## 2     0   0     0       0         0          0      0    0         0         0
## 3     0   0     0       0         0          0      0    0         0         0
## 4     0   0     0       0         0          0      0    0         0         0
## 5     0   0     0       0         0          0      0    0         0         0
## 6     0   0     0       0         0          0      0    0         0         0
##   So..Miss Auburn Oklahoma.State Illinois Florida.State Ga..Southern Iowa.State
## 1        0      0              0        0             0            0          0
## 2        0      0              0        0             0            0          0
## 3        0      0              0        0             0            0          0
## 4        0      0              0        0             1            0          0
## 5        0      0              0        0             0            0          0
## 6        0      0              0        0             0            0          0
```

```
##   Kansas.State Liberty Penn.State Wake.Forest Alabama Minnesota Oklahoma
## 1            0       0          0           0       0         0        0
## 2            0       0          0           0       0         0        0
## 3            0       0          0           0       0         0        0
## 4            0       0          0           0       0         0        0
## 5            0       0          0           0       0         0        1
## 6            0       0          0           0       0         0        0
##   South.Carolina San.Diego.State Virginia.Tech ULMonroe Tulane W..Kentucky
## 1              0              0             0        0      0            0
## 2              0              0             0        0      0            0
## 3              0              0             0        0      0            0
## 4              0              0             0        0      0            0
## 5              0              0             0        0      0            0
## 6              0              0             0        0      0            0
##   Washington Georgia.State Tennessee BYU Syracuse Georgia.Tech Pittsburgh FAU
## 1          0             0         0   0        0           0          0   0
## 2          0             0         0   0        0           0          0   0
## 3          0             0         0   0        0           0          0   0
## 4          0             0         0   0        0           0          0   0
## 5          0             0         0   0        0           0          0   0
## 6          0             0         0   0        0           0          0   0
##   TCU Wyoming Colorado.State Kansas Rutgers Kentucky Boston.College Duke
## 1   0       0             0      0       0        0             0    1
## 2   0       0             0      0       1        0             0    0
## 3   0       0             0      0       0        0             0    0
## 4   0       0             0      0       0        0             0    0
## 5   0       0             0      0       0        0             0    0
## 6   0       0             0      0       0        1             0    0
##   Maryland Miami..Fla.. Arkansas Miss..State USC Arizona.State Baylor Indiana
## 1        0           0        0           0   0            0     0       0
## 2        0           0        0           0   0            0     0       0
## 3        0           0        0           0   0            0     0       0
## 4        0           0        0           0   0            0     0       0
## 5        0           0        0           0   0            0     0       0
## 6        0           0        0           0   0            0     0       0
##   Michigan Georgia Missouri Michigan.State
## 1        0       0        0              0
## 2        0       0        0              0
## 3        0       0        0              0
## 4        0       0        0              0
```

```
## 5       0      0       0             0
## 6       0      0       0             0
```

Non-Penalized glm()

```
# BT Model that does not take into account the home field advantage
model <- glm(home_win ~. - 1 - Clemson , family = binomial, data = df) # Using Clemson as a reference team
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
summary(model)
##
## Call:
## glm(formula = home_win ~ . - 1 - Clemson, family = binomial,
##     data = df)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -2.34823  -0.39214   0.00038   0.66392   2.78608
##
## Coefficients:
##                    Estimate Std. Error z value Pr(>|z|)
## Northwestern        -0.93411    1.24346  -0.751 0.452519
## Oregon.State         0.50619    1.44023   0.351 0.725240
## LSU                  2.68921    1.83947   1.462 0.143756
## Arkansas.State      -3.15036    1.29215  -2.438 0.014765 *
## Ball.State          -4.49435    1.41763  -3.170 0.001523 **
## Bowling.Green       -2.04014    1.43901  -1.418 0.156265
## Colorado            -1.78940    1.27903  -1.399 0.161802
## East.Carolina       -7.52933    1.77863  -4.233 2.30e-05 ***
## Fresno.State        -1.95698    1.26566  -1.546 0.122053
## Louisiana.Tech      -6.52066    1.59755  -4.082 4.47e-05 ***
## N..Illinois         -3.78433    1.35104  -2.801 0.005094 **
## Utah.State          -2.81264    1.31221  -2.143 0.032078 *
## Virginia            -2.27490    1.17974  -1.928 0.053817 .
## Akron              -23.49708 1830.35810  -0.013 0.989757
## Boise.State         -1.94401    1.27609  -1.523 0.127657
## Buffalo             -5.13496    1.60334  -3.203 0.001362 **
## Ohio.State          32.83790 2231.49798   0.015 0.988259
## Rice                -4.90028    1.49241  -3.283 0.001025 **
## South.Florida       -5.18830    1.44935  -3.580 0.000344 ***
## UMass               -4.65338    1.40211  -3.319 0.000904 ***
## California          -0.67550    1.34788  -0.501 0.616262
```

```
## Nevada             -4.19165    1.35710  -3.089 0.002011 **
## Army               -3.52047    1.32492  -2.657 0.007881 **
## New.Mexico         -3.62433    1.32238  -2.741 0.006130 **
## UTSA               -3.32067    1.39943  -2.373 0.017651 *
## Texas.State        -3.15430    1.24364  -2.536 0.011202 *
## Washington.State   -1.41289    1.32072  -1.070 0.284718
## Middle.Tenn.       -5.62168    1.59671  -3.521 0.000430 ***
## Toledo             -0.29920    1.53230  -0.195 0.845186
## North.Carolina     -0.60623    0.99229  -0.611 0.541240
## West.Virginia      -0.99438    1.38670  -0.717 0.473322
## Old.Dominion       -2.54425    1.22610  -2.075 0.037979 *
## South.Alabama      -2.91705    1.32727  -2.198 0.027965 *
## Texas.Tech         -1.50708    1.33624  -1.128 0.259382
## Sam.Houston        -6.34771    1.62423  -3.908 9.30e-05 ***
## C..Carolina        -2.29604    1.22435  -1.875 0.060750 .
## C..Michigan        -4.32657    1.35773  -3.187 0.001439 **
## MiamiOH            -1.02967    1.42114  -0.725 0.468736
## Louisville          0.71973    1.16590   0.617 0.537024
## Stanford           -1.51399    1.35814  -1.115 0.264956
## Kent.State        -40.01068 2968.78582  -0.013 0.989247
## NC.State            0.20992    1.03140   0.204 0.838721
## Florida            -1.06428    1.44188  -0.738 0.460441
## Nebraska           -2.05098    1.24907  -1.642 0.100591
## Navy               -5.58447    1.57243  -3.551 0.000383 ***
## UTEP               -6.29562    1.65948  -3.794 0.000148 ***
## Ohio               -2.32541    1.38792  -1.675 0.093843 .
## San.Jose.State     -1.65900    1.28932  -1.287 0.198190
## FIU                -6.10043    1.50119  -4.064 4.83e-05 ***
## Hawaii             -3.32783    1.27127  -2.618 0.008852 **
## Vanderbilt         -3.70022    1.50953  -2.451 0.014237 *
## James.Madison      -0.03963    1.49922  -0.026 0.978909
## Notre.Dame          0.68734    1.13822   0.604 0.545932
## Purdue             -2.10161    1.20454  -1.745 0.081030 .
## Troy               -0.84478    1.45325  -0.581 0.561034
## Utah                0.69776    1.41973   0.491 0.623090
## Iowa                0.73636    1.48849   0.495 0.620812
## Ole.Miss            3.59838    2.04397   1.760 0.078325 .
## UNLV               -1.34877    1.38399  -0.975 0.329781
## W..Michigan        -3.90193    1.34323  -2.905 0.003674 **
## Texas.A.M          -0.28843    1.31917  -0.219 0.826927
```

```
## Marshall            -2.61751    1.24362  -2.105 0.035313 *
## Tulsa               -6.44304    1.57914  -4.080 4.50e-05 ***
## App..State          -1.92718    1.26380  -1.525 0.127281
## New.Mexico.State    -2.33522    1.44392  -1.617 0.105819
## SMU                 -1.70524    1.70306  -1.001 0.316691
## UAB                 -5.73126    1.50532  -3.807 0.000140 ***
## Louisiana           -3.66967    1.28377  -2.859 0.004256 **
## Cincinnati          -3.57665    1.38193  -2.588 0.009649 **
## North.Texas         -5.62375    1.50553  -3.735 0.000187 ***
## Houston             -3.42532    1.36196  -2.515 0.011904 *
## Jacksonville.State  -3.10245    1.42606  -2.176 0.029590 *
## Memphis             -1.50010    1.61637  -0.928 0.353373
## Oregon              17.93942 1889.75795   0.009 0.992426
## Texas               2.88779    1.97696   1.461 0.144092
## UCF                 -2.05615    1.36097  -1.511 0.130839
## UConn               -5.41085    1.47588  -3.666 0.000246 ***
## Arizona             0.66189    1.42256   0.465 0.641731
## Charlotte           -6.79250    1.63209  -4.162 3.16e-05 ***
## E..Michigan         -4.56390    1.34297  -3.398 0.000678 ***
## Temple              -7.19032    1.72876  -4.159 3.19e-05 ***
## UCLA                -0.75488    1.30743  -0.577 0.563685
## Wisconsin           -1.18469    1.25176  -0.946 0.343933
## Air.Force           -2.26648    1.29739  -1.747 0.080647 .
## So..Miss            -4.37751    1.41444  -3.095 0.001969 **
## Auburn              -0.96358    1.40486  -0.686 0.492781
## Oklahoma.State      -0.22555    1.36189  -0.166 0.868462
## Illinois            -1.69557    1.25756  -1.348 0.177561
## Florida.State       19.61152 2602.07595   0.008 0.993987
## Ga..Southern        -3.11902    1.26989  -2.456 0.014044 *
## Iowa.State          -0.62322    1.35093  -0.461 0.644567
## Kansas.State        -0.11052    1.36734  -0.081 0.935578
## Liberty             15.85118 2651.51665   0.006 0.995230
## Penn.State          17.08114 1554.48207   0.011 0.991233
## Wake.Forest         -2.58295    1.17615  -2.196 0.028085 *
## Alabama              3.89440    1.93004   2.018 0.043614 *
## Minnesota           -1.78110    1.24644  -1.429 0.153016
## Oklahoma             0.80338    1.51173   0.531 0.595120
## South.Carolina      -1.30316    1.18181  -1.103 0.270167
## San.Diego.State     -3.47113    1.30681  -2.656 0.007903 **
## Virginia.Tech       -1.67372    1.12742  -1.485 0.137662
```

```
## ULMonroe            -5.34559    1.53449  -3.484 0.000495 ***
## Tulane               1.29290    2.79501   0.463 0.643671
## W..Kentucky         -3.72704    1.56076  -2.388 0.016942 *
## Washington          34.38338 2944.00562   0.012 0.990682
## Georgia.State       -2.76769    1.30941  -2.114 0.034541 *
## Tennessee            0.38733    1.44991   0.267 0.789360
## BYU                 -2.13424    1.37034  -1.557 0.119364
## Syracuse            -2.10428    1.11611  -1.885 0.059381 .
## Georgia.Tech        -1.04568    1.07494  -0.973 0.330664
## Pittsburgh          -3.14053    1.26399  -2.485 0.012969 *
## FAU                 -6.16080    1.56351  -3.940 8.14e-05 ***
## TCU                 -2.04805    1.32495  -1.546 0.122163
## Wyoming             -1.52260    1.28702  -1.183 0.236794
## Colorado.State      -3.35307    1.30004  -2.579 0.009903 **
## Kansas              -0.32049    1.36032  -0.236 0.813742
## Rutgers             -0.98248    1.35676  -0.724 0.468983
## Kentucky            -0.14283    1.38300  -0.103 0.917742
## Boston.College      -2.26278    1.18002  -1.918 0.055164 .
## Duke                -0.48283    1.05552  -0.457 0.647362
## Maryland            -0.87467    1.37010  -0.638 0.523215
## Miami..Fla..        -0.43425    1.03428  -0.420 0.674589
## Arkansas            -2.66566    1.53597  -1.735 0.082654 .
## Miss..State         -1.50715    1.35037  -1.116 0.264379
## USC                 0.16186    1.31814   0.123 0.902268
## Arizona.State       -2.07547    1.37218  -1.513 0.130398
## Baylor              -3.57146    1.40311  -2.545 0.010916 *
## Indiana             -3.22613    1.54864  -2.083 0.037233 *
## Michigan            49.13577 3063.15329   0.016 0.987202
## Georgia             20.53306 2469.08872   0.008 0.993365
## Missouri             2.26424    1.76484   1.283 0.199503
## Michigan.State      -2.18110    1.42221  -1.534 0.125130
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1024.47  on 739  degrees of freedom
## Residual deviance:  529.53  on 607  degrees of freedom
## AIC: 793.53
```

```
##
## Number of Fisher Scoring iterations: 18
# BT_model that DOES take into account home field advantage
model_with_intercept <- glm(home_win ~. - Clemson, family = binomial, data = df) # Using Clemson as a reference team
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
summary(model_with_intercept)
##
## Call:
## glm(formula = home_win ~ . - Clemson, family = binomial, data = df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.49688  -0.46049   0.00036   0.56818   2.60247
##
## Coefficients:
##                   Estimate Std. Error z value Pr(>|z|)
## (Intercept)        0.37479    0.11654   3.216 0.001300 **
## Northwestern      -0.80763    1.26570  -0.638 0.523417
## Oregon.State       0.65032    1.44382   0.450 0.652410
## LSU                2.86485    1.89402   1.513 0.130386
## Arkansas.State    -3.04600    1.29980  -2.343 0.019107 *
## Ball.State        -4.18296    1.42653  -2.932 0.003365 **
## Bowling.Green     -1.80760    1.45395  -1.243 0.213780
## Colorado          -1.74429    1.29001  -1.352 0.176327
## East.Carolina     -7.47659    1.82047  -4.107 4.01e-05 ***
## Fresno.State      -1.77675    1.27924  -1.389 0.164861
## Louisiana.Tech    -6.50351    1.62162  -4.011 6.06e-05 ***
## N..Illinois       -3.65833    1.38566  -2.640 0.008287 **
## Utah.State        -2.67666    1.31870  -2.030 0.042380 *
## Virginia          -2.29595    1.18188  -1.943 0.052063 .
## Akron            -23.05185 1854.82717  -0.012 0.990084
## Boise.State       -1.84771    1.28150  -1.442 0.149352
## Buffalo           -5.11107    1.64143  -3.114 0.001847 **
## Ohio.State        32.41904 2193.53869   0.015 0.988208
## Rice              -4.75253    1.49173  -3.186 0.001443 **
## South.Florida     -4.96217    1.46900  -3.378 0.000730 ***
## UMass             -4.53611    1.41981  -3.195 0.001399 **
## California        -0.70711    1.36261  -0.519 0.603807
## Nevada            -4.04096    1.37073  -2.948 0.003198 **
```

```
## Army               -3.40183    1.32411  -2.569 0.010195 *
## New.Mexico         -3.50746    1.32525  -2.647 0.008130 **
## UTSA               -3.26743    1.40639  -2.323 0.020164 *
## Texas.State        -2.93589    1.25710  -2.335 0.019520 *
## Washington.State   -1.42193    1.32098  -1.076 0.281736
## Middle.Tenn.       -5.66445    1.62739  -3.481 0.000500 ***
## Toledo              0.06904    1.56576   0.044 0.964831
## North.Carolina     -0.68913    1.01635  -0.678 0.497743
## West.Virginia      -0.93235    1.39609  -0.668 0.504244
## Old.Dominion       -2.42520    1.23913  -1.957 0.050327 .
## South.Alabama      -2.85999    1.35566  -2.110 0.034887 *
## Texas.Tech         -1.37470    1.35381  -1.015 0.309900
## Sam.Houston        -6.39003    1.65891  -3.852 0.000117 ***
## C..Carolina        -2.06191    1.23835  -1.665 0.095904 .
## C..Michigan        -4.01884    1.38681  -2.898 0.003757 **
## MiamiOH            -0.79641    1.40724  -0.566 0.571439
## Louisville          0.64584    1.16927   0.552 0.580713
## Stanford           -1.42976    1.34374  -1.064 0.287321
## Kent.State        -39.19274 2985.84825  -0.013 0.989527
## NC.State            0.04382    1.02749   0.043 0.965981
## Florida            -1.04042    1.50129  -0.693 0.488302
## Nebraska           -1.92057    1.27280  -1.509 0.131317
## Navy               -5.53841    1.56346  -3.542 0.000396 ***
## UTEP               -6.10543    1.68173  -3.630 0.000283 ***
## Ohio               -2.12042    1.37446  -1.543 0.122897
## San.Jose.State     -1.49909    1.31273  -1.142 0.253469
## FIU                -5.92824    1.51289  -3.918 8.91e-05 ***
## Hawaii             -3.26493    1.27331  -2.564 0.010344 *
## Vanderbilt         -3.65552    1.54616  -2.364 0.018067 *
## James.Madison       0.30466    1.52893   0.199 0.842055
## Notre.Dame          0.85495    1.13862   0.751 0.452734
## Purdue             -2.17772    1.22738  -1.774 0.076018 .
## Troy               -0.62209    1.46694  -0.424 0.671514
## Utah                0.83492    1.47328   0.567 0.570914
## Iowa                0.87980    1.51736   0.580 0.562035
## Ole.Miss            3.62914    2.04935   1.771 0.076582 .
## UNLV               -1.26016    1.39588  -0.903 0.366648
## W..Michigan        -3.64746    1.36210  -2.678 0.007410 **
## Texas.A.M          -0.33390    1.35866  -0.246 0.805869
## Marshall           -2.46896    1.25457  -1.968 0.049072 *
```

```
## Tulsa               -6.35544    1.58672  -4.005 6.19e-05 ***
## App..State          -1.69569    1.29007  -1.314 0.188706
## New.Mexico.State    -2.04915    1.47551  -1.389 0.164901
## SMU                 -1.33970    1.73287  -0.773 0.439458
## UAB                 -5.58800    1.51039  -3.700 0.000216 ***
## Louisiana           -3.54598    1.31065  -2.706 0.006820 **
## Cincinnati          -3.47578    1.38625  -2.507 0.012165 *
## North.Texas         -5.39931    1.49173  -3.619 0.000295 ***
## Houston             -3.38754    1.36919  -2.474 0.013356 *
## Jacksonville.State  -2.86001    1.47191  -1.943 0.052008 .
## Memphis             -1.49268    1.58671  -0.941 0.346838
## Oregon              18.00413 1859.54945   0.010 0.992275
## Texas               2.86157    2.01983   1.417 0.156559
## UCF                 -1.91468    1.38120  -1.386 0.165672
## UConn               -5.26140    1.50075  -3.506 0.000455 ***
## Arizona             0.75157    1.40867   0.534 0.593662
## Charlotte           -6.66504    1.62510  -4.101 4.11e-05 ***
## E..Michigan         -4.33915    1.37436  -3.157 0.001593 **
## Temple              -6.88653    1.72627  -3.989 6.63e-05 ***
## UCLA                -0.70557    1.33890  -0.527 0.598208
## Wisconsin           -1.11958    1.26238  -0.887 0.375143
## Air.Force           -2.18607    1.30748  -1.672 0.094530 .
## So..Miss            -4.21431    1.43690  -2.933 0.003358 **
## Auburn              -0.85276    1.42907  -0.597 0.550690
## Oklahoma.State      -0.18730    1.36154  -0.138 0.890582
## Illinois            -1.63389    1.28394  -1.273 0.203173
## Florida.State       19.48083 2630.31638   0.007 0.994091
## Ga..Southern        -2.89432    1.28482  -2.253 0.024278 *
## Iowa.State          -0.55711    1.34080  -0.416 0.677770
## Kansas.State        -0.01317    1.39350  -0.009 0.992458
## Liberty             15.89233 2657.39574   0.006 0.995228
## Penn.State          17.08507 1539.00960   0.011 0.991143
## Wake.Forest         -2.51736    1.19347  -2.109 0.034920 *
## Alabama              3.63286    1.89825   1.914 0.055647 .
## Minnesota           -1.81501    1.28181  -1.416 0.156783
## Oklahoma             0.92522    1.54456   0.599 0.549161
## South.Carolina      -1.52422    1.19183  -1.279 0.200934
## San.Diego.State     -3.40848    1.31206  -2.598 0.009382 **
## Virginia.Tech       -1.68823    1.13321  -1.490 0.136283
## ULMonroe            -5.16995    1.53211  -3.374 0.000740 ***
```

```
## Tulane               1.24314    2.57731    0.482 0.629564
## W..Kentucky         -3.53582    1.59991  -2.210 0.027105 *
## Washington          34.02925 2886.92041    0.012 0.990595
## Georgia.State       -2.52665    1.31248  -1.925 0.054217 .
## Tennessee            0.37971    1.50629    0.252 0.800977
## BYU                 -2.04527    1.38683  -1.475 0.140271
## Syracuse            -2.13392    1.13065  -1.887 0.059113 .
## Georgia.Tech        -1.08854    1.09298  -0.996 0.319282
## Pittsburgh          -3.08096    1.27832  -2.410 0.015946 *
## FAU                 -5.92749    1.57698  -3.759 0.000171 ***
## TCU                 -2.02431    1.34494  -1.505 0.132290
## Wyoming             -1.43970    1.30567  -1.103 0.270180
## Colorado.State      -3.29120    1.31153  -2.509 0.012092 *
## Kansas              -0.32534    1.37804  -0.236 0.813363
## Rutgers             -0.99857    1.38967  -0.719 0.472407
## Kentucky            -0.17778    1.37829  -0.129 0.897366
## Boston.College      -2.24190    1.17654  -1.906 0.056714 .
## Duke                -0.59752    1.05358  -0.567 0.570620
## Maryland            -0.66138    1.41750  -0.467 0.640802
## Miami..Fla..        -0.58300    1.03945  -0.561 0.574881
## Arkansas            -2.86187    1.55342  -1.842 0.065431 .
## Miss..State         -1.42986    1.35266  -1.057 0.290480
## USC                 0.17460    1.33635    0.131 0.896049
## Arizona.State       -2.21552    1.38931  -1.595 0.110781
## Baylor              -3.53155    1.41117  -2.503 0.012330 *
## Indiana             -3.21854    1.56685  -2.054 0.039961 *
## Michigan            48.30056 3005.93886    0.016 0.987180
## Georgia             20.24076 2512.31799    0.008 0.993572
## Missouri             2.08798    1.78300    1.171 0.241578
## Michigan.State      -2.05826    1.44987  -1.420 0.155718
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1005.87  on 738  degrees of freedom
## Residual deviance:  518.94  on 606  degrees of freedom
## AIC: 784.94
##
## Number of Fisher Scoring iterations: 18
```

Penalized brglm()

```
# Penalized BT model that does not take into account home field advantage
penalized_model <- brglm(home_win ~. -1 - Clemson, family = binomial, data = df)
summary(penalized_model)
##
## Call:
## brglm(formula = home_win ~ . - 1 - Clemson, family = binomial,
##     data = df)
##
##
## Coefficients:
##                   Estimate Std. Error z value Pr(>|z|)
## Northwestern       -0.75633    1.15289  -0.656 0.511805
## Oregon.State        0.34715    1.28736   0.270 0.787424
## LSU                1.41695    1.31565   1.077 0.281483
## Arkansas.State     -2.42046    1.17597  -2.058 0.039565 *
## Ball.State         -3.47142    1.27388  -2.725 0.006429 **
## Bowling.Green      -1.64268    1.26882  -1.295 0.195443
## Colorado           -1.44405    1.18429  -1.219 0.222714
## East.Carolina      -5.47225    1.47382  -3.713 0.000205 ***
## Fresno.State       -1.51395    1.17508  -1.288 0.197617
## Louisiana.Tech     -4.80612    1.35664  -3.543 0.000396 ***
## N..Illinois        -2.92772    1.21049  -2.419 0.015579 *
## Utah.State         -2.18231    1.19431  -1.827 0.067661 .
## Virginia           -1.79017    1.07613  -1.664 0.096205 .
## Akron              -6.17265    1.67224  -3.691 0.000223 ***
## Boise.State        -1.49654    1.17662  -1.272 0.203410
## Buffalo            -3.91514    1.36040  -2.878 0.004003 **
## Ohio.State          2.79602    1.80840   1.546 0.122073
## Rice               -3.59054    1.28544  -2.793 0.005218 **
## South.Florida      -3.78229    1.24894  -3.028 0.002458 **
## UMass              -3.53024    1.24933  -2.826 0.004718 **
## California         -0.57176    1.24240  -0.460 0.645365
## Nevada             -3.27724    1.23550  -2.653 0.007988 **
## Army               -2.69462    1.20097  -2.244 0.024852 *
## New.Mexico         -2.82259    1.19995  -2.352 0.018660 *
## UTSA               -2.45756    1.22956  -1.999 0.045638 *
## Texas.State        -2.42410    1.14413  -2.119 0.034113 *
## Washington.State   -1.13777    1.22447  -0.929 0.352787
## Middle.Tenn.       -4.14329    1.35718  -3.053 0.002267 **
```

```
## Toledo             -0.35953    1.32768  -0.271 0.786547
## North.Carolina     -0.48211    0.94419  -0.511 0.609630
## West.Virginia      -0.80099    1.23553  -0.648 0.516794
## Old.Dominion       -1.96503    1.12290  -1.750 0.080125 .
## South.Alabama      -2.23098    1.19701  -1.864 0.062351 .
## Texas.Tech         -1.19123    1.21824  -0.978 0.328161
## Sam.Houston        -4.68364    1.36332  -3.435 0.000592 ***
## C..Carolina        -1.74969    1.12700  -1.553 0.120538
## C..Michigan        -3.32987    1.21868  -2.732 0.006288 **
## MiamiOH            -0.89294    1.27212  -0.702 0.482722
## Louisville          0.53326    1.08026   0.494 0.621558
## Stanford           -1.21486    1.24401  -0.977 0.328782
## Kent.State         -7.10789    2.15663  -3.296 0.000981 ***
## NC.State            0.16336    0.98039   0.167 0.867662
## Florida            -0.89375    1.26009  -0.709 0.478156
## Nebraska           -1.64407    1.15556  -1.423 0.154811
## Navy               -4.05467    1.33199  -3.044 0.002334 **
## UTEP               -4.63574    1.41061  -3.286 0.001015 **
## Ohio               -1.84216    1.24275  -1.482 0.138253
## San.Jose.State     -1.26641    1.17928  -1.074 0.282873
## FIU                -4.49765    1.28521  -3.500 0.000466 ***
## Hawaii             -2.58699    1.16544  -2.220 0.026435 *
## Vanderbilt         -2.78810    1.30051  -2.144 0.032044 *
## James.Madison      -0.06619    1.28269  -0.052 0.958848
## Notre.Dame          0.51628    1.05878   0.488 0.625823
## Purdue             -1.66828    1.12333  -1.485 0.137510
## Troy               -0.66894    1.26715  -0.528 0.597562
## Utah                0.47630    1.27407   0.374 0.708520
## Iowa                0.44083    1.25098   0.352 0.724549
## Ole.Miss            1.89673    1.41469   1.341 0.180006
## UNLV               -1.02358    1.25632  -0.815 0.415215
## W..Michigan        -3.01488    1.20174  -2.509 0.012115 *
## Texas.A.M          -0.34403    1.17993  -0.292 0.770616
## Marshall           -2.01858    1.14231  -1.767 0.077211 .
## Tulsa              -4.68950    1.33865  -3.503 0.000460 ***
## App..State         -1.47062    1.15972  -1.268 0.204771
## New.Mexico.State   -1.79248    1.25607  -1.427 0.153565
## SMU                -1.39163    1.43664  -0.969 0.332709
## UAB                -4.19542    1.28283  -3.270 0.001074 **
## Louisiana          -2.83206    1.16334  -2.434 0.014916 *
```

```
## Cincinnati         -2.79294    1.24498  -2.243 0.024873 *
## North.Texas        -4.09129    1.28621  -3.181 0.001468 **
## Houston            -2.65149    1.22041  -2.173 0.029808 *
## Jacksonville.State -2.33950    1.24967  -1.872 0.061193 .
## Memphis            -1.21853    1.35709  -0.898 0.369239
## Oregon              2.30792    1.65241   1.397 0.162504
## Texas               1.76747    1.46061   1.210 0.226244
## UCF                -1.62762    1.23472  -1.318 0.187435
## UConn              -4.01040    1.27765  -3.139 0.001696 **
## Arizona             0.47830    1.28793   0.371 0.710362
## Charlotte          -4.96499    1.37441  -3.612 0.000303 ***
## E..Michigan        -3.51424    1.21096  -2.902 0.003708 **
## Temple             -5.12906    1.37007  -3.744 0.000181 ***
## UCLA               -0.62307    1.20396  -0.518 0.604799
## Wisconsin          -0.94932    1.15370  -0.823 0.410597
## Air.Force          -1.74457    1.18700  -1.470 0.141634
## So..Miss           -3.36601    1.25974  -2.672 0.007540 **
## Auburn             -0.83321    1.24205  -0.671 0.502324
## Oklahoma.State     -0.21868    1.22643  -0.178 0.858484
## Illinois           -1.36243    1.15572  -1.179 0.238454
## Florida.State       2.63606    1.73741   1.517 0.129206
## Ga..Southern       -2.41904    1.15782  -2.089 0.036680 *
## Iowa.State         -0.54376    1.21526  -0.447 0.654555
## Kansas.State       -0.13060    1.22143  -0.107 0.914847
## Liberty             0.23247    1.70606   0.136 0.891617
## Penn.State          1.83913    1.57282   1.169 0.242273
## Wake.Forest        -2.01477    1.06374  -1.894 0.058219 .
## Alabama             2.27940    1.42277   1.602 0.109137
## Minnesota          -1.41792    1.15043  -1.233 0.217759
## Oklahoma            0.52413    1.30609   0.401 0.688199
## South.Carolina     -1.07672    1.08065  -0.996 0.319073
## San.Diego.State    -2.72010    1.19750  -2.271 0.023117 *
## Virginia.Tech      -1.29644    1.04302  -1.243 0.213878
## ULMonroe           -4.07822    1.32573  -3.076 0.002097 **
## Tulane              0.21851    1.71550   0.127 0.898644
## W..Kentucky        -2.74871    1.32752  -2.071 0.038399 *
## Washington          3.42864    1.96073   1.749 0.080351 .
## Georgia.State      -2.13531    1.19045  -1.794 0.072861 .
## Tennessee           0.16025    1.24752   0.128 0.897787
## BYU                -1.71040    1.23909  -1.380 0.167472
```

```
## Syracuse          -1.64897    1.02665  -1.606 0.108240
## Georgia.Tech       -0.83210    1.00116  -0.831 0.405896
## Pittsburgh        -2.45060    1.14250  -2.145 0.031957 *
## FAU               -4.50842    1.32433  -3.404 0.000663 ***
## TCU               -1.63558    1.20688  -1.355 0.175351
## Wyoming            -1.15312    1.18433  -0.974 0.330230
## Colorado.State     -2.62308    1.18643  -2.211 0.027043 *
## Kansas            -0.29813    1.22709  -0.243 0.808040
## Rutgers           -0.79564    1.24025  -0.642 0.521186
## Kentucky           -0.20569    1.22983  -0.167 0.867175
## Boston.College     -1.77254    1.08558  -1.633 0.102509
## Duke              -0.38577    0.99290  -0.389 0.697628
## Maryland           -0.71445    1.24604  -0.573 0.566389
## Miami..Fla..       -0.36808    0.98116  -0.375 0.707552
## Arkansas          -2.08515    1.33830  -1.558 0.119220
## Miss..State        -1.24629    1.20649  -1.033 0.301606
## USC                0.09955    1.20829   0.082 0.934338
## Arizona.State      -1.64567    1.26018  -1.306 0.191586
## Baylor            -2.78658    1.26877  -2.196 0.028072 *
## Indiana           -2.48130    1.35737  -1.828 0.067546 .
## Michigan           3.86411    2.21958   1.741 0.081698 .
## Georgia            3.22290    1.86128   1.732 0.083353 .
## Missouri           1.32884    1.34106   0.991 0.321741
## Michigan.State     -1.71814    1.29212  -1.330 0.183613
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 757.57  on 739  degrees of freedom
## Residual deviance: 552.90  on 607  degrees of freedom
## Penalized deviance: 515.259
## AIC:  816.9
# BT_model that DOES take into account home field advanatage
p_model_with_intercept <- brglm(home_win ~. - Clemson, family = binomial, data = df) #
Using Clemson as a reference team
summary(p_model_with_intercept)
##
## Call:
## brglm(formula = home_win ~ . - Clemson, family = binomial, data = df)
```

```
##
##
## Coefficients:
##                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)       0.29192    0.10238   2.851 0.004354 **
## Northwestern     -0.65093    1.16051  -0.561 0.574869
## Oregon.State      0.47421    1.28483   0.369 0.712062
## LSU               1.54904    1.34497   1.152 0.249433
## Arkansas.State   -2.31542    1.17765  -1.966 0.049283 *
## Ball.State       -3.19055    1.27277  -2.507 0.012184 *
## Bowling.Green    -1.42770    1.27401  -1.121 0.262444
## Colorado         -1.38979    1.18572  -1.172 0.241153
## East.Carolina    -5.39906    1.49970  -3.600 0.000318 ***
## Fresno.State     -1.35006    1.18003  -1.144 0.252587
## Louisiana.Tech   -4.76406    1.37435  -3.466 0.000527 ***
## N..Illinois      -2.79944    1.22526  -2.285 0.022326 *
## Utah.State       -2.04592    1.19405  -1.713 0.086635 .
## Virginia         -1.78676    1.07562  -1.661 0.096686 .
## Akron            -5.90232    1.65331  -3.570 0.000357 ***
## Boise.State      -1.39782    1.17583  -1.189 0.234519
## Buffalo          -3.81899    1.35923  -2.810 0.004959 **
## Ohio.State        2.89093    1.78007   1.624 0.104363
## Rice             -3.46574    1.28352  -2.700 0.006930 **
## South.Florida    -3.59964    1.26098  -2.855 0.004309 **
## UMass            -3.41152    1.25817  -2.711 0.006698 **
## California       -0.58362    1.24470  -0.469 0.639154
## Nevada           -3.12578    1.23633  -2.528 0.011462 *
## Army             -2.58102    1.19775  -2.155 0.031171 *
## New.Mexico       -2.69863    1.19868  -2.251 0.024364 *
## UTSA             -2.39588    1.23131  -1.946 0.051679 .
## Texas.State      -2.22930    1.14906  -1.940 0.052367 .
## Washington.State -1.12585    1.21831  -0.924 0.355432
## Middle.Tenn.     -4.15096    1.37559  -3.018 0.002548 **
## Toledo           -0.06124    1.34655  -0.045 0.963723
## North.Carolina   -0.53855    0.95490  -0.564 0.572761
## West.Virginia    -0.72576    1.24338  -0.584 0.559423
## Old.Dominion     -1.84977    1.13007  -1.637 0.101658
## South.Alabama    -2.15906    1.21436  -1.778 0.075413 .
## Texas.Tech       -1.06453    1.22451  -0.869 0.384656
## Sam.Houston      -4.68125    1.38805  -3.373 0.000745 ***
```

```
## C..Carolina        -1.55128    1.13242  -1.370 0.170726
## C..Michigan        -3.04366    1.23207  -2.470 0.013498 *
## MiamiOH            -0.68271    1.25854  -0.542 0.587500
## Louisville          0.47527    1.07925   0.440 0.659667
## Stanford           -1.13258    1.22800  -0.922 0.356375
## Kent.State         -6.62330    2.03984  -3.247 0.001166 **
## NC.State            0.04202    0.97212   0.043 0.965518
## Florida            -0.86439    1.28253  -0.674 0.500329
## Nebraska           -1.52330    1.16473  -1.308 0.190920
## Navy               -4.00966    1.33003  -3.015 0.002572 **
## UTEP               -4.45995    1.41645  -3.149 0.001640 **
## Ohio               -1.65104    1.23053  -1.342 0.179684
## San.Jose.State     -1.11316    1.18867  -0.936 0.349031
## FIU                -4.34013    1.28772  -3.370 0.000751 ***
## Hawaii             -2.50948    1.16223  -2.159 0.030835 *
## Vanderbilt         -2.72225    1.30937  -2.079 0.037612 *
## James.Madison       0.20261    1.29913   0.156 0.876068
## Notre.Dame          0.64048    1.05814   0.605 0.544991
## Purdue             -1.70582    1.13383  -1.504 0.132461
## Troy               -0.48712    1.27242  -0.383 0.701846
## Utah                0.55753    1.29464   0.431 0.666726
## Iowa                0.58019    1.27584   0.455 0.649286
## Ole.Miss            1.94010    1.41808   1.368 0.171276
## UNLV               -0.93625    1.25831  -0.744 0.456845
## W..Michigan        -2.78441    1.20918  -2.303 0.021294 *
## Texas.A.M          -0.36126    1.19930  -0.301 0.763240
## Marshall           -1.88473    1.14584  -1.645 0.100002
## Tulsa              -4.61172    1.34711  -3.423 0.000618 ***
## App..State         -1.27175    1.17437  -1.083 0.278841
## New.Mexico.State   -1.56045    1.27598  -1.223 0.221351
## SMU                -1.09677    1.47083  -0.746 0.455859
## UAB                -4.08250    1.28867  -3.168 0.001535 **
## Louisiana          -2.70496    1.17695  -2.298 0.021546 *
## Cincinnati         -2.68668    1.24127  -2.164 0.030430 *
## North.Texas        -3.91386    1.27543  -3.069 0.002150 **
## Houston            -2.59053    1.22253  -2.119 0.034091 *
## Jacksonville.State -2.14796    1.26401  -1.699 0.089259 .
## Memphis            -1.20531    1.33366  -0.904 0.366122
## Oregon              2.41596    1.68015   1.438 0.150450
## Texas               1.74801    1.46789   1.191 0.233720
```

```
## UCF              -1.49186    1.24523  -1.198 0.230894
## UConn            -3.87059    1.28292  -3.017 0.002553 **
## Arizona           0.55406    1.26926   0.437 0.662460
## Charlotte        -4.85257    1.37014  -3.542 0.000398 ***
## E..Michigan      -3.29623    1.22668  -2.687 0.007207 **
## Temple           -4.92152    1.36586  -3.603 0.000314 ***
## UCLA             -0.57013    1.21593  -0.469 0.639154
## Wisconsin        -0.87875    1.15467  -0.761 0.446636
## Air.Force        -1.65930    1.18702  -1.398 0.162154
## So..Miss         -3.20721    1.26785  -2.530 0.011418 *
## Auburn           -0.73402    1.24278  -0.591 0.554771
## Oklahoma.State   -0.16914    1.22431  -0.138 0.890121
## Illinois         -1.29399    1.16738  -1.108 0.267663
## Florida.State     2.54163    1.69151   1.503 0.132947
## Ga..Southern     -2.21994    1.16789  -1.901 0.057326 .
## Iowa.State       -0.46791    1.20347  -0.389 0.697422
## Kansas.State     -0.04077    1.23609  -0.033 0.973691
## Liberty           0.28295    1.69482   0.167 0.867410
## Penn.State        1.95091    1.57333   1.240 0.214978
## Wake.Forest      -1.95102    1.07187  -1.820 0.068728 .
## Alabama           2.10700    1.38739   1.519 0.128842
## Minnesota        -1.42208    1.16820  -1.217 0.223479
## Oklahoma          0.62972    1.32352   0.476 0.634224
## South.Carolina   -1.23494    1.08387  -1.139 0.254547
## San.Diego.State  -2.64495    1.19682  -2.210 0.027107 *
## Virginia.Tech    -1.29148    1.04447  -1.236 0.216275
## ULMonroe         -3.90907    1.32022  -2.961 0.003067 **
## Tulane            0.28052    1.67790   0.167 0.867224
## W..Kentucky      -2.59833    1.34196  -1.936 0.052842 .
## Washington        3.38046    1.89105   1.788 0.073838 .
## Georgia.State    -1.93161    1.18993  -1.623 0.104527
## Tennessee         0.15057    1.27101   0.118 0.905698
## BYU              -1.61597    1.24817  -1.295 0.195435
## Syracuse         -1.65723    1.03003  -1.609 0.107634
## Georgia.Tech     -0.85019    1.00820  -0.843 0.399074
## Pittsburgh       -2.38237    1.14518  -2.080 0.037495 *
## FAU              -4.32115    1.32803  -3.254 0.001139 **
## TCU              -1.59067    1.21713  -1.307 0.191247
## Wyoming          -1.07065    1.19259  -0.898 0.369320
## Colorado.State   -2.54557    1.18904  -2.141 0.032284 *
```

```
## Kansas            -0.28704    1.23149  -0.233 0.815696
## Rutgers           -0.78712    1.25633  -0.627 0.530974
## Kentucky          -0.24942    1.21449  -0.205 0.837285
## Boston.College    -1.73645    1.08026  -1.607 0.107960
## Duke              -0.47086    0.99020  -0.476 0.634412
## Maryland          -0.54381    1.26644  -0.429 0.667632
## Miami..Fla..      -0.47458    0.98113  -0.484 0.628591
## Arkansas          -2.21723    1.34653  -1.647 0.099634 .
## Miss..State       -1.18058    1.19886  -0.985 0.324745
## USC                0.12243    1.21304   0.101 0.919606
## Arizona.State     -1.72961    1.26531  -1.367 0.171642
## Baylor            -2.72314    1.26645  -2.150 0.031538 *
## Indiana           -2.44510    1.35938  -1.799 0.072067 .
## Michigan           3.86448    2.14488   1.802 0.071589 .
## Georgia            3.00074    1.79566   1.671 0.094701 .
## Missouri           1.18975    1.34692   0.883 0.377069
## Michigan.State    -1.59424    1.30151  -1.225 0.220606
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 740.85  on 738  degrees of freedom
## Residual deviance: 542.79  on 606  degrees of freedom
## Penalized deviance: 502.0224
## AIC:  808.79
# Assuming X and y are your design matrix and response variable
X <- model.matrix(home_win ~ . - 1 - Clemson, data = df)
y <- df$home_win

# Non-Penalized glm() model with intercept
model_with_intercept <- glm(home_win ~ . - Clemson, family = binomial, data = df)
cv_non_penalized_with_intercept <- cv.glmnet(x = X, y = y, alpha = 0, family = "binomial")
rmse_non_penalized_with_intercept <- sqrt(min(cv_non_penalized_with_intercept$cvm))

# Non-Penalized glm() model without intercept
model_without_intercept <- glm(home_win ~ . - 1 - Clemson, family = binomial, data = df)
cv_non_penalized_without_intercept <- cv.glmnet(x = X, y = y, alpha = 0, family = "binomial")
rmse_non_penalized_without_intercept <- sqrt(min(cv_non_penalized_without_intercept$cvm))
```

```
# Penalized brglm() model without intercept
penalized_model_without_intercept <- brglm(home_win ~ . - 1 - Clemson, family = binomial,
data = df)
cv_penalized_without_intercept <- cv.glmnet(x = X, y = y, alpha = 0, family = "binomial")
rmse_penalized_without_intercept <- sqrt(min(cv_penalized_without_intercept$cvm))

# Penalized brglm() model with intercept
X_with_intercept <- model.matrix(home_win ~ . - Clemson, data = df)
penalized_model_with_intercept <- brglm(home_win ~ . - Clemson, family = binomial, data =
df) # Using Clemson as a reference team
cv_penalized_with_intercept <- cv.glmnet(x = X_with_intercept, y = y, alpha = 0, family =
"binomial")
rmse_penalized_with_intercept <- sqrt(min(cv_penalized_with_intercept$cvm))

# Print the RMSE for each model
cat("RMSE (Non-Penalized with Intercept):", rmse_non_penalized_with_intercept, "\n")
## RMSE (Non-Penalized with Intercept): 1.050352
cat("RMSE (Non-Penalized without Intercept):", rmse_non_penalized_without_intercept, "\n")
## RMSE (Non-Penalized without Intercept): 1.05135
cat("RMSE (Penalized without Intercept):", rmse_penalized_without_intercept, "\n")
## RMSE (Penalized without Intercept): 1.049879
cat("RMSE (Penalized with Intercept):", rmse_penalized_with_intercept, "\n")
## RMSE (Penalized with Intercept): 1.047358
# Extract the coefficients and rannk them, 1:25
ranksings_unpenalized_with_intercept <- rank(-model_with_intercept$coefficients)
ranksings_penalized_with_intercept <- rank(-p_model_with_intercept$coefficients)
top_25_unpenalized_with_intercept <- ranksings_unpenalized_with_intercept[1:25]
top_25_penalized_with_intercept <- ranksings_penalized_with_intercept[1:25]

# Create a data frame sorted and ordered as (Team, Coefficient) from best to worst team
top_25_unpenalized_with_intercept_df <- data.frame(Team =
names(model_with_intercept$coefficients)[order(-model_with_intercept$coefficients)][1:25],
                                                   Coefficient = sort(model_with_intercept$coefficients,
decreasing = TRUE)[1:25])

top_25_penalized_with_intercept_df <- data.frame(Team =
names(p_model_with_intercept$coefficients)[order(-p_model_with_intercept$coefficients)][1:25
],
                                                 Coefficient = sort(p_model_with_intercept$coefficients,
```

```
decreasing = TRUE)[1:25])
print("Top 25 Unpenalized with Intercept:")
## [1] "Top 25 Unpenalized with Intercept:"
head(top_25_unpenalized_with_intercept_df, 25)
##                      Team Coefficient
## Michigan           Michigan  48.3005564
## Washington       Washington  34.0292535
## Ohio.State       Ohio.State  32.4190421
## Georgia             Georgia  20.2407578
## Florida.State Florida.State  19.4808285
## Oregon               Oregon  18.0041323
## Penn.State       Penn.State  17.0850724
## Liberty             Liberty  15.8923287
## Alabama             Alabama   3.6328609
## Ole.Miss           Ole.Miss   3.6291374
## LSU                     LSU   2.8648543
## Texas               Texas   2.8615695
## Missouri           Missouri   2.0879809
## Tulane             Tulane   1.2431422
## Oklahoma           Oklahoma   0.9252187
## Iowa                 Iowa   0.8798007
## Notre.Dame       Notre.Dame   0.8549458
## Utah                 Utah   0.8349161
## Arizona           Arizona   0.7515745
## Oregon.State   Oregon.State   0.6503180
## Louisville       Louisville   0.6458369
## Tennessee         Tennessee   0.3797097
## (Intercept)     (Intercept)   0.3747934
## James.Madison James.Madison   0.3046642
## USC                   USC   0.1745998
print("Top 25 Penalized with Intercept:")
## [1] "Top 25 Penalized with Intercept:"
head(top_25_penalized_with_intercept_df, 25)
##                      Team Coefficient
## Michigan           Michigan   3.8644796
## Washington       Washington   3.3804628
## Georgia             Georgia   3.0007424
## Ohio.State       Ohio.State   2.8909324
## Florida.State Florida.State   2.5416345
## Oregon               Oregon   2.4159640
```

Alabama Alabama 2.1070026
Penn.State Penn.State 1.9509141
Ole.Miss Ole.Miss 1.9401012
Texas Texas 1.7480099
LSU LSU 1.5490396
Missouri Missouri 1.1897500
Notre.Dame Notre.Dame 0.6404760
Oklahoma Oklahoma 0.6297162
Iowa Iowa 0.5801919
Utah Utah 0.5575310
Arizona Arizona 0.5540584
Louisville Louisville 0.4752742
Oregon.State Oregon.State 0.4742144
(Intercept) (Intercept) 0.2919154
Liberty Liberty 0.2829503
Tulane Tulane 0.2805216
James.Madison James.Madison 0.2026062
Tennessee Tennessee 0.1505728
USC USC 0.1224316

In terms of the differences in the top 25 between each model, Liberty is ranked 8th in the unpenalized model but 21st in the penalized model. Tulane is also ranked 14th in the un However, the top 10 seem to be very similar in ranking. Tulane is also ranked quite differently, 15th at the unpenalized model and 22nd at the penalized one. Further, in terms of the "strength" coefficients, the top 8 teams in the unpenalized model have extremely large values. For example, Michigan has a coefficient of 48.3, Washington at 34.02, etc. However at Alabama onward, coefficients taper off at 3.6 to 0.2. The penalized model does not have this feature. Michigan (also ranked #1) has a coefficient of 3.86, followed by Washington's 3.38 all the way to 10th place Texas at 1.74.

For the unpenalized model with intercept(taking home-field advantage into account), we see a stark difference in Liberty's ranking. It is ranked 8th here in the BT-model, but 24th in the CFP as of Nov 28, 2023. We also see Ole Miss ranked in the BT model while unranked in the CFP rankings. Tulane is also ranked higher at 14 in the model while 22 in the CFP rankings.

For the penalized model with intercept (taking home-field advantage into account), most rankings are not that different. However, we see several schools in the top 25 for the BT model with intercept brglm(home_win ~. - Clemson, family = binomial, data = df), but not in the CFP. Ole Miss, Utah, and USC, for example, are unraked in the CFP but show up in the top 25 from our model.

For Penn State

```
# Extract strengths for Michigan and Georgia
michigan_strength <- p_model_with_intercept$coefficients["Michigan"]
```

```
penn_strength <- p_model_with_intercept$coefficients["Penn.State"]

# Extract the covariance matrix
cov_matrix <- vcov(p_model_with_intercept)

# Calculate the standard errors for the differences in strengths
se_diff_penn <- sqrt(cov_matrix["Michigan", "Michigan"] + cov_matrix["Penn.State",
"Penn.State"])

# Calculate the t-statistic
t_stat_penn <- (michigan_strength - penn_strength) / se_diff_penn

# Calculate the degrees of freedom
df_penn <- length(p_model_with_intercept$coefficients) - 1  # Assuming one parameter for each
team

# Calculate the p-value
p_value_penn <- 2 * pt(abs(t_stat_penn), df = df_penn, lower.tail = FALSE)  # Fix the df
argument

# Print results
cat("T-statistic:", t_stat_penn, "\n")
## T-statistic: 0.7193723
cat("Degrees of freedom:", df_penn, "\n")
## Degrees of freedom: 132
cat("P-value:", p_value_penn, "\n")
## P-value: 0.4731832
For Iowa:
# Extract strengths for Michigan and USC
michigan_strength <- p_model_with_intercept$coefficients["Michigan"]
iowa_strength <- p_model_with_intercept$coefficients["Iowa"]

# Extract the covariance matrix
cov_matrix <- vcov(p_model_with_intercept)

# Calculate the standard errors for the differences in strengths
se_diff_iowa <- sqrt(cov_matrix["Michigan", "Michigan"] + cov_matrix["Iowa", "Iowa"])

# Calculate the t-statistic
t_stat_iowa <- (michigan_strength - iowa_strength) / se_diff_iowa
```

```
# Calculate the degrees of freedom
df_iowa <- length(p_model_with_intercept$coefficients) - 1  # Assuming one parameter for each team

# Calculate the p-value
p_value_iowa <- 2 * pt(abs(t_stat_iowa), df = df_iowa, lower.tail = FALSE)

# Apply Bonferroni correction
alpha <- 0.05

# Print results
cat("T-statistic:", t_stat_iowa, "\n")
## T-statistic: 1.316006
cat("Degrees of freedom:", df_iowa, "\n")
## Degrees of freedom: 132
cat("P-value:", p_value_iowa, "\n")
## P-value: 0.1904528
```

For Illionis:

```
# Extract strengths for Michigan and Georgia
michigan_strength <- p_model_with_intercept$coefficients["Michigan"]
illinois_strength <- p_model_with_intercept$coefficients["Illinois"]

# Extract the covariance matrix
cov_matrix <- vcov(p_model_with_intercept)

# Calculate the standard errors for the differences in strengths
se_diff_illinois <- sqrt(cov_matrix["Michigan", "Michigan"] + cov_matrix["Illinois", "Illinois"])

# Calculate the t-statistic
t_stat_illinois <- (michigan_strength - illinois_strength) / se_diff_illinois

# Calculate the degrees of freedom
df_illinois <- length(p_model_with_intercept$coefficients) - 1  # Assuming one parameter for each team

# Calculate the p-value
p_value_illinois <- 2 * pt(abs(t_stat_illinois), df = df_illinois, lower.tail = FALSE)
```

```
# Print results
cat("T-statistic:", t_stat_illinois, "\n")
## T-statistic: 2.112413
cat("Degrees of freedom:", df_illinois, "\n")
## Degrees of freedom: 132
cat("P-value:", p_value_illinois, "\n")
## P-value: 0.03653456
# Create a named vector
p_values <- c("Penn State" = p_value_penn, "Iowa" = p_value_iowa, "Illinois" =
p_value_illinois)

# Apply Bonferroni correction
adjusted <- p.adjust(p_values, "bonferroni")
adjusted
## Penn State.Michigan       Iowa.Michigan   Illinois.Michigan
##           1.0000000           0.5713583           0.1096037
# Extract coefficients for Michigan, USC, and Texas
michigan_coefficient <- p_model_with_intercept$coefficients["Michigan"]
illinois_coefficient <- p_model_with_intercept$coefficients["Illinois"]
penn_coefficient <- p_model_with_intercept$coefficients["Penn.State"]
iowa_coefficient <- p_model_with_intercept$coefficients["Iowa"]

penn_coefficient
## Penn.State
##   1.950914
# Create a data frame with team names and coefficients
teams <- c("Michigan", "Illinois", "Penn.State", "Iowa")
coefficients <- c(michigan_coefficient, illinois_coefficient, penn_coefficient, iowa_coefficient)
df_coefficients <- data.frame(Team = teams, Coefficient = coefficients)

# Create a bar plot
barplot(df_coefficients$Coefficient, names.arg = df_coefficients$Team, col = "skyblue",
        main = "Bradley-Terry Model Coefficients for Teams",
        xlab = "Teams", ylab = "Coefficient")
# Team names
teams <- c("Georgia", "Michigan", "Washington", "Florida State", "Oregon", "Ohio State",
           "Texas", "Alabama", "Missouri", "Penn State", "Mississippi", "Oklahoma",
           "LSU", "Louisville", "Arizona", "Iowa", "Notre Dame", "Oklahoma State",
           "NC State", "Oregon State", "Tennessee", "Tulane", "Clemson", "Liberty", "Kansas
State")
```

```
# Create a data frame with only the team names
team_table <- data.frame(Team = teams)

# Print the table
print(team_table)
##             Team
## 1        Georgia
## 2       Michigan
## 3     Washington
## 4   Florida State
## 5         Oregon
## 6     Ohio State
## 7          Texas
## 8        Alabama
## 9       Missouri
## 10    Penn State
## 11   Mississippi
## 12      Oklahoma
## 13           LSU
## 14    Louisville
## 15       Arizona
## 16          Iowa
## 17    Notre Dame
## 18 Oklahoma State
## 19      NC State
## 20   Oregon State
## 21     Tennessee
## 22        Tulane
## 23       Clemson
## 24       Liberty
## 25   Kansas State
```