IRTForests

Random Forest × Item Response Theory Diagnostics

Andrew T. Scott · Fall 2025

github.com/ascott02/IRTForests

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Random Forest × Item Response Theory

- Trees become respondents, images become items.
- Response matrix records per-tree correctness on held-out examples.
- Goal: explain RF behavior via IRT ability & difficulty signals.

GenAI In the Loop Scientific Exploration

- Started from a concise README spec describing goals, datasets, and desired diagnostics.
- Iterated with notebook + CLI automation to run every experiment end-to-end.
- Generated figures, tables, and reports that surfaced ability/difficulty patterns.
- Translated the workflow into this deck, tightening narrative and visuals each pass.

Motivation & Guiding Questions

- Random forests glue together many weak classifiers; from an IRT lens each tree is a respondent with latent ability (θ).
- Held-out images act as items whose difficulty (δ) we can infer by watching which trees succeed or fail.
- What can θ and δ tell us about dataset quality, backbone choice, and where ambiguity still lives?
- Can these signals guide future studies—pruning weak trees, curating hard items, and iterating faster with gen-AI tooling?

Story Arc

1. **Background:** IRT mechanics + RF diagnostics we rely on.
2. **Pipeline:** Datasets, embeddings, and response matrices powering the studies.
3. **Case Studies:** Baseline CIFAR, MobileNet upgrade, and MNIST control.
4. **Synthesis:** Cross-study comparisons, takeaways, and next steps.

Why Item Response Theory for Random Forests?

- Treat each tree as a “test taker” answering every held-out image.
- Latent **ability** (θ) separates reliable trees from drifted or shallow ones.
- Latent **difficulty** (δ) surfaces mislabeled or ambiguous images without manual review.
- Shared scale lets us compare studies, backbones, and curation strategies apples-to-apples.

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Item Response Theory Building Blocks

Core Terms

- Ability (θ): respondent skill; higher → better odds of a correct answer.
- Difficulty (δ): item hardness; higher → harder even for strong respondents.
- Discrimination (a): slope of the logistic curve near δ.
- Guessing (c): lower bound for multiple-choice exams (rare in our setup).

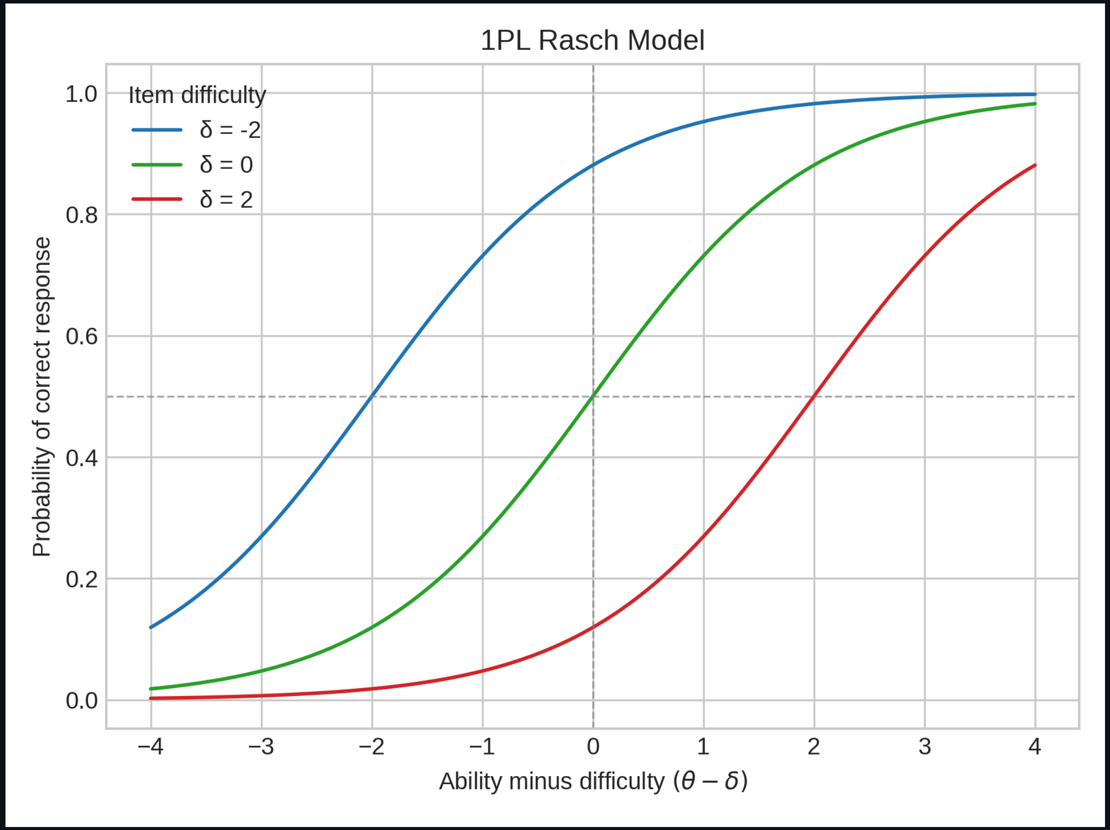
Ensemble Analogy

- Respondents → decision trees evaluated on a common test set.
- Items → images; responses are binary (tree got it right?).
- Response matrix $R_{ij} \in \{0, 1\}$ fuels variational IRT fitting.
- Outputs: distributions over θ_i and δ_j plus information curves.

Rasch (1PL) Model in One Picture

$$\Pr(R_{ij} = 1 \mid \theta_i, \delta_j) = \frac{1}{1 + e^{-(\theta_i - \delta_j)}}$$

- Single global slope ensures parameters live on a shared logit scale.
- $(\theta - \delta) = 0 \Rightarrow$ 50% chance of success; shifts left/right flip odds.
- Fisher information peaks where curves are steepest → ideal for spotting uncertain regions.
- IRT ICC Visualizer



1PL logistic curves for items of varying difficulty

What We Extract from IRT

- **Ability histograms**: flag inconsistent or low-skill trees worth pruning.
- **Difficulty ladders**: highlight ambiguous or mislabeled items for relabeling.
- **Wright maps**: overlay θ and δ to see coverage gaps.
- **Information curves**: identify where ensemble confidence is fragile.
- These diagnostics complement RF metrics by focusing on *who* struggles and *why*.

Random Forest Mechanics (Deeper Dive)

Bootstrap Aggregation

- Sample n examples with replacement per tree; leave-one-out OOB estimates follow.
- Feature subsampling at each split decorrelates trees and trims variance.
- Majority vote or probability averaging reduces variance relative to any single tree.

Hyperparameters that Matter

- n_{trees} controls variance reduction ceiling.
- Max depth / min samples per leaf shape bias vs variance.
- Max features governs how diverse each tree becomes.
- Class weights & sample balancing steer towards rare classes.

CART Splits & Gini Impurity

- Trees grow by selecting the feature/threshold minimizing impurity after the split.
- **Gini impurity** for node t with class proportions p_k:

$$G(t) = 1 - \sum_k p_k^2$$

- Split gain: $\Delta G = G(t) - \sum_{child} \frac{|t_{child}|}{|t|} G(t_{child})$.
- Alternative is entropy $H(t) = -\sum_k p_k \log p_k$; we track both for diagnostics.
- Deep nodes with tiny $|t|$ overfit; pruning or depth limits keep signals meaningful.

Margins, Entropy, and Ensemble Confidence

- **Margin**: $m(x) = P(\hat{y} = y_{true}) - \max_{c \neq y_{true}} P(\hat{y} = c)$.
  - Near 0 → ambiguous votes; negative → systematic misclassification.
- **Entropy** over class probabilities captures total disagreement across trees.
- Pairing $m(x)$ and entropy with δ spots mislabeled or out-of-distribution examples.
- Track margin trajectories per item to measure progress after curation.

Variable Importance Playbook

- **Mean decrease in impurity (MDI)**: sum of Gini drops attributable to each feature.
- **Permutation importance**: shuffle feature k, re-score; larger drops → higher reliance.
- **SHAP / local attributions**: optional, clarify per-item influence.
- Cross-study comparison of importance vectors reveals when new embeddings truly shift focus.
- Coupled with IRT, we can ask whether hard items lack salient features or trees misuse them.

Pipeline Overview

Data Prep (done)

- Stratified CIFAR-10 subset: 10k train / 2k val / 2k test.
- Resize 64×64, normalize, PCA → 128-D embeddings (plus MobileNet-V3 cache).
- MNIST mini: 4k / 800 / 800 digits, normalized 28×28 grayscale, flattened to vectors.
- Cached artifacts in `data/cifar10_subset.npz`, `data/cifar10_embeddings.npz`, and `data/mnist/mnist_split.npz`.

Modeling Status

- RF (200 trees) trained for each study; metrics + importances saved.
- Response matrices persisted: CIFAR `(200 × 2000)` for PCA & MobileNet, MNIST `(200 × 800)`.
- 1PL Rasch fit (SVI, 600 epochs) complete for CIFAR; MNIST run mirrors the pipeline with shared notebooks.

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Dataset Overview

Dataset	Train	Val	Test	Feature Pipeline	Notes
CIFAR-10 subset	10,000	2,000	2,000	64×64 RGB → PCA-128 or MobileNet-V3 (960-D)	Shared splits across Study I & II
MNIST mini	4,000	800	800	28×28 grayscale → raw pixels (no PCA)	Sanity check for clean handwriting

- All studies reuse cached artifacts under `data/` for reproducibility.
- CIFAR runs differ only in embedding backbone (PCA vs MobileNet); labels & splits stay fixed.
- MNIST mini-study mirrors the workflow to confirm signals transfer to simpler data.

Section I · Baseline Study (CIFAR + PCA)

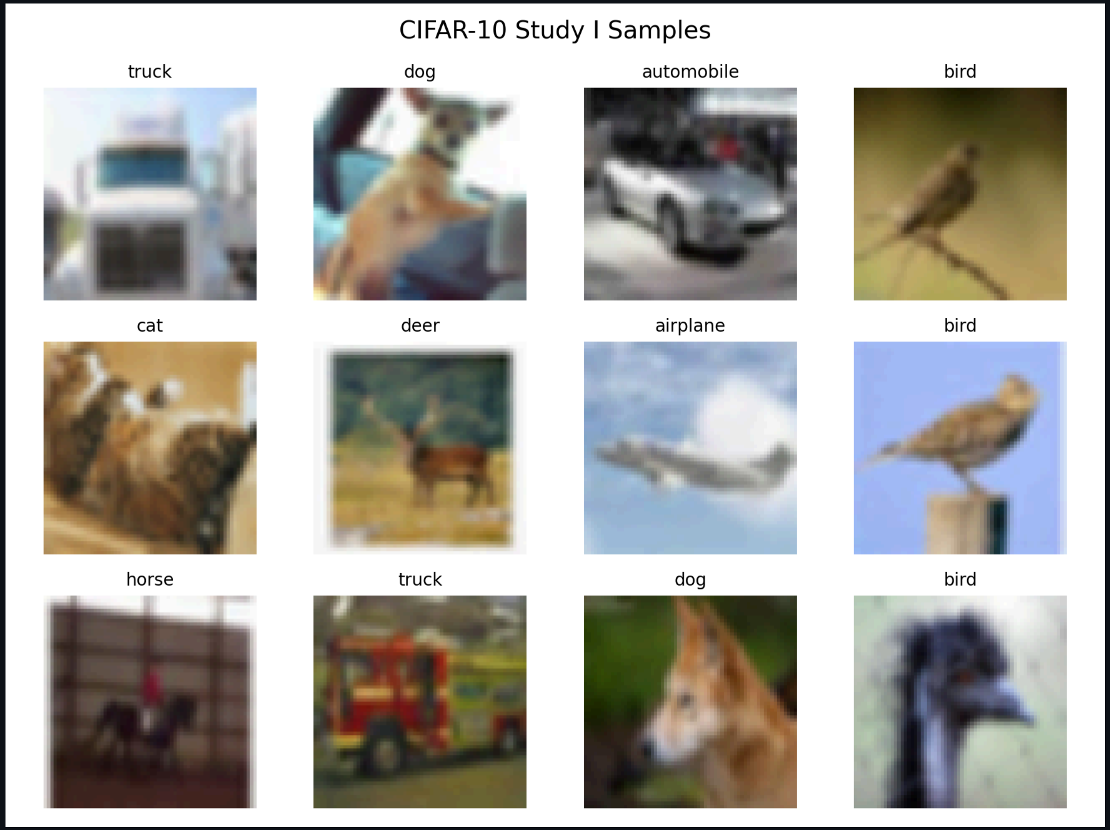
- Establish reference performance with lightweight PCA embeddings.
- Inspect how IRT parameters align with classic RF uncertainty signals.
- Identify pain points to motivate stronger features.

Study I: CIFAR-10 + PCA-128 Embeddings

- Baseline vision setup: 64×64 resize + PCA to 128 dims.
- 200-tree Random Forest trained on embeddings; response matrix size 200 × 2000.
- Use this run to introduce IRT diagnostics and identify weak spots.

Study I Setup: CIFAR-10 + PCA-128

- CIFAR-10 subset (10k / 2k / 2k) with stratified sampling and fixed seed.
- Preprocess: resize 64×64, normalize, PCA → 128-D embeddings (`data/cifar10_embeddings.npz`).
- Response matrix shape 200 × 2000 with mean tree accuracy 0.176.
- Artifacts: metrics, margins, entropy, and IRT outputs stored under `data/` & `figures/` root.



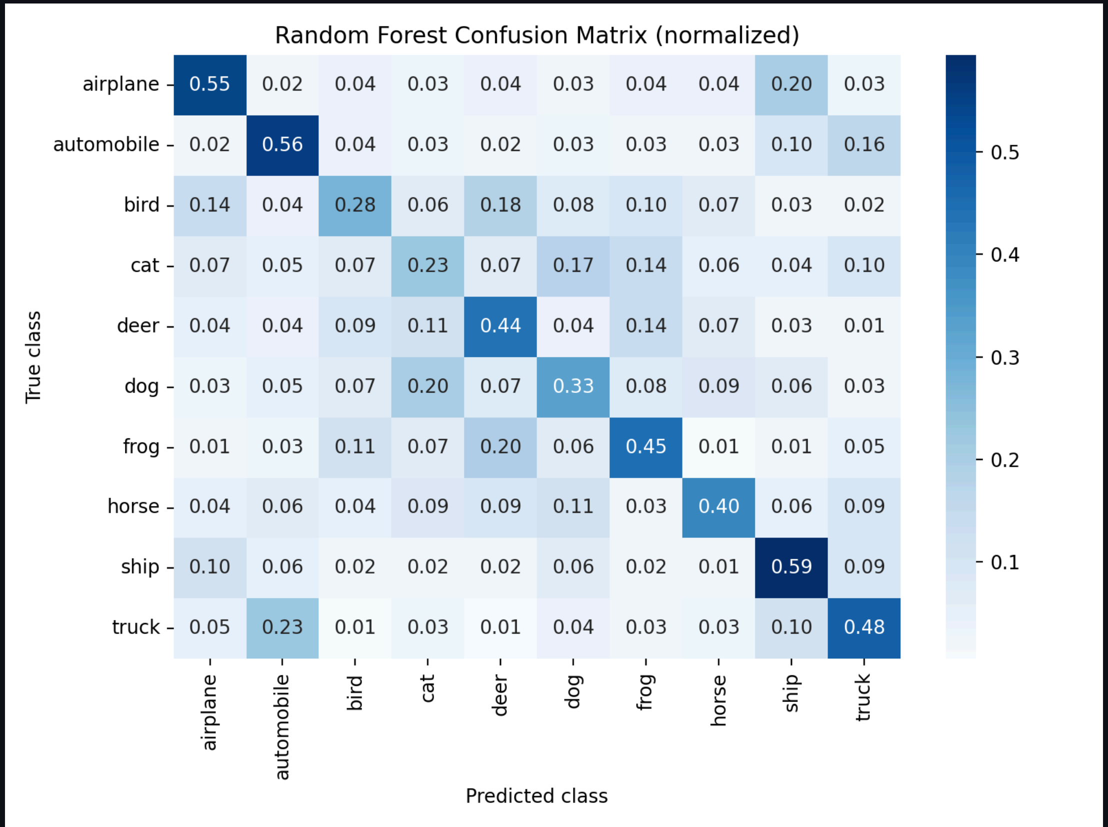
Study I sample grid — stratified CIFAR-10 slices

Study I Performance (PCA-128)

Metric	Value
Test / Val / OOB acc	0.4305 / 0.4145 / 0.3730
Per-class range	0.225 (cat) → 0.595 (ship)
Mean tree accuracy	0.1759
Mean margin / entropy	−0.0028 / 2.1503
δ ↔️ margin (Pearson)	−0.8286
δ ↔️ entropy (Pearson)	0.6782

- Baseline ensemble underperforms due to weak PCA features yet preserves δ alignment.
- Low mean margin + high entropy indicate broad tree disagreement → fertile ground for IRT.
- Artifacts: metrics (`data/rf_metrics.json`), confusion (`data/rf_confusion.npy`), importances, permutations.

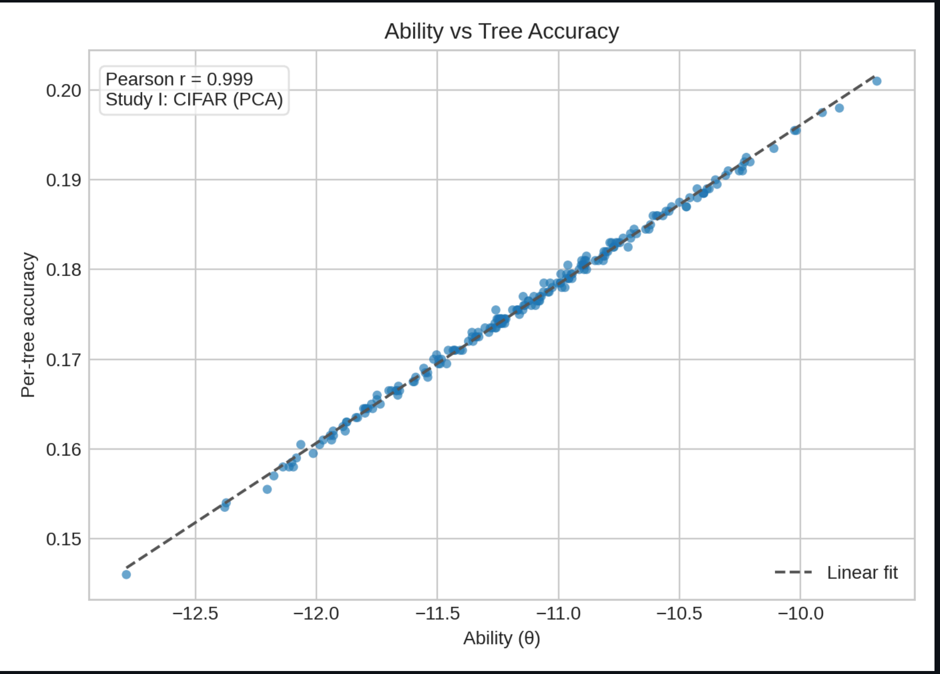
Study I Confusion Matrix



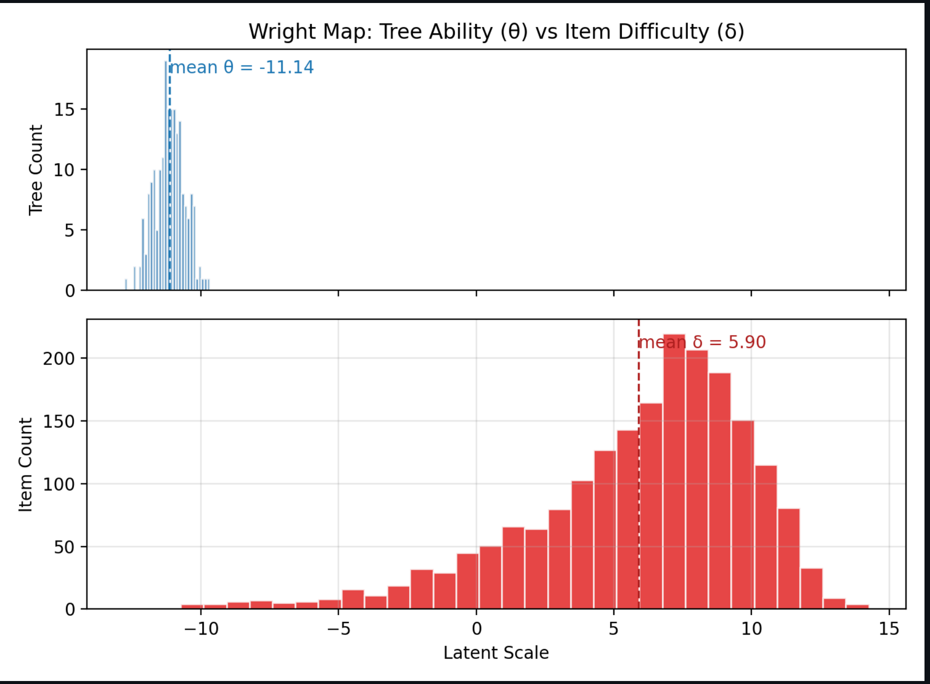
Reading the matrix

- High off-diagonal mass for cat ↔️ dog, bird ↔️ airplane, horse ↔️ deer.
- Ships and trucks maintain >80% normalized diagonal despite shared structure.
- Hotspots align with IRT δ spikes (slides that follow), signalling data curation targets.

Study I Diagnostics: Ability Profiles



Ability (θ) vs tree accuracy — Spearman ≈ 0.99



Wright map: θ cluster near −11; δ stretches to 14

- Trees with θ above −10 outperform peers by ~3 pp even with PCA features.
- Long tail of low-ability trees (< −11.5) drags ensemble accuracy; pruning candidates.
- Wright map shows limited θ spread versus broad δ tail → feature quality bottleneck.

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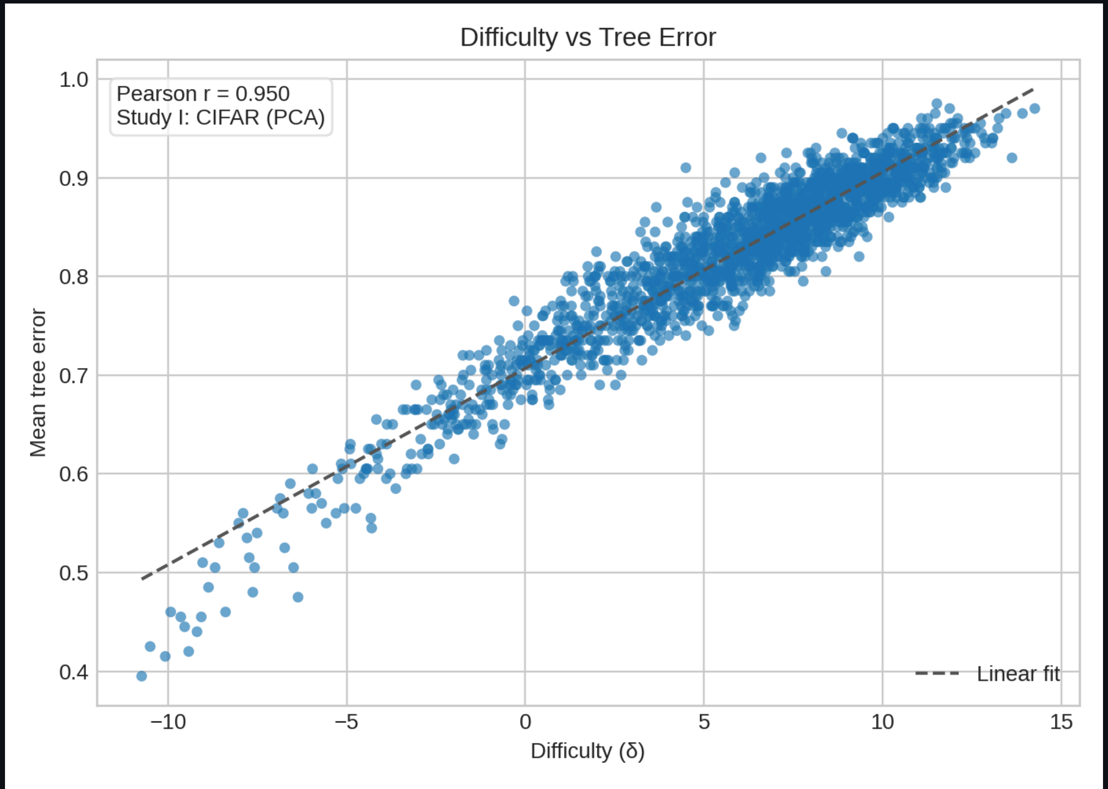
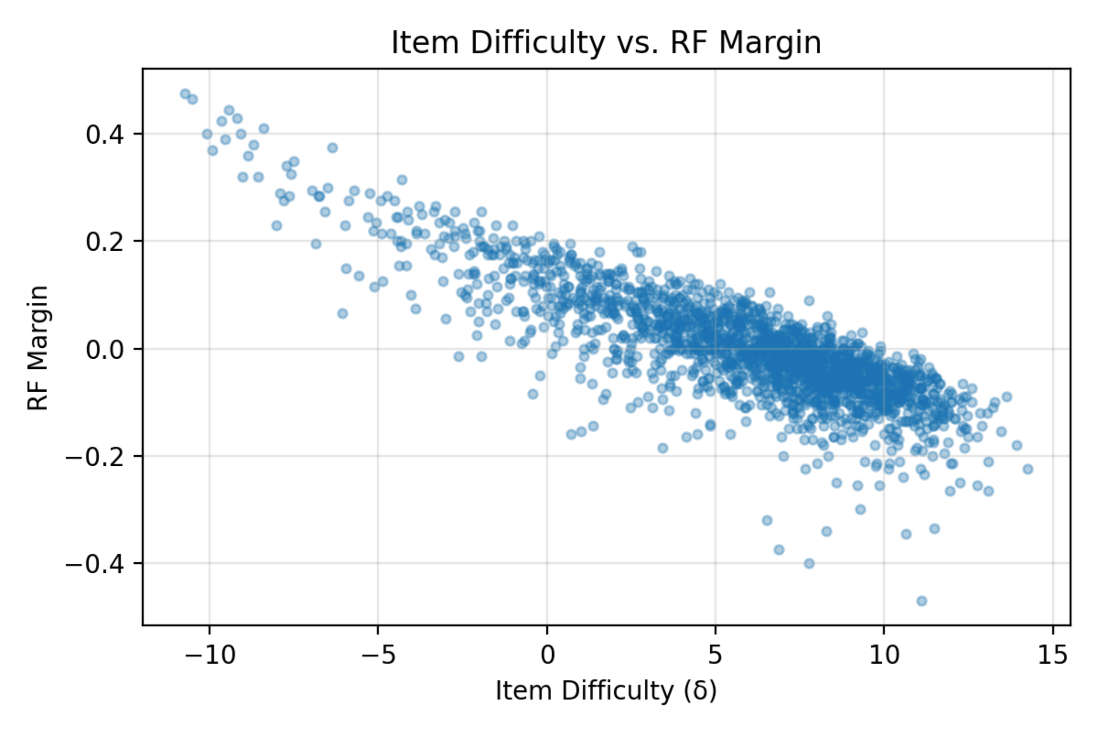
Study I Diagnostics: δ vs Error Rate

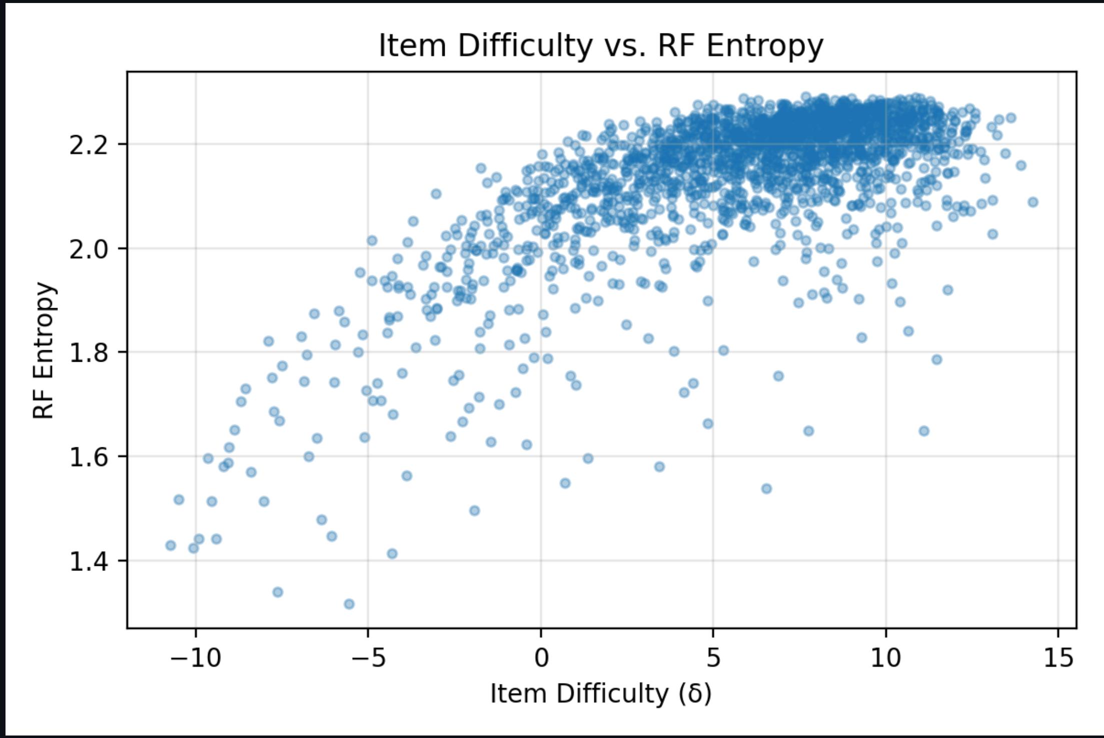


- δ > 10 corresponds to averaged tree error >80%, mostly ambiguous animals.
- Items with δ < 0 are “free points” — nearly every tree agrees.
- Pearson ≈ 0.95, Spearman ≈ 0.94. Difficulty doubles as an error heat-map.

Study I Diagnostics: δ vs RF Signals



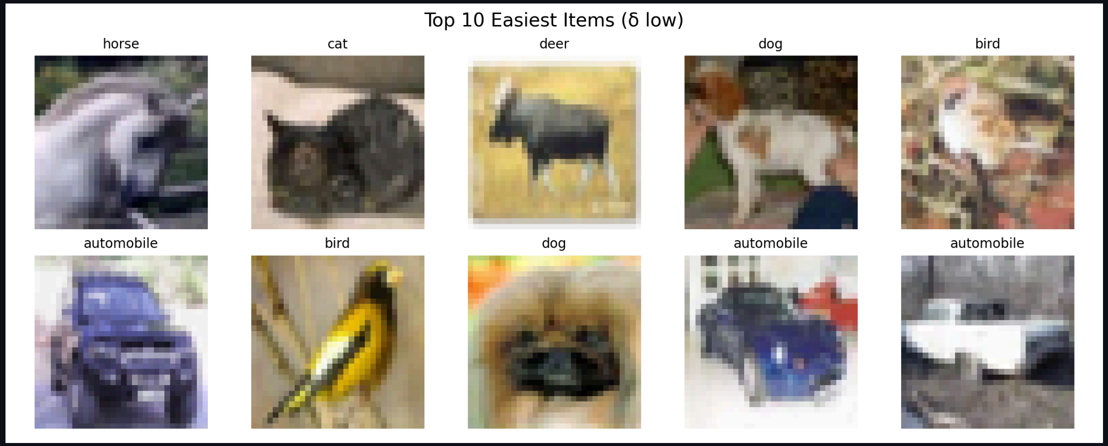
PCA run: δ vs margin (Pearson −0.83)



PCA run: δ vs entropy (Pearson 0.68)

- Hard items cluster bottom-right (low margin, high entropy) → ripe for relabeling or augmentation.
- Opposite corner contains “easy wins” with positive margin and low entropy.
- Study II mirrors these plots with MobileNet features, pushing |corr| above 0.80

Study I Evidence: Hard vs Easy Examples



- Hardest items skew toward ambiguous airplane/ship silhouettes and cluttered cat/dog scenes.
- Notice recurring mislabeled-looking ships ($\delta \approx 14$) flagged for manual review.
- Easy set dominated by deterministic cues (red fire trucks, high-contrast ships) → low δ and entropy.

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Study I Takeaways

- Weak PCA features create long tails in both ability (θ) and difficulty (δ), exposing erratic trees.
- Margin and entropy correlate with δ, but clusters of high-difficulty animals persist across diagnostics.
- Visual inspection confirms mislabeled or low-signal items driving high δ, motivating feature upgrades.

Section II · Feature-Rich CIFAR (MobileNet)

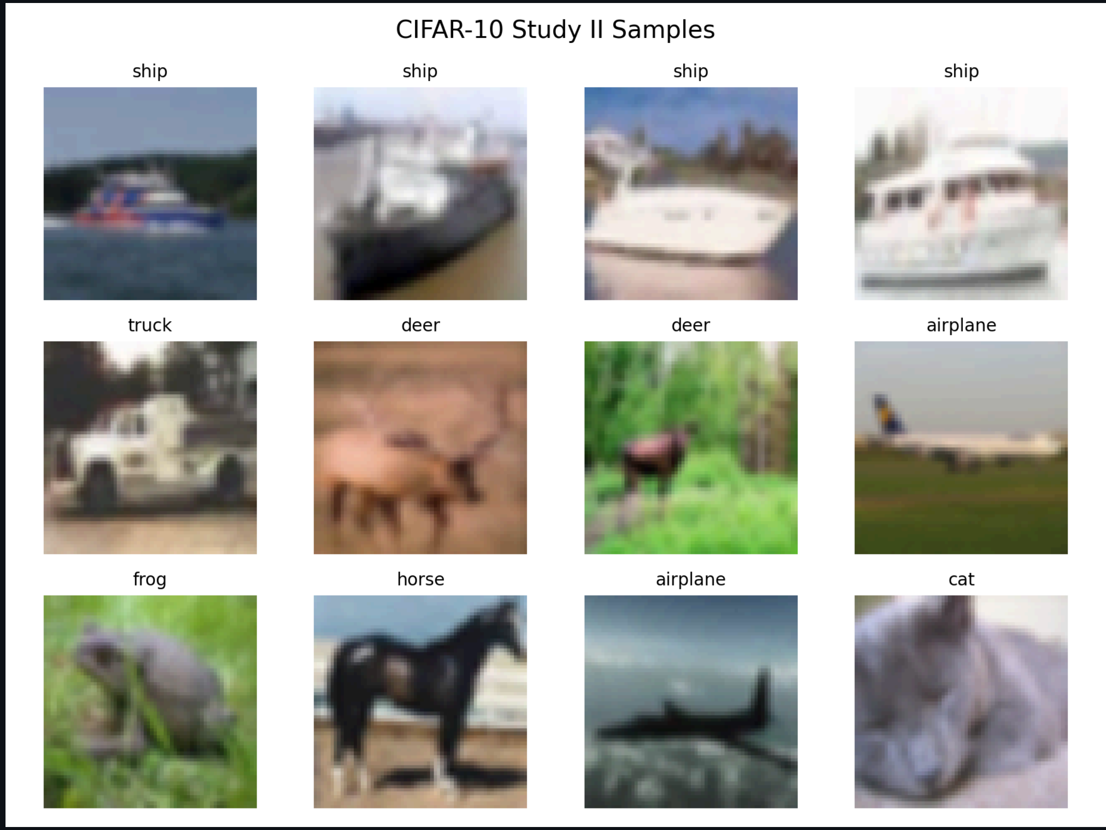
- Hold data splits constant to isolate backbone improvements.
- Expect tighter ability spread and stronger δ alignment with RF confidence.
- Validate whether ambiguous animal classes persist after feature upgrade.

Study II: CIFAR-10 + MobileNet Embeddings

- Swap PCA features for MobileNet-V3 (960-D) while keeping tree count and splits constant.
- Measure how richer features alter RF metrics, margins/entropy, and IRT parameter spreads.
- Use as a reality check before expanding to new datasets.

Study II Setup: CIFAR-10 + MobileNet-V3

- Reuse CIFAR-10 subset splits from Study I to isolate feature effects.
- Extract 960-D embeddings from pretrained MobileNet-V3 Small (`data/cifar10_mobilenet_embeddings.npz`).
- Response matrix shape 200 × 2000 with mean tree accuracy 0.482.
- Dedicated artifacts: `data/mobilenet/*`, plots in `figures/mobilenet/`.



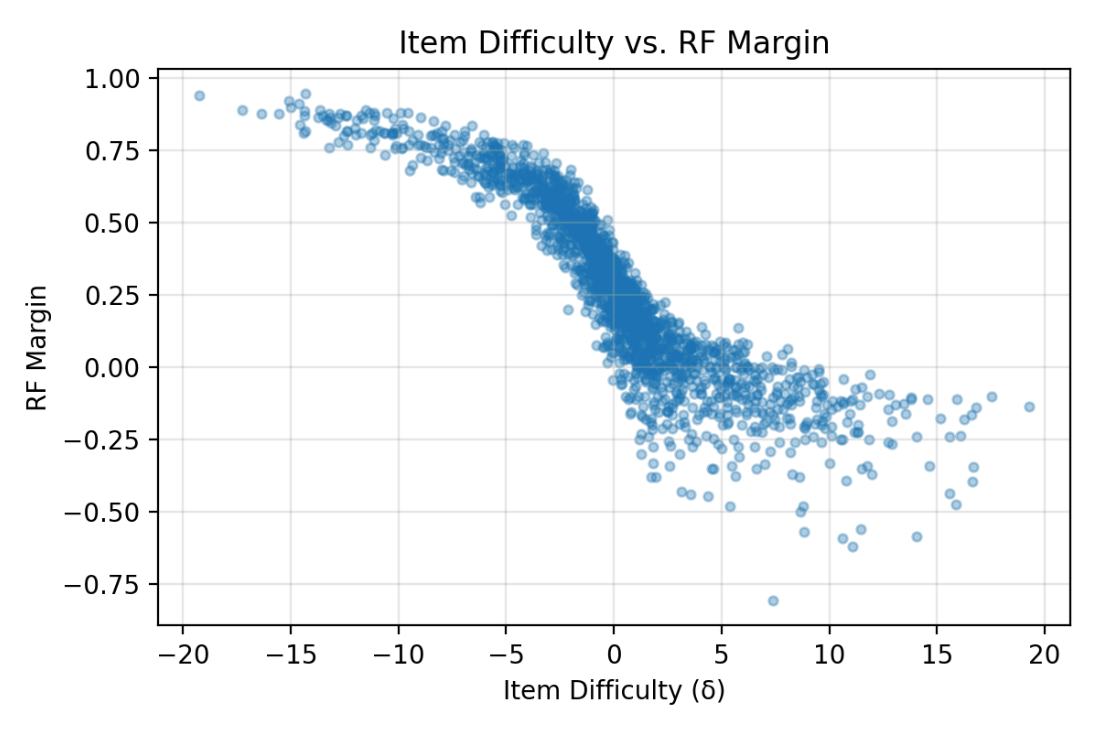
Study II sample grid — same splits, MobileNet embeddings

Study II Performance (MobileNet-V3)

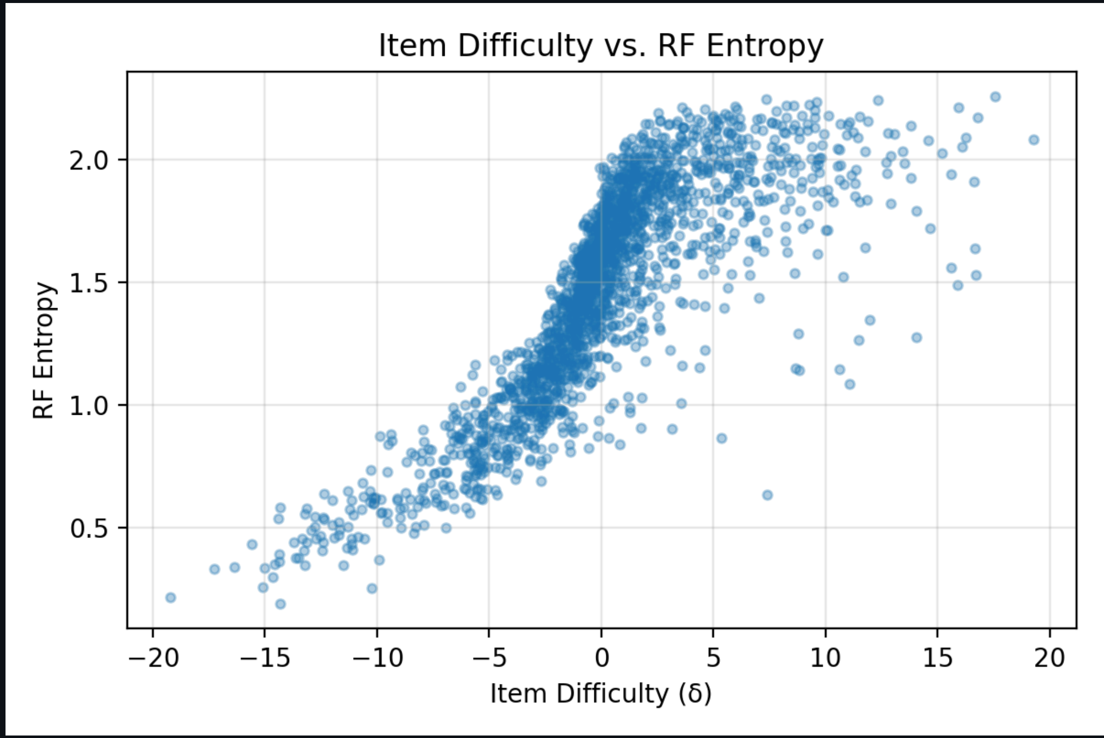
Metric	Value
Test / Val / OOB acc	0.8090 / 0.8135 / 0.7967
Per-class range	0.68 (cat) → 0.915 (ship)
Mean tree accuracy	0.4817
Mean margin / entropy	0.2806 / 1.4663
δ ↔️ margin (Pearson)	−0.8825
δ ↔️ entropy (Pearson)	0.8113

- Pretrained features boost accuracy by 37 pp while strengthening δ correlations.
- Higher margins + lower entropy show confidence gains except on stubborn animal classes.
- Artifacts live under `data/mobilenet/` (metrics, response matrix, signals, IRT outputs).

Study II Diagnostics: δ vs RF Signals



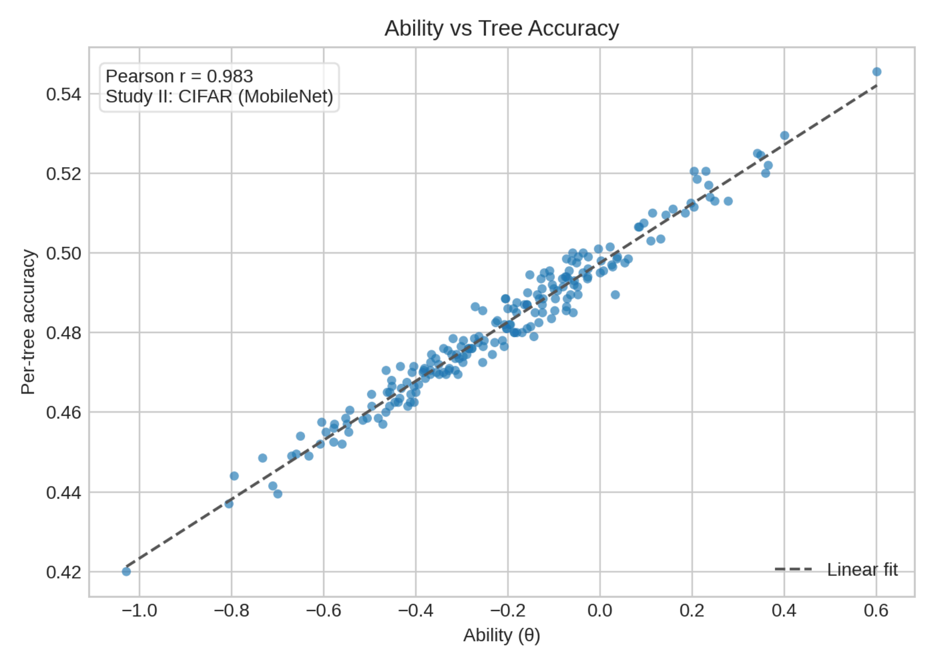
δ vs margin (Pearson −0.88)



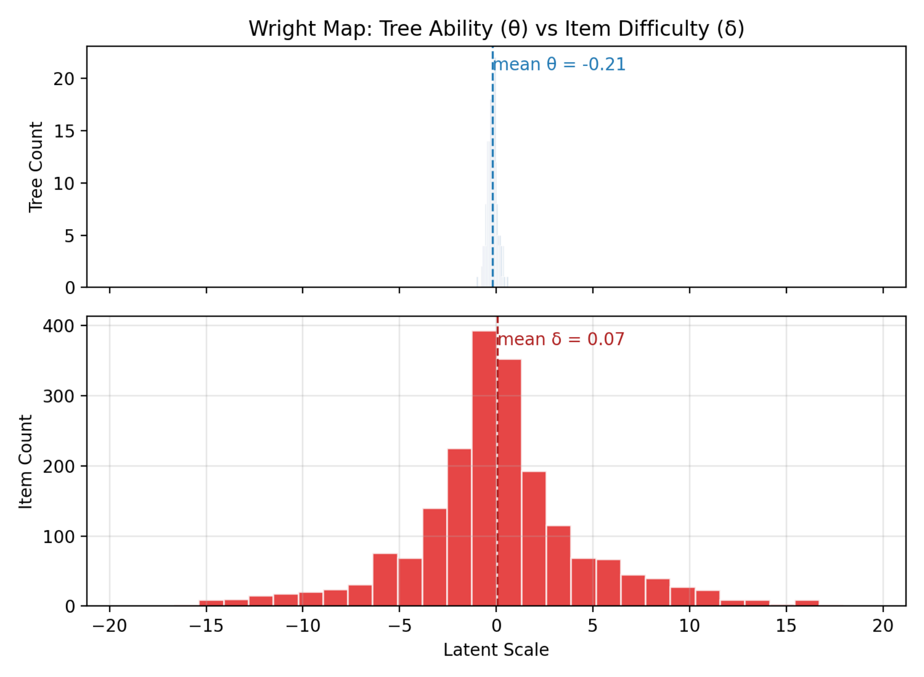
δ vs entropy (Pearson 0.81)

- MobileNet compresses the easy cluster (high margin, low entropy) while isolating true hard cases.
- Higher correlation magnitudes indicate better alignment between δ and RF uncertainty signals.
- Cat/dog confusions persist despite stronger features → candidates for targeted curation

Study II Diagnostics: Ability Profiles



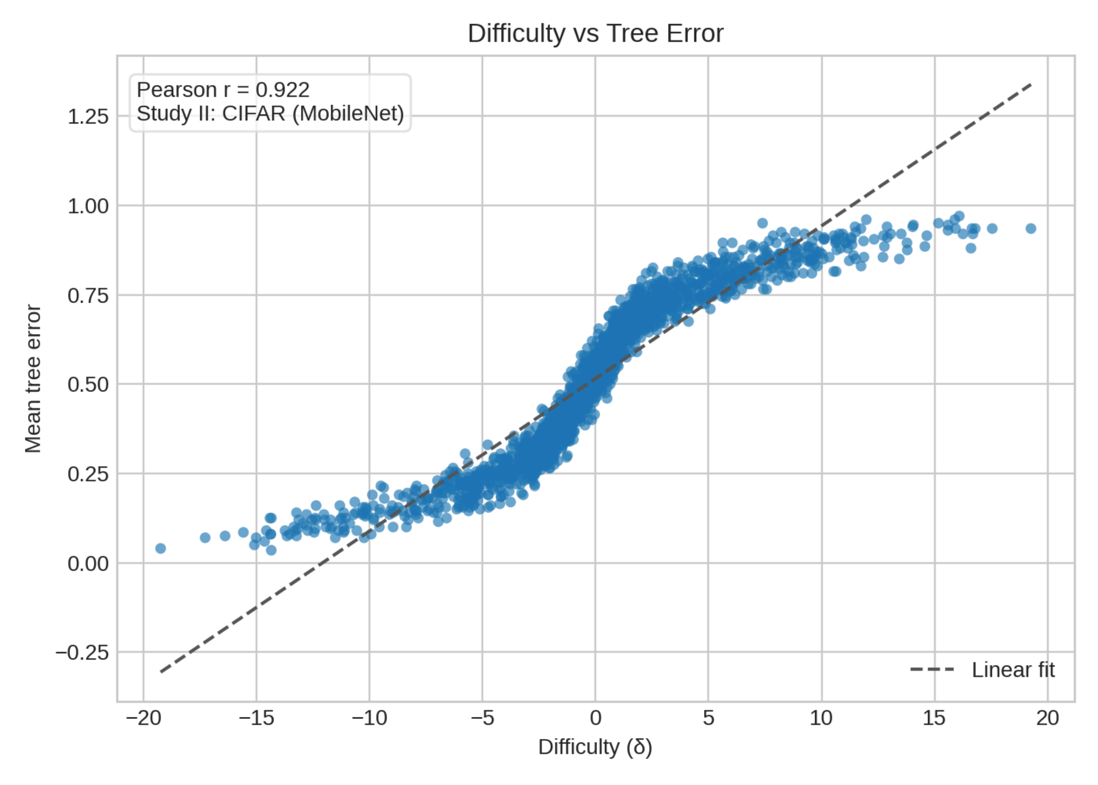
Ability (θ) vs tree accuracy — Pearson 0.983



Wright map: θ variance shrinks to 0.25

- θ mean −0.21 ± 0.25: trees cluster closer together than PCA baseline (σ 0.55 → 0.25).
- Ability remains tightly coupled to per-tree accuracy; even weakest trees clear 40%.
- Shared axis shows overlap where confident trees meet easy airplane/ship items.
- Ability compression signals that feature quality, not tree diversity, now limits performance.

Study II Diagnostics: δ vs Error Rate



- Pearson 0.922: δ remains strongly aligned with mean tree error despite higher accuracy ceiling.
- Hardest items (δ > 8) persist from PCA run — mostly cat/dog overlaps and ambiguous aircraft.
- Easy zone (δ < −3) expands, showing MobileNet features unlock more “free points.”

Study II Takeaways

- MobileNet embeddings boost accuracy by 37 pp while collapsing ability variance ($\sigma\theta$ 0.55 → 0.25).
- δ remains correlated with RF uncertainty, concentrating hard cases into a smaller ambiguous cluster.
- Residual cat/dog confusion suggests future gains must come from data curation, not just features.

Section III · Control Study (MNIST)

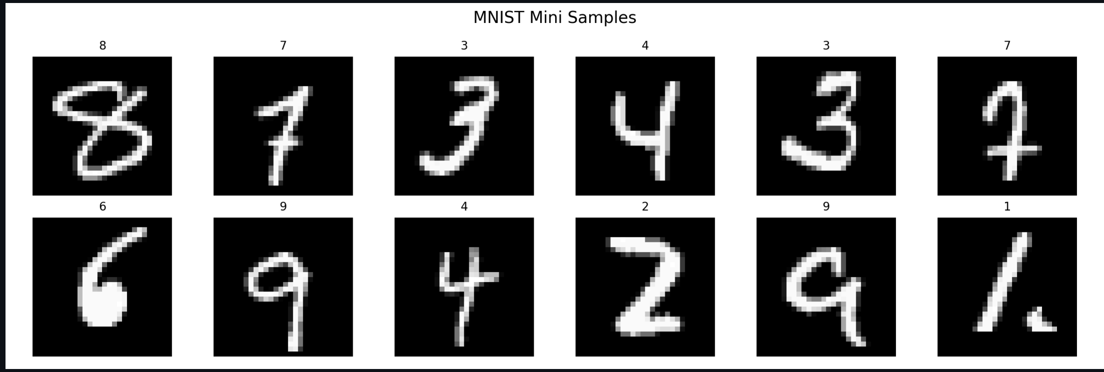
- Probe pipeline behavior on a high-signal, low-noise dataset.
- Confirm that IRT still mirrors RF uncertainty when accuracy is near perfect.
- Use as guardrail before applying to additional tabular or vision datasets.

Study III: MNIST Mini-Study

- Lightweight handwriting dataset to validate RF × IRT beyond CIFAR-10.
- Serves as control: simpler classes, higher accuracy, clearer δ separation.
- Highlights how pipeline behaves when ambiguity is rare but still detectable.

Study III Setup: MNIST Mini-Study

- Split 4k / 800 / 800 digits with stratified sampling and fixed seed.
- Minimal preprocessing: 28×28 grayscale flattened; no augmentation.
- Random Forest (200 trees) trained on raw pixels; response matrix shape 200 × 800.
- Artifacts stored under `data/mnist/` with plots in `figures/mnist/`.



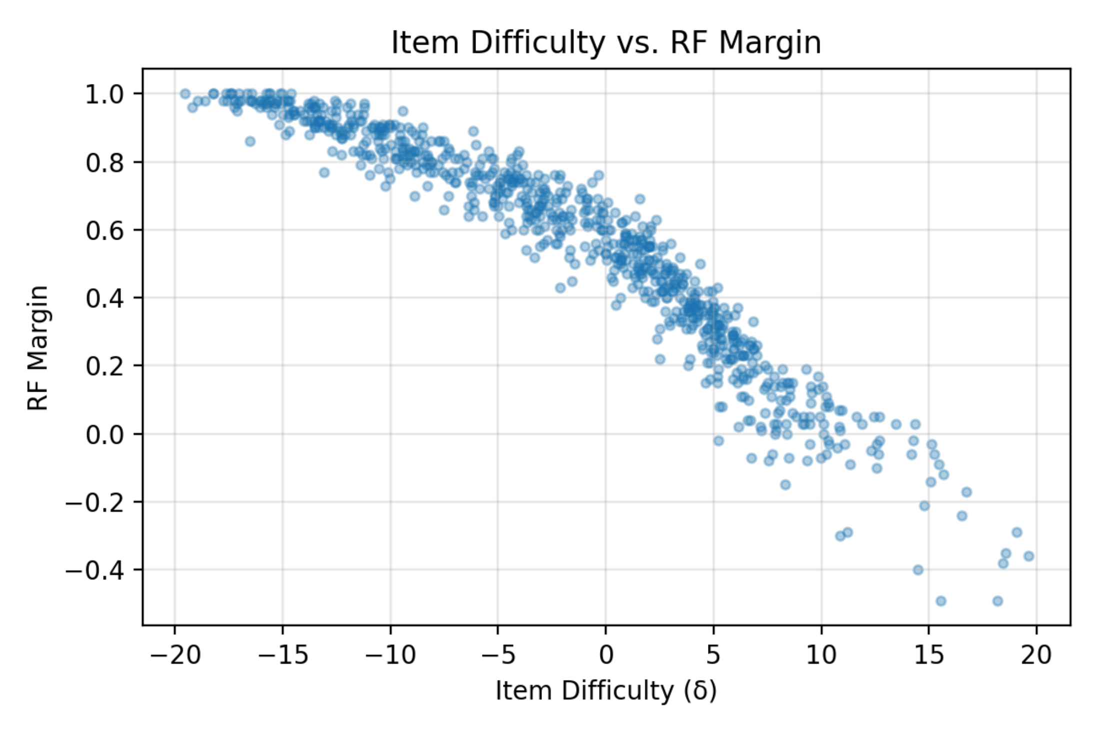
Study III sample grid — curated MNIST mini split

Study III Performance (MNIST)

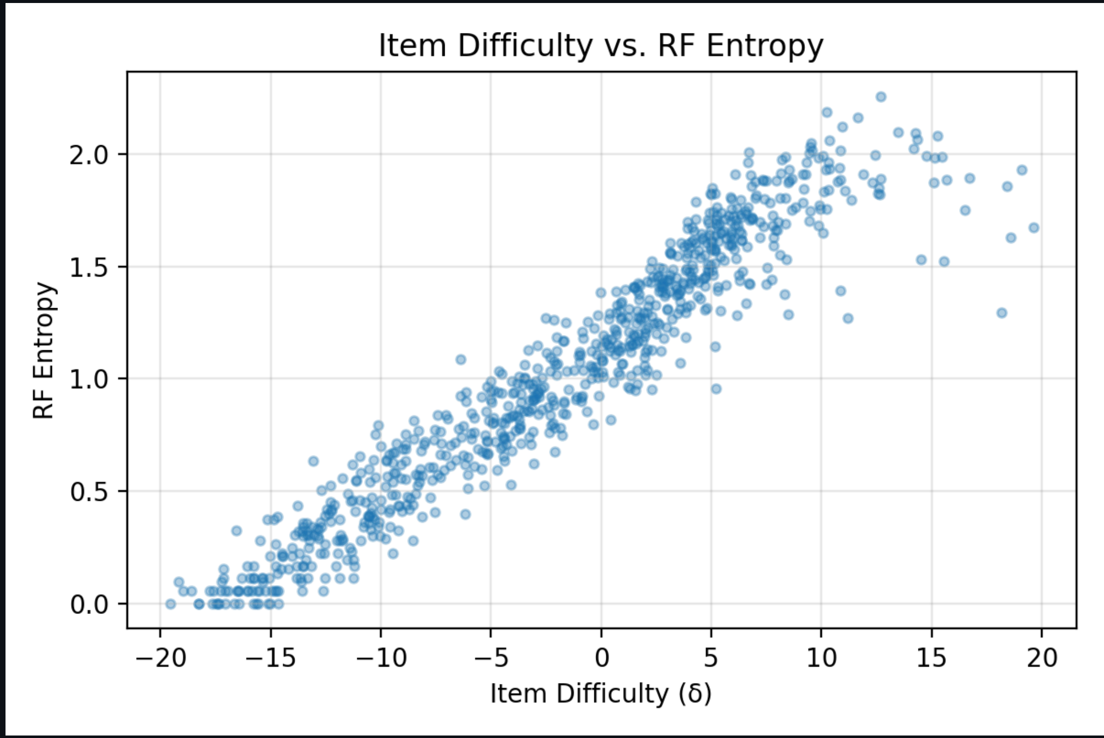
Metric	Value
Train / Val / Test	4000 / 800 / 800
RF test / val / OOB	0.9475 / 0.9413 / 0.9140
Mean margin / entropy	0.5546 / 1.0351
δ ↔️ margin (Pearson)	−0.950
δ ↔️ entropy (Pearson)	0.958
θ mean $\pm$ σ	4.23 ± 0.44
δ mean $\pm$ σ	−1.75 ± 8.19

- Ambiguous digits (e.g., brushed 5 vs 6) spike δ toward ±20; trees vote confidently elsewhere.
- Reinforces link between low entropy, high margin, and low δ on clean handwriting data.
- Provides a “sanity benchmark” to validate the RF × IRT pipeline outside CIFAR.

Study III Diagnostics: δ vs RF Signals



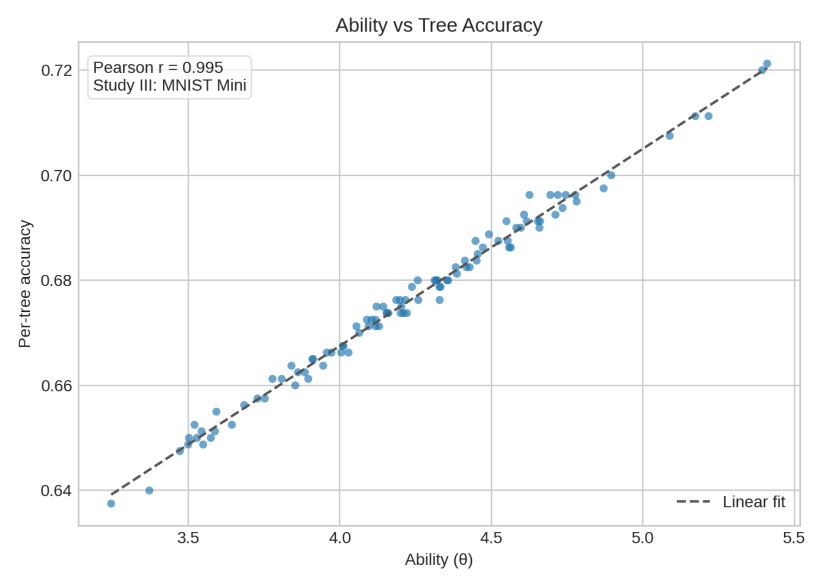
δ vs margin (Pearson −0.95)



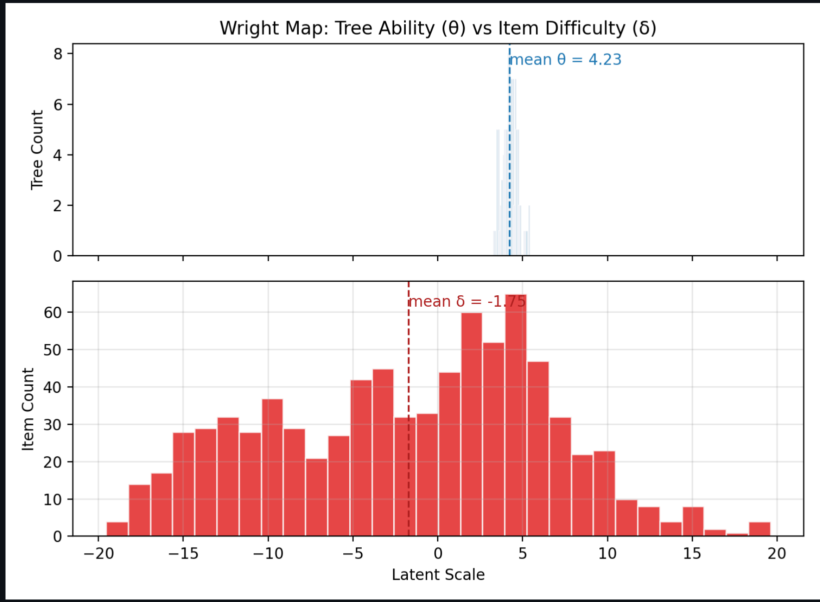
δ vs entropy (Pearson 0.96)

- Clean digits show near-perfect alignment between IRT difficulty and RF uncertainty signals.
- Low scatter indicates only a handful of items drive ensemble uncertainty.
- Marks δ > 12 digits for manual audit (stroke collisions, 3 vs 5, 4 vs 9).

Study III Diagnostics: Ability Profiles



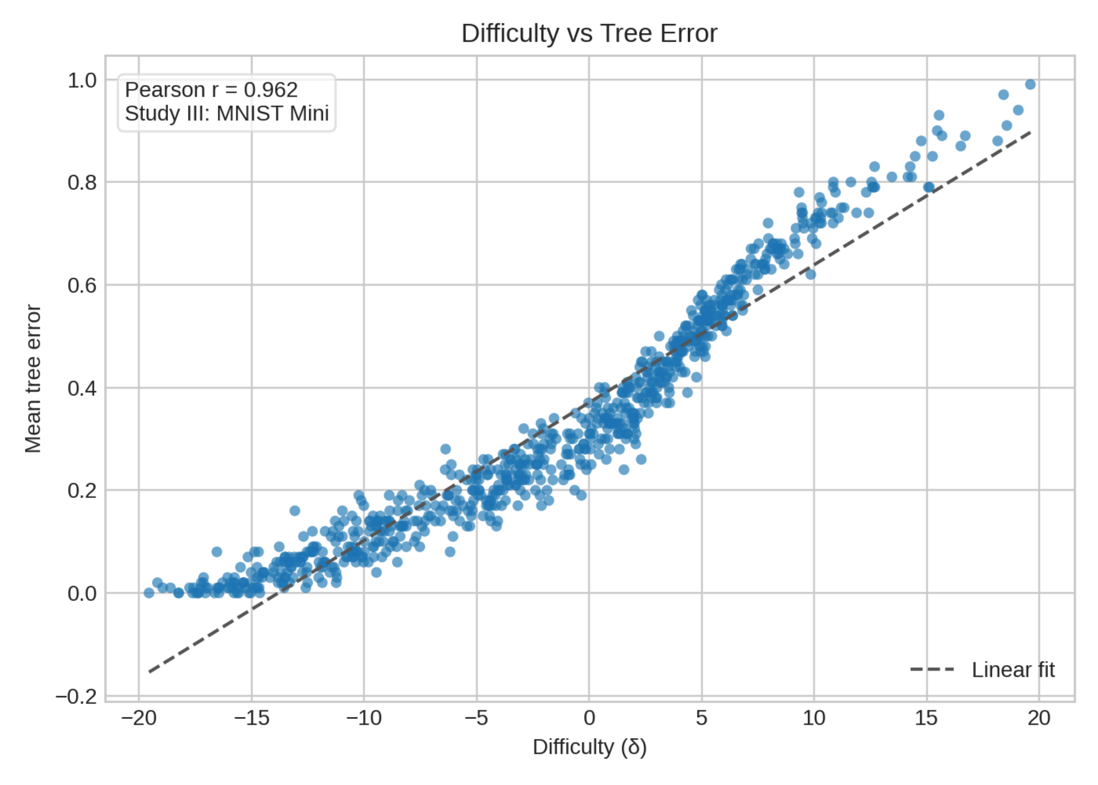
Ability (θ) vs tree accuracy — Pearson 0.995



Wright map: θ mean 4.23 ± 0.44; δ mean −1.75 ± 8.19

- θ mean 4.23 ± 0.44: trees quickly separate easy digits, reflecting high consensus.
- δ mean −1.75 ± 8.19 with heavy tails on ambiguous strokes.
- Shared scale shows abundant overlap → most items are easy wins with a few hard spikes.
- Provides contrast against CIFAR studies where ability mass sat below zero.

Study III Diagnostics: δ vs Error Rate



- Pearson 0.962: δ spikes pinpoint the rare ambiguous digits despite overall high accuracy.
- Outliers (δ > 12) correspond to stroke-collided 3/5/8 and 4/9 pairs flagged for curation.
- Long negative tail shows the majority of digits are trivial for the ensemble.

Study III Takeaways

- Clean digits yield near-perfect agreement between δ and RF uncertainty metrics.
- Ability scores stay high yet retain enough variance to flag the rare ambiguous strokes.
- Control study validates that the RF × IRT pipeline generalizes beyond noisy vision data.

Section IV · Cross-Study & Diagnostics

- Compare backbones and datasets on a shared θ/δ scale.
- Surface themes that repeat across studies before diving into supporting diagnostics.
- Set the stage for consolidated takeaways and action items.

Cross-Study Snapshot

Study	Feature Backbone	Test Acc	δ ↔️ margin (Pearson)	δ ↔️ entropy (Pearson)	θ σ	δ σ
Study I: CIFAR + PCA-128	PCA-128	0.4305	−0.8286	0.6782	0.55	4.10
Study II: CIFAR + MobileNet	MobileNet-V3 (960-D)	0.8090	−0.8825	0.8113	0.25	4.67
Study III: MNIST Mini	Raw pixels	0.9475	−0.950	0.958	0.44	8.19

- Feature backbone drives both accuracy gains and δ alignment strength.
- θ variance collapses with MobileNet (0.25) indicating tree consistency; MNIST keeps moderate spread despite high accuracy.
- MNIST δ σ expands to 8.19, highlighting rare but extreme digit ambiguities versus CIFAR's visual noise.

Key Takeaways

- IRT mirrors random forest uncertainty: θ aligns with per-tree accuracy and δ with item error across every study.
- Feature backbones reshape the θ/δ landscape—MobileNet curbs tree variance while preserving a hard-item tail.
- Combining δ with margins and entropy cleanly triages ambiguous animal classes without manual inspection.
- Control datasets like MNIST confirm the pipeline generalizes beyond noisy vision data before we branch out further.

Next Steps

- Expand notebooks to auto-export the comparison tables and montage panels featured here.
- Run planned 2PL/3PL experiments (see `reports/discrimination_analysis_plan.md`) to capture discrimination effects.
- Correlate tree ability with structural traits (depth, leaf count) to prioritize pruning or retraining.
- Scale the δ + margin triage to curate ambiguous CIFAR items and validate on upcoming tabular studies.

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Appendix · Extended Diagnostics

- Supplemental slides for reference during Q&A or deep dives.
- Includes tabular baselines, training curves, and class-level breakdowns.
- Safe to skip on first pass; revisit as questions arise.

IRT Fit (Study I Baseline, 1PL 600 epochs)

- Optimizer: Adam lr=0.05, SVI Trace_ELBO, seed=7.
- Final loss: **1.50M** (down from 165M at init).
- Tree ability (θ): mean −11.14, σ 0.55, range [−12.79, −9.68].
- Item difficulty (δ): mean 5.90, σ 4.10, range [−10.74, 14.26].
- Correlations — ability ↔️ tree accuracy **0.999**, difficulty ↔️ item error **0.950**.
- Cross-check: embedding & MNIST tables confirm these correlations persist across datasets.

Diagnostic JSON: `data/irt_summary.json`, extremes in `data/irt_extremes.json`.

Edge Cases Across Datasets

- **CIFAR-10 (PCA):** δ tail contains grayscale ships + occluded pets. Margin < −0.2, entropy > 2.2.
- **CIFAR-10 (MobileNet):** Outliers shrink but persist for cat/dog overlap; δ still > 8 despite cleaner features.
- **MNIST:** High δ digits stem from stroke noise (e.g., 9 vs 4). Entropy jumps above 1.9 only for these cases.
- Actionable: focus audits on items with δ > 8 + low margins; they recur across embeddings.

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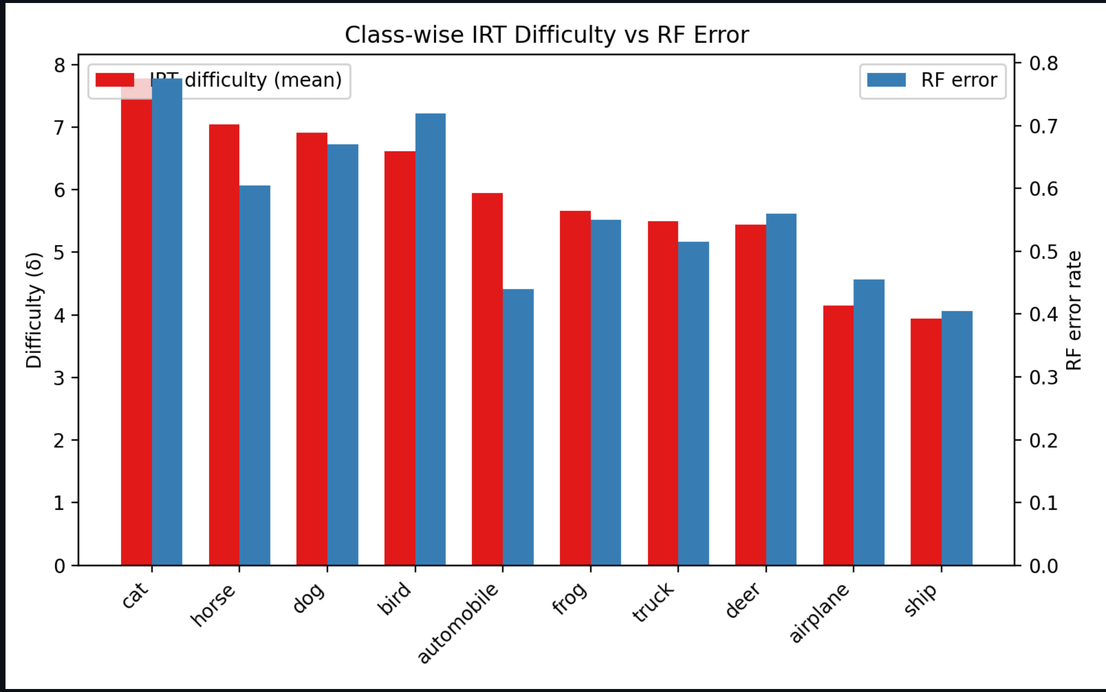
Class Difficulty vs RF Error

- Cats, horses, dogs exhibit δ ≈ 7–8 with RF error ≥ 0.60, marking priority classes for curation.
- Ships and airplanes remain easiest: δ ≈ 4 with RF error ≤ 0.46.
- Aligning δ with RF error spotlights where ensemble uncertainty mirrors misclassification hotspots.



Training Loss & Distributions