# IRTForests

**Random Forest × Item Response Theory Diagnostics**

Andrew T. Scott · Fall 2025

github.com/ascott02/IRTForests

# Random Forest × Item Response Theory

- Trees become respondents, images become items.
- Response matrix records per-tree correctness on held-out examples.
- Goal: explain RF behavior via IRT ability & difficulty signals.

## GenAI In the Loop Scientific Exploration

- Started from a focused README spec outlining goals, datasets, and diagnostics.
- Automated notebook + CLI runs to regenerate every experiment end-to-end.
- Promoted the resulting figures and tables into this deck, sharpening the story each loop.

# Motivation & Guiding Questions

- Random forests bundle weak learners; IRT recasts each tree as a respondent with latent ability ($\theta$).
- Held-out images become items whose difficulty ($\delta$) emerges from tree wins and losses.
- How do $\theta$ and $\delta$ steer backbone choices, surface label issues, and focus the next curation loop?

# Story Arc

1. **Background:** IRT mechanics + RF diagnostics we rely on.
2. **Pipeline:** Datasets, embeddings, and response matrices powering the studies.
3. **Case Studies:** Baseline CIFAR, MobileNet upgrade, and MNIST control.
4. **Synthesis:** Cross-study comparisons, takeaways, and next steps.

# Why Item Response Theory for Random Forests?

- Trees answer the same held-out images, so treat them as “test takers.”
- Latent **ability** ($\theta$) ranks trees; latent **difficulty** ($\delta$) flags ambiguous images.
- Shared scales let us compare studies, backbones, and curation tactics directly.

# Item Response Theory Building Blocks

**Core Terms**

- Ability ($\theta$): respondent skill; higher → higher success odds.
- Difficulty ($\delta$): item hardness; higher → harder even for strong respondents.
- Discrimination ($a$): slope near $\delta$.
- Guessing ($c$): floor for multiple-choice exams (rare here).

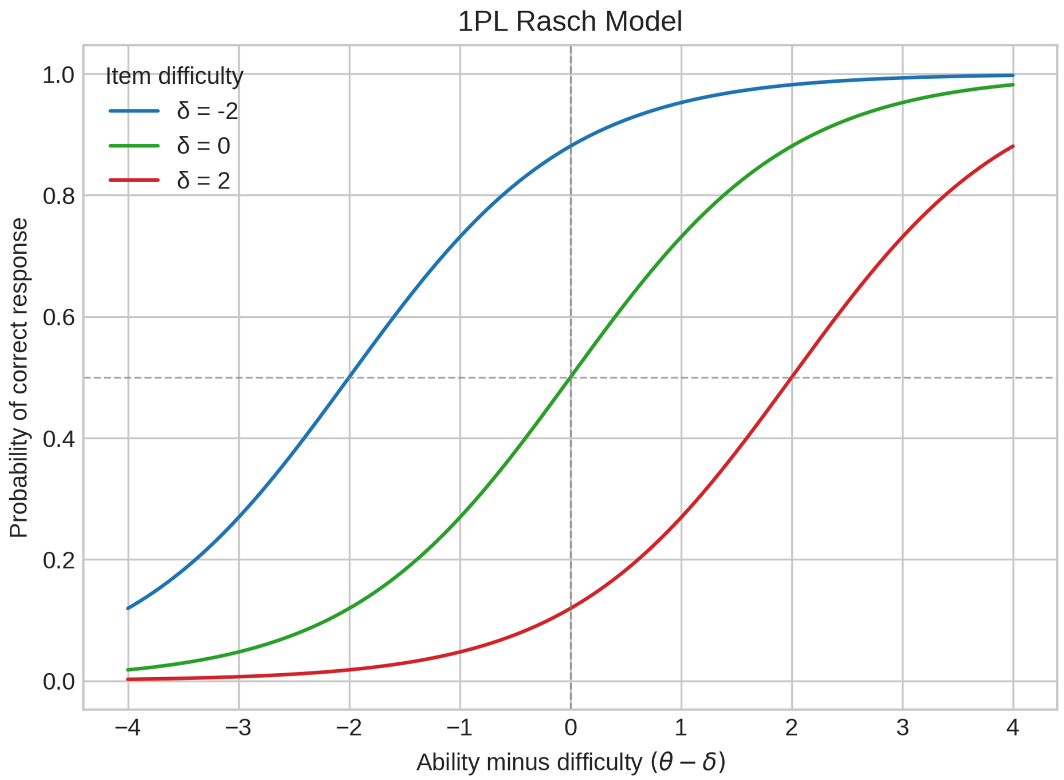
**Ensemble Analogy**

- Respondents → decision trees on a shared test set.
- Items → images; responses are binary (tree correct?).
- Response matrix $R_{ij} \in \{0, 1\}$ feeds variational IRT.
- Outputs: posteriors over $\theta_i$, $\delta_j$, and information curves.

# Rasch (1PL) Model in One Picture

$$\Pr(R_{ij} = 1 \mid \theta_i, \delta_j) = \frac{1}{1 + e^{-(\theta_i - \delta_j)}}$$

- Single global slope keeps parameters on a shared logit scale.
- $(\theta - \delta) = 0 \Rightarrow$ 50% success; shifts left/right change odds.
- Fisher information peaks where curves are steepest—prime for spotting uncertainty.
- IRT ICC Visualizer




1PL logistic curves for items of varying difficulty

# What We Extract from IRT

- **Ability histograms** flag low-skill trees worth pruning.
- **Difficulty ladders** highlight mislabeled or ambiguous items.
- **Wright maps** overlay $\theta$ and $\delta$ to expose coverage gaps.
- **Information curves** reveal where ensemble confidence is fragile.
- Together they explain *who* struggles and *why* beyond RF metrics.

# Margins, Entropy, and Ensemble Confidence

- Tree votes yield class probabilities we mine for uncertainty signals.
- **Margin** $m(x) = P(\hat{y} = y_{true}) - \max_{c \neq y_{true}} P(\hat{y} = c)$ near 0 marks ambiguity; negative marks systematic flips.
- **Entropy** captures ensemble disagreement; combining both with $\delta$ surfaces mislabeled or OOD items and tracks curation gains.

## Margins & Entropy — Why They Matter

- Aggregated tree votes turn into class probabilities, giving us raw material for uncertainty scoring.
- The margin gap shows whether the forest is decisive (large positive) or split/incorrect (near or below zero).
- Entropy summarizes how scattered those votes are; mixing it with $\delta$ spotlights mislabeled or out-of-distribution items and lets us watch them shrink after curation.

# Pipeline Overview

**Data Prep (done)**

- Stratified CIFAR-10 subset: 10k / 2k / 2k splits.
- Resize 64×64, normalize, PCA → 128-D embeddings (plus MobileNet-V3 cache).
- MNIST mini: 4k / 800 / 800 digits, normalized 28×28 grayscale.
- Artifacts cached in `data/cifar10_subset.npz`, `data/cifar10_embeddings.npz`, and `data/mnist/mnist_split.npz`.

**Modeling Status**

- RF (200 trees) trained for every study; metrics and importances saved.
- Response matrices persisted: CIFAR `(200 × 2000)` for PCA & MobileNet, MNIST `(200 × 800)`.
- 1PL Rasch (SVI, 600 epochs) complete for CIFAR; MNIST mirrors the same notebook.

## Dataset Overview

| Dataset | Train | Val | Test | Feature Pipeline | Notes |
|---|---|---|---|---|---|
| CIFAR-10 subset | 10,000 | 2,000 | 2,000 | 64×64 RGB → PCA-128 / MobileNet-V3 (960-D) | Shared splits across Study I & II |
| MNIST mini | 4,000 | 800 | 800 | 28×28 grayscale → raw pixels (no PCA) | Control for clean handwriting |

- All studies reuse cached artifacts under `data/`.
- CIFAR runs differ only by embeddings; labels and splits stay fixed.
- MNIST mirrors the workflow to confirm signals on cleaner data.

## Section I · Baseline Study (CIFAR + PCA)

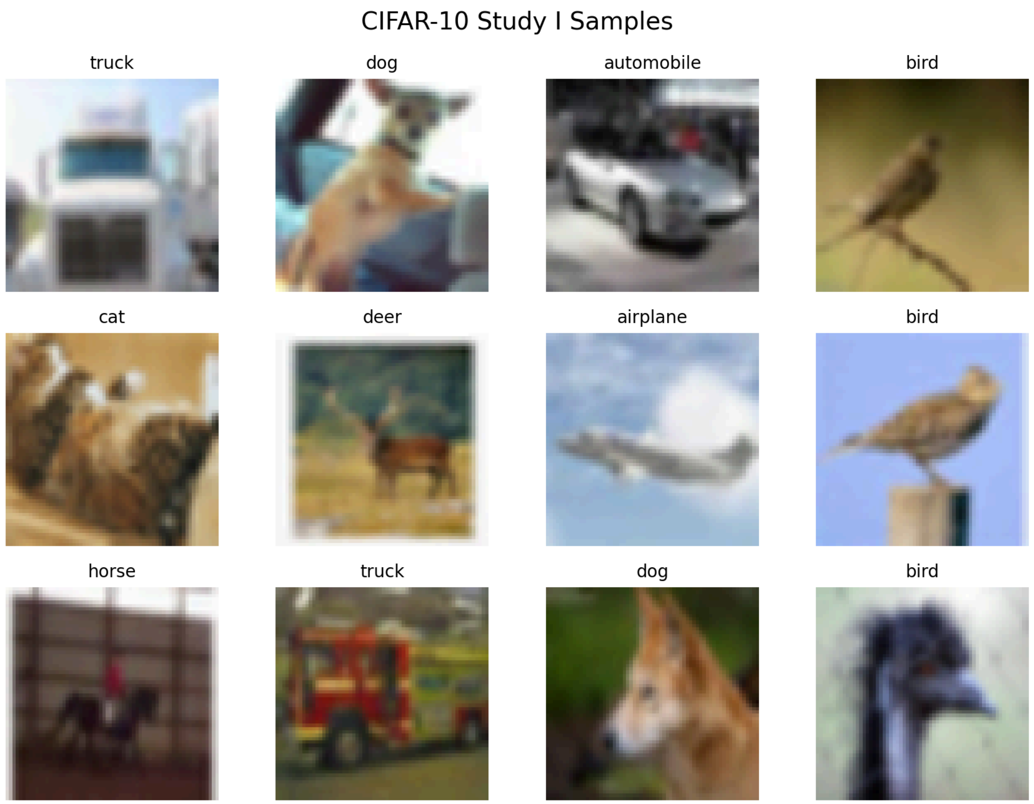
- Establish the PCA baseline and its uncertainty signals.
- Use IRT to pinpoint weak trees and hard items that motivate stronger features.

## Study I: CIFAR-10 + PCA-128 Embeddings

- Baseline vision setup: 64×64 resize + PCA to 128 dims.
- 200-tree Random Forest with a 200 × 2000 response matrix anchors the diagnostics.
- Use this run to surface weak trees and mislabeled items.

# Study I Setup: CIFAR-10 + PCA-128

- Fixed stratified CIFAR-10 split (10k / 2k / 2k).
- Resize 64×64, normalize, PCA → 128-D embeddings (`data/cifar10_embeddings.npz`).
- Response matrix 200 × 2000 with mean tree accuracy 0.176.
- Artifacts: metrics, margins, entropy, IRT outputs under `data/` and `figures/`.



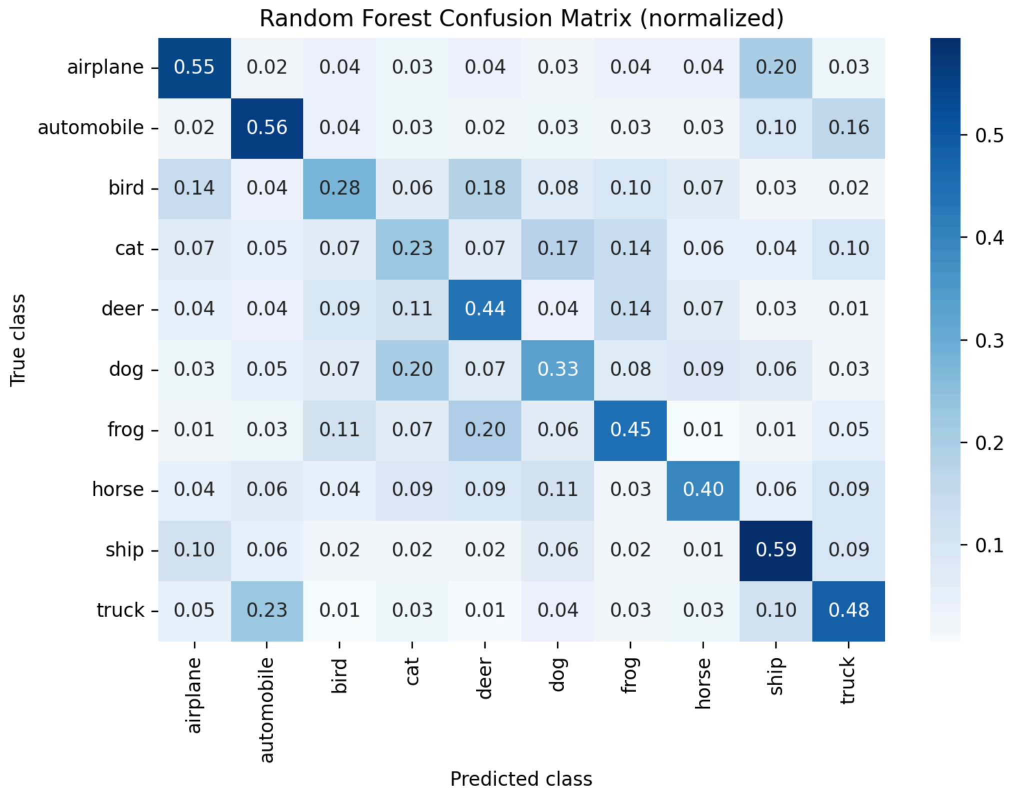

Study I sample grid — stratified CIFAR-10 slices

# Study I Performance (PCA-128)

| Metric | Value |
|---|---|
| Test / Val / OOB acc | 0.4305 / 0.4145 / 0.3730 |
| Per-class range | 0.225 (cat) → 0.595 (ship) |
| Mean tree accuracy | 0.1759 |
| Mean margin / entropy | −0.0028 / 2.1503 |
| δ ↔️ margin (Pearson) | −0.8286 |
| δ ↔️ entropy (Pearson) | 0.6782 |

- Baseline ensemble underperforms due to weak PCA features yet preserves δ alignment.
- Margins sit near zero and entropy stays high, signalling broad disagreement—prime for IRT.
- Artifacts: metrics (`data/rf_metrics.json`), confusion (`data/rf_confusion.npy`), importances, permutations.

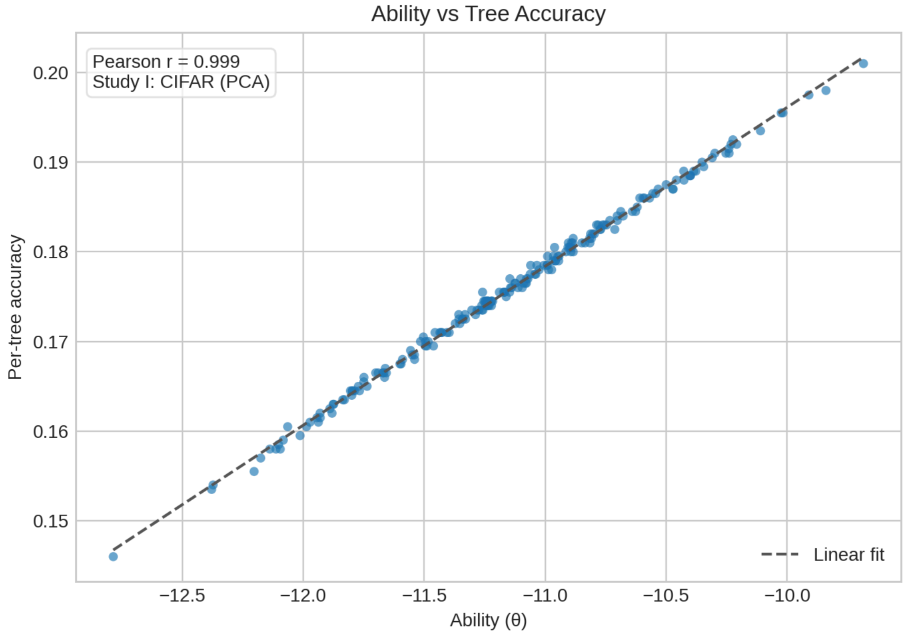
# Study I Confusion Matrix



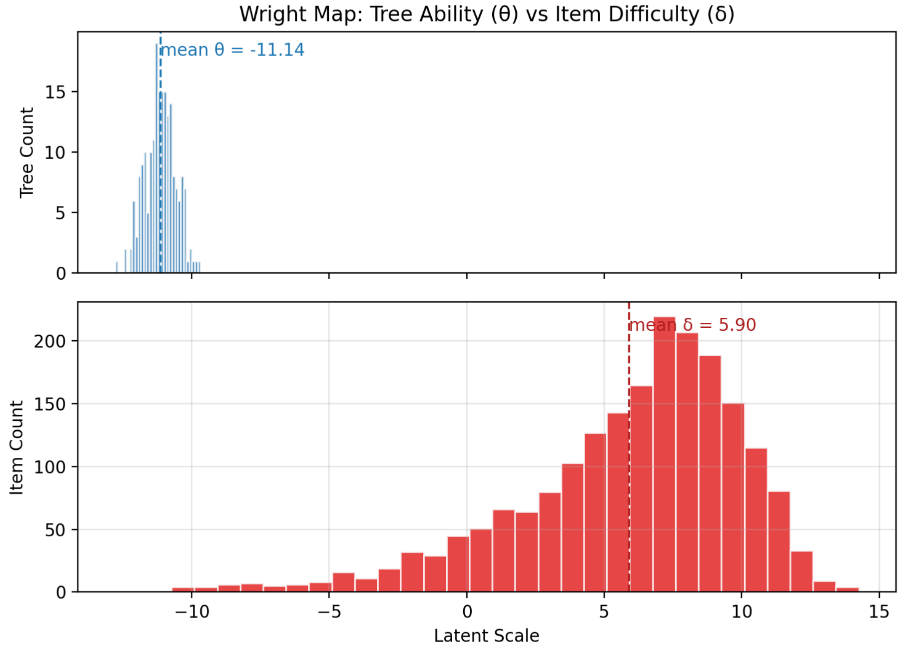

**Reading the matrix**

- Off-diagonal spikes (cat↔dog, bird↔airplane, horse↔deer) mirror high-δ items.
- Ships/trucks stay >80% on-diagonal; the highlighted hotspots mark curation targets.

# Study I Diagnostics: Ability Profiles



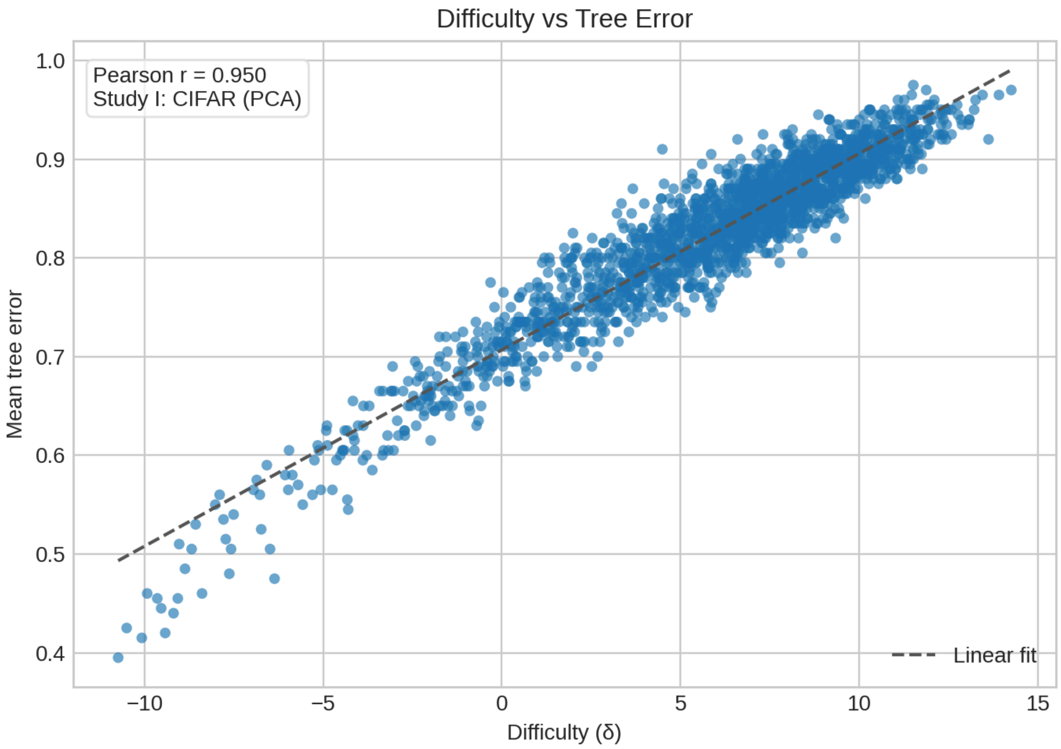

Ability (θ) vs tree accuracy — Spearman ≈ 0.99




Wright map: θ cluster near −11; δ stretches to 14

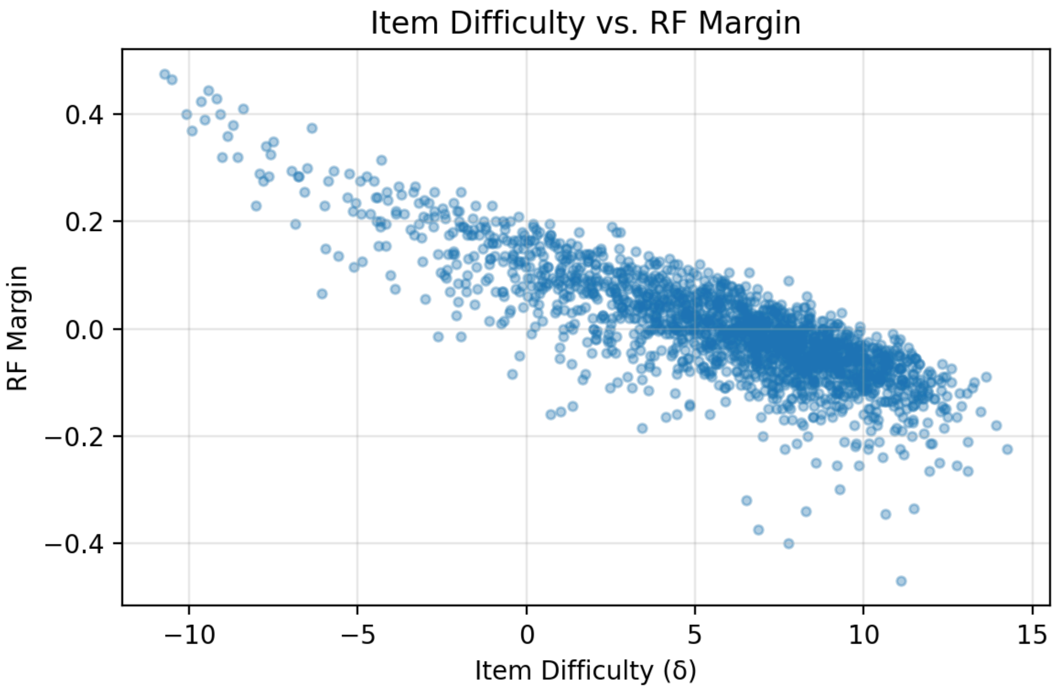
- Trees with θ above −10 beat peers by ~3 pp even with PCA features.
- Long-tail θ < −11.5 drags accuracy, and the Wright map shows δ stretching far beyond the compressed ability range.

# Study I Diagnostics: δ vs Error Rate

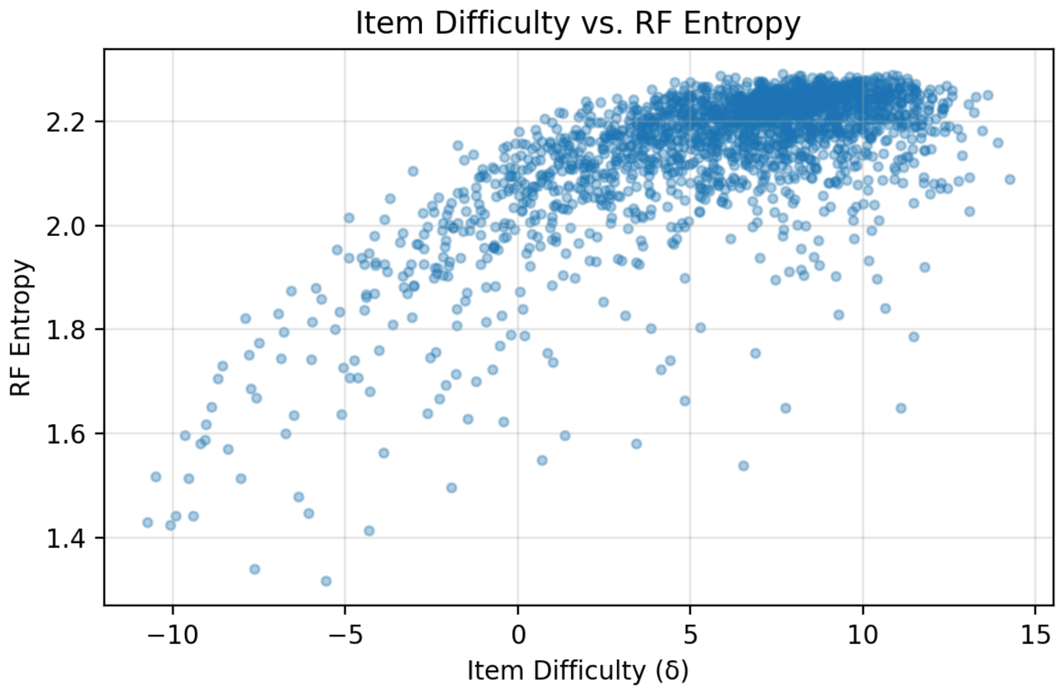



- δ > 10 maps to >80% tree error—mostly ambiguous animals—while δ < 0 becomes "free points."
- Pearson ≈ 0.95, Spearman ≈ 0.94: difficulty doubles as an error heat-map.

# Study I Diagnostics: δ vs RF Signals



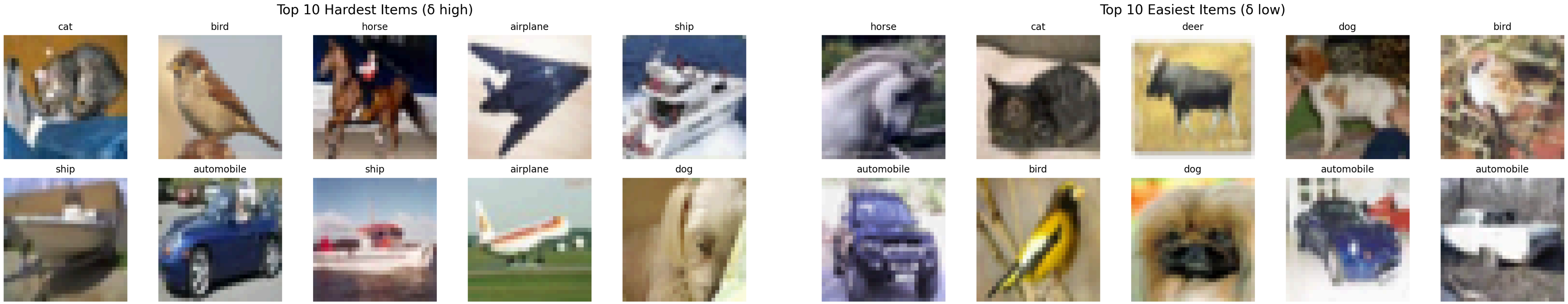

PCA run: δ vs margin (Pearson −0.83)




PCA run: δ vs entropy (Pearson 0.68)

- Hard items cluster bottom-right (low margin, high entropy); opposite corner houses easy wins.
- Study II mirrors the trend with even stronger correlations.

# Study I Evidence: Hard vs Easy Examples




- Hardest items skew toward ambiguous airplane/ship silhouettes and cluttered cat/dog scenes.
- Easy set is dominated by high-contrast cues (e.g., red fire trucks), yielding low δ and entropy.

## Study I Takeaways

- Weak PCA features create long tails in both ability ($\theta$) and difficulty ($\delta$), exposing erratic trees.
- Margin and entropy correlate with $\delta$, but clusters of high-difficulty animals persist across diagnostics.
- Visual inspection confirms mislabeled or low-signal items driving high $\delta$, motivating feature upgrades.

## Section II · Feature-Rich CIFAR (MobileNet)

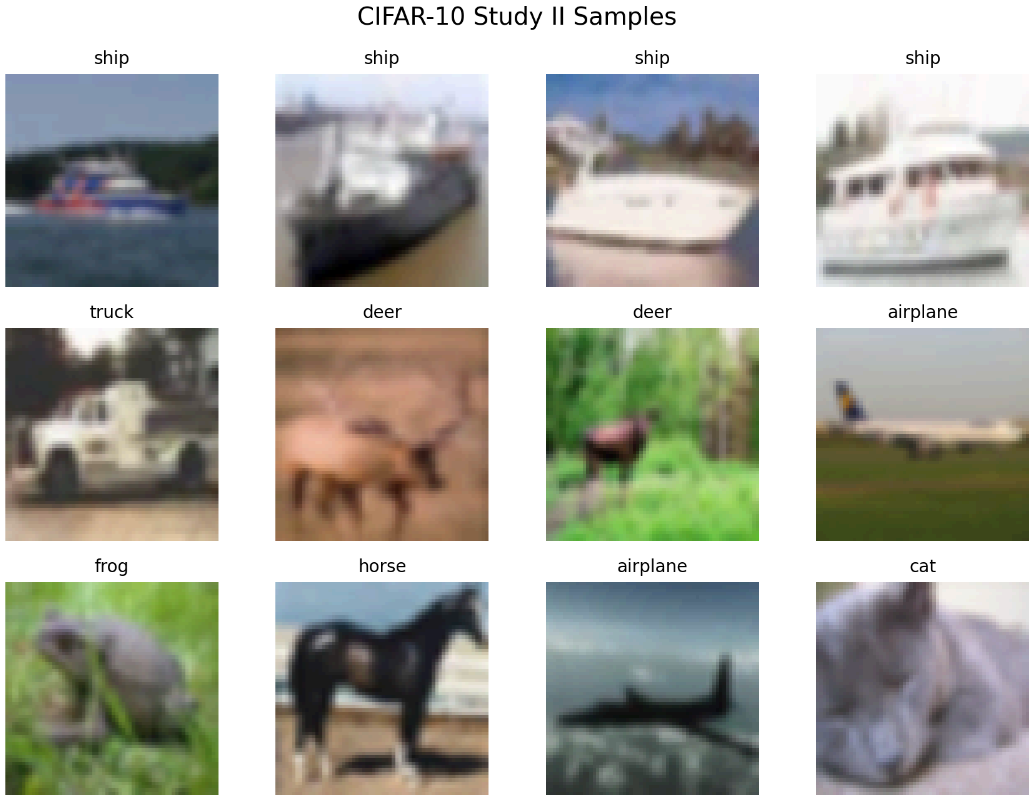
- Hold the splits fixed to isolate feature gains.
- Test whether richer embeddings tighten θ spread and retain δ alignment.

## Study II: CIFAR-10 + MobileNet Embeddings

- Swap PCA features for MobileNet-V3 (960-D) while keeping tree count and splits constant.
- Compare RF metrics, uncertainty signals, and IRT parameters against the baseline.

# Study II Setup: CIFAR-10 + MobileNet-V3

- Reuse Study I splits to isolate feature effects.
- Extract 960-D MobileNet-V3 Small embeddings (`data/cifar10_mobilenet_embeddings.npz`).
- Response matrix 200 × 2000 with mean tree accuracy 0.482.
- Artifacts live under `data/mobilenet/*` and `figures/mobilenet/`.



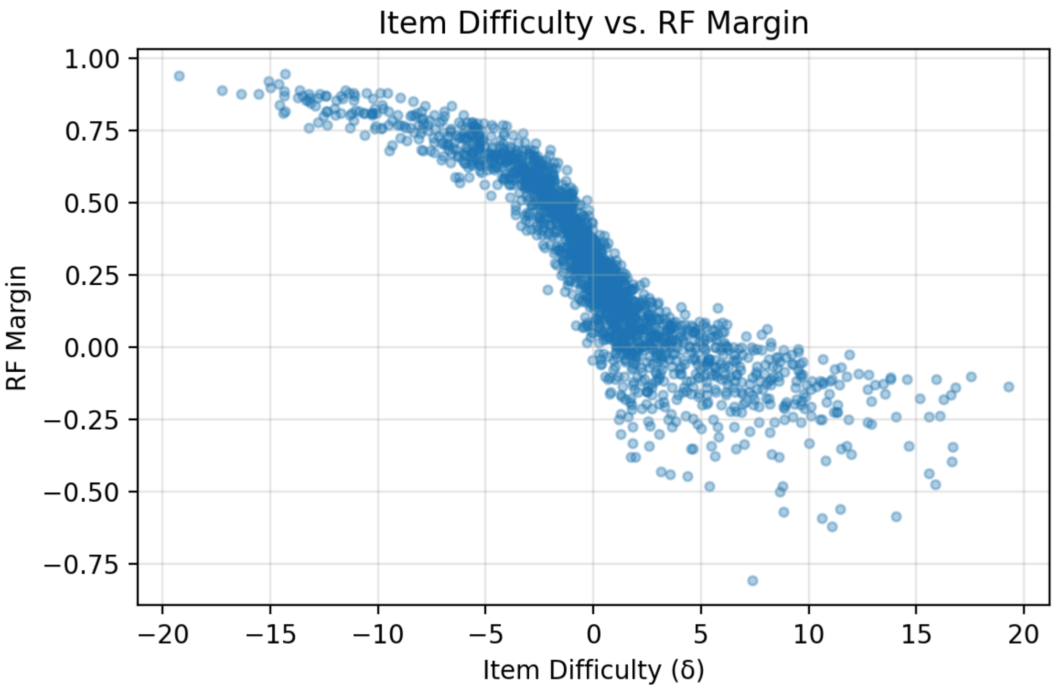

Study II sample grid — same splits, MobileNet embeddings

# Study II Performance (MobileNet-V3)

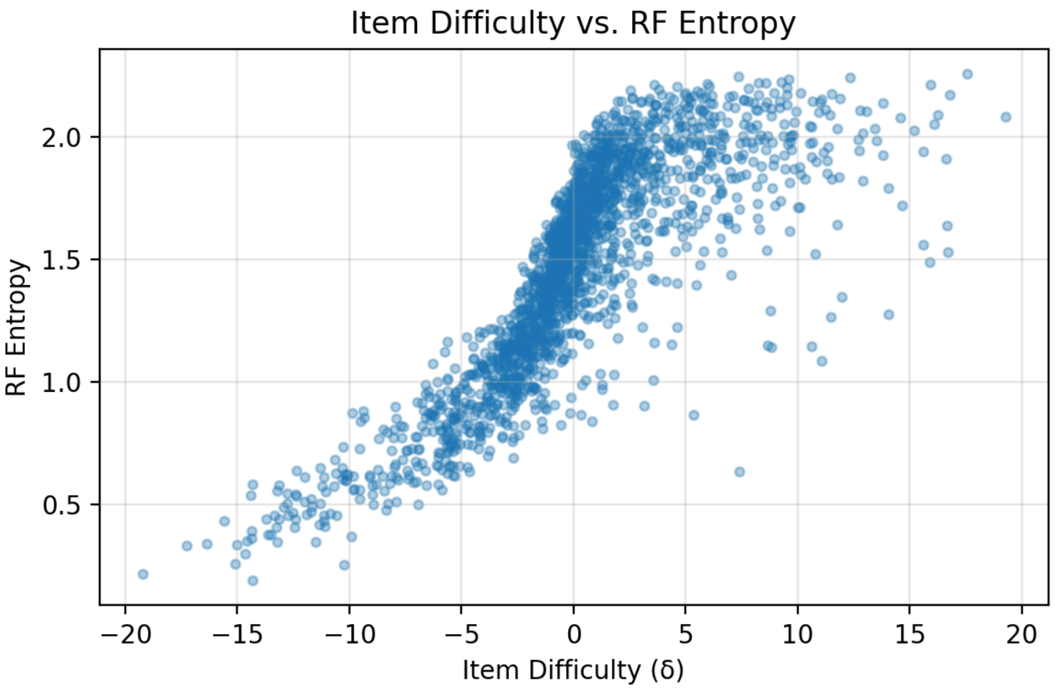
| Metric | Value |
|---|---|
| Test / Val / OOB acc | 0.8090 / 0.8135 / 0.7967 |
| Per-class range | 0.68 (cat) → 0.915 (ship) |
| Mean tree accuracy | 0.4817 |
| Mean margin / entropy | 0.2806 / 1.4663 |
| δ ↔️ margin (Pearson) | −0.8825 |
| δ ↔️ entropy (Pearson) | 0.8113 |

- Pretrained features boost accuracy by 37 pp while strengthening δ correlations.
- Higher margins and lower entropy show confidence gains except on stubborn animal classes.
- Artifacts: metrics, response matrix, signals, and IRT outputs under `data/mobilenet/`.

# Study II Diagnostics: δ vs RF Signals



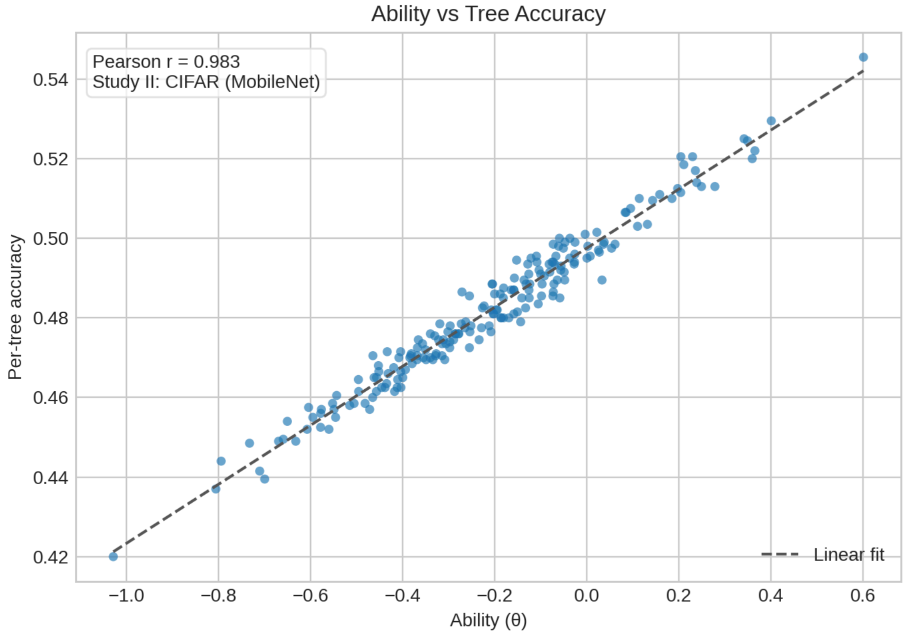

δ vs margin (Pearson −0.88)



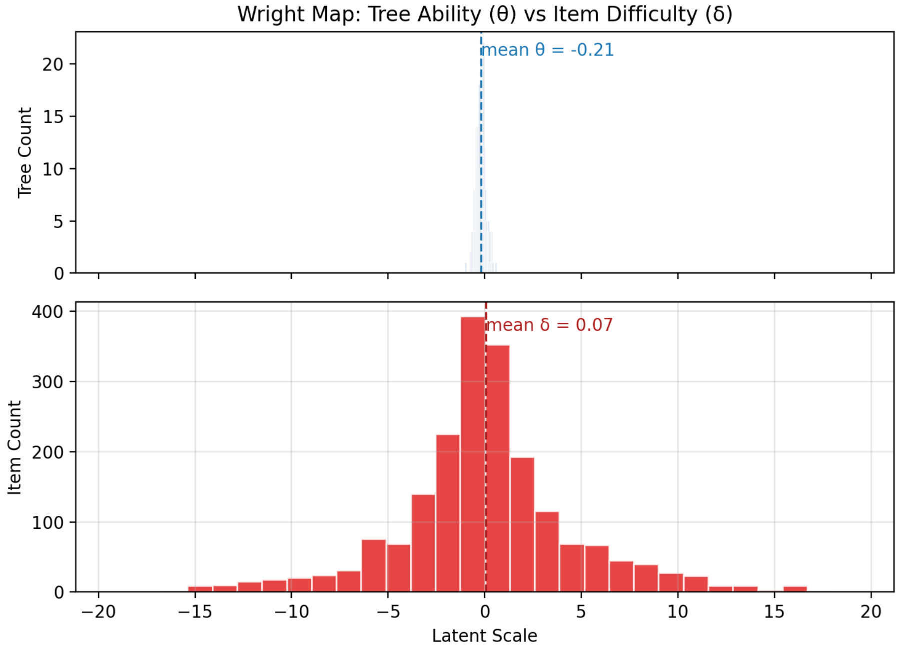

δ vs entropy (Pearson 0.81)

- MobileNet compresses the easy cluster (high margin, low entropy) while isolating true hard cases.
- Larger |corr| values show tighter agreement between δ and RF uncertainty.
- Cat/dog confusions persist, marking curation targets.

# Study II Diagnostics: Ability Profiles



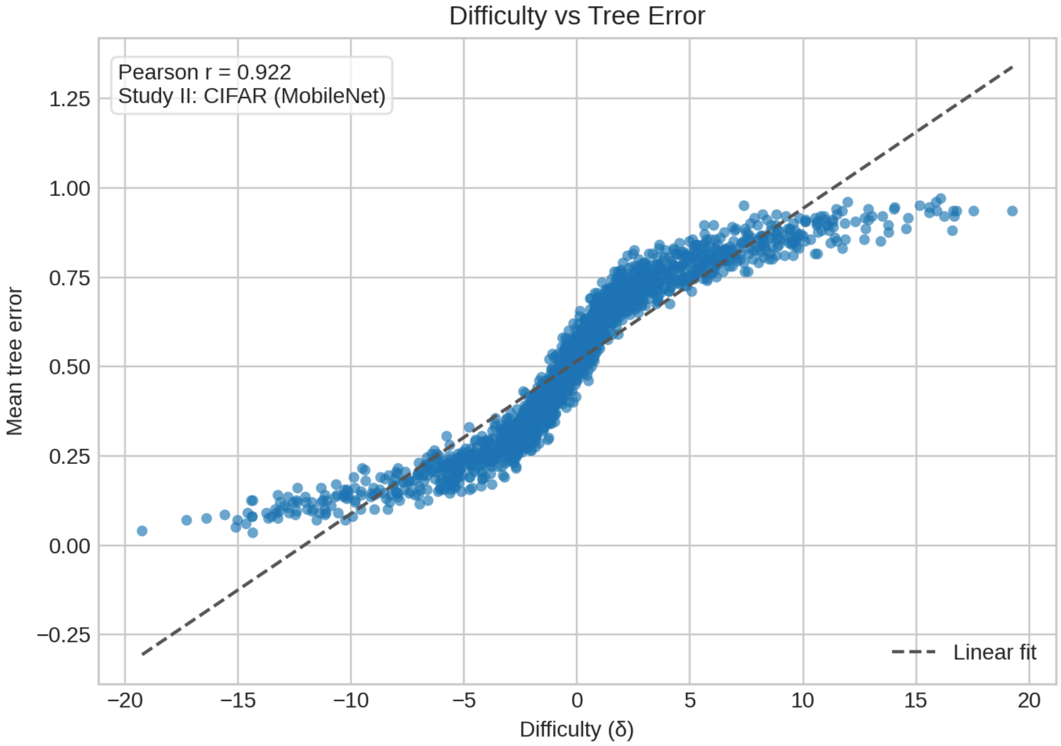

Ability (θ) vs tree accuracy — Pearson 0.983




Wright map: θ variance shrinks to 0.25

- θ mean −0.21 ± 0.25: trees cluster far tighter than the PCA baseline (σ 0.55 → 0.25).
- Ability remains tied to per-tree accuracy, so feature quality—rather than tree diversity—now caps gains.

# Study II Diagnostics: δ vs Error Rate




- Pearson 0.922 keeps δ aligned with mean tree error even at the higher accuracy ceiling.
- Hardest items ($\delta > 8$) persist—mostly cat/dog overlaps and ambiguous aircraft—while the easy zone ($\delta < -3$) expands.

## Study II Takeaways

- MobileNet embeddings add 37 pp of accuracy while collapsing ability variance ($\sigma\theta$ 0.55 → 0.25).
- $\delta$ stays aligned with RF uncertainty, isolating a smaller yet stubborn ambiguous cluster.
- Residual cat/dog confusion points to data curation as the next lever.

## Section III · Control Study (MNIST)

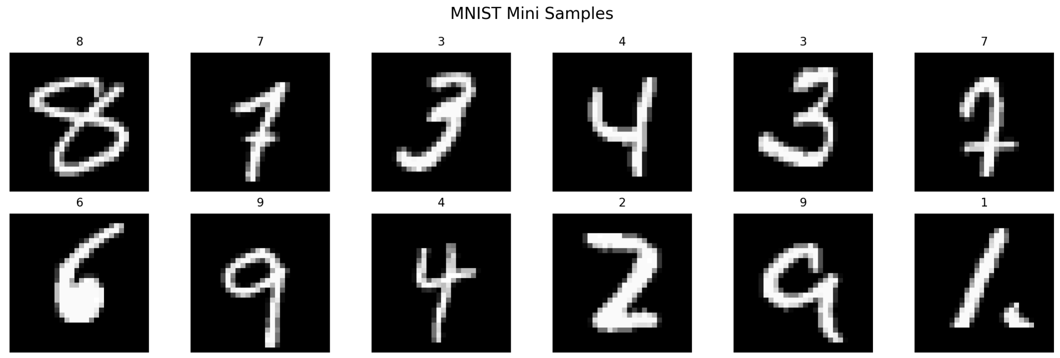
- Probe the pipeline on a high-signal, low-noise dataset.
- Confirm that IRT still mirrors RF uncertainty when accuracy is near perfect.

## Study III: MNIST Mini-Study

- Lightweight handwriting dataset to validate RF × IRT beyond CIFAR-10.
- Acts as a control where ambiguity is rare yet still detectable.

# Study III Setup: MNIST Mini-Study

- Split 4k / 800 / 800 digits with stratified sampling and a fixed seed.
- Flatten 28×28 grayscale digits; no augmentation.
- Train a 200-tree RF on raw pixels; response matrix 200 × 800.
- Artifacts land in `data/mnist/` with plots in `figures/mnist/`.



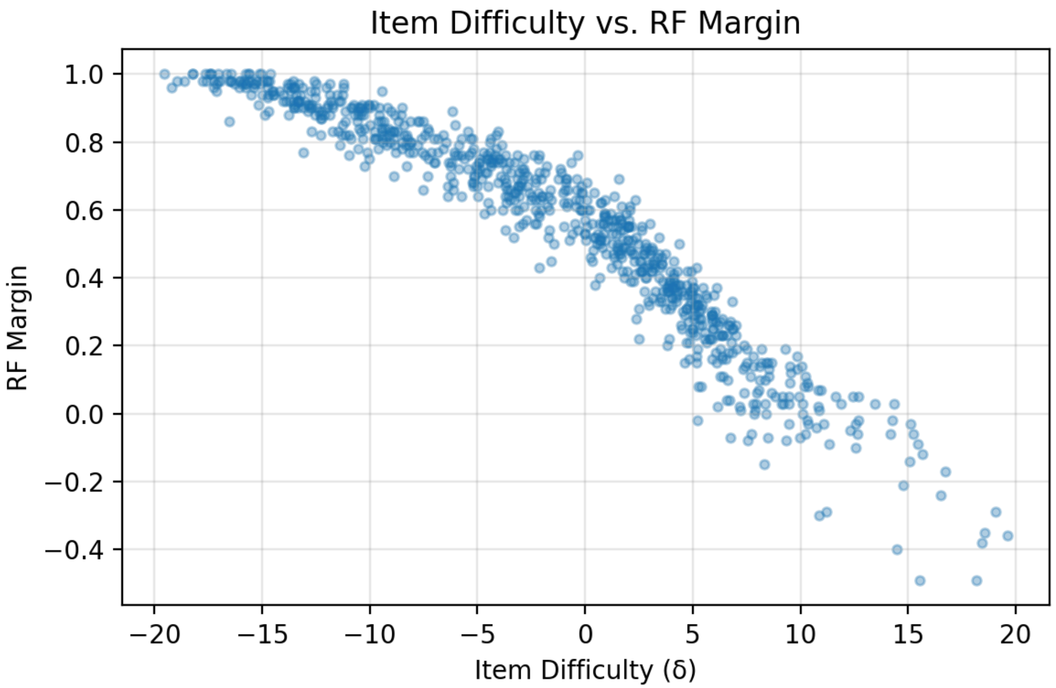

Study III sample grid — curated MNIST mini split

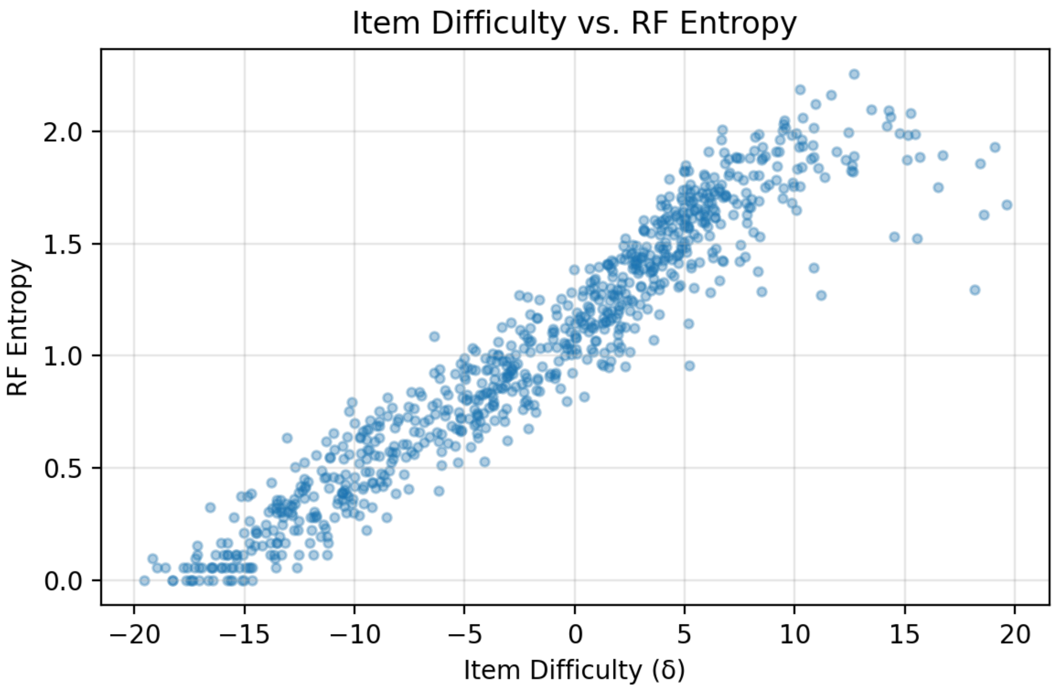
# Study III Performance (MNIST)

| Metric | Value |
|---|---|
| Train / Val / Test | 4000 / 800 / 800 |
| RF test / val / OOB | 0.9475 / 0.9413 / 0.9140 |
| Mean margin / entropy | 0.5546 / 1.0351 |
| δ ↔️ margin (Pearson) | −0.950 |
| δ ↔️ entropy (Pearson) | 0.958 |
| θ mean ± σ | 4.23 ± 0.44 |
| δ mean ± σ | −1.75 ± 8.19 |

- Ambiguous digits (e.g., brushed 5 vs 6) spike δ toward ±20; elsewhere the forest is decisive.
- Low entropy + high margin line up with low δ, giving a “sanity benchmark” beyond CIFAR.

# Study III Diagnostics: δ vs RF Signals



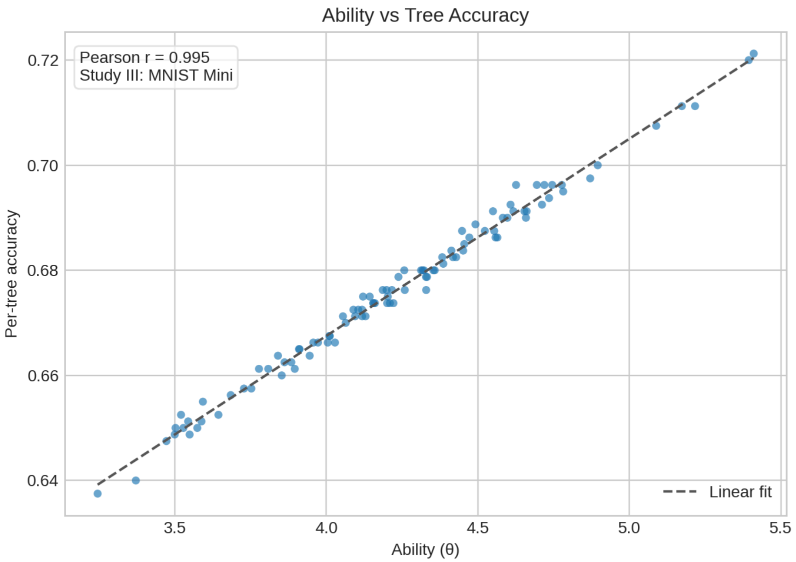




δ vs margin (Pearson −0.95)

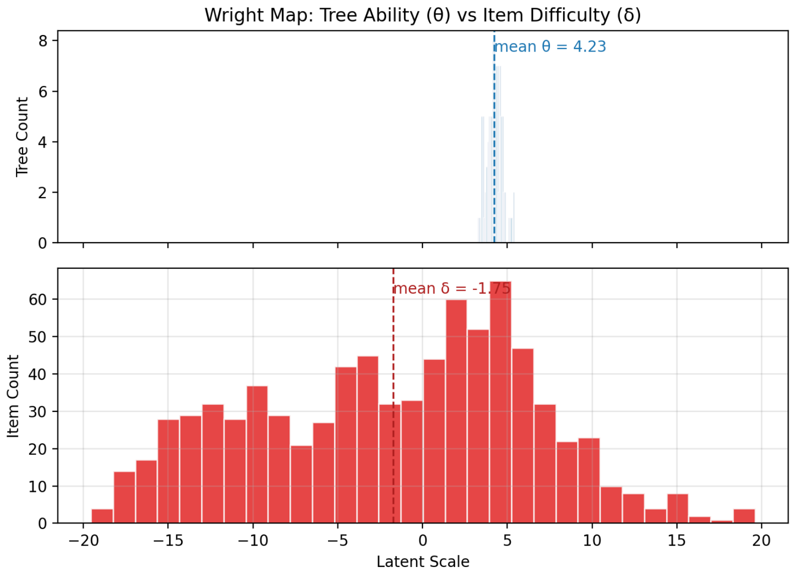
δ vs entropy (Pearson 0.96)

- Clean digits show near-perfect alignment between δ and RF uncertainty.
- Only a handful of δ > 12 digits drive the residual uncertainty (stroke collisions like 3/5, 4/9).

# Study III Diagnostics: Ability Profiles



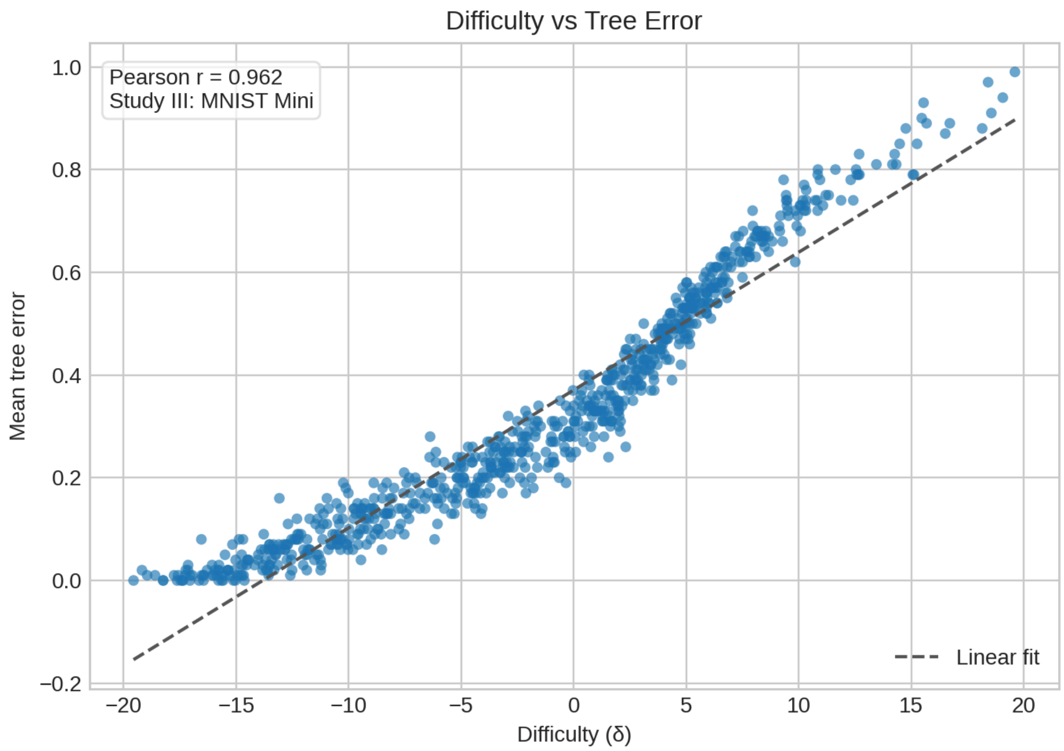

Ability (θ) vs tree accuracy — Pearson 0.995




Wright map: θ mean 4.23 ± 0.44; δ mean −1.75 ± 8.19

- θ mean 4.23 ± 0.44 shows strong consensus, while δ mean −1.75 ± 8.19 keeps heavy tails for ambiguous strokes.
- Shared scales expose plentiful easy wins with a few sharp spikes—opposite of the CIFAR baseline.

# Study III Diagnostics: δ vs Error Rate




- Pearson 0.962 keeps δ tied to mean tree error despite the high accuracy ceiling.
- δ > 12 corresponds to stroke-collided 3/5/8 and 4/9 pairs; the long negative tail is trivial for the ensemble.

## Study III Takeaways

- δ and RF uncertainty agree almost perfectly, while θ stays high yet still flags the rare ambiguous strokes.
- The control study confirms the RF × IRT pipeline holds outside noisy vision data.

# Section IV · Cross-Study & Diagnostics

- Compare backbones and datasets on a shared θ/δ scale.
- Surface recurring themes before the close.

# Cross-Study Snapshot

| Study | Feature Backbone | Test Acc | δ ↔️ margin (Pearson) | δ ↔️ entropy (Pearson) | θ σ | δ σ |
|---|---|---|---|---|---|---|
| Study I: CIFAR + PCA-128 | PCA-128 | 0.4305 | −0.8286 | 0.6782 | 0.55 | 4.10 |
| Study II: CIFAR + MobileNet | MobileNet-V3 (960-D) | 0.8090 | −0.8825 | 0.8113 | 0.25 | 4.67 |
| Study III: MNIST Mini | Raw pixels | 0.9475 | −0.950 | 0.958 | 0.44 | 8.19 |

- Feature backbone drives both accuracy gains and δ alignment strength.
- θ variance collapses with MobileNet (0.25) while MNIST keeps moderate spread despite high accuracy.
- MNIST δ σ expands to 8.19, highlighting rare but extreme digit ambiguities versus CIFAR's visual noise.

## Key Takeaways

- IRT mirrors RF uncertainty: θ tracks per-tree accuracy and δ tracks item error across studies.
- Feature backbones reshape the θ/δ landscape—MobileNet curbs variance yet preserves a hard-item tail.
- Pairing δ with margins and entropy cleanly triages ambiguous classes without manual inspection.
- MNIST confirms the pipeline before we branch to new domains.

## Next Steps

- Extend notebooks to auto-export the comparison tables and montages.
- Run the queued 2PL/3PL experiments (`reports/discrimination_analysis_plan.md`).
- Correlate θ with tree structure (depth, leaf count) to guide pruning.
- Scale the δ + margin triage on CIFAR before moving to tabular studies.