IRTForests

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github.com/ascott02/IRTForests

Item Response Theory + Random Forests

- Trees become respondents, images become items.
- Response matrix records per-tree correctness on held-out examples.
- Goal: explain RF behavior via IRT ability & difficulty signals and vice versa.

Agenda

- Background: IRT + RF primers
- Pipeline: datasets, embeddings, and response matrices
- Case studies: CIFAR (PCA), CIFAR (MobileNet), MNIST
- Cross-study comparison, 2PL/3PL updates, takeaways, next steps

Item Response Theory (IRT) (Wilson, 2005)

Why? Because performance != ability — but they're related.

- Classical Test Theory (CTT) tells us *how someone did on this test.*
- IRT models *how someone would perform on any set of items that measure the same underlying ability*.
- IRT doesn't replace CTT, it generalizes it with **portable, interpretable measurements** of capability.

CTT	IRT
Measures perf. on specific test	Estimates underlying ability
Test = sample of items	Items = samples from a calibrated continuum
Precision assumed constant	Precision varies with ability
Great for grading	Great for understanding and interpretability

> A joint calibration framework where ability and difficulty are inferred together, each defined only in relation to the other.
> It's less like grading individuals and more like synchronizing clocks — each calibrated against the ensemble.

Item Response Theory Building Blocks

Core Terms

- Ability (θ): respondent skill; higher → higher success odds (1PL).
- Difficulty (δ): item hardness; higher → harder even for strong respondents (1PL).
- Discrimination (a): slope near δ (2PL).
- Guessing (c): floor for multiple-choice exams (3PL).

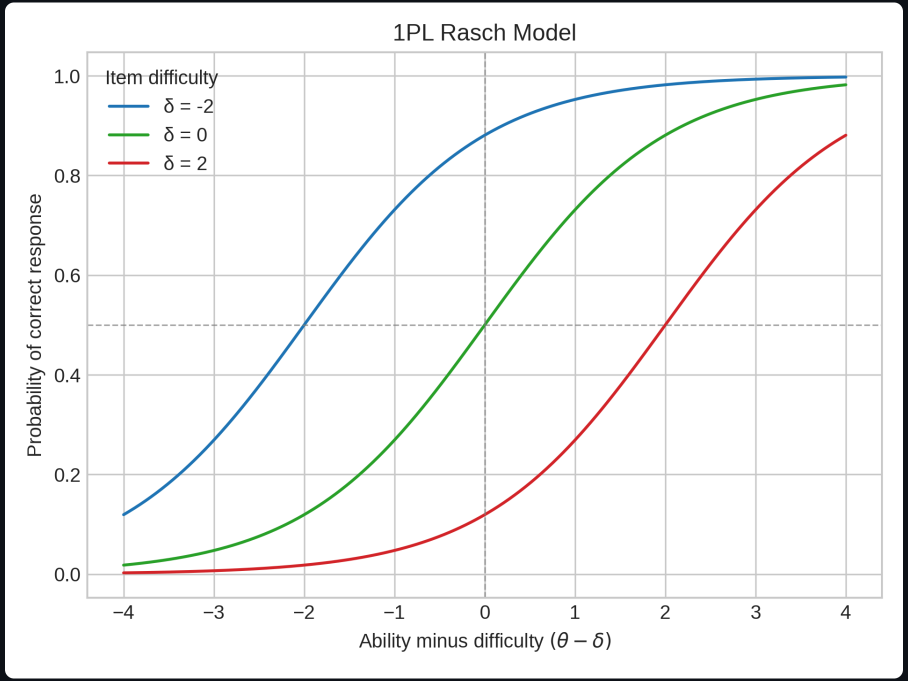
Forest Analogy

- Respondents → decision trees on a shared test set.
- Items → images; responses are binary (tree correct?).
- Response matrix $R_{ij} \in \{0, 1\}$ feeds variational IRT.
- Outputs: posteriors over θ_i, δ_j, and information curves.

Rasch (1PL) Model in One Picture

$$\Pr(R_{ij} = 1 \mid \theta_i, \delta_j) = \frac{1}{1 + e^{-(\theta_i - \delta_j)}}$$

- The probability a respondent gets the item correct, given their ability, and the item's difficulty.
- Single global slope keeps parameters on a shared logit scale.
- $\theta - \delta = 0 \Rightarrow 50\%$ success; shifts left/right change odds.
- Fisher information peaks where curves are steepest.
- See IRT ICC Visualizer for 2PL, 3PL, 4PL



1PL Item Characteristic Curves (ICC)

IRT Output

- **Ability histograms** flag low-skill trees worth pruning.
- **Difficulty ladders** highlight mislabeled or ambiguous items.
- **Wright maps** overlay θ and δ to expose coverage gaps.
- **Information curves** reveal where ensemble confidence is fragile.
- Together they explain *who* struggles and *why*, beyond RF metrics.

Random Forests — Many Noisy Trees, One Stable Voice (Breiman, 2001)

- Train trees on bootstrapped samples with random feature subsets to decorrelate their votes.
- Aggregate those votes by majority (classification) or mean (regression) to cut variance.
- **Margin:** gap between the correct class and the runner-up; **entropy:** dispersion of votes.
- Reading the two together exposes how confident—or conflicted—the forest is, especially once aligned with δ.

Random Forest Margins — How Confident Is the Crowd?

$$\text{margin}(x_i) = P_{\text{correct}}(x_i) - \max_{j \neq \text{true}} P_j(x_i)$$

The **margin** measures how far ahead the correct class is over its nearest competitor.

- **High margin:** trees vote strongly for the right class → confident.
- **Low or negative margin:** trees disagree or favor another class → uncertain.

> Think of it as the *vote gap* in an election — the wider the gap, the clearer the win.

Ensemble Entropy — How Much Do Trees Disagree?

$$H(x_i) = -\sum_j P_j(x_i) \log_2 P_j(x_i)$$

The **entropy** measures how dispersed the votes are across classes.

- **Low entropy:** trees nearly unanimous → decisive prediction.
- **High entropy:** votes spread out → uncertainty or class confusion.

> Within trees, entropy drives splits (purity).
> Across trees, entropy reveals disagreement — the forest’s collective uncertainty.

GenAI in the Loop Scientific Experimentation

- Recursive prompting (akin to context engineering) keeps each iteration scoped.
- Ground every cycle in the `README.md` spec—goals, datasets, diagnostics.
- Automate the CLI so runs regenerate figures and tables straight into the deck.
- Commit, push, repeat: github.com/ascott02/IRTForests

> Plastic tubes and pots and pans
> Bits and pieces and the magic from the hand - Oingo Boingo, "Weird Science" 1985

Pipeline Overview

Data preparation for three studies

1. Stratified CIFAR-10 subset: 10k / 2k / 2k splits. Resize 64×64, normalize, PCA → 128-D embeddings.
2. Stratified CIFAR-10 subset: 10k / 2k / 2k splits. Resize 64×64, normalize, MobileNet → 960-D embeddings.
3. MNIST mini: 4k / 800 / 800 digits, normalized 28×28 grayscale. Raw pixels.

Random forest training

- RF (2000 trees) trained for every study; metrics and importances saved.
- Response matrices saved: CIFAR `(2000 × 2000)` for PCA & MobileNet, MNIST `(2000 × 800)`.

IRT analysis

- 1PL Rasch (SVI, 600 epochs) complete for CIFAR+PCA, CIFAR+MobileNet, and MNIST.
- 2PL (SVI, 800 epochs) complete for CIFAR+PCA, CIFAR+MobileNet, and MNIST.
- 3PL (SVI, 1000 epochs) CIFAR MobileNet only.

Datasets Overview

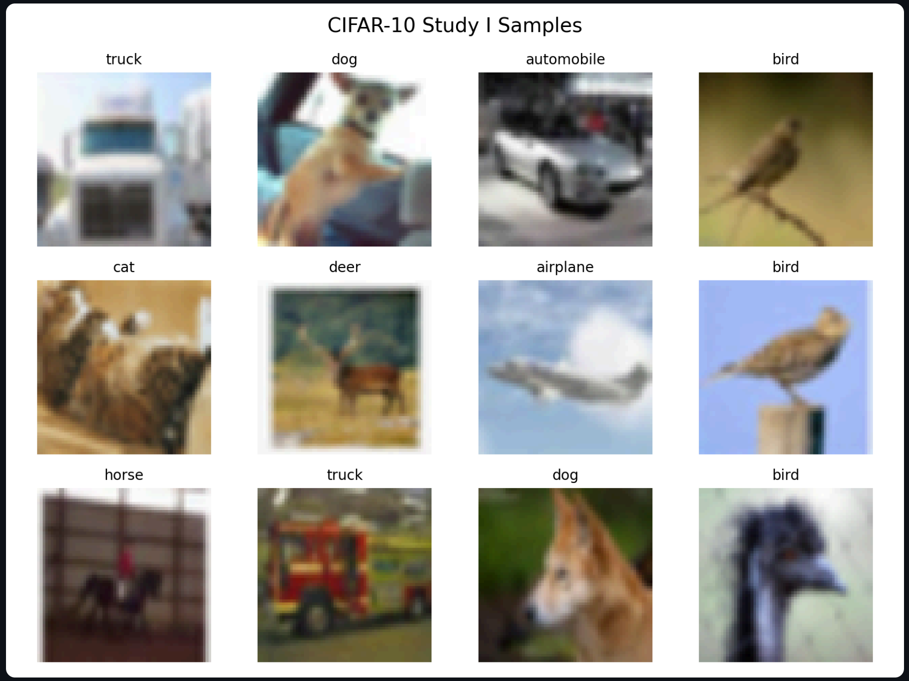
Dataset	Train	Val	Test	Feature Pipeline	Notes
CIFAR-10 subset	10,000	2,000	2,000	PCA-128 / MobileNet-V3 (960-D)	Shared splits Study I & II
MNIST mini	4,000	800	800	28×28 grayscale → raw pixels (no PCA)	Control for clean handwriting

- CIFAR runs differ only by embeddings; labels and splits stay fixed.
- MNIST mirrors the workflow to confirm signals on cleaner data.

Study I: CIFAR-10 + PCA-128 Embeddings

Study I Setup: CIFAR-10 + PCA-128

- Establish the PCA baseline and capture RF uncertainty signals.
- Use IRT to pinpoint weak trees and hard items that motivate stronger features.
- Fix a stratified CIFAR-10 split (10k / 2k / 2k).
- Train 2000 trees and score them on the shared test set.
- Build a 2000 × 2000 response matrix (mean tree accuracy ≈ 0.18).
- Artifacts: metrics, margins, entropy, IRT outputs.



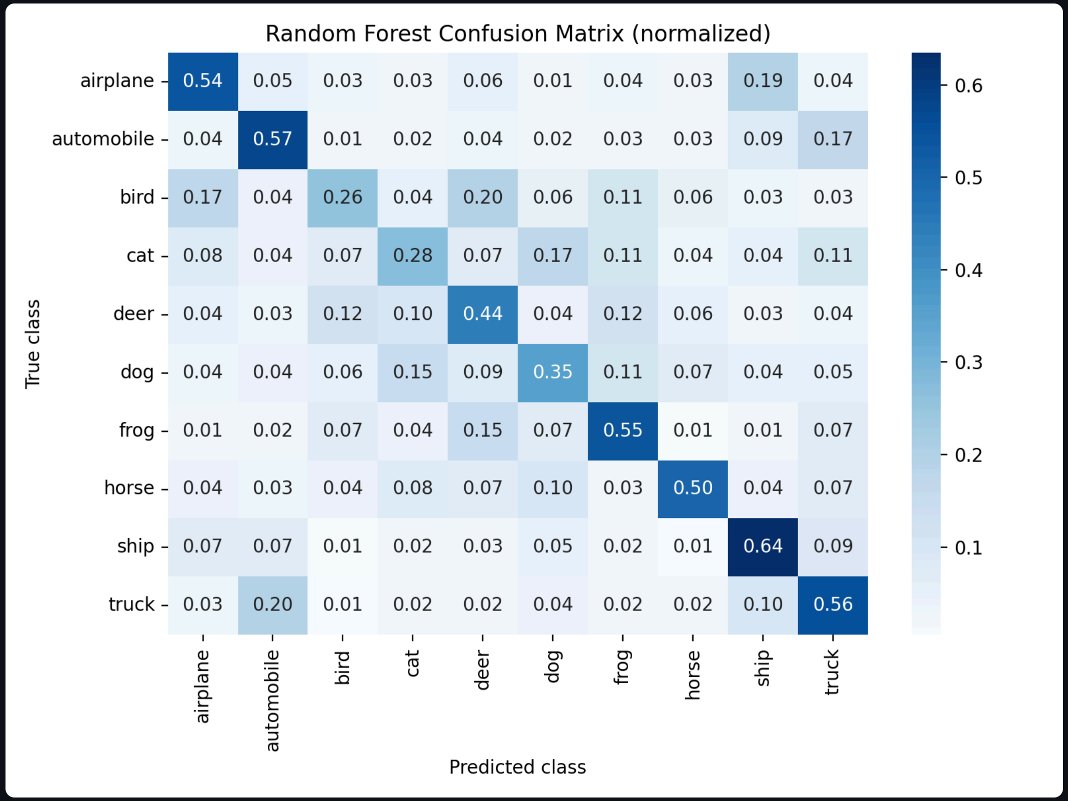
Study I sample grid — stratified CIFAR-10 slices

Study I Performance (PCA-128)

Metric	Value
Test / Val / OOB acc	0.468 / 0.470 / 0.442
Per-class range	0.260 (bird) → 0.635 (ship)
Mean tree accuracy	0.1763
Mean margin / entropy	0.0058 / 2.1723
δ negatively correlates with margin (Pearson)	−0.815
δ positively correlates with entropy (Pearson)	0.687

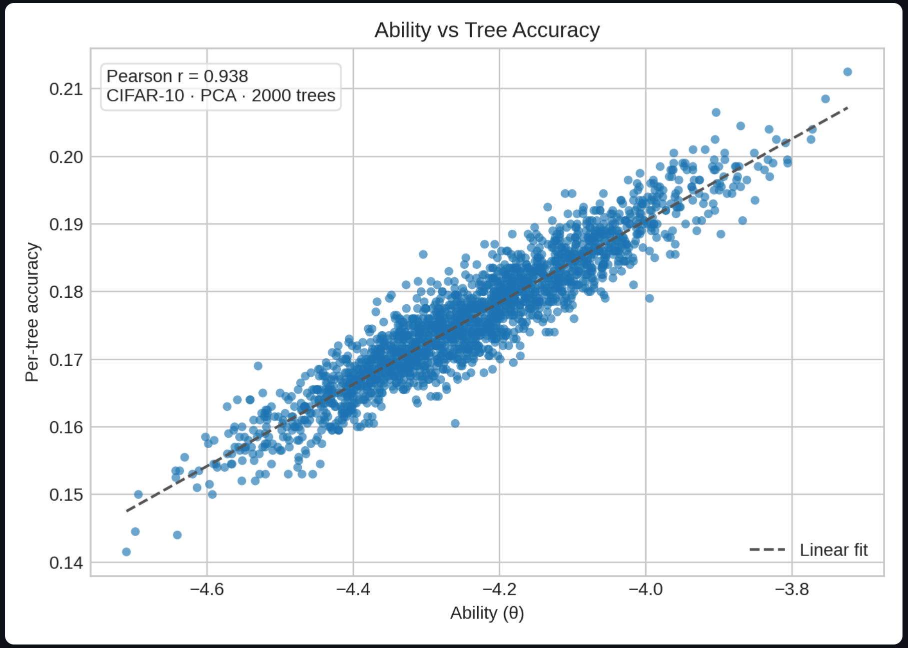
- Baseline ensemble still underperforms due to weak PCA features yet preserves δ alignment.
- Margins hover near zero (mean ≈0.006) and entropy stays high (2.17), signalling broad disagreement—prime for IRT.

Study I Confusion Matrix

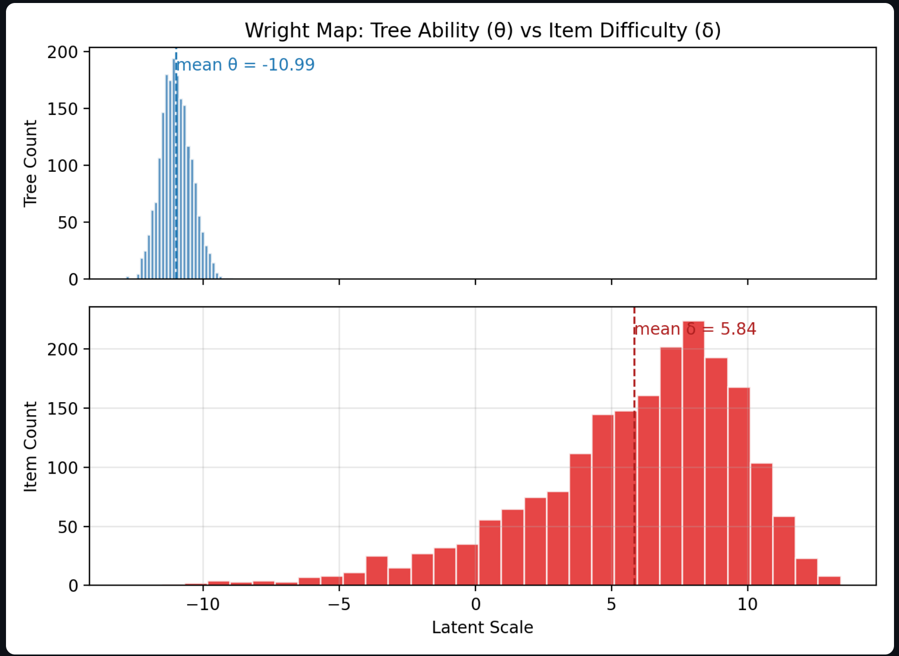


- Off-diagonal spikes (cat vs dog, bird vs airplane, horse vs deer) mirror high-δ items.
- Ships and trucks still lead the diagonal (≈64% / 56% accuracy), yet well short of a clean block—further underscoring the curation need.

Study I Diagnostics: Ability Profiles



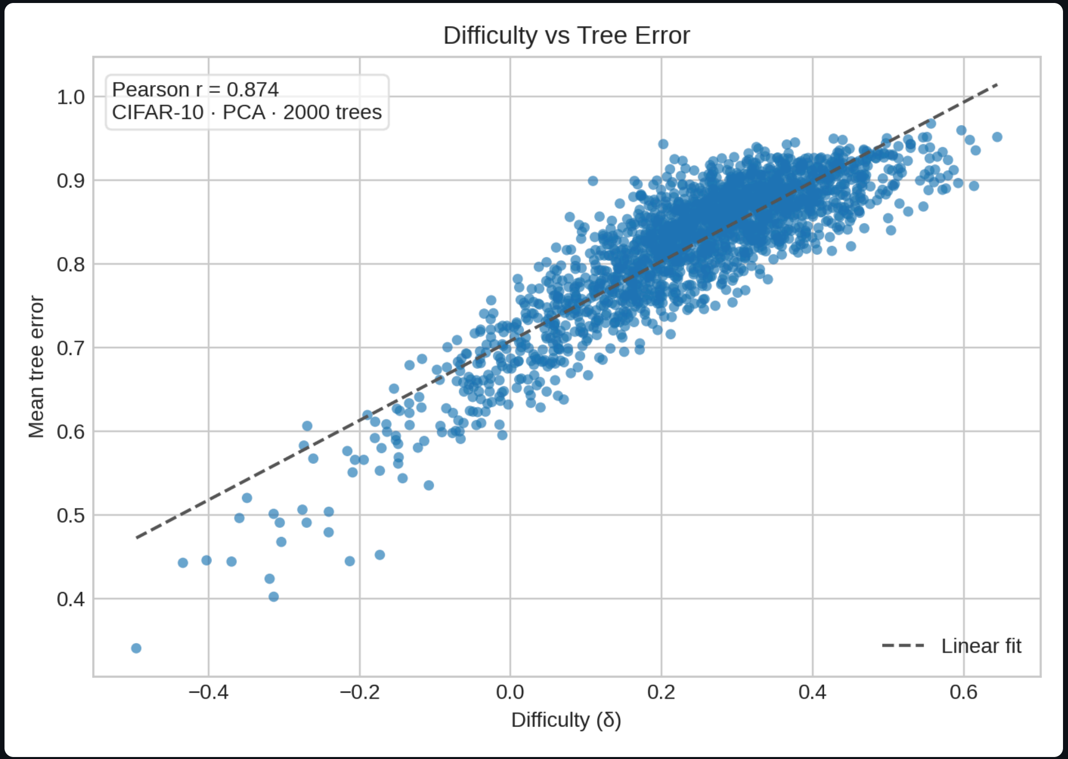
Ability (θ) vs tree accuracy — Spearman ≈ 0.99



Wright map: θ mean ≈ −11.0 (σ ≈ 0.56); δ mean ≈ 5.8 with a wide tail

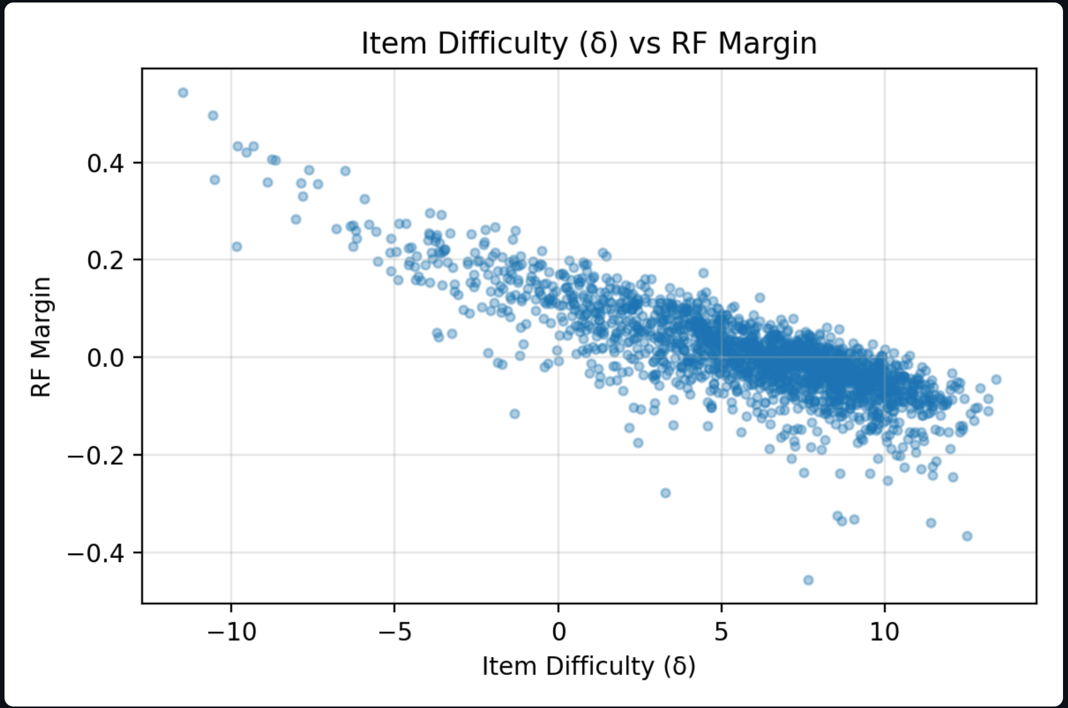
- θ ranges from about −12.8 to −8.9 (mean ≈ −11.0 ± 0.56), so even small shifts separate stronger trees by a few percentage points.
- δ centers near 5.8 but stretches from roughly −11.5 to 13.4, highlighting how ambiguous animal items sit far from the easy tail.

Study I Diagnostics: δ vs Error Rate

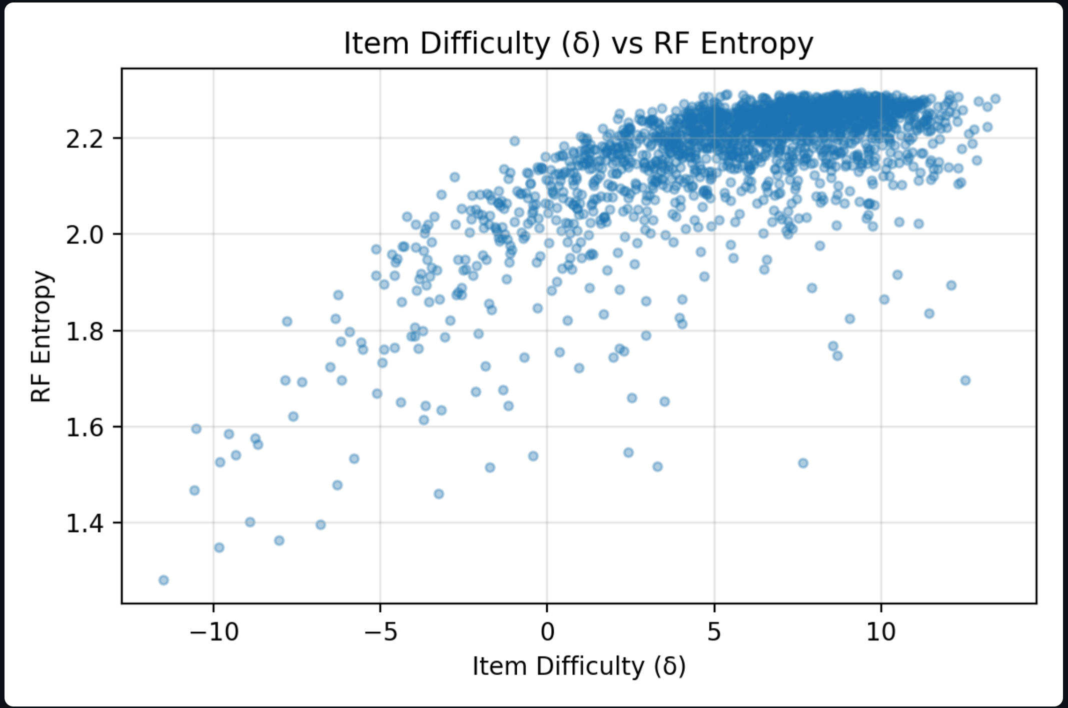


- δ > 0.4 maps to >80% tree error—mostly ambiguous animals—while δ < −0.3 becomes “free points.”
- Pearson ≈ 0.87, Spearman ≈ 0.86: difficulty doubles as an error heat-map.

Study I Diagnostics: δ vs RF Signals



PCA run: δ vs margin (Pearson −0.82)



PCA run: δ vs entropy (Pearson 0.69)

- Hard items cluster bottom-right (low margin, high entropy); opposite corner houses easy wins.
- Study II mirrors the trend with even stronger correlations.

Study I Evidence: Hard vs Easy Examples

- Hardest items skew toward ambiguous airplane/ship silhouettes and cluttered cat/dog scenes.
- Easy set is dominated by high-contrast cues (e.g., red fire trucks), yielding low δ and entropy.

Study I Fit Checks & Edge Cases

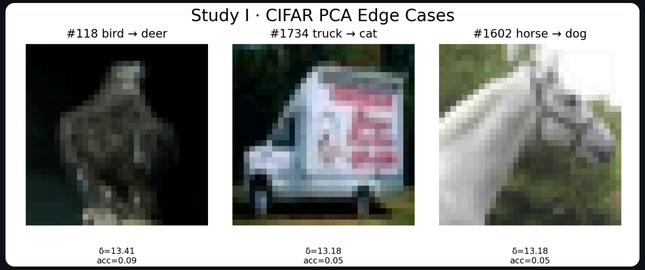
Fit diagnostics

Metric	Value
Item infit μ / p95	0.18 / 0.35
Item outfit μ / p95	0.18 / 0.34
Tree infit μ / p95	0.35 / 0.48
Tree outfit μ / p95	0.18 / 0.19

- MSQs well below 1 show tree responses are steadier than a pure Rasch prior; |z| never exceeds 0.05.

Edge cases worth a look

- `#118` bird → deer votes (δ ≈ 13.4, margin ≈ −0.05, entropy ≈ 2.28).
- `#1734` truck → cat/frog split (δ ≈ 13.2, margin ≈ −0.09, entropy ≈ 2.27).
- `#1602` horse → dog/horse tie (δ ≈ 13.2, margin ≈ −0.11, entropy ≈ 2.22).
- Each item sits below 9% tree accuracy—prime targets for relabeling or curated augmentations.



Study I edge cases · IDs 118, 1734, 1602

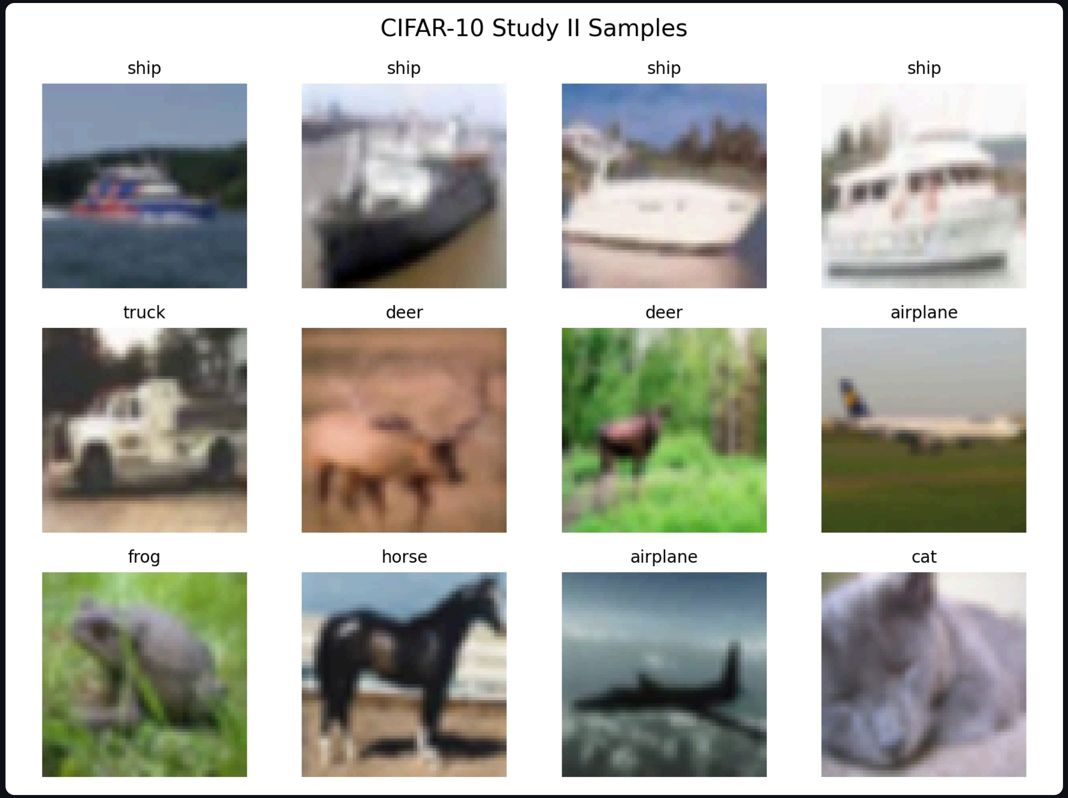
Study I Takeaways

- Weak PCA features create long tails in both ability (θ) and difficulty (δ), exposing erratic trees.
- Margin and entropy correlate with δ, but clusters of high-difficulty animals persist across diagnostics.
- Visual inspection confirms mislabeled or low-signal items driving high δ, motivating feature upgrades.

Study II: CIFAR-10 + MobileNet Embeddings

Study II Setup: CIFAR-10 + MobileNet-V3

- Hold the splits fixed to isolate feature gains.
- Swap PCA features for MobileNet-V3 (960-D) while keeping tree count and splits constant.
- Test whether richer embeddings tighten θ spread and retain δ alignment.
- Compare RF metrics, uncertainty signals, and IRT parameters against the baseline.



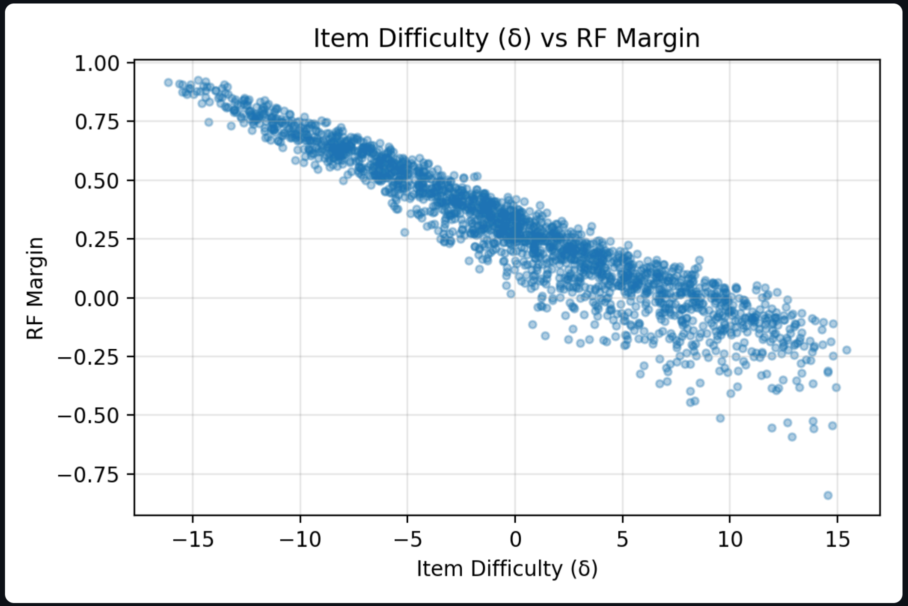
Study II sample grid — same splits, MobileNet embeddings

Study II Performance (MobileNet-V3)

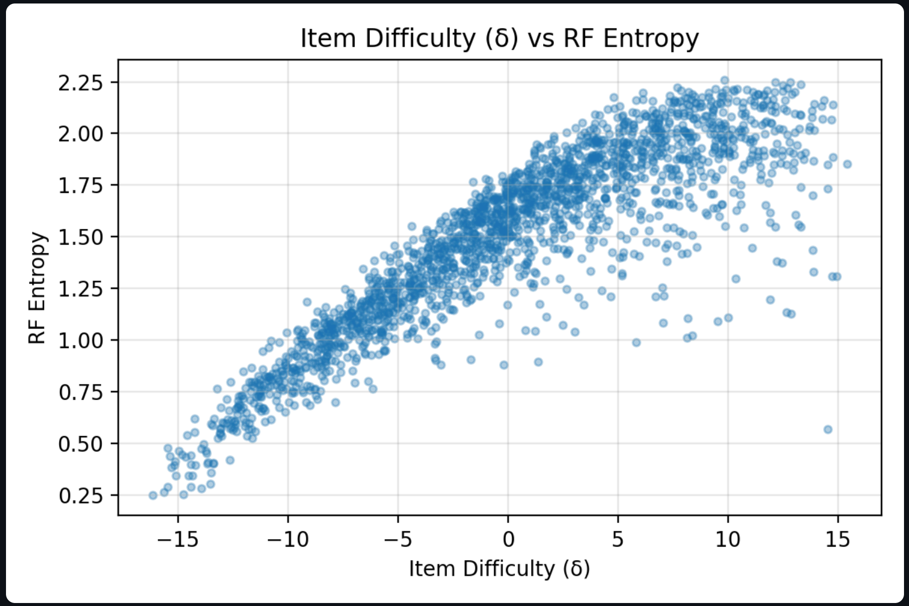
Metric	Value
Test / Val / OOB acc	0.819 / 0.820 / 0.812
Per-class range	0.695 (bird) → 0.925 (ship)
Mean tree accuracy	0.4792
Mean margin / entropy	0.2806 / 1.4929
δ negatively correlates with margin (Pearson)	−0.950
δ positively correlates with entropy (Pearson)	0.881

- Pretrained features boost accuracy by 35 pp while strengthening δ correlations.
- Higher margins and lower entropy show confidence gains except on stubborn animal classes.
- Artifacts: metrics, response matrix, signals, and IRT outputs under `data/mobilenet/`.

Study II Diagnostics: δ vs RF Signals



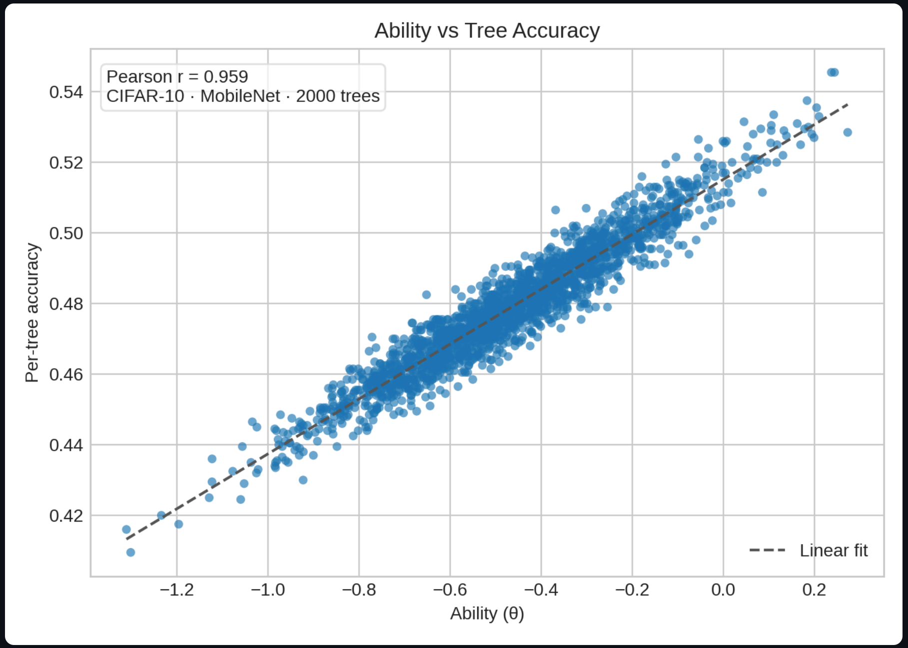
δ vs margin (Pearson −0.95)



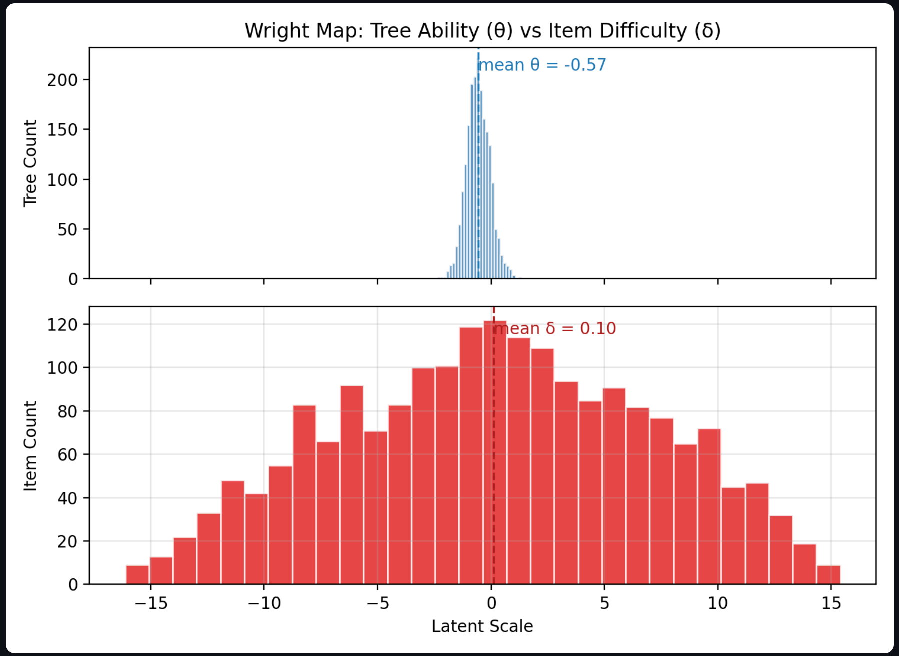
δ vs entropy (Pearson 0.88)

- MobileNet compresses the easy cluster (high margin, low entropy) while isolating true hard cases.
- Larger |corr| values show tighter agreement between δ and RF uncertainty.
- Cat/dog confusions persist, marking curation targets.

Study II Diagnostics: Ability Profiles



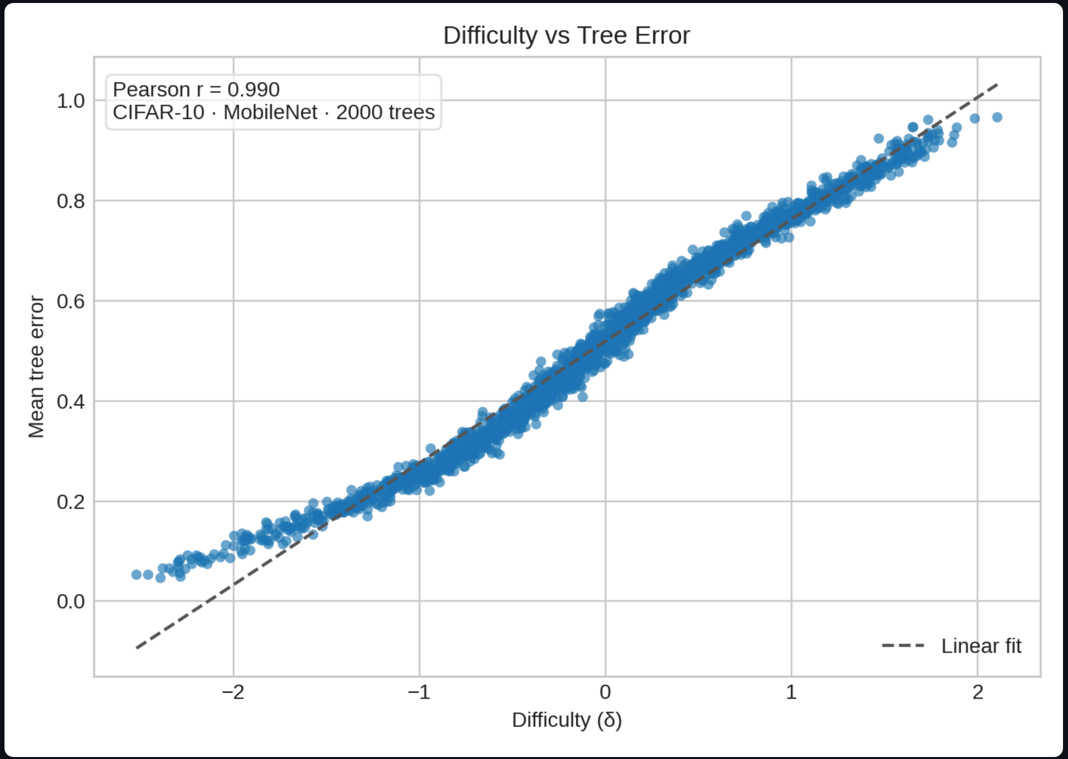
Ability (θ) vs tree accuracy — Pearson 0.96



Wright map: θ ≈ −0.46 ± 0.23; δ spans ±2.1

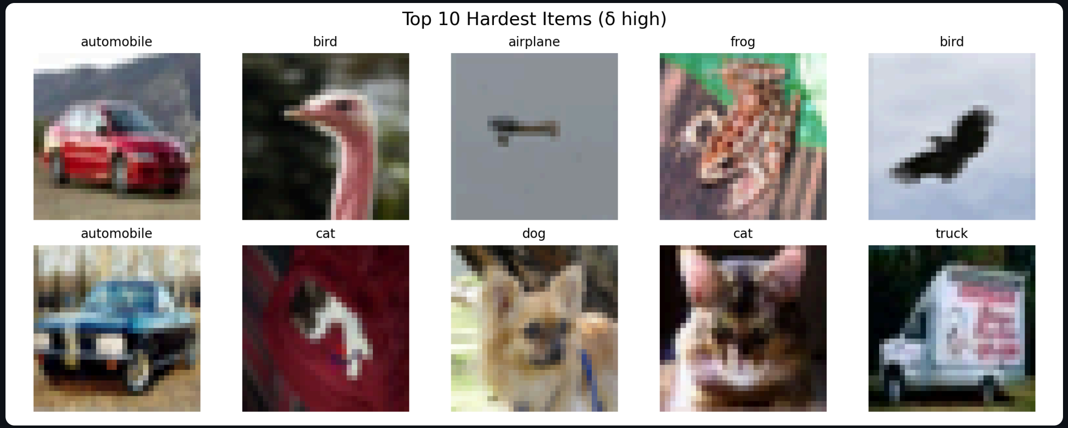
- θ mean −0.46 ± 0.23 keeps the ensemble tightly banded while still ranking trees cleanly.
- Ability remains tied to per-tree accuracy, so feature quality—rather than tree diversity—now caps gains.

Study II Diagnostics: δ vs Error Rate



- Pearson 0.99 keeps δ aligned with mean tree error even at the higher accuracy ceiling.
- Hardest items (δ > 1.5) persist—mostly cat/dog overlaps and ambiguous aircraft—while the easy zone (δ < −1) expands.

Study II Evidence: Hard vs Easy Examples



- MobileNet tightens easy clusters yet the same cat/dog outliers survive with $\delta > 1.5$.
- Easy wins sharpen into high-contrast ships and trucks, showing how feature upgrades cleanly separate low-δ items.

Study II Fit Checks & Edge Cases

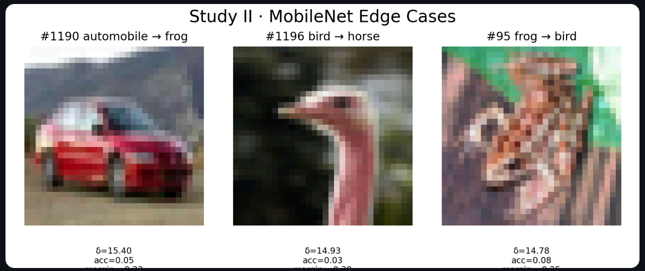
Fit diagnostics

Metric	Value
Item infit μ / p95	0.27 / 0.37
Item outfit μ / p95	0.27 / 0.37
Tree infit μ / p95	0.29 / 0.31
Tree outfit μ / p95	0.27 / 0.29

- Narrow MSQ spread (≤0.37) confirms MobileNet trees behave consistently; no misfit flags at $|z| > 0.05$.

Edge cases worth a look

- `#1190` automobile → frog votes (δ ≈ 15.4, margin ≈ −0.22, entropy ≈ 1.85; top probs frog 0.28, deer 0.27).
- `#1196` bird → horse (δ ≈ 14.9, margin ≈ −0.38, entropy ≈ 1.31; horse 0.41, deer 0.41, bird 0.08).
- `#95` frog → bird (δ ≈ 14.8, margin ≈ −0.25, entropy ≈ 1.89; bird 0.32, deer 0.20, frog 0.17).
- These persistent outliers survive the feature upgrade—queue them for image/label review next.



Study II edge cases · IDs 1190, 1196, 95

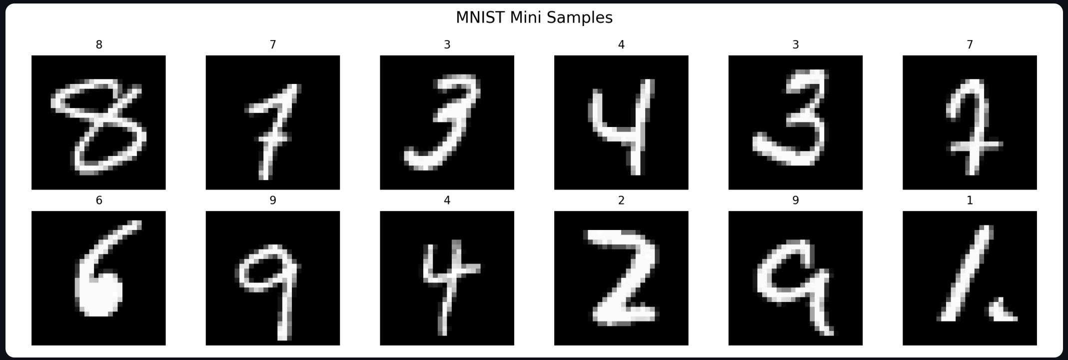
Study II Takeaways

- MobileNet embeddings add 35 pp of accuracy while maintaining a focused ability band (Std(θ) ≈ 0.23).
- δ stays aligned with RF uncertainty, isolating a smaller yet stubborn ambiguous cluster.
- Residual cat/dog confusion points to data curation as the next lever.

Section III · Control Study (MNIST)

Study III Setup: MNIST Mini-Study

- Probe the pipeline on a high-signal, low-noise dataset.
- Use a lightweight handwriting set to validate RF × IRT beyond CIFAR-10.
- Confirm that IRT still mirrors RF uncertainty when accuracy is near perfect.
- Treat it as a control case where ambiguity is rare yet still detectable.



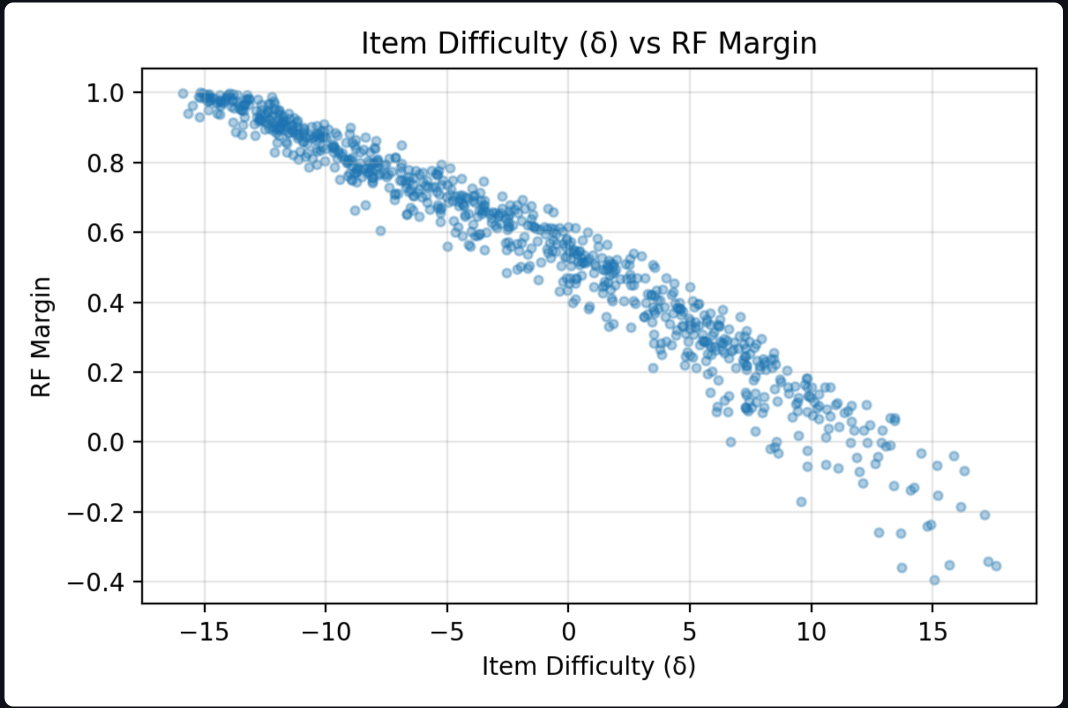
Study III sample grid — curated MNIST mini split

Study III Performance (MNIST)

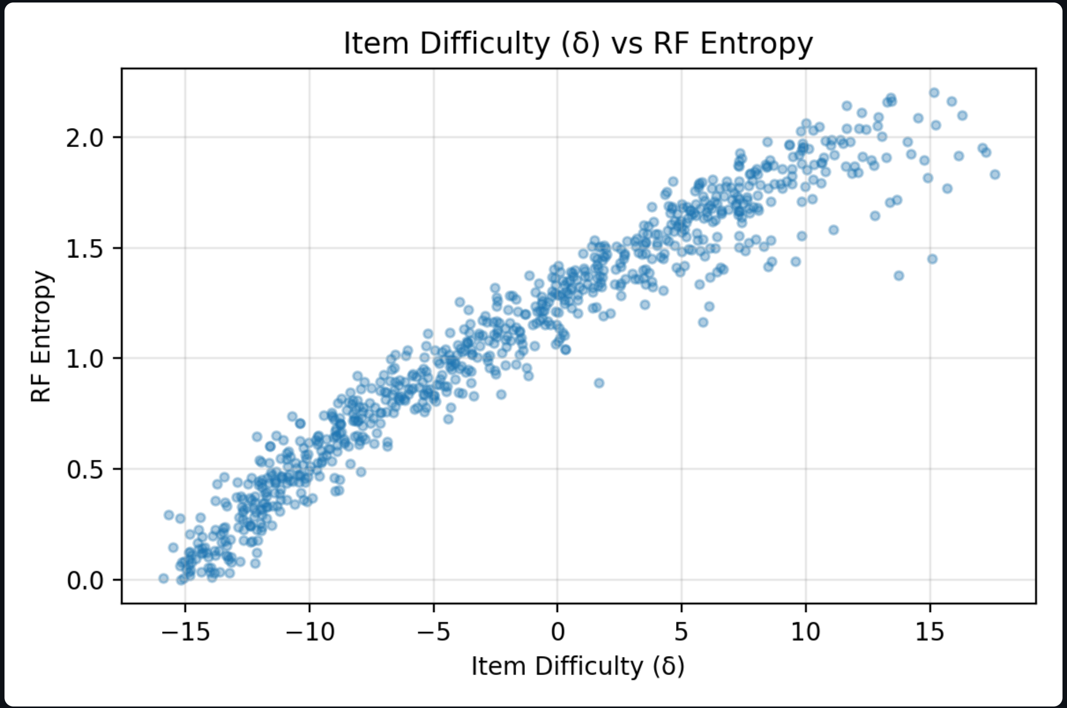
Metric	Value
Train / Val / Test	4000 / 800 / 800
RF test / val / OOB	0.954 / 0.944 / 0.939
Mean margin / entropy	0.5644 / 1.0768
δ negatively correlates with margin (Pearson)	−0.975
δ positively correlates with entropy (Pearson)	0.970
θ mean ± std	3.04 ± 0.29
δ mean ± std	−0.13 ± 0.47

- Ambiguous digits (e.g., brushed 5 vs 6) still spike δ toward the positive tail; elsewhere the forest is decisive.
- Low entropy + high margin line up with low δ, giving a “sanity benchmark” beyond CIFAR.

Study III Diagnostics: δ vs RF Signals



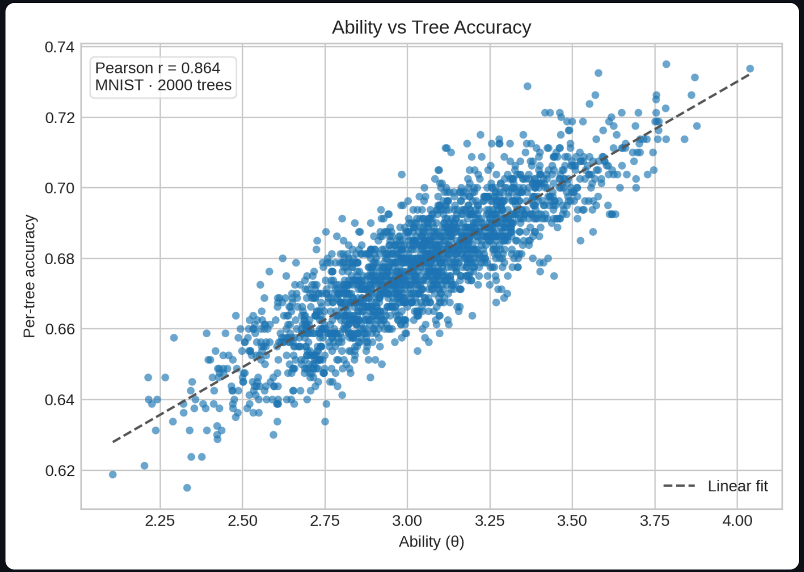
δ vs margin (Pearson −0.97)



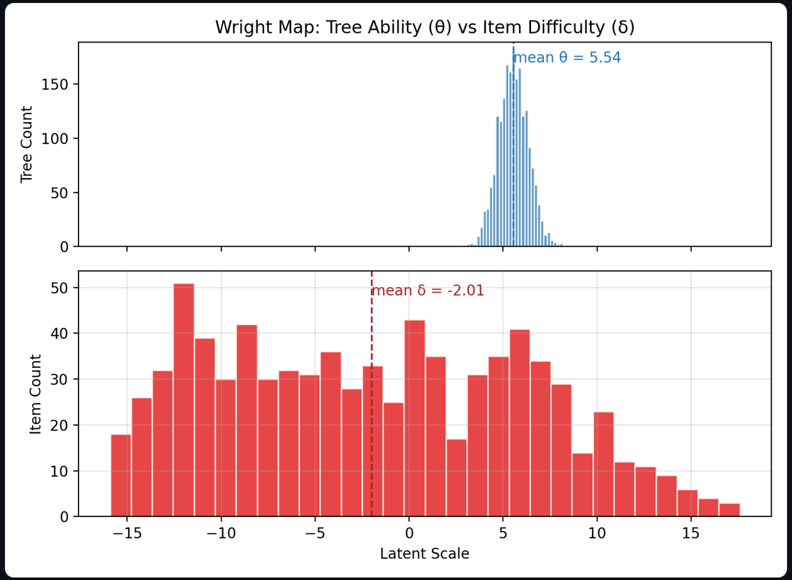
δ vs entropy (Pearson 0.97)

- Clean digits show near-perfect alignment between δ and RF uncertainty.
- Only a handful of δ > 1.2 digits drive the residual uncertainty (stroke collisions like 3/5, 4/9).

Study III Diagnostics: Ability Profiles



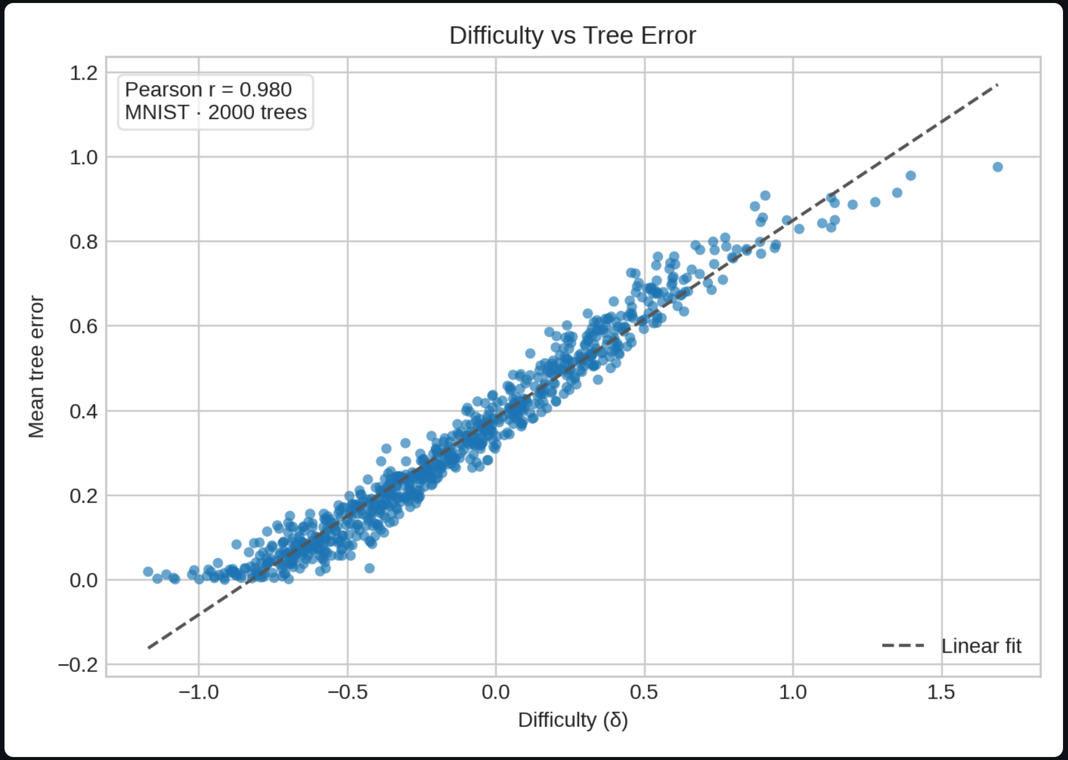
Ability (θ) vs tree accuracy — Pearson 0.98



Wright map: θ mean 3.04 ± 0.29; δ mean −0.13 ± 0.47

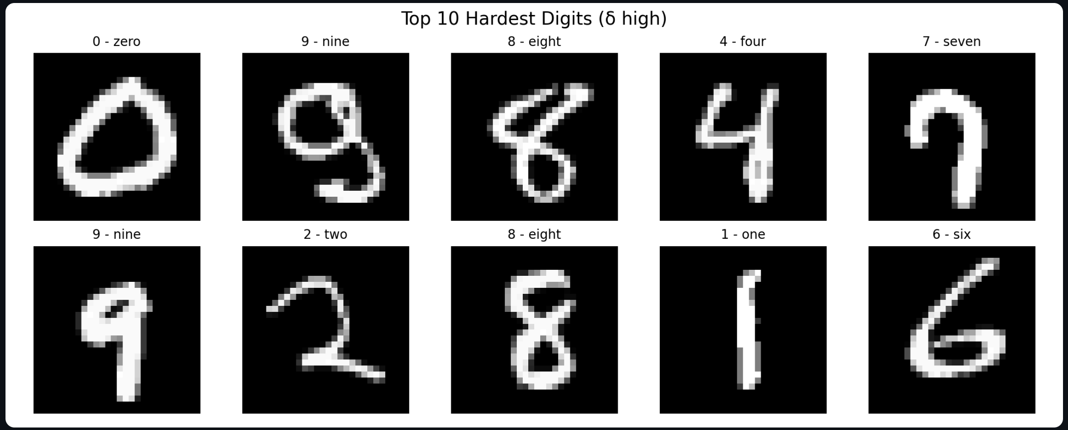
- θ mean 3.04 ± 0.29 shows strong consensus, while δ mean −0.13 ± 0.47 keeps a modest positive tail for ambiguous strokes.
- Shared scales expose plentiful easy wins with only a few sharp spikes—opposite of the CIFAR baseline.

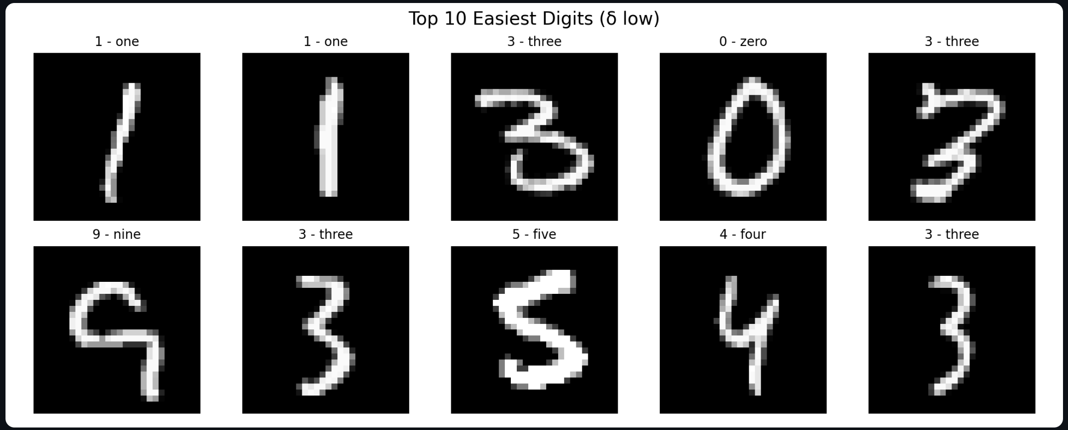
Study III Diagnostics: δ vs Error Rate



- Pearson 0.98 keeps δ tied to mean tree error despite the high accuracy ceiling.
- δ > 1.2 corresponds to stroke-collided 3/5/8 and 4/9 pairs; the long negative tail is trivial for the ensemble.

Study III Evidence: Hard vs Easy Digits





- Hardest digits show stroke collisions (3 vs 5, 4 vs 9) that push δ above 1 despite high margins elsewhere.
- Easy digits are crisp, centered strokes—useful anchors when explaining why δ plunges on most of the dataset.

Study III Fit Checks & Edge Cases

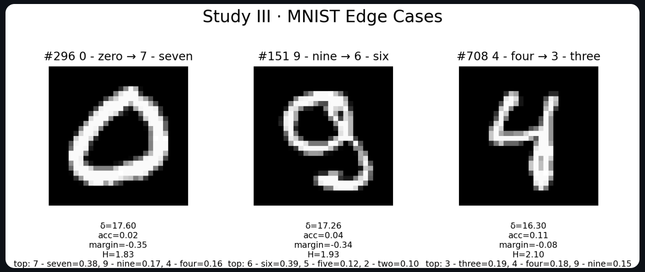
Fit diagnostics

Metric	Value
Item infit μ / p95	0.23 / 0.38
Item outfit μ / p95	0.22 / 0.37
Tree infit μ / p95	0.30 / 0.32
Tree outfit μ / p95	0.22 / 0.25

- Rasch residuals stay tight ($|z| < 0.07$), confirming the control study's consistency.

Edge cases worth a look

- `#296` digit 0 → vote 7 (δ ≈ 17.6, margin ≈ −0.35, entropy ≈ 1.83; top probs 7=0.38, 9=0.18, 4=0.16).
- `#151` digit 9 → vote 6 (δ ≈ 17.3, margin ≈ −0.34, entropy ≈ 1.93; top probs 6=0.39, 5=0.12, 2=0.10).
- `#708` digit 4 → vote 3 (δ ≈ 16.3, margin ≈ −0.08, entropy ≈ 2.10; top probs 3=0.19, 4=0.18, 9=0.15).
- Archive these strokes for a "confusing digits" gallery or curation playbook.



Study III edge cases · IDs 296, 151, 708

Study III Takeaways

- δ and RF uncertainty agree almost perfectly, while θ stays high yet still flags the rare ambiguous strokes.
- The control study confirms the RF × IRT pipeline holds outside noisy vision data.

Section IV · Cross-Study & Diagnostics

- Compare backbones and datasets on a shared θ/δ scale.
- Surface recurring themes before the close.

Cross-Study Snapshot

Study	Feature Backbone	Test Acc	δ negatively correlates with margin (Pearson)	δ positively correlates with entropy (Pearson)	Std(θ)	Std(δ)
Study I: CIFAR + PCA-128	PCA-128	0.468	−0.815	0.687	0.154	0.150
Study II: CIFAR + MobileNet	MobileNet-V3 (960-D)	0.819	−0.950	0.881	0.228	0.871
Study III: MNIST Mini	Raw pixels	0.954	−0.975	0.970	0.289	0.472

- *Std(θ) measures tree ability spread; Std(δ) measures item difficulty spread.*
- δ stays negative with margin and positive with entropy for every study (−0.82/−0.95/−0.98 vs +0.69/+0.88/+0.97).
- θ spread remains compact (Std(θ) ≈ 0.15–0.29); MobileNet is only slightly wider as headroom grows.
- Difficulty variance jumps on MobileNet (Std(δ) ≈ 0.87) while MNIST stays moderate, highlighting how rich features surface nuanced “hard” digits.

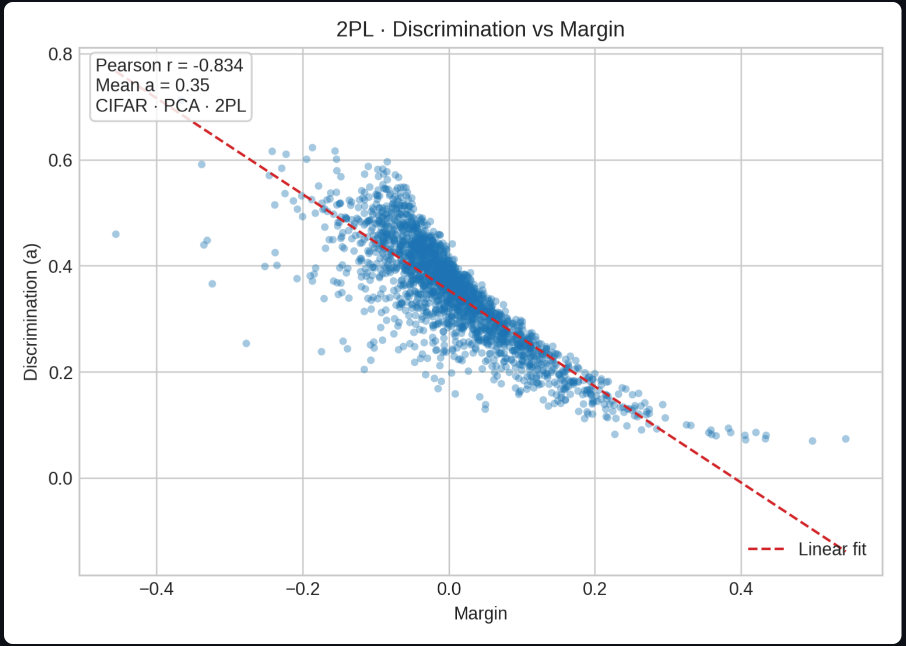
Cross-Study Fit Snapshot

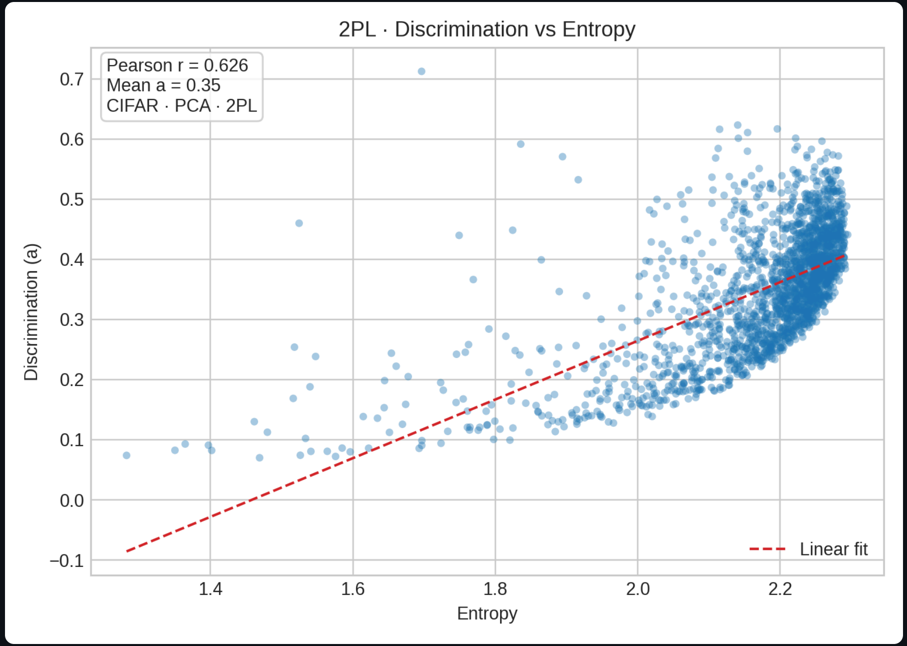
Study	Item infit μ / p95	Item outfit μ / p95	Tree infit μ / p95	Tree outfit μ / p95
CIFAR + PCA	0.18 / 0.35	0.18 / 0.34	0.35 / 0.48	0.18 / 0.19
CIFAR + MobileNet	0.27 / 0.37	0.27 / 0.37	0.29 / 0.31	0.27 / 0.29
MNIST mini	0.23 / 0.38	0.22 / 0.37	0.30 / 0.32	0.22 / 0.25

- All MSQs stay well below 1, indicating over-dispersed errors are rare and Rasch assumptions hold after 2000-tree scaling.
- MobileNet's slight lift in item MSQ reflects richer feature diversity, while MNIST keeps both item and tree fits exceptionally tight.

2PL Discrimination (CIFAR + PCA)

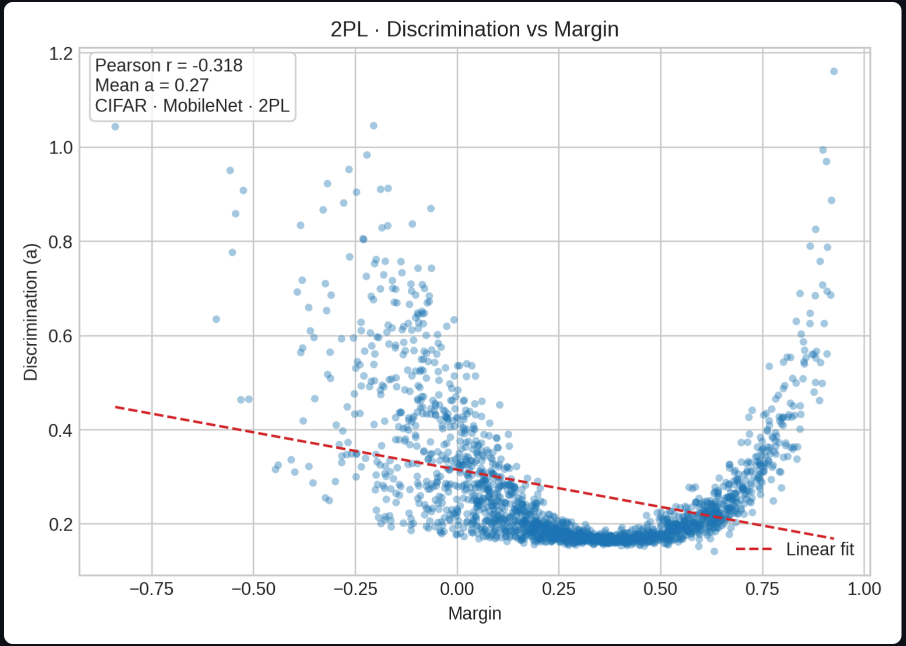
- 800-epoch 2PL fit (lr 0.02) yields mean $a \approx$ **0.35 ± 0.10** (range 0.07–0.71).
- a correlates with margin at **−0.83** and with entropy at **+0.63**, aligning slope with RF uncertainty signals.
- Discrimination peaks on the low-margin, high-entropy animal items and steadily tapers for easier scenes, leaving high-margin images with softer slopes.

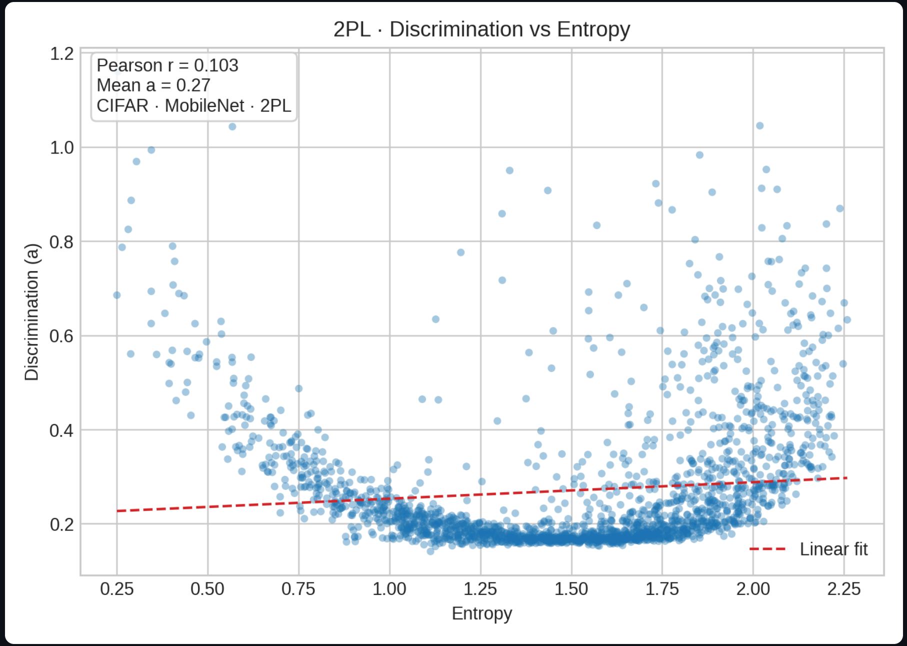




2PL Discrimination (CIFAR + MobileNet)

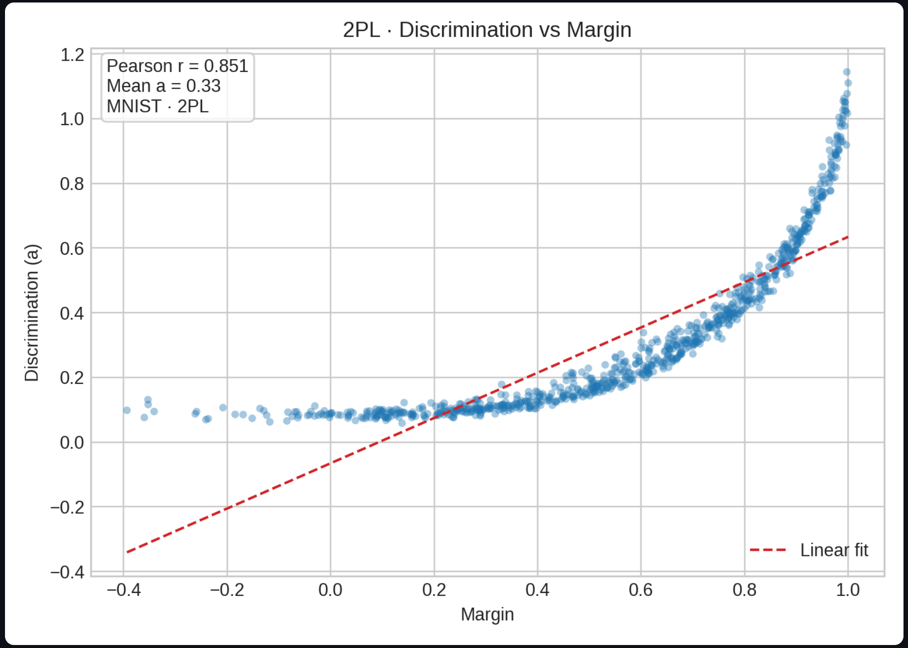
- Mean a settles at **0.27 ± 0.15** with a modest tail (max ≈1.16).
- a correlates with margin at **−0.32** and with entropy at **+0.10**, keeping residual cat/dog confusion in focus while the easy cluster sharpens.
- Discrimination concentrates in the tails: hard animal confusions and trivially easy scenes separate trees, while mid-uncertainty items contribute little.

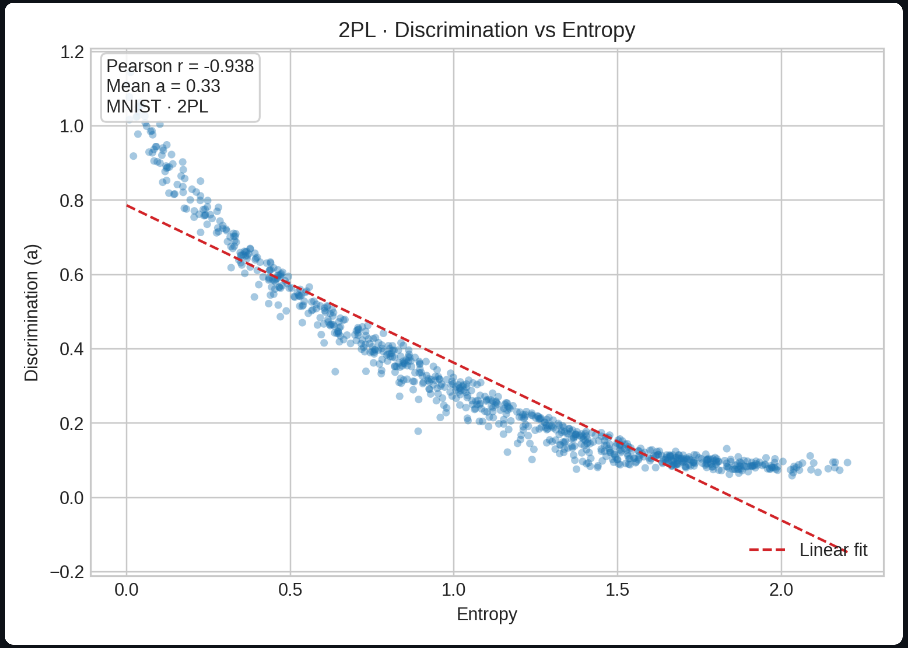




2PL Discrimination (MNIST)

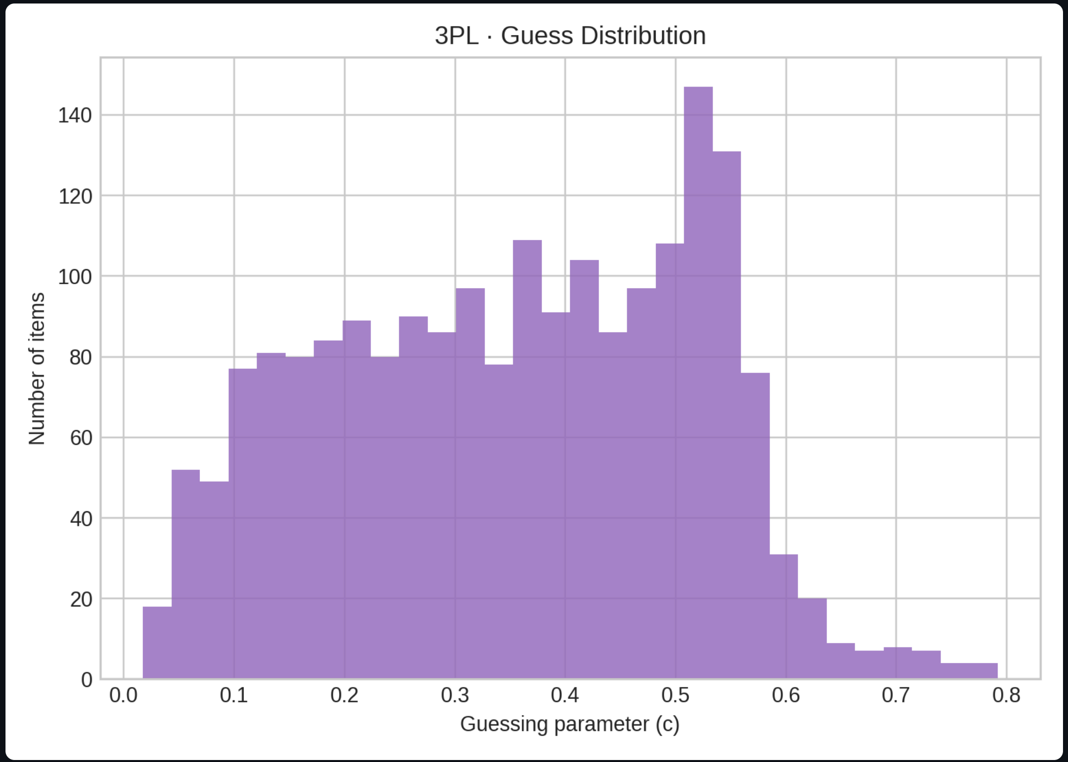
- Mean a lifts to **0.33 ± 0.25**, so only a modest slice of digits remains truly separating despite the high accuracy ceiling.
- a correlates with margin at **+0.89** while its correlation with entropy flips to **−0.96**—uncertainty vanishes outside the awkward strokes.
- Discrimination climbs with margin and falls with entropy: crisp, easy digits carry the steepest slopes while ambiguous stroke collisions stay much flatter.



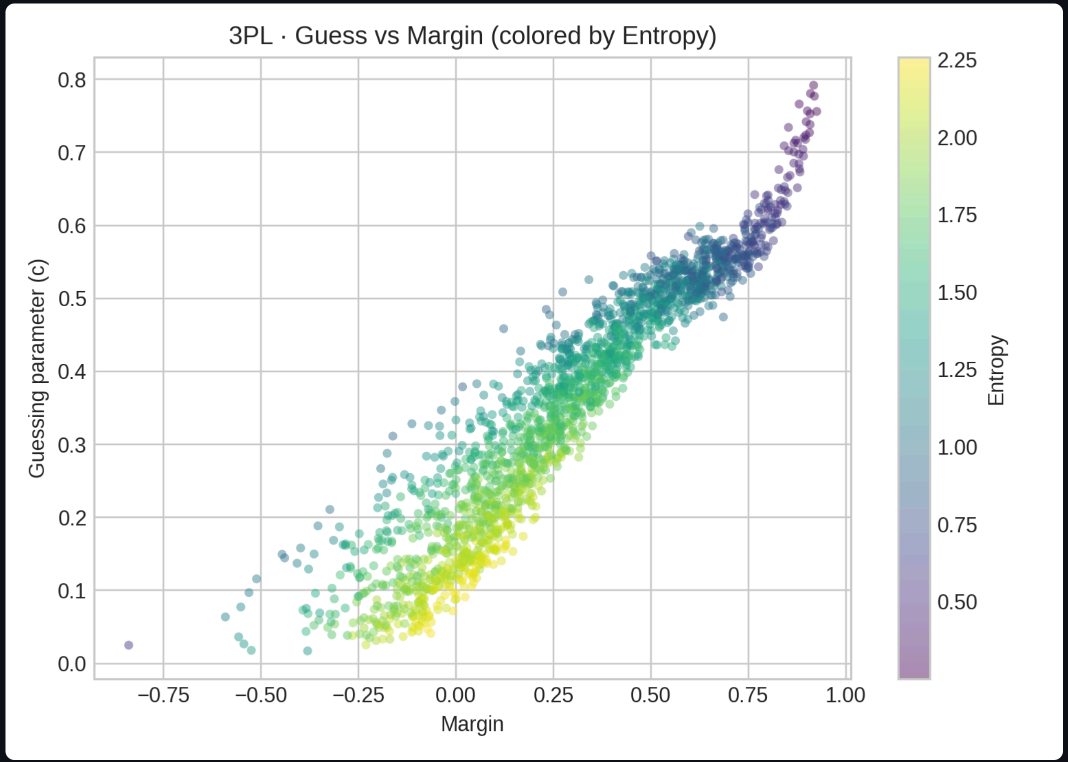


3PL Pilot · MobileNet

- 1k-epoch 3PL run (lr 0.01) lands at guess mean **0.35 ± 0.16**.
- θ vs accuracy stays tight (Pearson **0.98**); slopes average **0.32 ± 0.08** with a broader tail.
- High guess mass piles onto the ambiguous animal scenes (low margin, high entropy), reinforcing the “guessing” narrative.



3PL MobileNet · Guess distribution

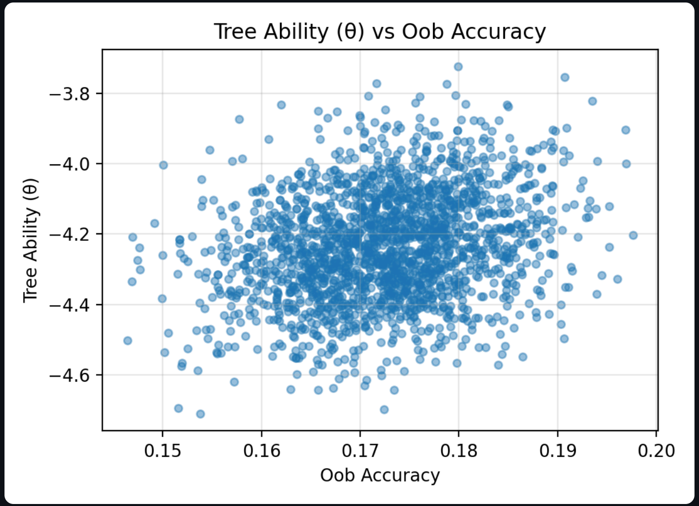


3PL MobileNet · Guess vs Margin (colored by entropy)

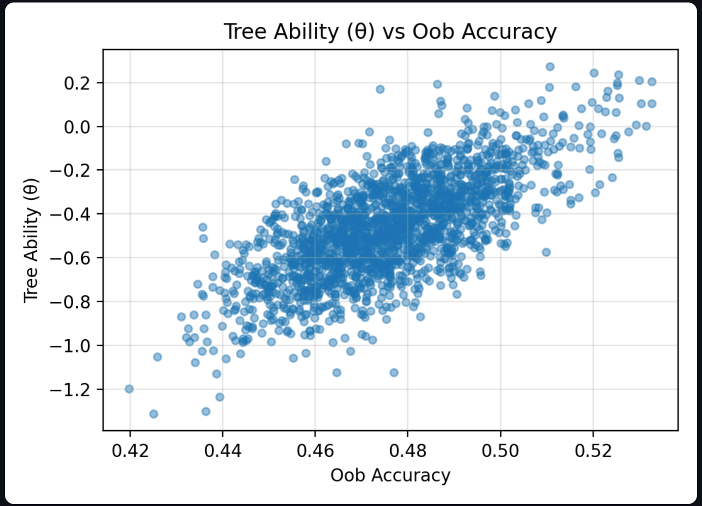
Tree Attribute Correlations · OOB Accuracy vs θ

- `scripts/analyze_tree_attribute_correlations.py` merges each tree's depth/leaves/OOB stats with θ and discrimination aggregates.
- Pearson r (OOB accuracy, θ): PCA **+0.25**, MobileNet **+0.70**, MNIST **+0.39** — reliable trees earn higher ability across every study.
- CSV/JSON exports: `data/*/tree_attributes_with_signals.csv`, `data/*/tree_attribute_correlations*.json` for deeper dives.

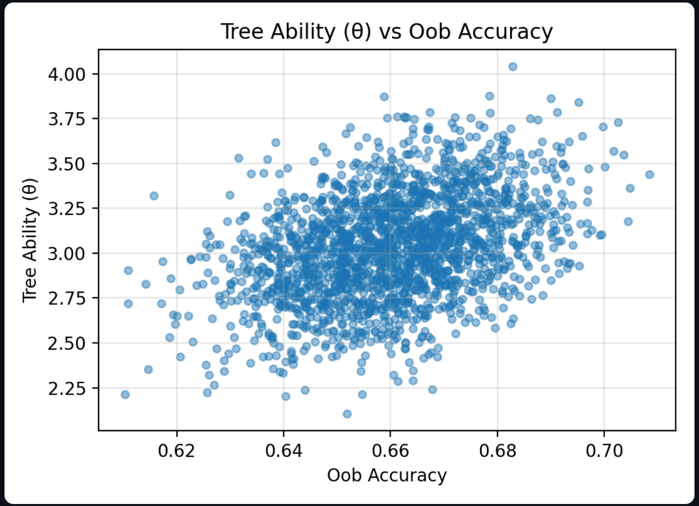
PCA · OOB accuracy vs θ (r = +0.25)



MobileNet · OOB acc vs θ (r = +0.70)



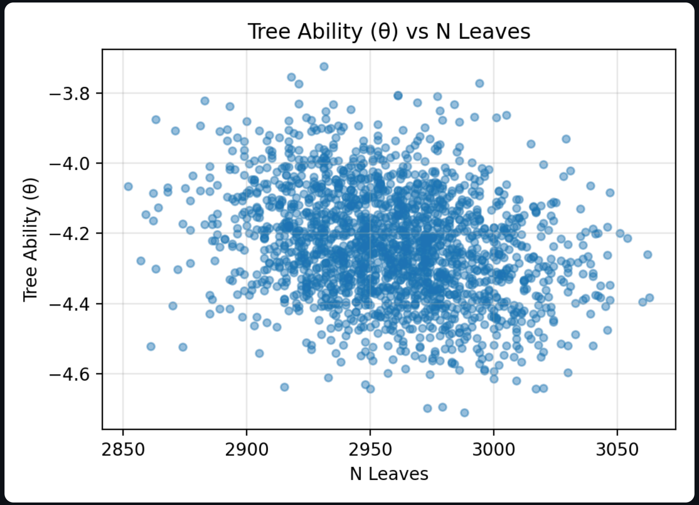
MNIST · OOB accuracy vs θ (r = +0.39)



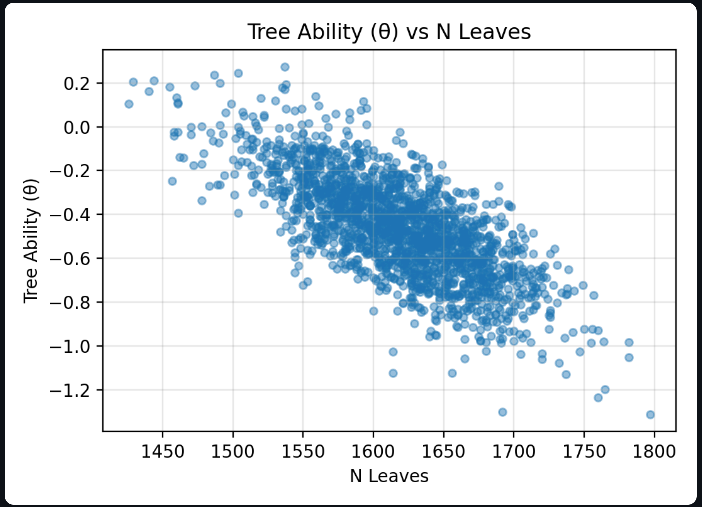
Tree Attribute Correlations · Leaf Count vs θ

- Pearson r (leaf count, θ): PCA **−0.27**, MobileNet **−0.73**, MNIST **−0.38** — pruning shallower trees boosts ability rankings.
- Leaf count penalizes overfitting branches; MobileNet shows the steepest drop because high-quality features reward compact trees.

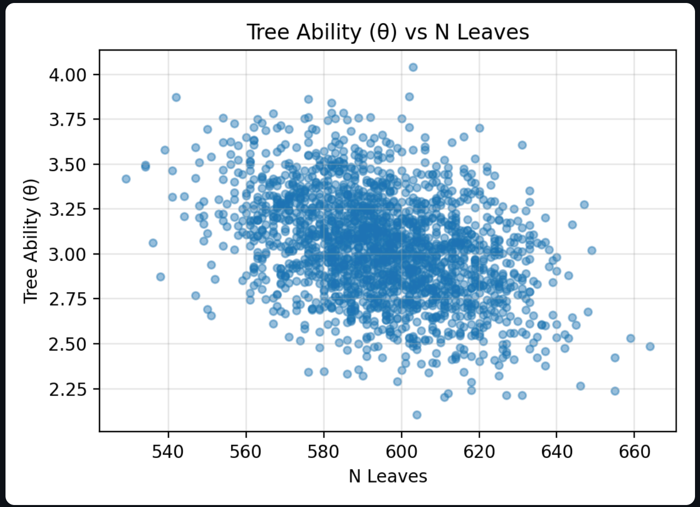
PCA · Leaf count vs θ (r = −0.27)



MobileNet · Leaf count vs θ (r = −0.73)



MNIST · Leaf count vs θ (r = −0.38)



Key Takeaways

- IRT and RF still move in lockstep: θ tracks per-tree accuracy, while δ and a surface stubborn item pockets.
- MobileNet’s discrimination tail isolates animal confusions despite stronger features; MNIST flips signs because mistakes are rare.
- 3PL adds a modest guessing floor (~0.25) without upsetting θ–accuracy alignment.
- Tree attributes expose pruning cues: shallow, high-OOB trees consistently land higher θ.

Next Steps

- Run stability sweeps (50/100 trees, alternate seeds) to quantify variance in a and θ.
- Decide whether 3PL merits extension to PCA/MNIST or documenting as MobileNet-only.
- Finish item-tier overlays (high/medium/low a) and align them with the qualitative grids.

Decision Trees — From Data to Splits

Idea: recursively split data to increase *purity* of labels (Breiman et al., 1984).

Example:

> “PetalLength < 2.5?” → all *Setosa* left, others right.

At each node:

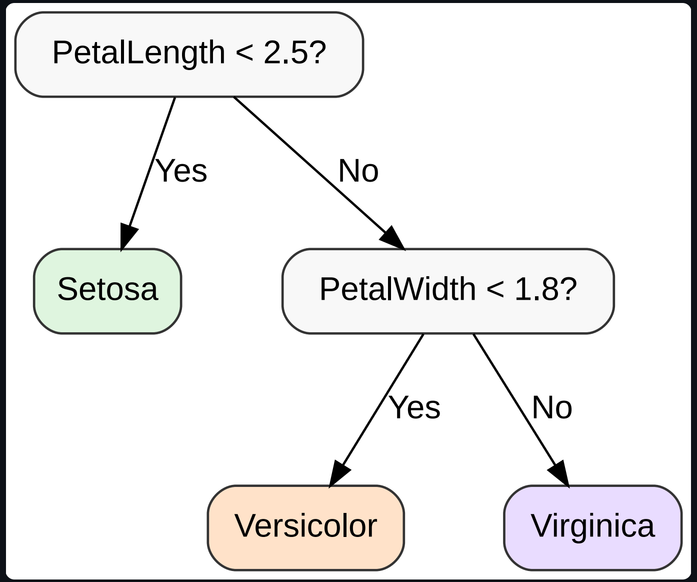
- compute **impurity** (e.g., *entropy* or *Gini*):

$$H = -\sum_i p_i \log_2 p_i$$

- choose the split that **maximally reduces impurity** — i.e. makes groups more uniform.

A single tree = a set of *if–then* rules that classify or predict.

PetalLength	PetalWidth	Species
1.4	0.2	Setosa
4.7	1.4	Versicolor
5.5	2.0	Virginica



Gini vs. Entropy — Two Lenses on Node Impurity

Entropy (Information Theory):

$$H = -\sum_i p_i \log_2 p_i$$

Measures **uncertainty** — expected information (in bits) needed to classify a random sample. *High when classes are evenly mixed.*

Gini Impurity (Probability of Misclassification):

$$G = 1 - \sum_i p_i^2$$

Measures **chance of error** — probability that two randomly drawn samples from the node belong to different classes.

Metric	Theoretical Lens	Interpretation	Typical Use
Entropy	Information theory	“How surprised would I be?”	ID3, C4.5 trees
Gini	Probability theory	“How often would I be wrong?”	CART trees, scikit-learn default

> Both peak when classes are perfectly mixed (p = 0.5).
> Gini is slightly flatter — faster to compute, less sensitive to extremes.

References

- Wilson, M. (2005). *Constructing Measures: An Item Response Modeling Approach*. Lawrence Erlbaum Associates.
- Breiman, L., Friedman, J. H., Olshen, R. A., & Stone, C. J. (1984). *Classification and Regression Trees*. Wadsworth.
- Breiman, L. (2001). "Random Forests." *Machine Learning*, 45(1), 5–32.