Spectroscopic Factor Calculation for ^{15}N(^{6}Li, d)^{19}F

Erin Good

Spring 2019

1 Outline of project

This spring we will be measuring the ^{15}Ne(^{6}Li, d)^{19}F cross section using HELIOS at Argonne National Lab, during which we will find the spectroscopic factor of the 3.908 MeV state in ^{19}F; this state is the mirror of the 4.033 MeV state in ^{19}Ne. Because the particles are mirrors, their spectroscopic factors should be the same; once we find the spectroscopic factor of the mirror state we can use it to calculate the α partial width of the ^{19}Ne state we are interested in.

For this project, I will first start by re-calculating the spectroscopic factor used in an often maligned paper by Mao *et al* (Ref. [1]) that measures the cross section and corresponding spectroscopic factor but makes a bad choice of normalization. Using the measured cross section, we can calculate the spectroscopic factor using:

$$S_\alpha = \frac{\sigma_{expt}}{\sigma_{calc}} \frac{1}{N} \frac{(2J_i+1)(2L+1)}{(2J_f+1)} \tag{1}$$

where σ_{expt} is the experimental cross section, σ_{calc} is the cross section calculated from an optical potential, N is the normalization factor, J_i is the spin of the target nucleus, L is the angular momentum of the transferred alpha, and J_f is the spin of the residual nucleus. To recalculate this I will need to reproduce σ_{calc} using Fresco from Mao *et al*'s potential parameters as an input.

After re-calculating the spectroscopic factor for the data given in the Mao *et al* paper, I will evaluate the calculated cross section and recalculate it using another optical potential model I feel is appropriate. I may calculate several of these and do some sort of sensitivity study.

Lastly, I will use the NuShellX code to calculate the alpha particle width via the relation $\Gamma_\alpha = S_\alpha \Gamma_{sp}$, where Γ_{sp} is the single particle partial width calculated in NuShellX. I will most likely use several different models for single particle widths and make a choice on the best option.

2 Optical potential formalism in Fresco

Fresco calculations require five optical potentials as input, as well as overlaps and couplings. Here the formalism behind these choices is described in terms of a (d,p) reaction: $A(d,p)B$. For transfer reactions, the 1-step DWBA formalism is typically written in terms of the T-matrix in either post- or prior-form:

$$\begin{aligned} T_{\text{post}} &= \langle \phi_{nA}\chi^-_{pB} | V_{np}(r_1) + U_{pA}(r_p) - U_{pB}(R_2) | \phi_{np}\chi^+_{dA} \rangle \\ &= \langle \phi_{nA}\chi^-_{pB} | V_{np}(r_1) | \phi_{np}\chi^+_{dA} \rangle + \langle \phi_{nA}\chi^-_{pB} | U_{pA}(r_p) - U_{pB}(R_2) | \phi_{np}\chi^+_{dA} \rangle \end{aligned} \tag{2}$$

$$\begin{aligned} T_{\text{prior}} &= \langle \phi_{nA}\chi^-_{pB} | U_{nA}(r_n) + U_{pA}(r_p) - U_{dA}(R_1) | \phi_{np}\chi^+_{dA} \rangle \\ &= \langle \phi_{nA}\chi^-_{pB} | U_{nA}(r_n) | \phi_{np}\chi^+_{dA} \rangle + \langle \phi_{nA}\chi^-_{pB} | U_{pA}(r_p) - U_{dA}(R_1) | \phi_{np}\chi^+_{dA} \rangle \end{aligned} \tag{3}$$

where the ϕ_{np} and ϕ_{nA} wave functions are the bound states of the deuteron and the neutron in the composite system of B particles, respectively. The χ^-_{pB} and χ^+_{dA} are distorted waves associated with the outgoing and incoming channels, respectively.

The five required potentials for Fresco calcuations are as follows: Potential 1 is U_{dA}, an optical potential for deuteron scattering on the target nucleus, which is chosen at the deuteron beam energy. Potential 2 is

U_{pB}, an optical potential for proton scattering on the composite final nucleus, chosen at the deuteron beam energy plus the Q-value (note that the Q-value must be added after the deuteron beam energy is converted to the center of mass frame, then converted back to the lab frame). Potential 3 is U_{nA}, which binds the neutron to particle A in the composite B-particle system; this is not typically a traditional scattering optical potential but a combination of gaussian and other interactions describing the nucleus. Potential 4 is V_{np}, the deuteron binding potential, which encapsulates the breakup of the projectile into its core and valence parts (here the proton is the core and the neutron is the valence as the neutron is transferred to the final composite nucleus); this can be modeled as a gaussian system or otherwise, but is also not a typical optical model potential. Lastly, potential 5 is U_{pA}, an optical potential for proton scattering on the target nucleus; this can be calculated at the beam energy or the beam energy plus Q-value, but does not always make a difference. The resultant cross section is independent of the choice of post- or prior-form.

Finally, we must define the couplings between the incoming and outgoing partitions that we need to consider to describe the transfer reaction. In FRESCO, this is done using an `&Coupling` namelist. For the post-form 1-step DWBA calculation, we define coupling to the n + ^{25}Mg partition to the d + ^{24}Mg partition. We consider the zero-range approximation[1] (FRESCO `&Coupling` namelist flag `kind=5`), as well as a finite-range calculation (FRESCO `&Coupling` namelist flag `kind=7`). For the finite-range calculations, we consider the approximation with no remnant term (FRESCO `&Coupling` namelist flag `ip2=0`) and with a full complex remnant term. We also set the coefficients of fractional parentage for the two couplings to `a=1.00`, each.

3 Theoretical Cross Section Calculations

I chose to recalculate the spectroscopic factor for the Mao paper [1] for a ^{6}Li beam at 22 MeV and to do prep work for the upcoming HELIOS experiment, currently scheduled to run a ^{15}N beam at 100.5 MeV. With a few calculations to find the correct energies for each, the same optical model potentials can be used for both theoretical cross section calculations. Here I will discuss the five optical potentials required for the FRESCO calculations, and discuss the energies chosen (and how they were chosen) for these optical models in each subsection. The subsections will also discuss the other input choices required by FRESCO and how those were decided upon.

Optical Model Potentials

Potential 1 is the U_{dA} potential, here representing ^{15}N(^{6}Li,^{6}Li) scattering at the incident beam energy. Because ^{6}Li scattering data is scarce, I was unable to find a potential for specific beam energies, but was able to find a potential by Wu et al. in Ref. [2] that does derive its OMP parameters from actual ^{15}N(^{6}Li,^{6}Li) scattering data at 34.5 MeV. This potential contains only a volume term `type=1` with a Woods-Saxon squared form `shape=1` for the real part `p1, p2, p3` and a regular Woods-Saxon form type=1 shape=0 for the imaginary part `p4, p5, p6`. They did investigate the spin-orbit potential but found that they were thought to have little or no influence on the cross section. Surface derivative terms are not included or discussed.

Potential 2 is the U_{pB} potential, for ^{19}F(d,d) scattering at $E_{beam} + Q$. Potential 5 is the U_{pA} potential, here representing ^{15}N(d,d) scattering at 22 MeV (this can also be calculated at the beam + Q value energy but in previous work I have found that changing these values makes minimal to no difference in the final cross-section outcome). For deuteron scattering off of ^{15}N and ^{19}F (the U_{pB} and U_{pA} denoted by potentials

[1] Note that FRESCO requires different inputs in the `&Coupling` namelist than the other DWBA approximations. This requires new parameter flags `p1` and `p2` that describe the depth and width of our approximated Dirac delta function, described in detail in the full FRESCO manual.

2 and 5, respectively) I chose to use the global deuteron optical potential by Han et al. in Ref. [3], as it had the best fits in the Z and A region of our nuclei of interest, a more narrow range around our Z and A of interest, and built on the Bojowald et al. model in the RIPL-3 database. The volume `type=1`, surface derivative `type=2`, and spin-orbit `type=3` potentials are of the Woods-Saxon shapes `shape=0` defined for each potential type in FRESCO.

Potential 3 is the V_{np} potential, or the potential of our projectile's core and valence particles. Here ^{6}Li can be viewed as a deuteron core and an alpha valence particle, as the alpha is the particle getting transferred to our final compound nucleus ^{19}F. After conducting a literature search for a ^{6}Li potential modeled as $d+\alpha$, the only potential I was able to parse in FRESCO was a potential by Kubo and Hirata from 1972, in Ref. [4]. This potential models the $d+\alpha$ system as an analytic function of Woods-Saxon shape with real parameters $V_{WS}, R_0,$ and a, interpreted in FRESCO as a type=1 shape=0 potential with parameters `p1, p2, p3`. The potential depth V_{WS} was determined to give a separation energy of the deuteron and alpha of 1.47 MeV, 1 node, and $l_a = 0$ for the relative motion of the two clusters in the ground state of ^{6}Li. Because of how FRESCO calculates parameter `p2`, the value from the Kubo paper ($R_0 = 1.9$ fm) must be manipulated to get the actual parameter value of `p2`:

$$R_0 = \texttt{p2}(A_T^{1/3} + A_p^{1/3}) \rightarrow \texttt{p2} = \frac{R_0}{(A_T^{1/3} + A_p^{1/3})} = \frac{1.9}{(4^{1/3} + 2^{1/3})} = 0.67 \tag{4}$$

where A_T and A_p are the number of nucleons in the deuteron and alpha constituent parts of ^{6}Li. There is another potential by Dubovichenko from 1994 that I would like to try but its implementation in FRESCO is not straightforward.

Potential 4 is the U_{nA} potential, or the potential of target A and the transferred particle. Here this is the potential between the transferred α particle and the ^{15}N nucleus. I chose a potential modeled in a paper by Avila et al. from 2014 in Ref. [5] for $^{16}\text{O}+\alpha$, as I understood all of the inputs.[2] The potential they used gave just a real volume radius and diffuseness term, or `p2` and `p3`, and allowed fresco to fit the potential depth to reproduce the experimental binding energy given in the overlap term associated with this potential using `isc=1`. I gave an initial term of `p1=40.0` just to give FRESCO somewhere to start. Additionally, I used a real spin-orbit term as we did in Homework 3, though after testing its effects it seems it has no influence on the cross section. I left it in anyways. Lastly, I included a Coulomb term with a radius of 1.25. This term does affect the cross section, particularly the shape, and so I will leave it in for now.

Overlaps

We will be coding in overlaps for the projectile overlap, $\langle ^6\text{Li}|d\rangle$, and the target overlap, $\langle ^{15}\text{O}|^{19}\text{F}\rangle$.

For the $\langle ^6\text{Li}|d\rangle$ overlap we have a spin of 1+ for the ^{6}Li, a spin of 1+ for the deuteron, and a spin of 0+ for the transferred alpha particle. To start, we have an index `kn1=???` and `kn2=???` that. Our indices `ic1=1` and `ic2=2` indicate the partitions that our ^{6}Li and deuteron come from, respectively. The `in=1` indicates that we are looking at projectiles. For our projectile, the overlap assumes an inert core and take $|(l, sn)j\rangle$ coupling (`kind=0`). This takes the inputs of quantum numbers `nn=1 l=0 sn=0 j=0`, which correspond to the radial quantum number 1 (since FRESCO starts at `n=1`), angular momentum transfer $l = 0$, an intrinsic spin of our transferred alpha particle of $s = 0$ and a total angular momentum $j = l + s = 0$. This coupling is paired to potential 3, or V_{np}, and describes the potential between our core deuteron and our valence alpha particle with `kbpot=3`. We then declare the binding energy of the $^6\text{Li} = d + \alpha$ system, which is `be=1.474`. will we use the isc flag? depends on chosen potential i suppose

[2] I found a very nice potential by Otani et al from 2014 that modeled this potential by fitting scattering data of $\alpha + ^{15}$N in the range of $E_\alpha = 6 \sim 55$ MeV. It provides parameters for the real part of the Woods-Saxon volume potential and does not disclose what it claims to be energy dependent imaginary volume components for the potential. It also defines a Coulomb term. Because not everything was disclosed, I went with a different potential.

For the $\langle {}^{15}\mathrm{O}|{}^{19}\mathrm{F}\rangle$ overlap we have a spin of `nn=3 l=2 sn=0.5 j=2.5`, and we allow FRESCO to vary the potential that binds the alpha particle in ^{19}F to reproduce the binding energy `be=0.106` through the flag `isc=1`. need to choose potential and fix this entire section

Couplings

The `&Coupling` section starts with general parameters and then contains two coefficients of fractional parentage sections, `&CFP`. The main section calls `icto=2` and `icfrom=1`, which couples all the states in partition `icfrom` to all the states in partition `icto`. Here we are coupling all the states in partition 2 to partition 1. The `kind=7` indicates a finite range DWBA transfer (`kind=5` would indicate a zero-range DWBA transfer requiring different parameters `p1` and `p2`, which correspond to the D_0 zero range coupling constant and the effective range parameter FNRNG, respectively). For `kind=7` you must choose `ip1=0` for post form (1 for prior) and `ip2=0,1,-1` for no remnant, full real remnant, and full complex remnant. Lastly, `ip3=5` indicates the index of the core-core optical potential, which here is potential 5 as it is the potential between the ^{6}Li's deuteron core and the ^{15}N core.

3.1 Mao: 22MeV ^{6}Li beam paper reproduction

The Mao paper was relatively straightforward to reproduce, as it was a regular kinematics experiment. The beam was ^{6}Li at 22MeV incident on a ^{15}N gas target. However, because they used DWUCK to calculate the cross section, they only required three optical potentials as opposed to the five required in FRESCO. Additionally, because DWUCK is a zero-range DWBA code, that is what we will be using to reproduce their results.

Potential 1 should be at 22 MeV, but as discussed in the previous section, this potential is at 34.5 MeV; that is not too bad for a potential that is the correct particle scattering. Potential 2 is the U_{pB} potential, for ^{19}F(d,d) scattering at 22 MeV+Q. The Q-value for ^{15}N(^{6}Li,d)^{19}F to create the 3.908MeV state of interest is -1.368 MeV, so once E_{beam} is converted into the center of mass frame, Q is added, and converted back to the lab frame, potential 2 will be calculated at 20.5 MeV.Because potentials 3 and 4 are calculated as binding potentials, their potentials do not require incident energies. Potential 5 is the U_{pA} potential, here representing ^{15}N(d,d) scattering at 22 MeV (this can also be calculated at the beam + Q value energy but in previous work I have found that changing these values makes minimal to no difference in the final cross-section outcome). The values for each of these potentials can be seen in Table 1.

Reaction	Value [MeV]
$^{15}\mathrm{N} + {}^{6}\mathrm{Li} \rightarrow d + {}^{19}\mathrm{F}$	-1.368
$\alpha + d \rightarrow {}^{6}\mathrm{Li}$	1.474
$^{15}\mathrm{N} + \alpha \rightarrow {}^{19}\mathrm{F}$	0.106

Table 1: Q-values of interest for 3.908 MeV ^{19}F state in the Mao paper that are required by FRESCO. Note that the middle line does not depend on the value of the ^{19}F state.

3.2 Calculations for HELIOS Experiment

The experiment at HELIOS will be in inverse kinematics, which does not change the optical potentials required from the Mao paper, but does require a flag in FRESCO. Just before the beam energy is declared,

	Potential	Energy [MeV]	V_V [MeV]	r_V [fm]	a_V [fm]	W_V [MeV]	a_{WV} [fm]	a_V [fm]	V_S [MeV]	r_{VS} [fm]	a_S [fm]	W_S [MeV]	r_{WS} [fm]	a_S [fm]	V_{SO} [MeV]	r_{VSO} [fm]	a_{SO} [fm]	W_{SO} [MeV]	r_{WSO} [fm]	a_{SO} [fm]	r_C [fm]
1	^{15}N(^{6}Li, ^{6}Li)	34.5?	132.0	0.901	1.37	31.3	0.945	0.918	0	0	0	0	0	0	0	0	0	0	0	0	1.30
2	^{19}F(d,d)	20.5	80.5	1.17	0.81	0	1.56	0.82	0	0	0	17.1	1.33	0.58	3.7	1.23	0.81	0	1.23	0.81	1.70
3	$d+\alpha = {}^6$Li	-	77.2	0.67	0.65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-
4	^{15}N$+\alpha = {}^{19}$F	-	40.0	0.77	0.8	0	0	0	0	0	0	0	0	0	7.0	1.2	0.65	0	0	0	1.25
5	^{15}N(d,d)	22.0	79.2	1.17	0.81	0	1.56	0.81	0	0	0	16.3	1.33	0.58	3.7	1.23	0.81	0	1.23	0.81	1.70

Table 2: Optical potential parameters used as input to FRESCO for the 22 MeV ^{15}N(^{6}Li,d)^{19}F measured by Mao in regular kinematics.

the `lin=2` flag must be declared to tell FRESCO that the experiment is in inverse kinematics. This will allow the `elab` energy to be declared as the beam energy of the heavy projectile without any change to the optical potentials.

For this experiment, a beam of ^{15}N at an energy of 6.7 MeV/u will hit a LiF target. This equates to an incident beam energy of `elab=100.5` MeV. To calculate the appropriate energy of our optical potentials without having to locate inverse kinematic optical potentials, we must convert this beam energy into the center of mass frame and then convert into the lab frame for a ^{6}Li beam (adding in any Q values in the center of mass frame as required):

$$E_1' = (\frac{m_2}{m_1+m_2})E \qquad E_2' = (\frac{m_1}{m_1+m_2})E \tag{5}$$

where the primed values represent center of mass values and the non-primed values represent lab frames. In this way we will be able to use the more easily located light-projectile-on-heavy-target optical potentials. Converting our energy to the center of mass frame, we find that $E_{com} = 28.714$ MeV.

This center of mass energy corresponds to a ^{6}Li beam energy of 40.2 MeV (which is still 6.7 MeV/u). Thus we would like our ^{15}N(^{6}Li,^{6}Li) scattering potential, or potential 1, to have an energy of 40.2 MeV. However, due to scarcity of ^{6}Li scattering potentials on our nucleus of interest, we were able to find a potential at 34.5 MeV (the chart says 37.7 in the paper and I am worried about a typo) by Wu et al. for ^{15}N(^{6}Li,^{6}Li) scattering, which is the only potential for this specific reaction that exists as far as I can tell. Similarly to the Mao paper, potential 2 should be at Ebeam+Q, which is 38.3 MeV. Because potentials 3 and 4 are calculated as binding potentials, their potentials do not require incident energies. Once again, the ^{15}N(d,d) scattering of potential 5 will be calculated at 40.2 MeV, though it could be calculated at 38.3 MeV for similar reasons to potential 2. The values for all these potential parameters can be seen in Table 3.

	Potential	Energy [MeV]	V_V [MeV]	r_V [fm]	a_V [fm]	W_V [MeV]	a_{WV} [fm]	a_V [fm]	V_S [MeV]	r_{VS} [fm]	a_S [fm]	W_S [MeV]	r_{WS} [fm]	a_S [fm]	V_{SO} [MeV]	r_{VSO} [fm]	a_{SO} [fm]	W_{SO} [MeV]	r_{WSO} [fm]	a_{SO} [fm]	r_C [fm]
1	^{15}N(^{6}Li, ^{6}Li)	34.5?	132.0	0.901	1.37	31.3	0.945	0.918	0	0	0	0	0	0	0	0	0	0	0	0	1.30
2	^{19}F(d,d)	38.3	76.9	1.17	0.81	0	1.56	0.82	0	0	0	15.6	1.33	0.58	3.7	1.23	0.81	0	1.23	0.81	1.70
3	$d+\alpha = {}^6$Li	-	77.2	0.67	0.65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-
4	^{15}N$+\alpha = {}^{19}$F	-	40.0	0.77	0.8	0	0	0	0	0	0	0	0	0	7.0	1.2	0.65	0	0	0	1.25
5	^{15}N(d,d)	40.2	75.5	1.17	0.81	0	1.56	0.81	0	0	0	14.9	1.33	0.58	3.7	1.23	0.81	0	1.23	0.81	1.70

Table 3: Optical potentials used as input to FRESCO for the 3.9 MeV (d,p) reaction. The deuteron scattering potential (row 1) is described in detail in Ref. [3], and the two proton scattering potentials (rows 2,3,4) are described in Ref. [**?**]. The deuteron binding potential (row 5) is a real volume potential with Gaussian shape. Finally, the n $-$ ^{24}Mg potential is strictly real with Woods-Saxon form and a spin-orbit component; FRESCO rescales this potential as needed to reproduce the binding energy of the neutron in ^{25}Mg during the calculation.

4 Partial width calculations in NuShellX

Because we require a partial width and not just a spectroscopic factor, we must calculate a theoretical single particle width for the alpha particle to obtain our alpha partial width:

$$\Gamma_\alpha = (CS^2)\Gamma_{s.p} \tag{6}$$

where (CS^2) is the spectroscopic factor and $\Gamma_{s.p.}$ is the single particle width that must be calculated in NuShellX.

Calculate single particle width in NuShellX.

	Potential	Energy [MeV]	V_V [MeV]	r_V [fm]	a_V [fm]	W_V [MeV]	a_{WV} [fm]	a_V [fm]	V_S [MeV]	r_{VS} [fm]	a_S [fm]	W_S [MeV]	r_{WS} [fm]	a_S [fm]	V_{SO} [MeV]	r_{VSO} [fm]	a_{SO} [fm]	W_{SO} [MeV]	r_{WSO} [fm]	a_{SO} [fm]	r_C [fm]
1	^{15}N(^{6}Li, ^{6}Li)	34.5	132.0	0.901	1.37	31.3	0.945	0.918	0	0	0	0	0	0	0	0	0	0	0	0	1.30
2	^{19}F(d,d)	20.5	80.5	1.17	0.81	0	1.56	0.82	0	0	0	17.1	1.33	0.58	3.7	1.23	0.81	0	1.23	0.81	1.70
2	^{19}F(d,d)	38.3	76.9	1.17	0.81	0	1.56	0.82	0	0	0	15.6	1.33	0.58	3.7	1.23	0.81	0	1.23	0.81	1.70
3	$d+\alpha=$ ^{6}Li	-	77.2	0.67	0.65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-
4	^{15}N$+\alpha=$ ^{19}F	-	40.0	0.77	0.8	0	0	0	0	0	0	0	0	0	7.0	1.2	0.65	0	0	0	1.25
5	^{15}N(d,d)	22.0	79.2	1.17	0.81	0	1.56	0.81	0	0	0	16.3	1.33	0.58	3.7	1.23	0.81	0	1.23	0.81	1.70
5	^{15}N(d,d)	40.2	75.5	1.17	0.81	0	1.56	0.81	0	0	0	14.9	1.33	0.58	3.7	1.23	0.81	0	1.23	0.81	1.70

```
hcm=0.1 rnl=7.00 centre=-0.2 cutl=-1.6 lin=1

hcm=0.07 rnl=23.900 centre=-0.15 cutl=-3.0 lin=2
```

References

[1] Z.Q. Mao, H.T. Fortune, and A.G. Lacaze. *Physical Review C*, 53, 1996.

[2] Z.D. Wu et al. *Physical Review C*, 89, 2014.

[3] Y. Han, Y. Shi, and Q. Shen. *Physical Review C*, 74:044615, 2006.

[4] K.I. Kubo and M. Hirata. *Nuclear Physics A*, 187, 1972.

[5] M.L. Avila et al. *Physical Review C*, 90:042801(R), 2014.