Chapter 4 Pure Bending v5

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Mechanics of Materials 8E

4

Pure Bending

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Objectives

In this chapter, we will:

- **Consider** the general principles of bending behavior.
- **Define** the deformations, strains, and normal stresses in beams subject to pure bending.
- **Describe** the behavior of composite beams made of more than one material.
- **Review** stress concentrations and how they are included in the design of beams.
- **Study** plastic deformations to determine how to evaluate beams made of elastoplastic materials.
- **Analyze** members subject to eccentric axial loading, involving both axial stresses and bending stresses.
- **Review** beams subject to unsymmetric bending, i.e., where bending does not occur in a plane of symmetry.
- **Study** bending of curved members.

4 Pure Bending

Introduction

This chapter is devoted to the analysis of prismatic members subjected to equal and opposite couples **M** and **M′** acting in the same longitudinal plane (**Fig. 4.1**).

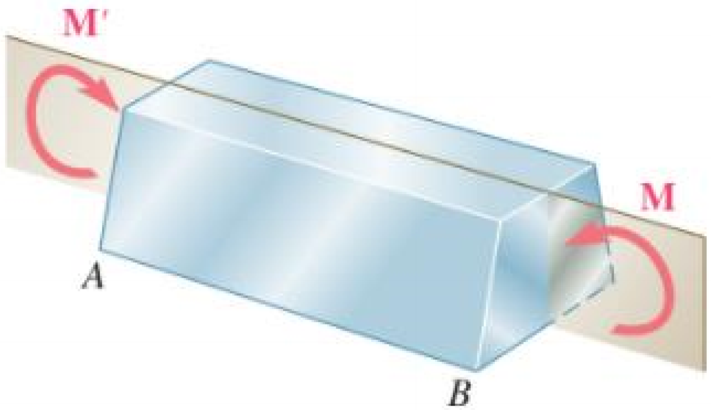

Fig. 4.1 Member in pure bending

4 Pure Bending

Introduction

Because of the symmetry of the free- body diagram of the bar (**Fig. 4.2*a***)

Therefore, as far as the middle portion *CD* of the bar is concerned, the weights and the reactions can be replaced by two equal and opposite 120-N · m couples (**Fig. 4.2*b***)

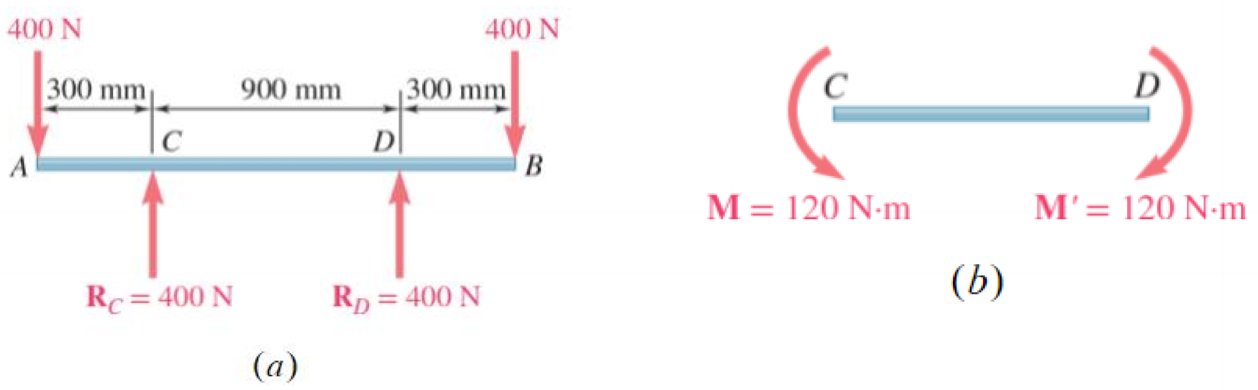

Fig. 4.2 (*a*) Free-body diagram of the barbell pictured in the chapter opening photo and (*b*) free-body diagram of the center portion of the bar, which is in pure bending.

4 Pure Bending

Introduction

Photo 4.2 shows a 300-mm steel bar clamp used to exert 600-N forces on two pieces of lumber as they are being glued together.

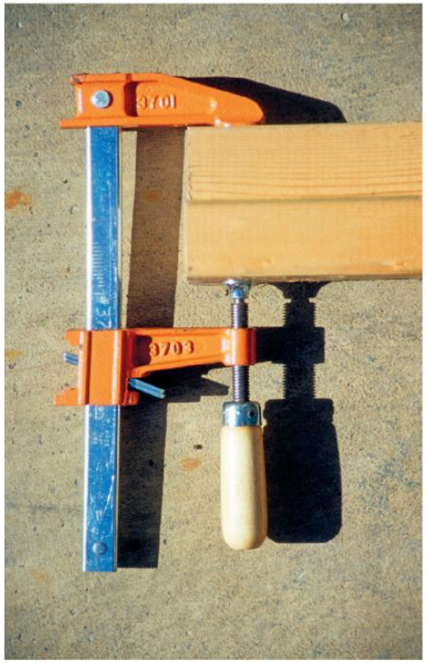

Photo 4.2 Clamp used to glue lumber pieces together.

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4 Pure Bending

Introduction

In Fig. 4.3*b*, a section CC' has been passed through the clamp and a free-body diagram has been drawn of the upper half of the clamp.

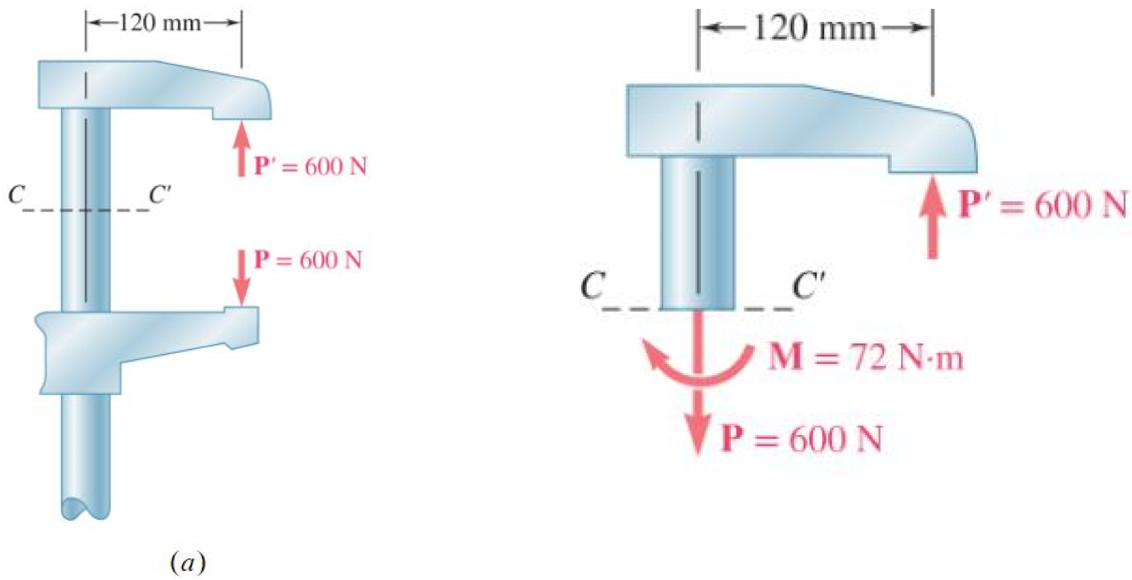


Fig. 4.3 (*a*) Free-body diagram of a clamp, (*b*) free-body diagram of the upper portion of the clamp.

4 Pure Bending

Introduction

Consider a cantilever beam AB supporting a concentrated load **P** at its free end (Fig. 4.4*a*).

If a section is passed through C at a distance x from A, the free-body diagram of AC (Fig. 4.4*b*) shows that the internal forces in the section consist of a force **P′** equal and opposite to **P** and a couple **M** of magnitude $M = Px$.

- The distribution of normal stresses in the section can be obtained from the couple **M** as if the beam were in pure bending. The shearing stresses in the section depend on the force **P’**, and their distribution over a given section is discussed in Chap. 6.

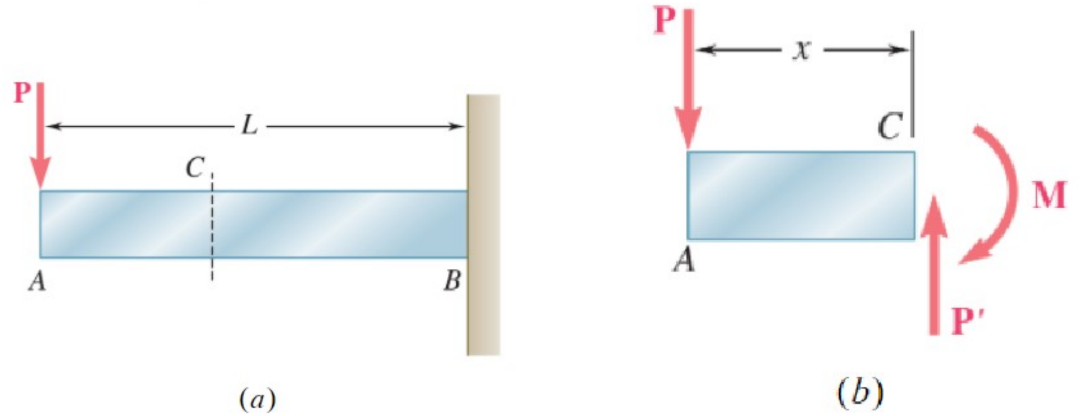


Fig. 4.4 (*a*) Cantilevered beam with end loading. (*b*) As portion *AC* shows, beam is not in pure bending.

4 Pure Bending

Introduction

- The first part of this chapter covers the analysis of stresses and deformations caused by pure bending in a homogeneous member possessing a plane of symmetry and made of a material following Hooke's law. The methods of statics are used in Sec. 4.1A to derive three fundamental equations which must be satisfied by the normal stresses in any given cross section of the member. In Sec. 4.1B, it will be proved that transverse sections remain plane in a member subjected to pure bending, while in Sec. 4.2, formulas are developed to determine the normal stresses and radius of curvature for that member within the elastic range.
- Sec. 4.4 covers the stresses and deformations in composite members made of more than one material.
- You will learn to draw a transformed section representing a member made of a homogeneous material that undergoes the same deformations as the composite member under the same loading. The transformed section is used to find the stresses and deformations in the original composite member.

4 Pure Bending

Introduction

- Section 4.5 is devoted to the determination of stress concentrations occurring where the cross section of a member undergoes a sudden change.
- In Sec. 4.7, you will analyze an eccentric axial loading in a plane of symmetry (Fig. 4.3) by superposing the stresses due to pure bending and a centric axial loading.
- The study of the bending of prismatic members concludes with the analysis of unsymmetric bending (Sec. 4.8), and the study of the general case of eccentric axial loading (Sec. 4.9).

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1A Internal Moment and StressRelations

Consider a prismatic member *AB* possessing a plane of symmetry and subjected to equal and opposite couples **M** and **M′** acting in that plane (**Fig. 4.5*a***).

Following the usual convention, a positive sign is assigned to *M* when the member

is assigned to *M* when the member is bent as shown in **Fig. 4.5*a*** (i.e., when the concavity of the beam faces upward) and a negative sign otherwise.

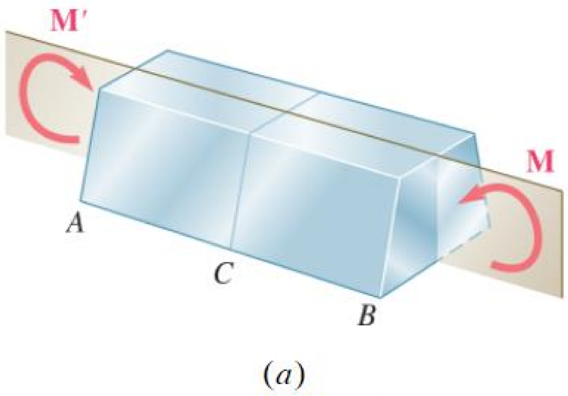
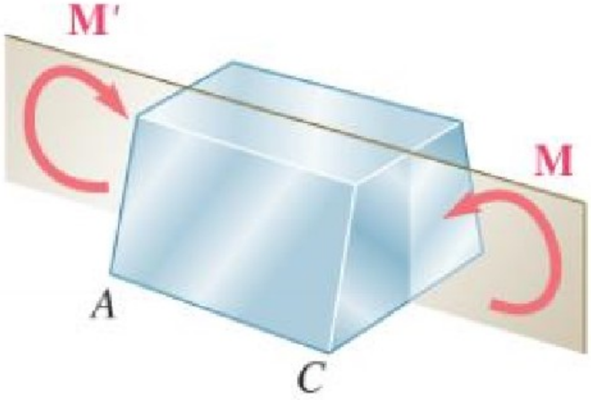


Fig. 4.5 (*a*) A member in a state of pure bending.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1A Internal Moment and StressRelations

If a section is passed through the member AB at some arbitrary point C, the conditions of equilibrium of the portion AC of the member require the internal forces in the section to be equivalent to the couple **M** (**Fig. 4.5*b***).



(*b*)

Fig. 4.5 (*b*) Any intermediate portion of *AB* will also be in pure bending.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1A Internal Moment and StressRelations

On any point on the cross section (**Fig. 4.6*a***), we have σ_x, the normal stress, and τ_{xy} and τ_{xz}, the components of the shearing stress. The system of these elementary internal forces exerted on the cross section is equivalent to the couple **M** (**Fig. 4.6**).

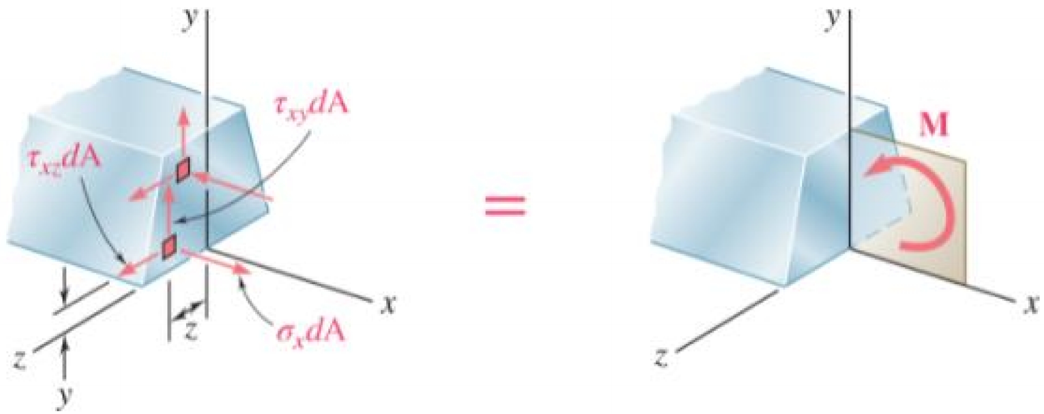


Fig. 4.6 Stresses resulting from pure bending moment **M**.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1A Internal Moment and StressRelations

Selecting arbitrarily the z axis shown in **Fig. 4.6**, the equivalence of the elementary internal forces and the couple **M** is expressed by writing that the sums of the components and moments of the forces are equal to the corresponding components and moments of the couple **M:**

τ_{xy} x components : $\int \sigma_x dA = 0$ **(4.1)**

τ_{xz} Moments about$y$axis : $\int z\sigma_x dA = 0$ **(4.2)**

Moments about$z$axis : $\int (-y\sigma_x dA) = M$ **(4.3)**

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1A Internal Moment and StressRelations

Two remarks should be made at this point:

1. The minus sign in **Eq. (4.3)** is due to the fact that a tensile stress ($\sigma_x > 0$) leads to a negative moment (clockwise) of the normal force $\sigma_x \, dA$ about the z axis.

2. **Equation (4.2)** could have been anticipated, since the application of couples in the plane of symmetry of member AB result in a distribution of normal stresses symmetric about the y axis.

- Once more, note that the actual distribution of stresses in a given cross section cannot be determined from statics alone. It is *statically indeterminate* and may be obtained only by analyzing the *deformations* produced in the member.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1B Deformations

The member will bend under the action of the couples, but will remain symmetric

symmetric with respect to that plane (**Fig. 4.7**).

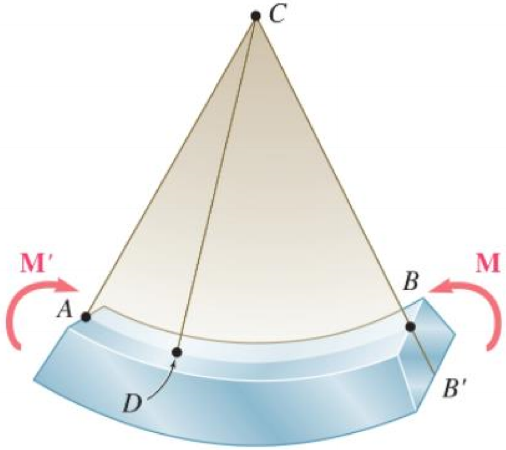


Fig. 4.7 Initially straight members in pure bending deform into a circular arc.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1B Deformations

If this were not the case, we could find a point E of the original section through D (**Fig. 4.8*a***) which, after the member has been bent, would *not* lie in the plane perpendicular to the plane of symmetry that contains line CD (**Fig. 4.8*b***).

Let us assume that, after the beam has been bent, both points would be located to the left of the plane defined by CD, as shown in **Fig. 4.8*b***.

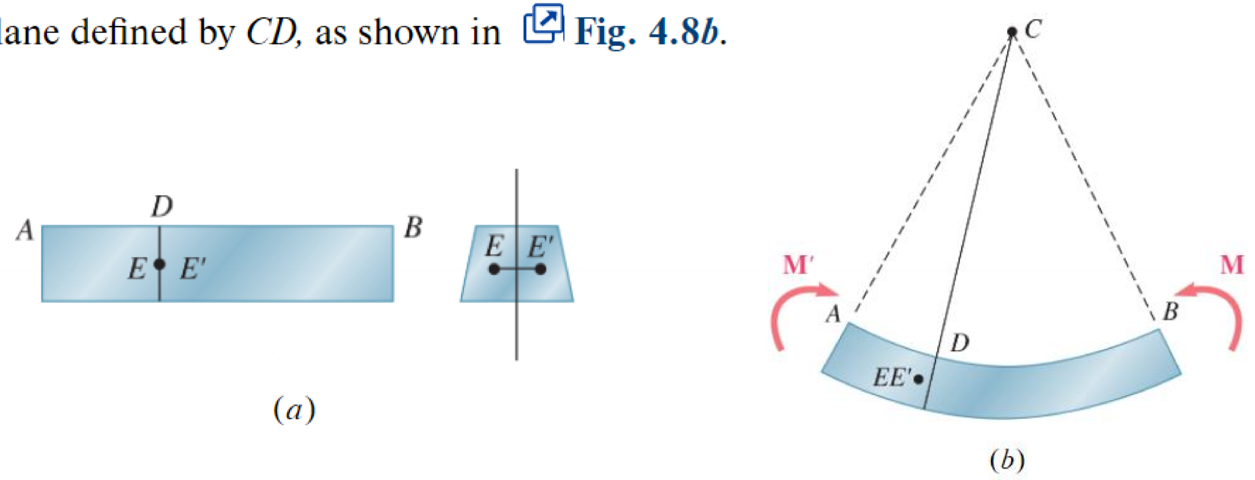


Fig. 4.8 (*a*) Two points in a cross section at *D* that is perpendicular to the member's axis. (*b*) Considering the possibility that these points do not remain in the cross section after bending.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1B Deformations

The property we have established requires that these elements be transformed as shown in **Fig. 4.9** when the member is subjected to the couples **M** and **M′**.

Since all the faces represented in the two projections of **Fig. 4.9** are at 90° to each other, we conclude that $\gamma_{xy} = \gamma_{zx} = 0$ and, thus, that $\tau_{xy} = \tau_{xz} = 0$.

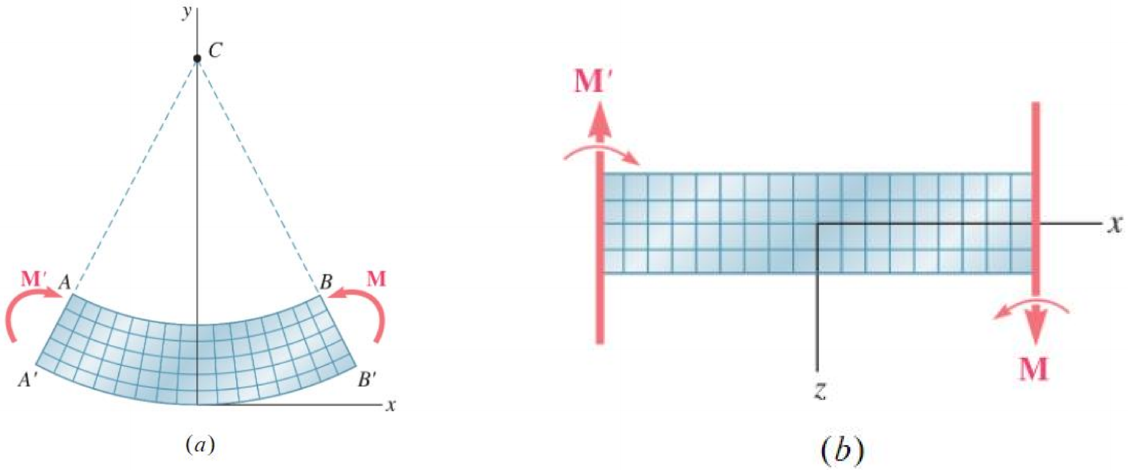


Fig. 4.9 Member subject to pure bending shown in two views. (*a*) Longitudinal, vertical section (plane of symmetry). (*b*) Longitudinal, horizontal section.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1B Deformations

The neutral surface intersects the plane of symmetry along an arc of circle *DE* (**Fig. 4.10*a***),

and it intersects a transverse section along a straight line called the *neutral axis* of

the section (**Fig. 4.10*b***).

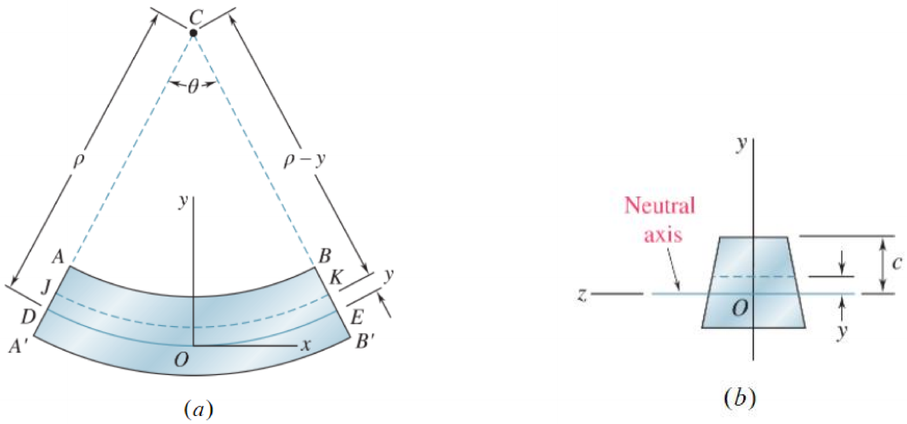


Fig. 4.10 Establishment of neutral axis. (*a*) Longitudinal, vertical section (plane of symmetry). (*b*) Transverse section at origin.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1B Deformations

Denoting by ρ the radius of arc *DE* (**Fig. 4.10*a***), by θ the central angle corresponding to *DE*, and observing that the length of *DE* is equal to the length L of the undeformed member, we write

$$L = \rho\theta \tag{4.4}$$

Considering the arc *JK* located at a distance y above the neutral surface, its length L' is

$$L' = (\rho - y)\theta \tag{4.5}$$

Since the original length of arc *JK* was equal to L, the deformation of *JK* is

$$\delta = L' - L \tag{4.6}$$

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1B Deformations

or, substituting from **Eqs. (4.4)** and **(4.5)** into **Eq. (4.6)**,

$$\delta = (\rho - y)\theta - \rho\theta = -y\theta \qquad \textbf{(4.7)}$$

The longitudinal strain ε_x in the elements of JK is obtained by dividing δ by the original length L of JK. Write

$$\varepsilon_x = \frac{\delta}{L} = \frac{-y\theta}{\rho\theta}$$

or

$$\boxed{\varepsilon_x = -\frac{y}{\rho}} \qquad \textbf{(4.8)}$$

- Because of the requirement that transverse sections remain plane, identical deformations occur in all planes parallel to the plane of symmetry. Thus, the value of the strain given by Eq. (4.8) is valid anywhere, and the *longitudinal normal strain εx varies linearly with the distance y from the neutral surface*.

4.1 SYMMETRIC MEMBERS IN PURE BENDING

4.1B Deformations

The strain ε_x reaches its maximum absolute value when y is largest.

Denoting the largest distance from the neutral surface as c (corresponding to either the upper or the lower surface of the member) and the *maximum absolute value* of the strain as ε_m, we have

$$\epsilon_m = \frac{c}{\rho} \tag{4.9}$$

Solving **Eq. (4.9)** for ρ and substituting into **Eq. (4.8)**,

$$\varepsilon_x = -\frac{y}{c}\varepsilon_m \tag{4.10}$$

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

- We now consider the case when the bending moment M is such that the normal stresses in the member remain below the yield strength σY. This means that the stresses in the member remain below the proportional limit and the elastic limit as well. There will be no permanent deformation, and Hooke's law for uniaxial stress applies.

Assuming the material to be homogeneous and denoting its modulus of elasticity by E, the normal stress in the longitudinal x direction is

$$\sigma_x = E\varepsilon_x \tag{4.11}$$

Recalling **Eq. (4.10)** and multiplying both members by E, we write

$$E\varepsilon_x = -\frac{y}{c}(E\varepsilon_m)$$

or using **Eq. (4.11)**,

$$\sigma_x = -\frac{y}{c}\sigma_m \tag{4.12}$$

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

This result shows that, *in the elastic range, the normal stress varies linearly with the distance from the neutral surface* (**Fig. 4.11**).

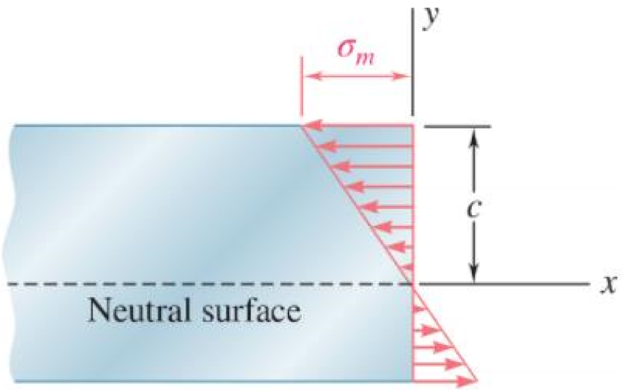


Fig. 4.11 Bending stresses vary linearly with distance from the neutral axis.

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

Note that neither the location of the neutral surface nor the maximum value σ_m of the stress have yet to be determined. Both can be found using Eqs. (4.1) and (4.3). Substituting for σ_x from Eq. (4.12) into Eq. (4.1), write

$$\Sigma F = \int \sigma_x dA = \int \left(-\frac{y}{c}\sigma_m\right) dA = -\frac{\sigma_m}{c}\int y\, dA = 0$$

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from which

$$\int y\, dA = 0 \qquad \textbf{(4.13)}$$

中心轴通過形心

x components :	$\int \sigma_x dA = 0$	**(4.1)**
Moments aboutyaxis :	$\int z\sigma_x dA = 0$	**(4.2)**
Moments aboutzaxis :	$\int (-y\sigma_x dA) = M$	**(4.3)**

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

$$\int y \, dA = 0 \tag{4.13}$$

This equation shows that the first moment of the cross section about its neutral axis must be zero.† Thus, for a member subjected to pure bending and *as long as the stresses remain in the elastic range, the neutral axis passes through the centroid of the section.*

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

Recall **Eq. (4.3)**, which was developed with respect to an *arbitrary* horizontal z axis:

$$\int (-y\sigma_x dA) = M \tag{4.3}$$

Specifying that the z axis coincides with the neutral axis of the cross section, substitute σ_x

from **Eq. (4.12)** into **Eq. (4.3)**:

$$\int (-y)\left(-\frac{y}{c}\sigma_m\right) dA = M$$

or

$$\frac{\sigma_m}{c}\int y^2 dA = M \tag{4.14}$$

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

Specifying that the z axis coincides with the neutral axis of the cross section,

substitute σ_x from **Eq. (4.12)** into **Eq. (4.3)**:

$$\int (-y)\left(-\frac{y}{c}\sigma_m\right)dA = M$$

or

$$\frac{\sigma_m}{c}\int y^2\,dA = M \qquad \textbf{(4.14)}$$

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

Solving Eq. (4.14) for σ_m,‡

$$\boxed{\sigma_m = \frac{Mc}{I}} \tag{4.15}$$

Substituting for σ_m from Eq. (4.15) into Eq. (4.12), we obtain the normal stress σ_x

at any distance y from the neutral axis:

$$\boxed{\sigma_x = -\frac{y}{c}\sigma_m} \tag{4.12}$$

$$\boxed{\sigma_x = -\frac{My}{I}} \tag{4.16}$$

Equations (4.15) and (4.16) are called the *elastic flexure formulas*, and the normal stress σ_x caused by the bending or "flexing" of the member is often referred to as the *flexural stress*. The stress is compressive ($\sigma_x < 0$) above the neutral axis ($y > 0$) when the bending moment M is positive and tensile ($\sigma_x > 0$) when M is negative.

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

This ratio is defined as the *elastic section modulus* S, where

$$\text{Elastic section modulus} = S = \frac{I}{c} \tag{4.17}$$

Substituting S for I/c into **Eq. (4.15)**, this equation in alternative form is

$$\boxed{\sigma_m = \frac{M}{S}} \tag{4.18}$$

Since the maximum stress σ_m is inversely proportional to the elastic section modulus S, beams should be designed with as large a value of S as is practical. For example, a wooden beam with a rectangular cross section of width b and depth h has

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^2 = \frac{1}{6}Ah \qquad S \propto h \tag{4.19}$$

where A is the cross-sectional area of the beam.

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

For two beams with the same cross-sectional area A (**Fig. 4.12**), the beam with the larger depth h will have the larger section modulus and will be the more effective in resisting bending.†

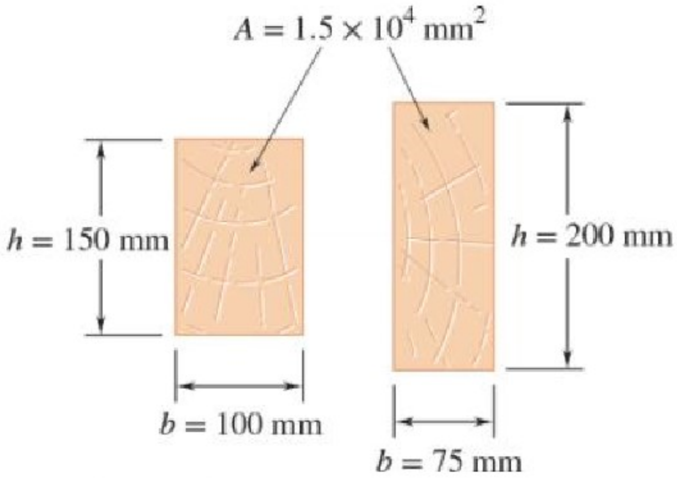

Fig. 4.12 Wood beam cross sections.

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

In the case of structural steel (**Photo 4.3**), American standard beams (S-beams) and wide-flange beams (W-beams) are preferred to other shapes because a large portion of their cross section is located far from the neutral axis (**Fig. 4.13**).

Photo 4.3 Wide-flange steel beams are used in the frame of this building.

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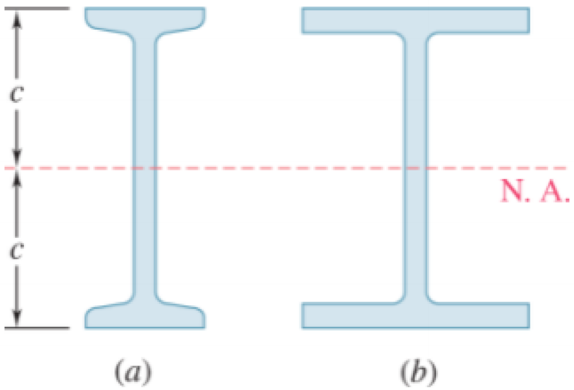


Fig. 4.13 Two types of steel beam cross sections: (*a*) American Standard beam (S), (*b*) wide-flange beam (W).

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

Thus, for a given cross-sectional area and a given depth, their design provides large values of I and S. Values of the elastic section modulus of commonly manufactured beams can be obtained from tables listing the various geometric properties of such beams. To determine the maximum stress σ_m in a given section of a standard beam, the engineer needs only to read the value of the elastic section modulus S in such a table and divide the bending moment M in the section by S.

Photo 4.3 Wide-flange steel beams are used in the frame of this building.

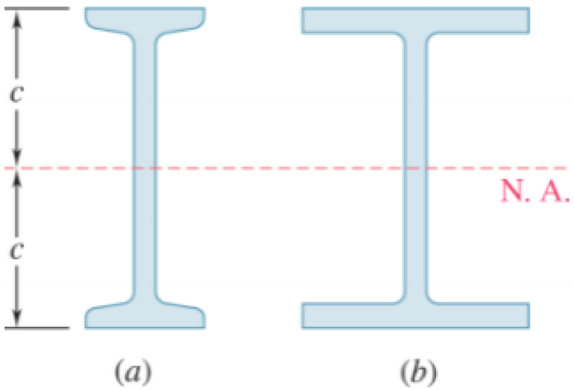


Fig. 4.13 Two types of steel beam cross sections: (*a*) American Standard beam (S), (*b*) wide-flange beam (W).

4.2 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

The curvature is defined as the reciprocal of the radius of curvature ρ and can be obtained by solving **Eq. (4.9)** for $1/\rho$: $\epsilon_m = \frac{c}{\rho}$ **(4.9)**

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} \qquad \textbf{(4.20)}$$

In the elastic range, $\varepsilon_m = \sigma_m / E$. Substituting for εm into **Eq. (4.20)** and recalling **Eq. (4.15)**, write

$$\frac{1}{\rho} = \frac{\sigma_m}{Ec} = \frac{1}{Ec}\frac{Mc}{I}$$

or

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \textbf{(4.21)}$$

4.3 DEFORMATIONS IN A TRANSVERSE CROSS SECTION

The normal strains ε_y and εz depend upon Poisson's ratio ε_z for the material used and are expressed as

$$\varepsilon_y = -\upsilon\varepsilon_x \qquad \varepsilon_z = -\upsilon\varepsilon_x$$

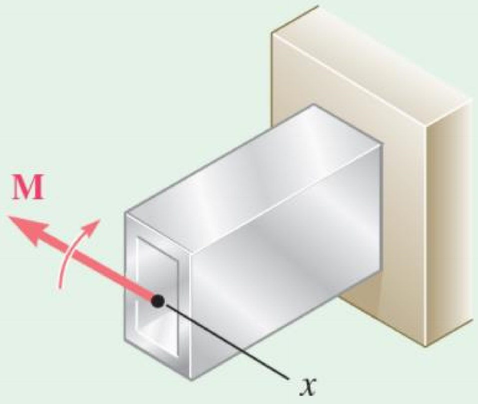
or recalling **Eq. (4.8)**,

$$\varepsilon_x = -\frac{y}{\rho} \tag{4.8}$$

$$\varepsilon_y = \frac{\upsilon y}{\rho} \qquad \varepsilon_z = \frac{\upsilon y}{\rho} \tag{4.22}$$

Sample Problem 4.1

The rectangular tube shown is extruded from an aluminum alloy for which $\sigma_Y = 275$ MPa, $\sigma_U = 415$ MPa, and $E = 73$ GPa. Neglecting the effect of fillets, determine (a) the bending moment M for which the factor of safety will be 3.00 and (b) the corresponding radius of curvature of the tube.



STRATEGY:

Use the factor of safety to determine the allowable stress. Then calculate the bending moment and radius of curvature using **Eqs. (4.15)** and **(4.21)**.

Sample Problem 4.1

MODELING and ANALYSIS:

Moment of Inertia.

Considering the cross-sectional area of the tube as the difference between the two rectangles shown in **Fig. 1** and recalling the formula for the centroidal moment of inertia of a rectangle, write

$$I = \frac{1}{12}(80\text{ mm})(120\text{ mm})^3 - \frac{1}{12}(68\text{ mm})(108\text{ mm})^3 \qquad I = 4.382 \times 10^6\,\text{mm}^4$$

$$= 4.382 \times 10^6\,\text{m}^4$$

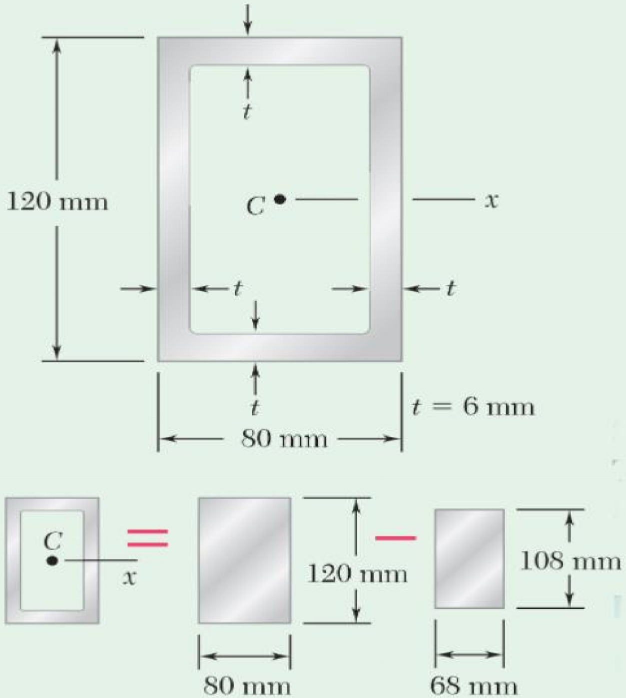


Fig. 1 Superposition for calculating moment of inertia.

Sample Problem 4.1

MODELING and ANALYSIS:

Allowable Stress.

For a factor of safety of 3.00 and an ultimate stress of 415 MPa, we have

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S} = \frac{415 \text{ MPa}}{3.00} = 138.3 \text{ MPa}$$

Since $\sigma_{\text{all}} < \sigma_Y$, the tube remains in the elastic range and we can apply the results of **Sec. 4.2**.

a. Bending Moment.

With $c = \frac{1}{2}(120 \text{ mm}) = 60 \text{ mm}$ we write

$$\sigma_{\text{all}} = \frac{Mc}{I} \quad M = \frac{I}{c}\sigma_{\text{all}} = \frac{4.382 \times 10^{-6} \text{m}^4}{0.06 \text{ m}}(138.33 \text{ MPa})$$

$M = 10.1 \text{ kN} \cdot \text{m}$

Sample Problem 4.1

MODELING and ANALYSIS:

Allowable Stress.

b. Radius of Curvature.

Using **Fig. 2** and recalling that $E = 73$ GPa, we substitute this value and the values obtained for I and M into **Eq. (4.21)** and find

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{10.1 \text{ kN} \cdot \text{m}}{(73 \text{ GPa})\left(4.382 \times 10^{-6} \text{m}^4\right)} = 0.0316 \text{ m}^{-1}$$

$$\rho = 31.7 \text{ m}$$

$\rho = 31.7$ m

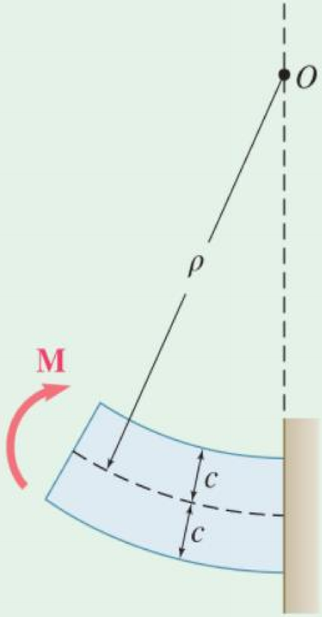
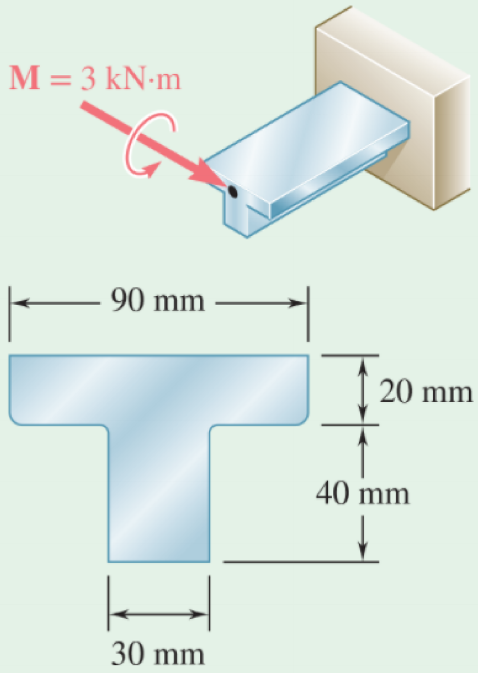


Fig. 2 Deformed shape of beam.

Sample Problem 4.2

A cast-iron machine part is acted upon by the $3\ \mathrm{kN} \cdot \mathrm{m}$ couple shown. Knowing that $E = 165$ GPa and neglecting the effect of fillets, determine (*a*) the maximum tensile and compressive stresses in the casting and (*b*) the radius of curvature of the casting.



STRATEGY:

The moment of inertia is determined, recognizing that it is first necessary to determine the location of the neutral axis. Then **Eqs. (4.15)** and **(4.21)** are used to determine the stresses and radius of curvature.

Sample Problem 4.2

MODELING and ANALYSIS:

Centroid.

Divide the T-shaped cross section into two rectangles as shown in **Fig. 1** and write

	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$(20)(90) = 1800$	50	90×10^3
2	$(40)(30) = 1200$	20	24×10^3
	$\Sigma A = 3000$		$\Sigma \bar{y}A = 114 \times 10^3$

$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$
$$\bar{Y}(3000) = 114 \times 10^6$$
$$\bar{Y} = 38 \text{ mm}$$

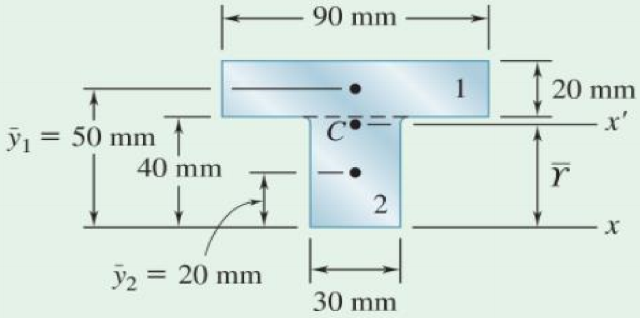


Fig. 1 Composite areas for calculating centroid.

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Sample Problem 4.2

MODELING and ANALYSIS:

Centroidal Moment of Inertia.

The parallel-axis theorem is used to determine the moment of inertia of each rectangle (**Fig. 2**) with respect to the axis x' that passes through the centroid of the composite section. Adding the moments of inertia of the rectangles, write

$$\begin{aligned}
I_{x'} &= \sum\left(\bar{I} + Ad^2\right) = \sum\left(\tfrac{1}{12}nh^3 + Ad^2\right) \\
&= \tfrac{1}{12}(90)(20)^3 + (90 \times 20)(12)^2 + \tfrac{1}{12}(30)(40)^3 + (30 \times 40)(18)^2 \\
&= 868 \times 10^3\,\text{mm}^4 \\
I &= 868 \times 10^{-9}\,\text{m}^4
\end{aligned}$$

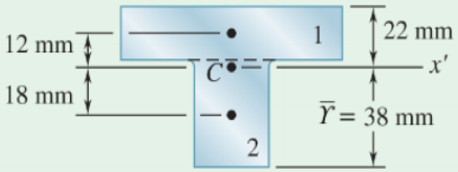


Fig. 2 Composite areas for calculating moment of inertia.

Sample Problem 4.2

MODELING and ANALYSIS:

a. Maximum Tensile Stress.

Since the applied couple bends the casting downward, the center of curvature is located below the cross section. The maximum tensile stress occurs at point A (**Fig. 3**), which is farthest from the center of curvature.

$$\sigma_A = \frac{Mc_A}{I} = \frac{(3\ \text{kN}\cdot\text{m})\,(0.22\ \text{m})}{868\times 10^{-9}\,\text{m}^4}$$

$\sigma_A = +76.0$ MPa

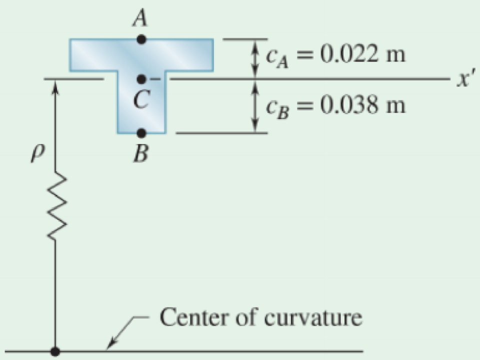


Fig. 3 Radius of curvature is measured to the centroid of the cross

Sample Problem 4.2

MODELING and ANALYSIS:

Maximum Compressive Stress.

This occurs at point B (**Fig. 3**):

$$\sigma_B = \frac{Mc_B}{I} = \frac{(3\text{ kN}\cdot\text{m})(0.038\text{ m})}{868\times10^{-9}\text{m}^4} \qquad \sigma_B = -131.3\text{ MPa}$$

b. Radius of Curvature.

From **Eq. (4.21)**, using **Fig. 3**, we have

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{3\text{ kN}\cdot\text{m}}{(165\text{ MPa})\left(868\times10^{-9}\text{m}^4\right)}$$
$$= 20.95\times10^{-3}\text{m}^{-1} \qquad \rho = 47.7\text{ m}$$

REFLECT and THINK:

Note the T section has a vertical plane of symmetry, with the applied moment in that plane. Thus the couple of this applied moment lies in the plane of symmetry, resulting in symmetrical bending. Had the couple been in another plane, we would have unsymmetric bending and thus would need to apply the principles of **Sec. 4.8**.

4.4 MEMBERS MADE OF COMPOSITE MATERIALS

Consider a bar consisting of two portions of different materials bonded together

as shown in **Fig. 4.18**.

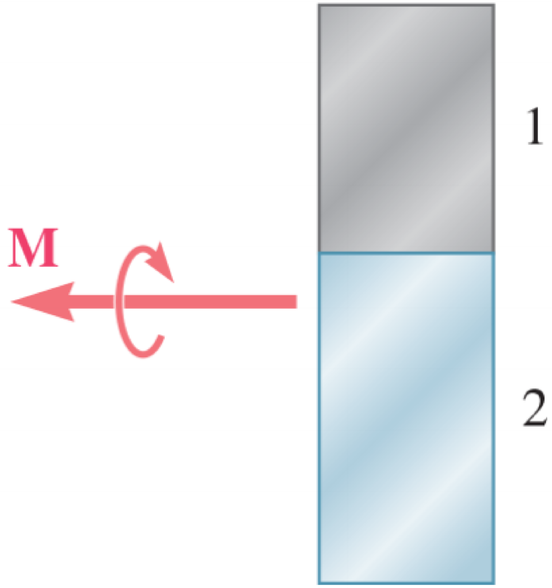


Fig. 4.18 Cross section made with different materials.

4.4 MEMBERS MADE OF COMPOSITE MATERIALS

the normal strain ε_x still varies linearly with the distance y from the neutral axis of the section (**Fig. 4.19*a*** and ***b***), and **Eq. (4.8)** holds:

$$\varepsilon_x = -\frac{y}{\rho} \tag{4.8}$$

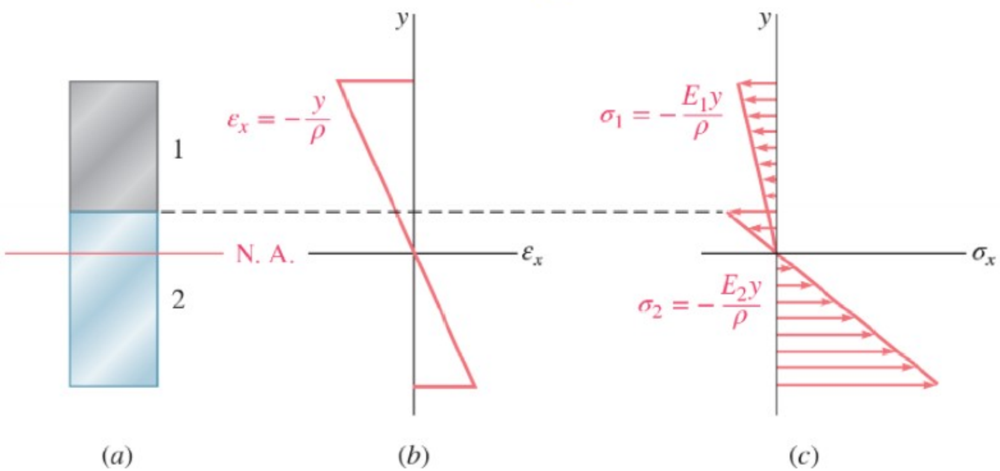


Fig. 4.19 Stress and strain distributions in bar made of two materials. (a) Neutral axis shifted from centroid. (b) Strain distribution. (c) Corresponding stress distribution.

4.4 MEMBERS MADE OF COMPOSITE MATERIALS

However, it cannot be assumed that the neutral axis passes through the centroid of the composite section, and one of the goals of this analysis is to determine the location of this axis.

Since the moduli of elasticity E_1 and E_2 of the two materials are different, the equations for the normal stress in each material are

$$\sigma_1 = E_1 \varepsilon_x = -\frac{E_1 y}{\rho}$$
$$\sigma_2 = E_2 \varepsilon_x = -\frac{E_2 y}{\rho} \tag{4.24}$$

It follows from **Eqs. (4.24)** that the force dF_1 exerted on an element of area dA of the upper portion of the cross section is

$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA \tag{4.25}$$

4.4 MEMBERS MADE OF COMPOSITE MATERIALS

while the force dF_2 exerted on an element of the same area dA of the lower portion is

$$dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA \qquad \textbf{(4.26)}$$

Denoting the ratio E_2/E_1 of the two moduli of elasticity by n, we can write

$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho} (n\, dA) \qquad \textbf{(4.27)}$$

$\sigma_2 = n\,\sigma_1$

4.4 MEMBERS MADE OF COMPOSITE MATERIALS

Comparing Eqs. (4.25) and (4.27), we note that the same force dF_2 would be exerted on an element of area $n\,dA$ of the first material. Thus, the resistance to bending of the bar would remain the same if both portions were made of the first material, provided that the width of each element of the lower portion were multiplied by the factor n. Note that this widening (if $n > 1$) or narrowing (if $n < 1$) must be *in a direction parallel to the neutral axis of the section,* since it is essential that the distance y of each element from the neutral axis remain the same. This new cross section is called the *transformed section* of the member (Fig. 4.20).

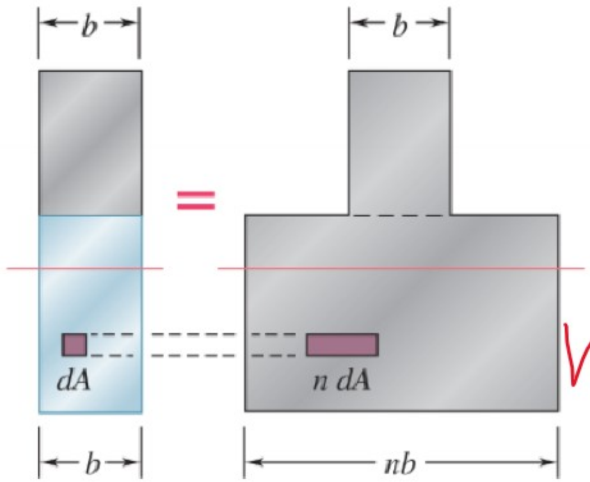


Fig. 4.20 Transformed section based on replacing lower material with that used on top.

$n =$ 杨氏系数比值

4.4 MEMBERS MADE OF COMPOSITE MATERIALS

Since the transformed section represents the cross section of a member made of a *homogeneous material* with a modulus of elasticity E_1, the method described in Sec. 4.2 can be used to determine the neutral axis of the section and the normal stress at various points. The neutral axis is drawn *through the centroid of the transformed section* (Fig. 4.21), and the stress σ_x at any point of the corresponding homogeneous member obtained from Eq. (4.16) is

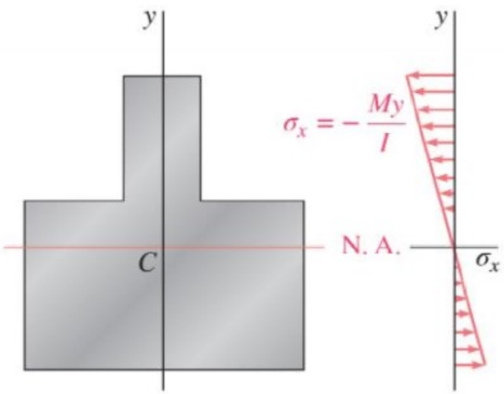


Fig. 4.21 Distribution of stresses in transformed section.

$$\sigma_x = -\frac{My}{I} \qquad \textbf{(4.16)}$$

where y is the distance from the neutral surface and I is *the moment of inertia of the transformed section* with respect to its centroidal axis.

4.4 MEMBERS MADE OF COMPOSITE

To obtain the stress σ_1 at a point located in the upper portion of the cross section of the original composite bar, compute the stress σ_x at the corresponding point of the transformed section. However, to obtain the stress σ_2 at a point in the lower portion of the cross section, we must *multiply by n* the stress σ_x computed at the corresponding point of the transformed section. Indeed, the same elementary force dF_2 is applied to an element of area $n\,dA$ of the transformed section and to an element of area dA of the original section. Thus, the stress σ_2 at a point of the original section must be n times larger than the stress at the corresponding point of the transformed section.

The deformations of a composite member can also be determined by using the transformed section. We recall that the transformed section represents the cross section of a member, made of a homogeneous material of modulus E_1, which deforms in the same manner as the composite member. Therefore, using Eq. (4.21), we write that the curvature of the composite member is

$$\frac{1}{\rho} = \frac{M}{E_1 I}$$

where I is the moment of inertia of the transformed section with respect to its neutral axis.

4.5 STRESS CONCENTRATIONS

The formula $\sigma_m = Mc/I$ for a member with a plane of symmetry and a uniform cross section is accurate throughout the entire length of the member only if the couples **M** and **M**$'$ are applied through the use of rigid and smooth plates. Under other conditions of application of the loads, stress concentrations exist near the points where the loads are applied.

Higher stresses also occur if the cross section of the member undergoes a sudden change. Two particular cases are a flat bar with a sudden change in width and a flat bar with grooves. Since the distribution of stresses in the critical cross sections depends only upon the geometry of the members, stress-concentration factors can be determined for various ratios of the parameters involved and recorded, as shown in Figs. 4.24 and 4.25. The value of the maximum stress in the critical cross section is expressed as

$$\sigma_m = K\frac{Mc}{I} \tag{4.29}$$

where K is the stress-concentration factor and c and I refer to the critical section (i.e., the section of width d). Figures 4.24 and 4.25 clearly show the importance of using fillets and grooves of radius r as large as practical.

4.5 STRESS CONCENTRATIONS

Finally, as for axial loading and torsion, the values of the factors K are computed under the assumption of a linear relation between stress and strain. In many applications, plastic deformations occur and result in values of the maximum stress lower than those indicated by Eq. (4.29).

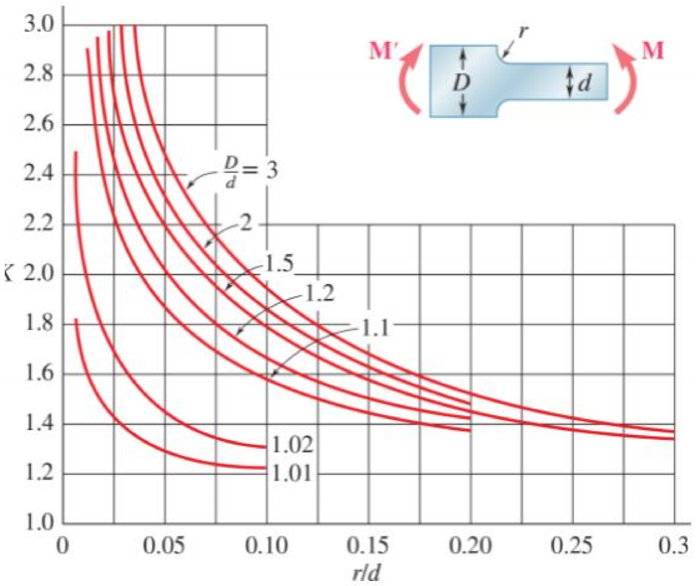


Fig. 4.24 Stress-concentration factors for *flat bars* with fillets under pure bending. (Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress Concentration Factors*, 3rd ed., John Wiley & Sons, New York, 2008.)

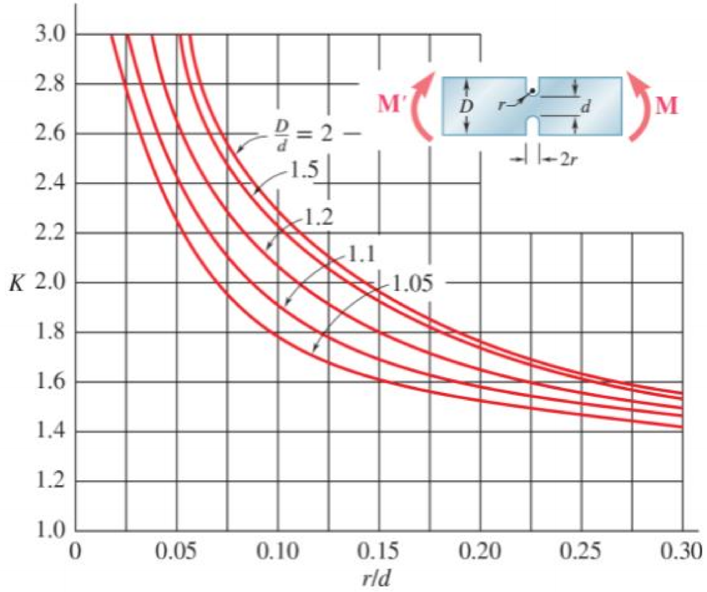


Fig. 4.25 Stress-concentration factors for *flat bars* with grooves (notches) under pure bending. (Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress Concentration Factors*, 3rd ed., John Wiley & Sons, New York, 2008.)

Concept Application 4.4

Grooves 10 mm deep are to be cut in a steel bar which is 60 mm wide and 9 mm thick (Fig. 4.26). Determine the smallest allowable width of the grooves if the stress in the bar is not to exceed 150 MPa when the bending moment is equal to 180 N·m.

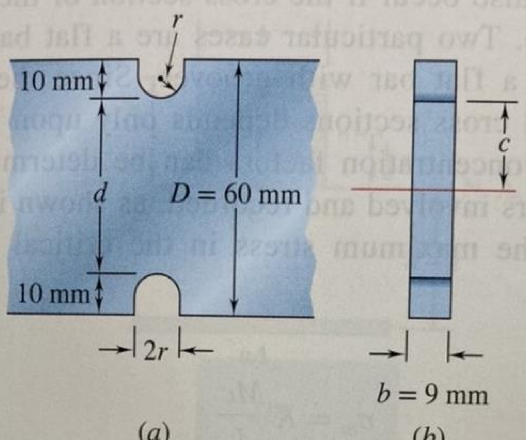


Fig. 4.26 (*a*) Notched bar dimensions. (*b*) Cross section.

Note from Fig. 4.26*a* that

$$d = 60\text{ mm} - 2(10\text{ mm}) = 40\text{ mm}$$

$$c = \tfrac{1}{2}d = 20\text{ mm} \qquad b = 9\text{ mm}$$

The moment of inertia of the critical cross section about its neutral axis is

$$I = \tfrac{1}{12}bd^3 = \tfrac{1}{12}(9 \times 10^{-3}\text{ m})(40 \times 10^{-3}\text{ m})^3$$
$$= 48 \times 10^{-9}\text{ m}^4$$

The value of the stress Mc/I is

$$\frac{Mc}{I} = \frac{(180\ \text{N}\cdot\text{m})(20 \times 10^{-3}\ \text{m})}{48 \times 10^{-9}\ \text{m}^4} = 75\ \text{MPa}$$

Substituting this value for Mc/I into Eq. (4.29) and making $\sigma_m = 150$ MPa, write

$$150\ \text{MPa} = K(75\ \text{MPa})$$

$$K = 2$$

On the other hand,

$$\frac{D}{d} = \frac{60\ \text{mm}}{40\ \text{mm}} = 1.5$$

Using the curve of Fig. 4.25 corresponding to $D/d = 1.5$, we find that the value $K = 2$ corresponds to a value r/d equal to 0.13. Therefore,

$$\frac{r}{d} = 0.13$$

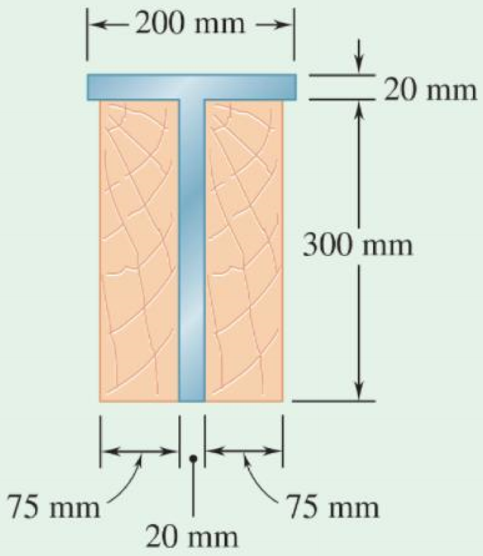
$$r = 0.13d = 0.13(40\ \text{mm}) = 5.2\ \text{mm}$$

The smallest allowable width of the grooves is

$$2r = 2(5.2\ \text{mm}) = 10.4\ \text{mm}$$

Sample Problem 4.3

Two steel plates have been welded together to form a beam in the shape of a T that has been strengthened by securely bolting to it the two oak timbers shown in the figure. The modulus of elasticity is 12.5 GPa for the wood and 200 GPa for the steel. Knowing that a bending moment $M = 50\ \mathrm{kN \cdot m}$ is applied to the composite beam, determine (*a*) the maximum stress in the wood and (*b*) the stress in the steel along the top edge.



STRATEGY:

The beam is first transformed to a beam made of a single material (either steel or wood). The moment of inertia is then determined for the transformed section, and this is used to determine the required stresses, remembering that the actual stresses must be based on the original material.

Sample Problem 4.3

MODELING:

Transformed Section.

First compute the ratio

$$n = \frac{E_s}{E_w} = \frac{200\text{ GPa}}{12.5\text{ GPa}} = 16$$

Multiplying the horizontal dimensions of the steel portion of the section by $n = 16$, a transformed section made entirely of wood is obtained.

Sample Problem 4.3

MODELING:

Neutral Axis.

Fig. 1 shows the transformed section. The neutral axis passes through the centroid of the transformed section. Since the section -consists of two rectangles,

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{(0.160\text{ m})(3.2\text{ m} \times 0.020\text{ m}) + 0}{3.2\ m \times 0.020\ m + 0.470\text{ m} \times 0.300\text{ m}} = 0.050\text{ m}$$

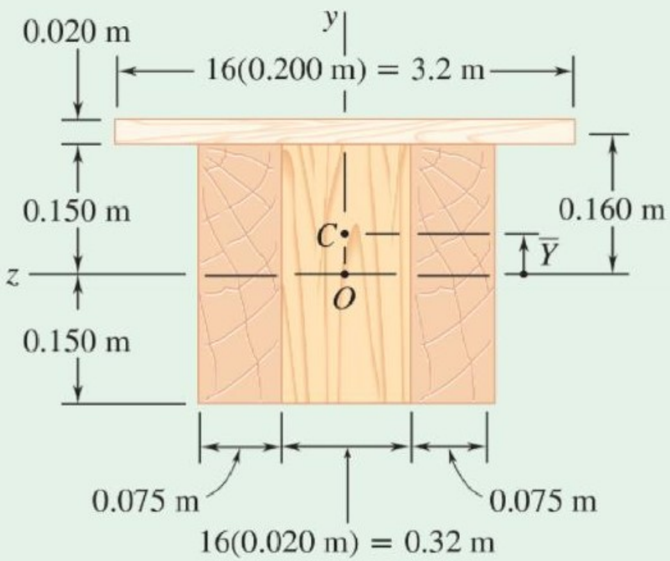


Fig. 1 Transformed cross section.

Sample Problem 4.3

MODELING:

Centroidal Moment of Inertia.

Using **Fig. 2** and the parallel-axis theorem,

$$
\begin{aligned}
I &= \tfrac{1}{12}(0.470)(0.300)^3 + (0.470 \times 0.300)(0.050)^2 \\
&\quad + \tfrac{1}{12}(3.2)(0.020)^3 + (3.2 \times 0.020)(0.160 - 0.050)^2 \\
&= 2.19 \times 10^{-3}\,\mathrm{m}^4
\end{aligned}
$$

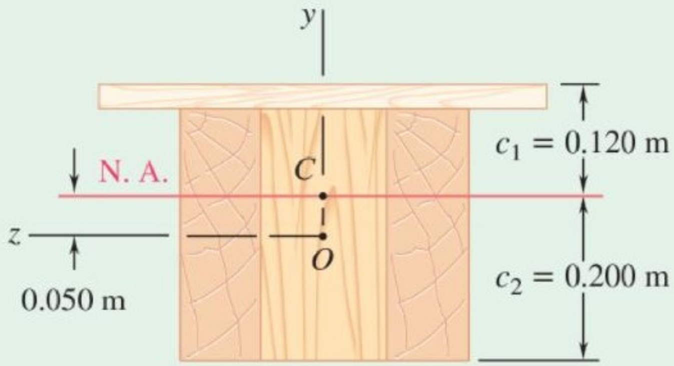


Fig. 2 Transformed section showing neutral axis and distances to extreme fibers.

Sample Problem 4.3

ANALYSIS:

a. Maximum Stress in Wood.

The wood farthest from the neutral axis is located along the bottom edge, where $c_2 = 0.200$ m.

$$\sigma_w = \frac{Mc_2}{I} = \frac{\left(50 \times 10^3 \text{N} \cdot \text{m}\right)(0.200 \text{ m})}{2.19 \times 10^{-3} \text{m}^4}$$

$\sigma_w = 4.57$ MPa ◀

Sample Problem 4.3

b. Stress in Steel.

Along the top edge, $c_1 = 0.120$ m. From the transformed section we obtain an equivalent stress in wood, which must be multiplied by n to obtain the stress in steel.

$$\sigma_s = n\frac{Mc_1}{I} = (16)\frac{\left(50 \times 10^3 \mathrm{N \cdot m}\right)(0.120 \mathrm{~m})}{2.19 \times 10^{-3} \mathrm{m}^4}$$

$\sigma_s = 43.8$ MPa ◀

4.7 ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

We saw in Sec. 1.2A that the distribution of stresses in the cross section of a member under axial loading can be assumed uniform only if the line of action of the loads **P** and **P′** passes through the centroid of the cross section. Such a loading is said to be *centric*. Let us now analyze the distribution of stresses when the line of action of the loads does *not* pass through the centroid of the cross section, i.e., when the loading is *eccentric*.

Two examples of an eccentric loading are shown in Photos 4.5 and 4.6. In Photo 4.5, the weight of the lamp causes an eccentric loading on the post. Likewise, the vertical forces exerted on the press in Photo. 4.6 cause an eccentric loading on the back column of the press.

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Photo 4.5 Walkway light.

Photo 4.6 Bench press.

Courtesy of John DeWolf

4.7 ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

In this section, our analysis will be limited to members that possess a plane of symmetry, and it will be assumed that the loads are applied in the plane of symmetry of the member (Fig. 4.39*a*). The internal forces acting on a given cross section may then be represented by a force **F** applied at the centroid *C* of the section and a couple **M** acting in the plane of symmetry of the member (Fig. 4.39*b*). The conditions of equilibrium of the free body *AC* require that the force **F** be equal and opposite to **P′** and that the moment of the couple **M** be equal and opposite to the moment of **P′** about *C*. Denoting by *d* the distance from the centroid *C* to the line of action *AB* of the forces **P** and **P′**, we have

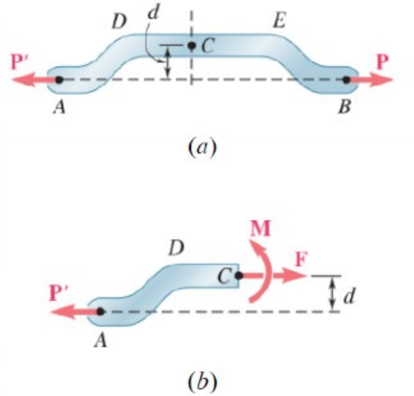

Fig. 4.39 (*a*) Member with eccentric loading. (*b*) Free body diagram of the member with internal forces at section *C*.

$$F = P \qquad \text{and} \qquad M = Pd \tag{4.49}$$

We now observe that the internal forces in the section would have been represented by the same force and couple if the straight portion *DE* of member *AB* had been detached from *AB* and subjected simultaneously to the centric loads **P** and **P′** and to the bending couples **M** and **M′** (Fig. 4.40). Thus, the stress distribution due to the original eccentric loading can be obtained by superposing the uniform stress distribution corresponding to the centric loads **P** and **P′** and the linear distribution corresponding to the bending couples **M** and **M′** (Fig. 4.41). Write

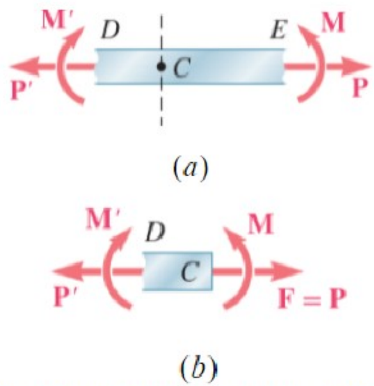

Fig. 4.40 (*a*) Free-body diagram of straight portion *DE*. (*b*) Free-body diagram of portion *CD*.

4.7 ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$

or recalling Eqs. (1.5) and (4.16),

$$\sigma_x = \frac{P}{A} - \frac{My}{I} \tag{4.50}$$

where A is the area of the cross section and I its centroidal moment of inertia and y is measured from the centroidal axis of the cross section. This relationship shows that the distribution of stresses across the section is *linear but not uniform.* Depending upon the geometry of the cross section and the eccentricity of the load, the combined stresses may all have the same sign, as shown in Fig. 4.41, or some may be positive and others negative, as shown in Fig. 4.42. In the latter case, there will be a line in the section, along which $\sigma_x = 0$. This line represents the *neutral axis* of the section. We note that the neutral axis does *not* coincide with the centroidal axis of the section, since $\sigma_x \neq 0$ for $y = 0$.

4.7 ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

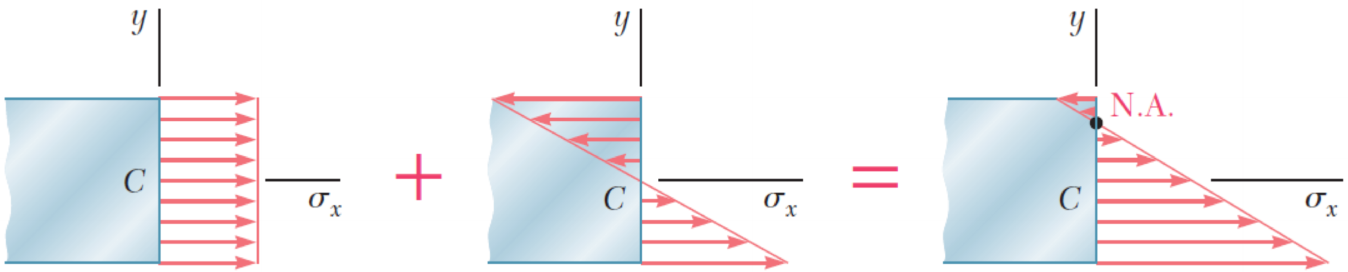

Fig. 4.42 Alternative stress distribution for eccentric loading that results in zones of tension and compression.

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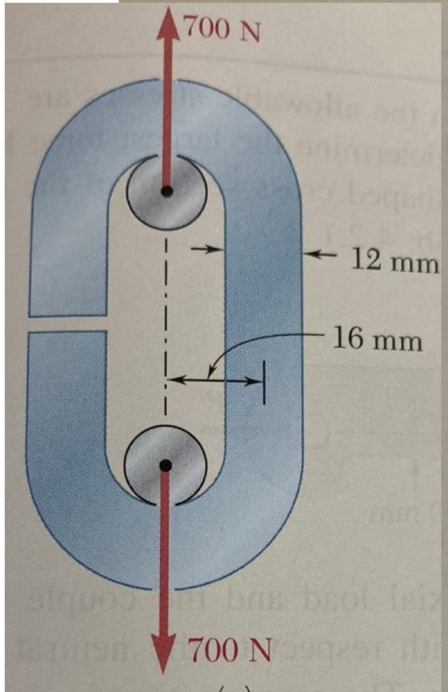
The results obtained are valid only to the extent that the conditions of applicability of the superposition principle (Sec. 2.5) and of Saint-Venant's principle (Sec. 2.10) are met. This means that the stresses involved must not exceed the proportional limit of the material. The deformations due to bending must not appreciably affect the distance d in Fig. 4.39a, and the cross section where the stresses are computed must not be too close to points D or E. The first of these requirements clearly shows that the superposition method cannot be applied to plastic deformations.

Concept Application 4.7

An open-link chain is obtained by bending low-carbon steel rods of 12-mm diameter into the shape shown (Fig. 4.43*a*). Knowing that the chain carries a load of 700 N, determine (*a*) the largest tensile and compressive stresses in the straight portion of a link, (*b*) the distance between the centroidal and the neutral axis of a cross section.



(*a*)

a. Largest Tensile and Compressive Stresses. The internal forces in the cross section are equivalent to a centric force **P** and a bending couple **M** (Fig. 4.43*b*) of magnitudes

$$P = 700 \text{ N}$$

$$M = Pd = (700 \text{ N})(0.016 \text{ mm}) = 11.2 \text{ N·m}$$

The corresponding stress distributions are shown in Fig. 4.43*c* and *d*. The distribution due to the centric force **P** is uniform and equal to $\sigma_0 = P/A$. We have

$$A = \pi c^2 = \pi(6 \text{ mm})^2 = 113.1 \text{ mm}^2$$

$$\sigma_0 = \frac{P}{A} = \frac{700 \text{ N}}{113.1 \text{ mm}^2} = 6.189 \text{ MPa}$$

The distribution due to the bending couple **M** is linear with a maximum stress $\sigma_m = Mc/I$. We write

$$I = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi(6 \text{ mm})^4 = 1017.9 \text{ mm}^4$$

$$\sigma_m = \frac{Mc}{I} = \frac{(11.2 \text{ N} \cdot \text{m})(6 \times 10^{-3} \text{ m})}{1017.9 \times 10^{-12} \text{ m}^4} = 66.02 \text{ MPa}$$

Superposing the two distributions, we obtain the stress distribution corresponding to the given eccentric loading (Fig. 4.43*e*). The largest tensile and compressive stresses in the section are found to be, respectively,

$$\sigma_t = \sigma_0 + \sigma_m = 6.189 + 66.02 = 72.2 \text{ MPa}$$

$$\sigma_c = \sigma_0 - \sigma_m = 6.189 - 66.02 = -59.8 \text{ MPa}$$

b. Distance Between Centroidal and Neutral Axes. The distance y_0 from the centroidal to the neutral axis of the section is obtained by setting $\sigma_x = 0$ in Eq. (4.50) and solving for y_0:

$$0 = \frac{P}{A} - \frac{My_0}{I}$$

$$y_0 = \left(\frac{P}{A}\right)\left(\frac{I}{M}\right) = (6.189 \times 10^6 \text{ Pa})\frac{(1017.9 \times 10^{-12} \text{ m}^4)}{11.2 \text{ N} \cdot \text{m}}$$

$$y_0 = 5.62 \times 10^{-4} \text{ m} = 0.562 \text{ mm}$$

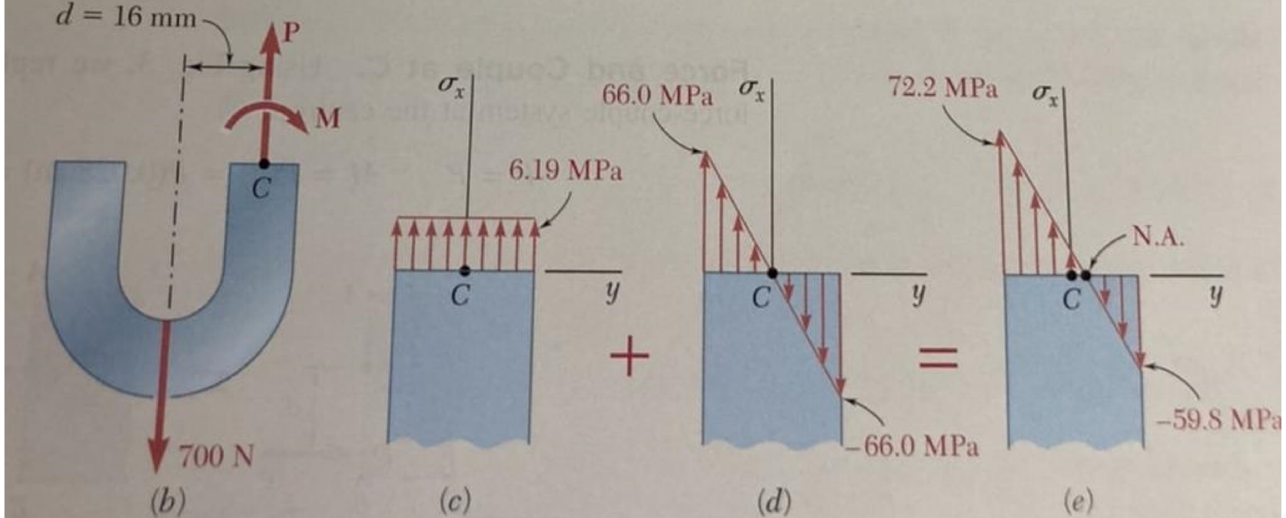
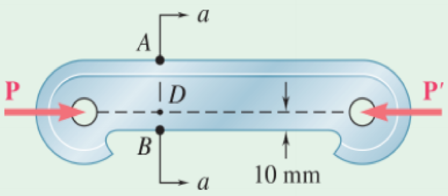


Fig. 4.43 (*a*) Open chain link under loading. (*b*) Free-body diagram for section at *C*. (*c*) Axial stress at section *C*. (*d*) Bending stress at *C*. (*e*) Superposition of stresses.

Sample Problem 4.8

Knowing that for the cast-iron link shown the allowable stresses are 30 MPa in tension and 120 MPa in compression, determine the largest force **P** which can be applied to the link. (*Note:* The T-shaped cross section of the link has previously been considered in **Sample Prob. 4.2**.)



STRATEGY:

The stresses due to the axial load and the couple resulting from the eccentricity of the axial load with respect to the neutral axis are superposed to obtain the maximum stresses. The cross section is singly symmetric, so it is necessary to determine both the maximum compression stress and the maximum tension stress and compare each to the corresponding allowable stress to find **P**.

Sample Problem 4.8

MODELING and ANALYSIS:

Properties of Cross Section.

The cross section is shown in **Fig. 1**. From **Sample Prob. 4.2**, we have

$$A = 3000\ \text{mm}^2 = 3 \times 10^{-3}\ \text{m}^2 \quad \overline{Y} = 38\ \text{mm} = 0.038\ \text{m}$$
$$I = 868 \times 10^{-9}\ \text{m}^4$$

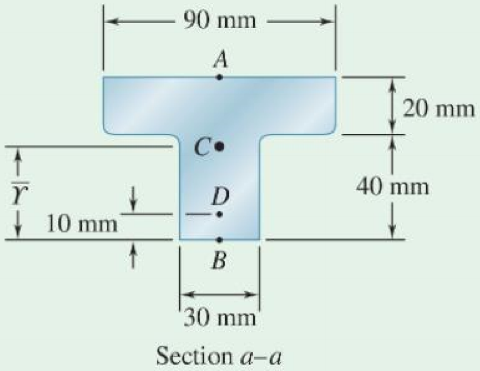


Fig. 1 Section geometry to find centroid location.

Sample Problem 4.8

MODELING and ANALYSIS:

We now write (**Fig. 2**):

$$d = (0.038\text{ m}) - (0.010\text{ m}) = 0.028\text{ m}$$

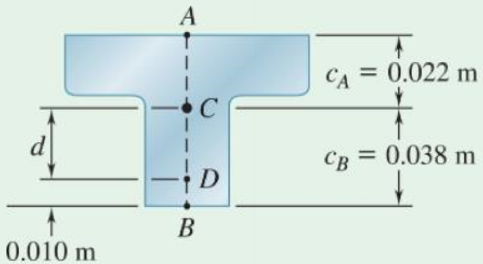


Fig. 2 Dimensions for finding *d*.

Force and Couple at *C*.

Using **Fig. 3**, we replace **P** by an equivalent force-couple system at the centroid *C*.

$$P = P \qquad M = P(d) = P(0.028\text{ m}) = 0.028P$$

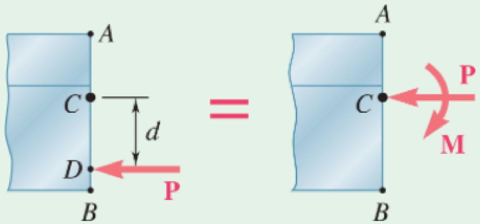


Fig. 3 Equivalent force-couple system at centroid *C*.

Sample Problem 4.8

MODELING and ANALYSIS:

The force **P** acting at the centroid causes a uniform stress distribution (**Fig. 4*a***). The bending couple **M** causes a linear stress distribution (**Fig. 4*b***).

$$\sigma_0 = \frac{P}{A} = \frac{P}{3 \times 10^{-3}} = 333P \qquad \text{(Compression)}$$

$$\sigma_1 = \frac{Mc_A}{I} = \frac{(0.028P)(0.022)}{868 \times 10^{-9}} = 710P \qquad \text{(Tension)}$$

$$\sigma_2 = \frac{Mc_B}{I} = \frac{(0.028P)(0.038)}{868 \times 10^{-9}} = 1226P \qquad \text{(Compression)}$$

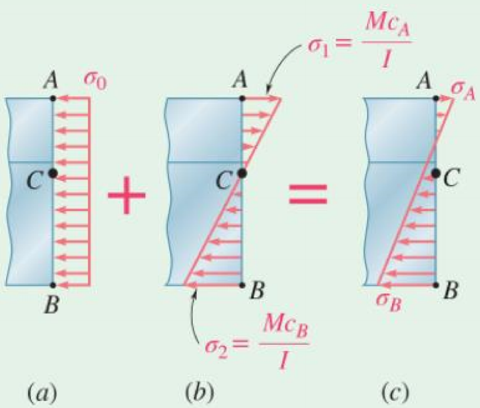


Fig. 4 Stress distribution at section *C* is superposition of axial and bending distributions.

Sample Problem 4.8

MODELING and ANALYSIS:

Superposition.

The total stress distribution (**Fig. 4*c***) is found by superposing the stress distributions caused by the centric force **P** and by the couple **M**. Since tension is positive, and compression negative, we have

$$\sigma_A = -\frac{P}{A} + \frac{Mc_A}{I} = -333P + 710P = +377P \qquad \text{(Tension)}$$
$$\sigma_B = -\frac{P}{A} - \frac{Mc_B}{I} = -333P - 1226P = -1559P \qquad \text{(Compression)}$$

Largest Allowable Force.

The magnitude of **P** for which the tensile stress at point A is equal to the allowable tensile stress of 30 MPa is found by writing

$$\sigma_A = 377P = 30 \text{ MPa} \qquad P = 79.6 \text{ kN} \blacktriangleleft$$

We also determine the magnitude of **P** for which the stress at B is equal to the allowable compressive stress of 120 MPa.

$$\sigma_B = -1559P = -120 \text{ MPa} \qquad P = 77.0 \text{ kN} \blacktriangleleft$$

The magnitude of the largest force **P** that can be applied without exceeding either of the allowable stresses is the smaller of the two values we have found.

$$P = 77.0 \text{ kN} \blacktriangleleft$$

Review and Summary

Normal Strain in Bending

In members possessing a plane of symmetry and subjected to couples acting in that plane, it was proven that *transverse sections remain plane* as a member is deformed. A member in pure bending also has a *neutral surface* along which normal strains and stresses are zero. The longitudinal *normal strain* ϵ_x *varies linearly* with the distance y from the neutral surface:

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$$\epsilon_x = -\frac{y}{\rho} \tag{4.8}$$

where ρ is the *radius of curvature* of the neutral surface (Fig. 4.64). The intersection of the neutral surface with a transverse section is known as the *neutral axis* of the section.

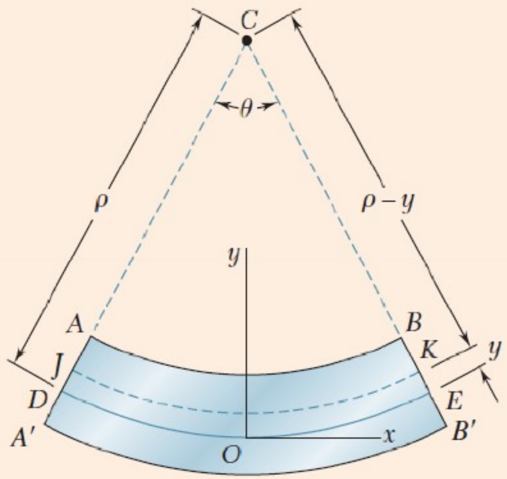

Fig. 4.64 Deformation with respect to neutral axis

Normal Stress in Elastic Range

For members made of a material that follows Hooke's law, the *normal stress* σ_x *varies linearly* with the distance from the neutral axis (Fig. 4.65). Using the maximum stress σ_m, the normal stress is

$$\sigma_x = -\frac{y}{c}\sigma_m \tag{4.12}$$

where c is the largest distance from the neutral axis to a point in the section.

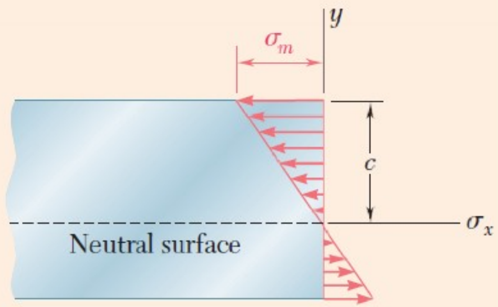

Fig. 4.65 Stress distribution for the elastic flexure formula.

Review and Summary

Elastic Flexure Formula

By setting the sum of the elementary forces $\sigma_x\,dA$ equal to zero, we proved that the *neutral axis passes through the centroid* of the cross section of a member in pure bending. Then by setting the sum of the moments of the elementary forces equal to the bending moment, the *elastic flexure formula* is

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$$\sigma_m = \frac{Mc}{I} \tag{4.15}$$

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where I is the moment of inertia of the cross section with respect to the neutral axis. The normal stress at any distance y from the neutral axis is

$$\sigma_x = -\frac{My}{I} \tag{4.16}$$

Review and Summary

Elastic Section Modulus

Noting that I and c depend only on the geometry of the cross section we introduced the *elastic section modulus*

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$$S = \frac{I}{c} \tag{4.17}$$

Use the section modulus to write an alternative expression for the maximum normal stress:

$$\sigma_m = \frac{M}{S} \tag{4.18}$$

Curvature of Member

The *curvature* of a member is the reciprocal of its radius of curvature, and may be found by

$$\frac{1}{\rho} = \frac{M}{EI} \tag{4.21}$$

Review and Summary

Members Made of Several Materials

We considered the bending of members made of several materials with *different moduli of elasticity*. While transverse sections remain plane, the *neutral axis does not pass through the centroid* of the composite cross section (Fig. 4.66). Using the ratio of the moduli of elasticity of the materials, we obtained a *transformed section* corresponding to an equivalent member made entirely of one material. The methods previously developed are used to determine the stresses in this equivalent homogeneous member (Fig. 4.67), and the ratio of the moduli of elasticity is used to determine the stresses in the composite beam.

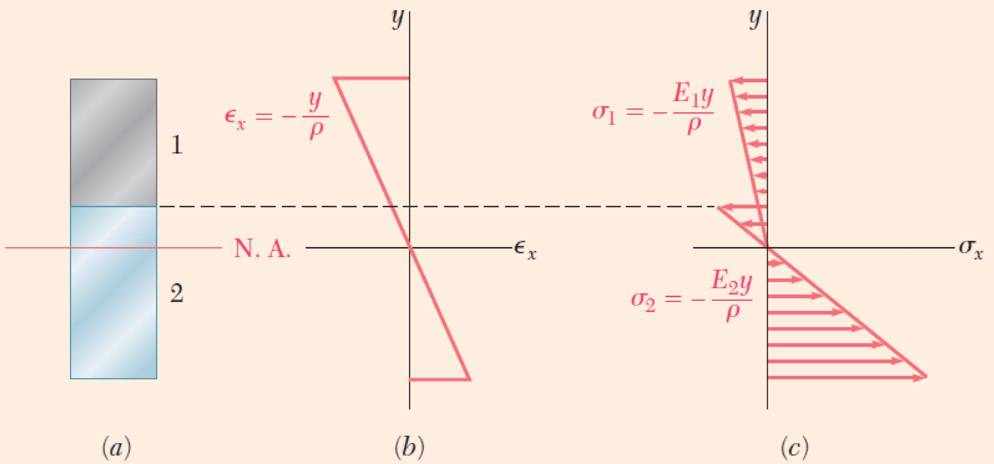

Fig. 4.66 (*a*) Composite section. (*b*) Strain distribution. (*c*) Stress distribution.

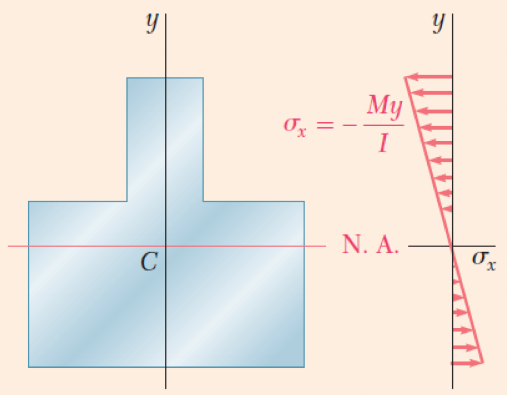

Fig. 4.67 Transformed section.

Review and Summary

Stress Concentrations

Stress concentrations occur in members in pure bending and were discussed; charts giving stress-concentration factors for flat bars with fillets and grooves also were presented in Figs. 4.24 and 4.25.

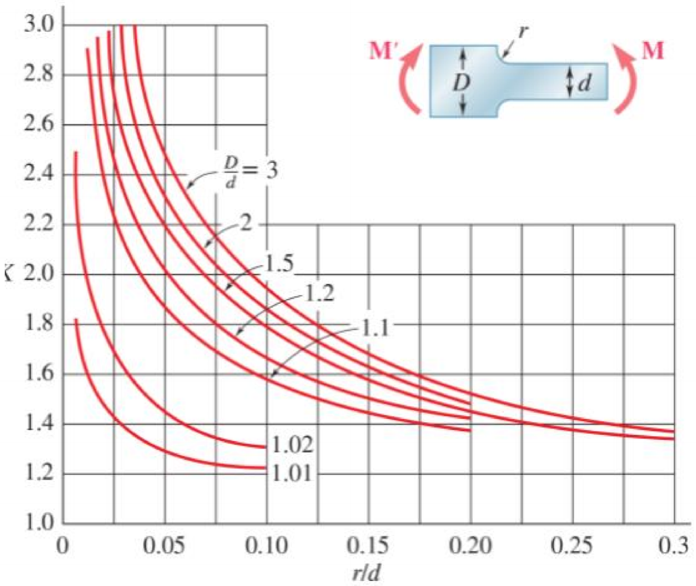

Fig. 4.24 Stress-concentration factors for *flat bars* with fillets under pure bending. (Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress Concentration Factors*, 3rd ed., John Wiley & Sons, New York, 2008.)

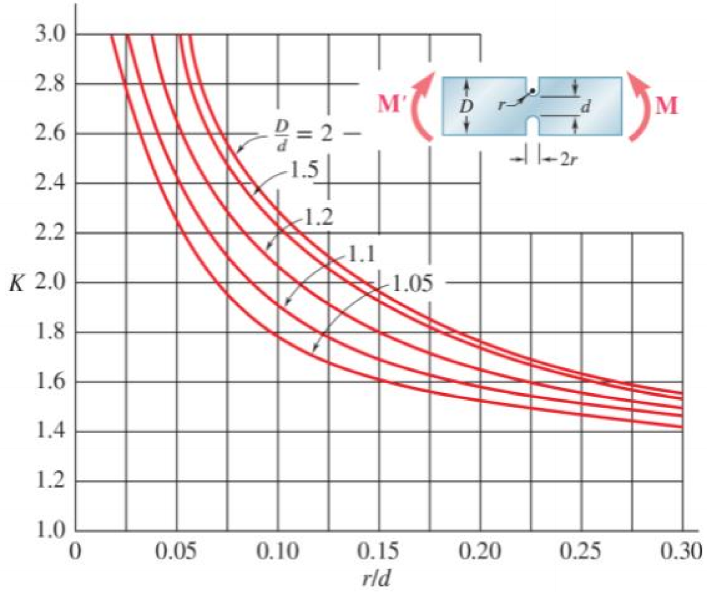

Fig. 4.25 Stress-concentration factors for *flat bars* with grooves (notches) under pure bending. (Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress Concentration Factors*, 3rd ed., John Wiley & Sons, New York, 2008.)

Review and Summary

Eccentric Axial Loading

When a member is loaded *eccentrically in a plane of symmetry*, the *eccentric load* is replaced with a force-couple system located at the centroid of the cross section (Fig. 4.70). The stresses from the centric load and the bending couple are superposed (Fig. 4.71):

$$\sigma_x = \frac{P}{A} - \frac{My}{I} \tag{4.50}$$

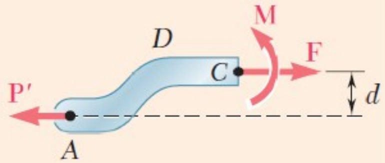

Fig. 4.70 Section of an eccentrically loaded member.

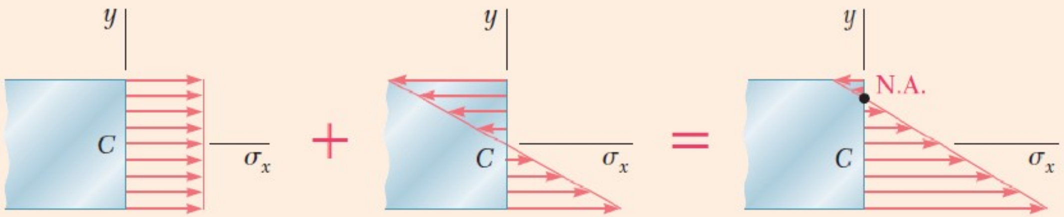

Fig. 4.71 Stress distribution for eccentric loading is obtained by superposing the axial and pure bending distributions.

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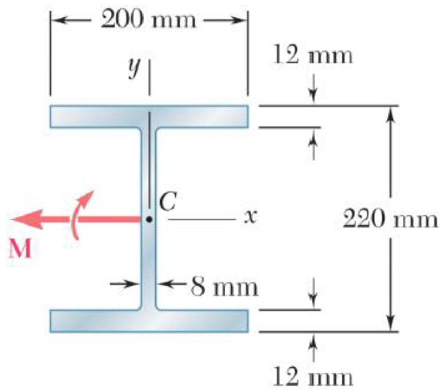
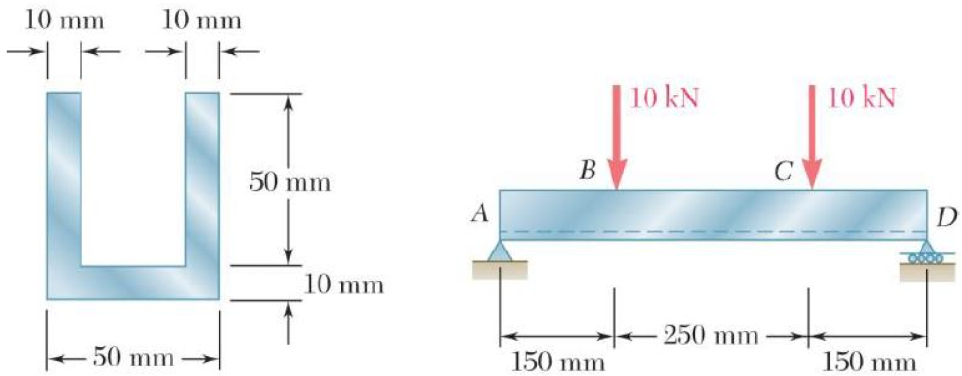


PROBLEM 4.3

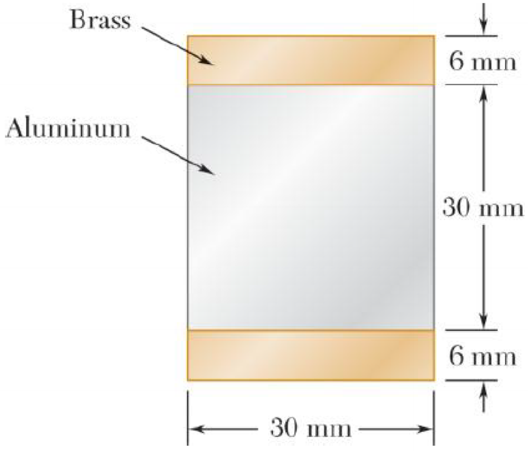
Using an allowable stress of 155 MPa, determine the largest bending moment **M** that can be applied to the wide-flange beam shown. Neglect the effect of fillets.



PROBLEM 4.11

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

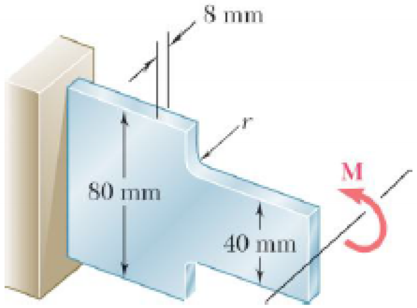
HW



PROBLEM 4.33

A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa



PROBLEM 4.61

Knowing that $M = 250\ \mathrm{N \cdot m}$, determine the maximum stress in the beam shown when the radius r of the fillets is (*a*) 4 mm, (*b*) 8 mm.