Supplementary Material for "Less is More: A Small-Scale Learning Particle Swarm Optimization for Large-Scale Optimization"

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TABLE S.I
EXPERIMENTAL COMPARISON RESULTS OF SSLPSO WITH THE TOP ALGORITHMS OF THE CEC2010 AND CEC2012 COMPETITIONS

FUN	SSLPSO	MA-SW-Chains	MOS	jDElsgo	CCGS
	Mean±Std	Mean±Std	Mean±Std	Mean±Std	Mean±Std
f_1	**0.00E+00 ± 0.00E+00**	2.10E–14 ± 1.99E–14 (+)	**0.00E+00 ± 0.00E+00 (≈)**	8.86E-20 ± 4.51E-20 (+)	1.83E-22 ± 3.68E-22 (+)
f_2	5.43E+02 ± 3.05E+01	8.10E+02 ± 5.88E+01 (+)	1.97E+02 ± 1.59E+01 (-)	1.25E-01 ± 3.45E-01 (-)	**4.44E-02 ± 1.99E-01 (-)**
f_3	**4.53E-14 ± 3.61E-15**	7.28E–13 ± 3.40E–13 (+)	1.12E+00 ± 1.00E+00 (+)	3.81E-12 ± 5.02E-12 (+)	1.91E-01 ± 4.49E-01 (+)
f_4	4.01E+10 ± 1.10E+10	3.53E+11 ± 3.12E+10 (+)	**1.91E+10 ± 8.08E+09 (-)**	8.06E+10 ± 3.08E+10 (+)	1.79E+12 ± 7.62E+11 (+)
f_5	2.79E+08 ± 7.87E+06	1.68E+08 ± 1.04E+08 (-)	6.81E+08 ± 1.42E+08 (+)	9.72E+07 ± 1.44E+07 (-)	**1.97E+07 ± 4.69E+06 (-)**
f_6	**4.00E-09 ± 7.04E-15**	8.14E+04 ± 2.84E+05 (+)	1.99E+07 ± 5.67E+04 (+)	1.70E-08 ± 4.03E-08 (+)	2.88E+06 ± 4.87E+05 (+)
f_7	2.90E-15 ± 1.32E-14	1.03E+02 ± 8.70E+01 (+)	**0.00E+00 ± 0.00E+00 (-)**	1.31E-02 ± 6.82E-02 (+)	1.37E+02 ± 1.16E+02 (+)
f_8	**3.98E+02 ± 5.76E+02**	1.41E+07 ± 3.68E+07 (+)	1.12E+06 ± 1.79E+06 (+)	3.15E+06 ± 3.27E+06 (+)	2.81E+07 ± 3.14E+07 (+)
f_9	**6.17E+06 ± 5.91E+05**	1.41E+07 ± 1.15E+06 (+)	8.78E+06 ± 1.01E+06 (+)	3.11E+07 ± 5.00E+06 (+)	5.53E+07 ± 9.60E+06 (+)
f_{10}	**6.69E+02 ± 4.43E+01**	2.07E+03 ± 1.44E+02 (+)	7.86E+03 ± 2.43E+02 (+)	2.64E+03 ± 3.19E+02 (+)	4.74E+03 ± 2.45E+03 (+)
f_{11}	**1.18E-13 ± 2.95E-15**	3.80E+01 ± 7.35E+00 (+)	1.99E+02 ± 4.52E-01 (+)	2.20E+01 ± 1.53E+01 (+)	2.99E+01 ± 3.98E+00 (+)
f_{12}	5.45E+02 ± 3.11E+02	3.62E–06 ± 5.92E–07 (-)	**0.00E+00 ± 0.00E+00 (-)**	1.21E+04 ± 2.04E+03 (+)	5.35E+03 ± 4.39E+02 (+)
f_{13}	**1.39E+02 ± 4.72E+01**	1.25E+03 ± 5.72E+02 (+)	1.36E+03 ± 9.37E+02 (+)	7.11E+02 ± 1.37E+02 (+)	1.51E+03 ± 6.94E+02 (+)
f_{14}	1.90E+07 ± 1.14E+06	3.11E+07 ± 1.93E+06 (+)	**1.82E+07 ± 1.18E+06 (-)**	1.69E+08 ± 2.08E+07 (+)	1.35E+08 ± 9.05E+06 (+)
f_{15}	1.01E+04 ± 6.62E+01	2.74E+03 ± 1.22E+02 (-)	1.54E+04 ± 5.36E+02 (+)	5.84E+03 ± 4.48E+02 (-)	**1.74E+03 ± 8.94E+01 (-)**
f_{16}	**1.60E-13 ± 3.26E-15**	9.98E+01 ± 1.40E+01 (+)	3.97E+02 ± 2.10E-01 (+)	1.44E+02 ± 3.43E+01 (+)	3.11E+01 ± 5.22E+00 (+)
f_{17}	2.44E+04 ± 6.31E+03	1.24E+00 ± 1.25E–01 (-)	**4.66E-05 ± 6.24E–06 (-)**	1.02E+05 ± 1.26E+04 (+)	1.48E+04 ± 1.02E+03 (-)
f_{18}	**5.84E+02 ± 1.18E+02**	1.30E+03 ± 4.36E+02 (+)	3.91E+03 ± 2.18E+03 (+)	1.85E+03 ± 3.18E+02 (+)	3.13E+03 ± 1.01E+03 (+)
f_{19}	1.51E+07 ± 1.11E+06	2.85E+05 ± 1.78E+04 (-)	**3.41E+04 ± 2.63E+03 (-)**	2.74E+05 ± 2.12E+04 (-)	5.93E+05 ± 4.21E+04 (-)
f_{20}	8.43E+02 ± 6.31E+00	1.07E+03 ± 7.29E+01 (+)	**8.31E+02 ± 3.76E+02 (-)**	1.53E+03 ± 1.32E+02 (+)	1.31E+03 ± 2.14E+02 (+)
+(SSLPSO is significantly better)		15	11	16	15
-(SSLPSO is significantly worse)		5	8	4	5
≈		0	1	0	0

TABLE S.II
OPTIMIZATION RESULTS ON THE 1000-D IEEE CEC2013 TEST SUITE WITH DIFFERENT SETTING OF β

FUN	β=0.6	β=0.2	β=0.4	β=0.8	β=1.0
	Mean±Std	Mean±Std	Mean±Std	Mean±Std	Mean±Std
F_1	1.03E-29 ± 5.55E-29	1.40E-28 ± 6.48E-28 (≈)	1.41E-29 ± 7.58E-29 (≈)	**0.00E+00 ± 0.00E+00 (≈)**	**0.00E+00 ± 0.00E+00 (≈)**
F_2	6.55E+02 ± 4.43E+01	9.27E+02 ± 7.59E+01 (+)	7.17E+02 ± 5.82E+01 (+)	5.95E+02 ± 3.58E+01 (-)	**5.73E+02 ± 2.88E+01 (-)**
F_3	**2.16E+01 ± 5.55E-03**	2.16E+01 ± 5.41E-03 (+)	2.16E+01 ± 3.60E-03 (+)	**2.16E+01 ± 4.91E-03 (≈)**	2.16E+01 ± 4.35E-03 (+)
F_4	3.01E+08 ± 5.92E+07	2.83E+08 ± 6.03E+07 (≈)	**2.65E+08 ± 5.03E+07 (-)**	3.18E+08 ± 6.57E+07 (≈)	3.30E+08 ± 4.94E+07 (+)
F_5	3.70E+06 ± 3.25E+06	**2.48E+06 ± 2.99E+06 (≈)**	3.20E+06 ± 3.18E+06 (≈)	3.77E+06 ± 3.26E+06 (≈)	3.83E+06 ± 3.18E+06 (≈)
F_6	**1.06E+06 ± 1.29E+03**	**1.06E+06 ± 1.18E+03 (≈)**	**1.06E+06 ± 1.52E+03 (≈)**	1.06E+06 ± 9.83E+02 (+)	1.06E+06 ± 8.94E+02 (+)
F_7	2.93E+05 ± 1.77E+05	2.11E+06 ± 8.20E+05 (+)	7.28E+05 ± 3.64E+05 (+)	1.38E+05 ± 1.12E+05 (-)	**8.75E+04 ± 4.65E+04 (-)**
F_8	1.28E+13 ± 2.68E+12	**1.18E+13 ± 3.75E+12 (≈)**	1.28E+13 ± 2.45E+12 (≈)	1.41E+13 ± 3.14E+12 (+)	1.46E+13 ± 3.54E+12 (+)
F_9	6.57E+07 ± 1.16E+08	4.11E+07 ± 7.12E+06 (+)	**5.03E+07 ± 7.53E+07 (≈)**	5.31E+07 ± 9.08E+07 (≈)	8.31E+07 ± 1.46E+08 (≈)
F_{10}	**9.40E+07 ± 2.23E+05**	**9.40E+07 ± 2.21E+05 (≈)**	**9.40E+07 ± 2.66E+05 (≈)**	9.41E+07 ± 1.81E+05 (≈)	**9.40E+07 ± 2.22E+05 (≈)**
F_{11}	6.61E+06 ± 5.18E+06	9.80E+06 ± 5.33E+06 (+)	7.28E+06 ± 4.39E+06 (≈)	**5.82E+06 ± 1.84E+06 (≈)**	8.01E+06 ± 4.61E+06 (+)
F_{12}	8.99E+02 ± 1.88E+01	**8.95E+02 ± 2.16E+01 (≈)**	8.99E+02 ± 1.76E+01 (≈)	9.05E+02 ± 2.13E+01 (≈)	9.05E+02 ± 1.19E+01 (+)
F_{13}	**4.60E+06 ± 2.03E+06**	1.54E+07 ± 1.18E+07 (+)	7.18E+06 ± 4.99E+06 (+)	5.10E+06 ± 2.17E+06 (≈)	6.07E+06 ± 3.38E+06 (≈)
F_{14}	**8.54E+06 ± 1.42E+06**	1.90E+07 ± 1.12E+07 (+)	1.04E+07 ± 1.90E+06 (+)	8.74E+06 ± 1.27E+06 (≈)	8.70E+06 ± 1.32E+06 (≈)
F_{15}	9.68E+06 ± 1.22E+07	**4.92E+06 ± 5.52E+05 (-)**	5.60E+06 ± 1.05E+06 (-)	2.04E+07 ± 3.22E+07 (+)	1.26E+08 ± 4.95E+07 (+)
+(β=0.6 is significantly better)		7	5	3	7
-(β=0.6 is significantly worse)		1	2	2	2
≈		7	8	10	6

TABLE S.III
OPTIMIZATION RESULTS ON THE 2000-D IEEE CEC2010 TEST SUITE

FUN	SSLPSO	RCI-PSO	HCLPSO	SDLSO	AGLDPSO	SLPSO-ARS
	Mean±Std	Mean±Std	Mean±Std	Mean±Std	Mean±Std	Mean±Std
f_1	**6.26E-23 ± 2.14E-23**	9.05E-21 ± 6.87E-22 (+)	1.24E-18 ± 2.27E-19 (+)	1.72E-16 ± 2.17E-17 (+)	3.38E-20 ± 1.80E-21 (+)	1.47E-17 ± 8.28E-19 (+)
f_2	1.14E+03 ± 4.08E+01	1.83E+03 ± 4.89E+01 (+)	1.52E+03 ± 5.23E+01 (+)	1.05E+03 ± 3.45E+01 (-)	6.13E+03 ± 3.02E+02 (+)	6.92E+03 ± 1.12E+03 (+)
f_3	7.96E-14 ± 4.01E-15	**5.79E-14 ± 3.49E-15 (-)**	1.56E-12 ± 1.75E-13 (+)	2.84E-11 ± 1.46E-12 (+)	4.41E+00 ± 3.72E-01 (+)	1.57E-12 ± 1.47E-14 (+)
f_4	1.20E+11 ± 1.33E+10	1.55E+11 ± 1.91E+10 (+)	1.08E+12 ± 1.77E+11 (+)	5.95E+11 ± 6.83E+10 (+)	3.06E+11 ± 7.24E+10 (+)	1.06E+12 ± 1.49E+11 (+)
f_5	6.98E+08 ± 1.45E+07	1.86E+07 ± 3.25E+06 (-)	1.39E+07 ± 2.83E+06 (-)	**9.72E+06 ± 2.35E+06 (-)**	1.44E+08 ± 2.19E+08 (-)	1.55E+07 ± 3.07E+06 (-)
f_6	**7.99E-09 ± 1.26E-14**	7.99E-09 ± 3.02E-14 (+)	2.66E-08 ± 2.66E-09 (+)	2.42E-08 ± 2.36E-09 (+)	3.98E+01 ± 2.27E-02 (+)	6.71E-07 ± 2.38E-08 (+)
f_7	**8.13E-03 ± 4.34E-02**	1.42E+03 ± 4.89E+03 (+)	1.31E+04 ± 4.76E+03 (+)	8.76E+04 ± 1.22E+04 (+)	2.20E+06 ± 4.27E+05 (+)	4.03E+05 ± 7.39E+04 (+)
f_8	**3.73E+03 ± 1.99E+03**	1.92E+04 ± 4.15E+03 (+)	7.21E+07 ± 1.49E+05 (+)	5.53E+07 ± 2.33E+05 (+)	3.75E+05 ± 9.91E+05 (+)	6.50E+07 ± 1.72E+05 (+)
f_9	**1.97E+07 ± 1.39E+06**	2.39E+07 ± 8.82E+05 (+)	1.21E+08 ± 6.93E+06 (+)	8.76E+07 ± 4.21E+06 (+)	6.23E+07 ± 4.62E+06 (+)	1.77E+08 ± 9.50E+06 (+)
f_{10}	1.38E+03 ± 4.66E+01	1.83E+03 ± 4.85E+01 (+)	1.54E+03 ± 4.84E+01 (+)	1.72E+04 ± 4.80E+02 (+)	6.59E+03 ± 3.65E+02 (+)	6.67E+03 ± 9.26E+02 (+)
f_{11}	**2.87E-13 ± 8.00E-15**	4.71E-13 ± 1.62E-14 (+)	4.13E-11 ± 7.74E-12 (+)	3.92E-10 ± 3.80E-11 (+)	5.77E+01 ± 1.30E+01 (+)	1.48E-11 ± 3.48E-13 (+)
f_{12}	5.36E+03 ± 6.37E+02	**2.43E+03 ± 1.89E+02 (-)**	1.60E+05 ± 5.77E+03 (+)	2.94E+05 ± 1.36E+04 (+)	1.15E+04 ± 1.05E+03 (+)	2.55E+05 ± 8.36E+03 (+)
f_{13}	**4.87E+02 ± 3.82E+01**	7.65E+02 ± 8.88E+01 (+)	1.19E+03 ± 1.80E+02 (+)	1.05E+03 ± 1.28E+02 (+)	1.39E+03 ± 1.90E+02 (+)	1.25E+03 ± 3.49E+02 (+)
f_{14}	6.54E+07 ± 3.37E+06	7.09E+07 ± 2.41E+06 (+)	3.62E+08 ± 1.83E+07 (+)	2.97E+08 ± 9.94E+06 (+)	1.72E+08 ± 8.88E+06 (+)	9.22E+08 ± 4.00E+07 (+)
f_{15}	2.10E+04 ± 9.58E+01	1.83E+03 ± 6.02E+01 (-)	1.69E+03 ± 6.14E+01 (-)	2.16E+04 ± 4.67E+02 (+)	7.13E+03 ± 3.05E+02 (-)	6.54E+03 ± 9.45E+02 (-)
f_{16}	**4.82E-13 ± 7.66E-15**	9.48E-02 ± 3.55E-01 (+)	1.03E+00 ± 1.31E+00 (+)	3.42E-02 ± 1.84E-01 (+)	1.93E+02 ± 3.69E+01 (+)	2.51E-11 ± 5.10E-13 (+)
f_{17}	9.46E+04 ± 1.03E+04	5.31E+04 ± 2.35E+03 (-)	8.80E+05 ± 3.06E+04 (+)	2.16E+06 ± 1.13E+05 (+)	1.17E+05 ± 6.88E+03 (+)	1.04E+06 ± 3.77E+04 (+)
f_{18}	**1.49E+03 ± 1.35E+02**	2.11E+03 ± 2.43E+02 (+)	4.16E+03 ± 9.63E+02 (+)	2.74E+03 ± 5.40E+02 (+)	3.81E+03 ± 4.92E+02 (+)	3.38E+03 ± 1.02E+03 (+)
f_{19}	5.72E+07 ± 3.70E+06	4.25E+06 ± 1.56E+05 (-)	1.21E+07 ± 6.89E+05 (-)	1.42E+08 ± 1.37E+08 (+)	3.27E+06 ± 1.33E+05 (-)	5.66E+06 ± 3.85E+05 (-)
f_{20}	**1.79E+03 ± 2.10E+01**	2.00E+03 ± 8.04E+01 (+)	2.94E+03 ± 2.48E+02 (+)	2.06E+03 ± 9.34E+01 (+)	3.49E+03 ± 2.15E+02 (+)	2.04E+03 ± 1.77E+02 (+)
+(SSLPSO is significantly better)		14	17	18	17	17
-(SSLPSO is significantly worse)		6	3	2	3	3
≈		0	0	0	0	0
FUN	TPLSO	DLLSO	SPLSO	CCPSO2	DDG	EADG
	Mean±Std	Mean±Std	Mean±Std	Mean±Std	Mean±Std	Mean±Std
f_1	3.16E-17 ± 9.82E-18 (+)	1.66E-20 ± 7.10E-22 (+)	5.42E-08 ± 1.27E-08 (+)	3.54E+04 ± 2.71E+03 (+)	4.26E+07 ± 3.38E+07 (+)	5.77E+07 ± 6.54E+07 (+)
f_2	2.30E+03 ± 2.08E+02 (+)	1.40E+03 ± 4.29E+01 (+)	1.01E+04 ± 1.39E+03 (+)	**5.10E-02 ± 1.43E-02 (-)**	1.26E+04 ± 5.50E+02 (+)	1.28E+04 ± 7.40E+02 (+)
f_3	3.20E+00 ± 2.62E-01 (+)	5.83E-14 ± 1.32E-15 (-)	3.94E-07 ± 5.82E-08 (+)	2.94E+01 ± 1.24E+01 (+)	2.68E+01 ± 6.39E-01 (+)	2.10E+01 ± 1.52E+00 (+)
f_4	8.06E+11 ± 1.79E+11 (+)	1.42E+12 ± 2.55E+11 (+)	1.40E+12 ± 1.77E+11 (+)	1.96E+13 ± 1.20E+13 (+)	1.56E+12 ± 3.45E+11 (+)	**4.80E+10 ± 2.99E+10 (-)**
f_5	5.00E+07 ± 1.17E+08 (-)	1.44E+07 ± 3.56E+06 (-)	6.96E+08 ± 1.35E+07 (≈)	1.05E+09 ± 1.68E+08 (+)	1.82E+08 ± 3.05E+07 (-)	1.32E+08 ± 1.64E+07 (-)
f_6	3.52E+00 ± 9.77E-01 (+)	8.30E-09 ± 7.95E-11 (+)	1.62E-06 ± 1.74E-07 (+)	3.91E+07 ± 2.56E+06 (+)	2.38E+06 ± 1.09E+06 (+)	5.86E+04 ± 2.19E+05 (+)
f_7	1.48E+05 ± 6.95E+04 (+)	8.51E+04 ± 2.34E+04 (+)	8.01E+05 ± 1.08E+05 (+)	7.75E+09 ± 7.88E+09 (+)	5.62E+06 ± 6.18E+05 (+)	2.01E+05 ± 3.58E+04 (+)
f_8	5.31E+07 ± 1.35E+07 (+)	6.99E+07 ± 1.20E+05 (+)	7.97E+07 ± 9.26E+04 (+)	4.82E+08 ± 4.92E+08 (+)	1.63E+08 ± 5.36E+07 (+)	3.00E+06 ± 2.35E+06 (+)
f_9	1.07E+08 ± 7.92E+06 (+)	1.03E+08 ± 4.85E+06 (+)	2.09E+08 ± 1.21E+07 (+)	6.46E+08 ± 2.65E+08 (+)	5.28E+08 ± 1.48E+08 (+)	3.62E+08 ± 9.57E+07 (+)
f_{10}	1.17E+04 ± 7.56E+03 (+)	**1.17E+03 ± 2.44E+01 (-)**	2.00E+04 ± 1.81E+02 (+)	1.13E+04 ± 1.70E+03 (+)	1.95E+04 ± 2.62E+03 (+)	7.93E+03 ± 2.81E+02 (+)
f_{11}	6.56E+00 ± 1.97E+00 (+)	5.60E-13 ± 1.97E-14 (+)	5.22E-06 ± 5.16E-07 (+)	4.35E+02 ± 9.67E+00 (+)	4.12E+02 ± 1.14E+01 (+)	2.31E+01 ± 2.04E+00 (+)
f_{12}	7.63E+04 ± 4.73E+03 (+)	1.03E+05 ± 4.82E+03 (+)	1.15E+06 ± 1.60E+05 (+)	5.45E+05 ± 2.21E+05 (+)	3.44E+05 ± 3.36E+04 (+)	1.91E+05 ± 2.66E+04 (+)
f_{13}	1.84E+03 ± 2.93E+02 (+)	1.21E+03 ± 2.05E+02 (+)	1.21E+03 ± 3.43E+02 (+)	1.24E+04 ± 6.20E+03 (+)	1.74E+07 ± 3.66E+07 (+)	1.85E+05 ± 3.12E+04 (+)
f_{14}	2.98E+08 ± 1.46E+07 (+)	2.82E+08 ± 1.14E+07 (+)	7.56E+08 ± 3.02E+07 (+)	1.74E+09 ± 8.40E+08 (+)	1.18E+09 ± 7.46E+07 (+)	**4.36E+07 ± 2.61E+06 (-)**
f_{15}	2.11E+04 ± 7.93E+01 (+)	2.06E+04 ± 7.47E+01 (-)	2.10E+04 ± 8.30E+01 (+)	2.38E+04 ± 1.72E+03 (+)	1.80E+04 ± 3.40E+03 (≈)	6.53E+03 ± 2.01E+02 (-)
f_{16}	4.27E+01 ± 1.10E+01 (+)	1.00E+00 ± 1.14E+00 (+)	9.08E-06 ± 9.92E-07 (+)	8.01E+02 ± 6.36E+00 (+)	8.20E+02 ± 2.15E+00 (+)	2.20E-01 ± 5.10E-01 (+)
f_{17}	4.07E+05 ± 1.89E+04 (+)	5.89E+05 ± 1.75E+04 (+)	3.12E+06 ± 1.60E+05 (+)	1.31E+06 ± 5.34E+05 (+)	8.03E+05 ± 6.12E+04 (+)	**3.24E+01 ± 3.66E+01 (-)**
f_{18}	6.19E+03 ± 1.15E+03 (+)	4.47E+03 ± 1.12E+03 (+)	5.09E+03 ± 3.02E+03 (+)	4.43E+04 ± 4.45E+04 (+)	5.24E+09 ± 5.64E+09 (+)	1.97E+03 ± 1.92E+02 (+)
f_{19}	1.08E+07 ± 5.36E+05 (-)	2.55E+07 ± 1.31E+06 (-)	4.18E+07 ± 2.06E+06 (-)	8.96E+06 ± 1.40E+06 (-)	2.96E+06 ± 1.80E+05 (-)	**1.94E+06 ± 1.05E+05 (-)**
f_{20}	4.43E+03 ± 2.56E+02 (+)	2.86E+03 ± 3.64E+02 (+)	2.22E+03 ± 3.51E+02 (+)	2.56E+04 ± 2.71E+04 (+)	5.46E+09 ± 4.35E+09 (+)	1.30E+08 ± 1.68E+08 (+)
+	18	15	18	18	17	14
-	2	5	1	2	2	6
≈	0	0	1	0	1	0

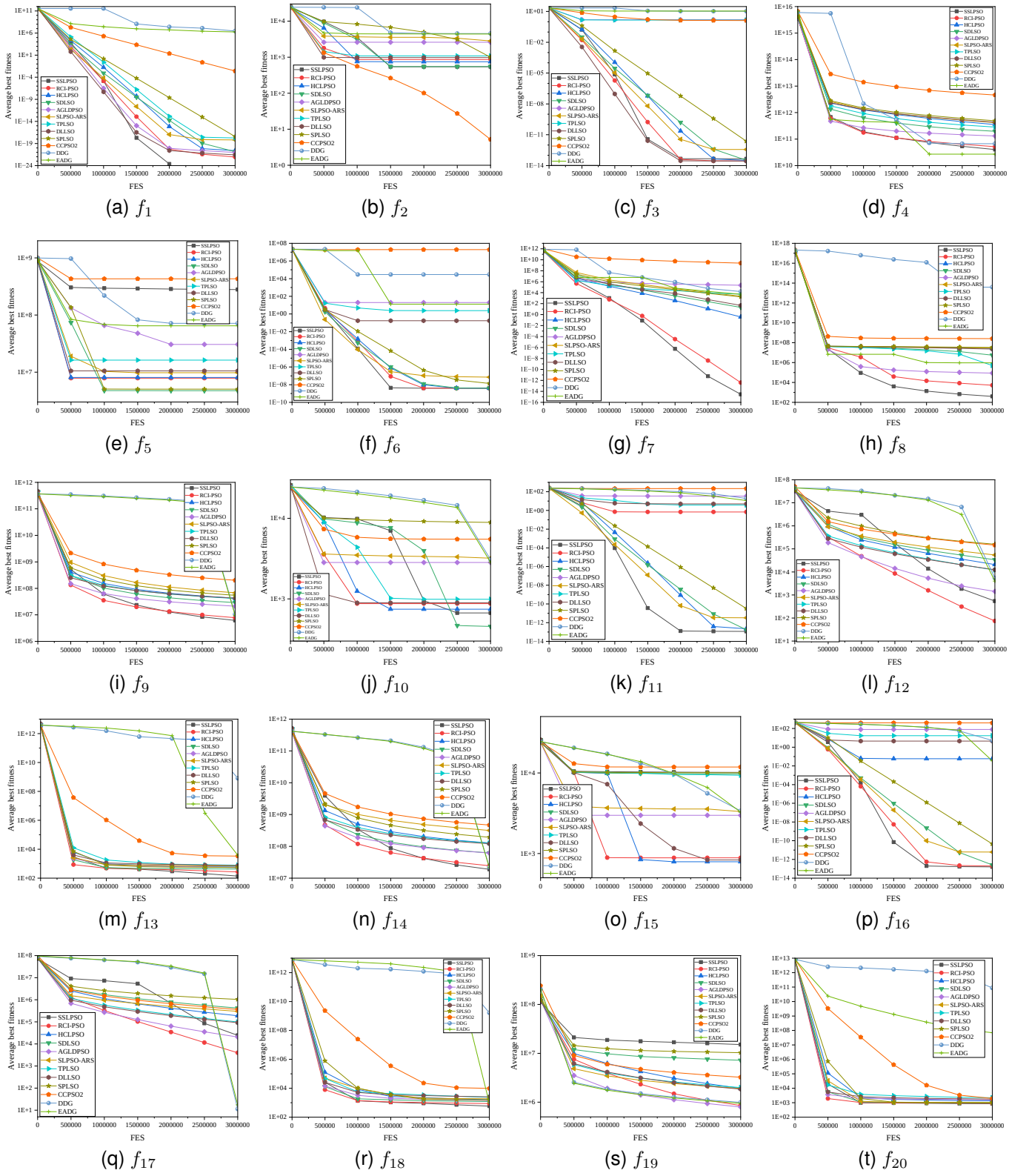
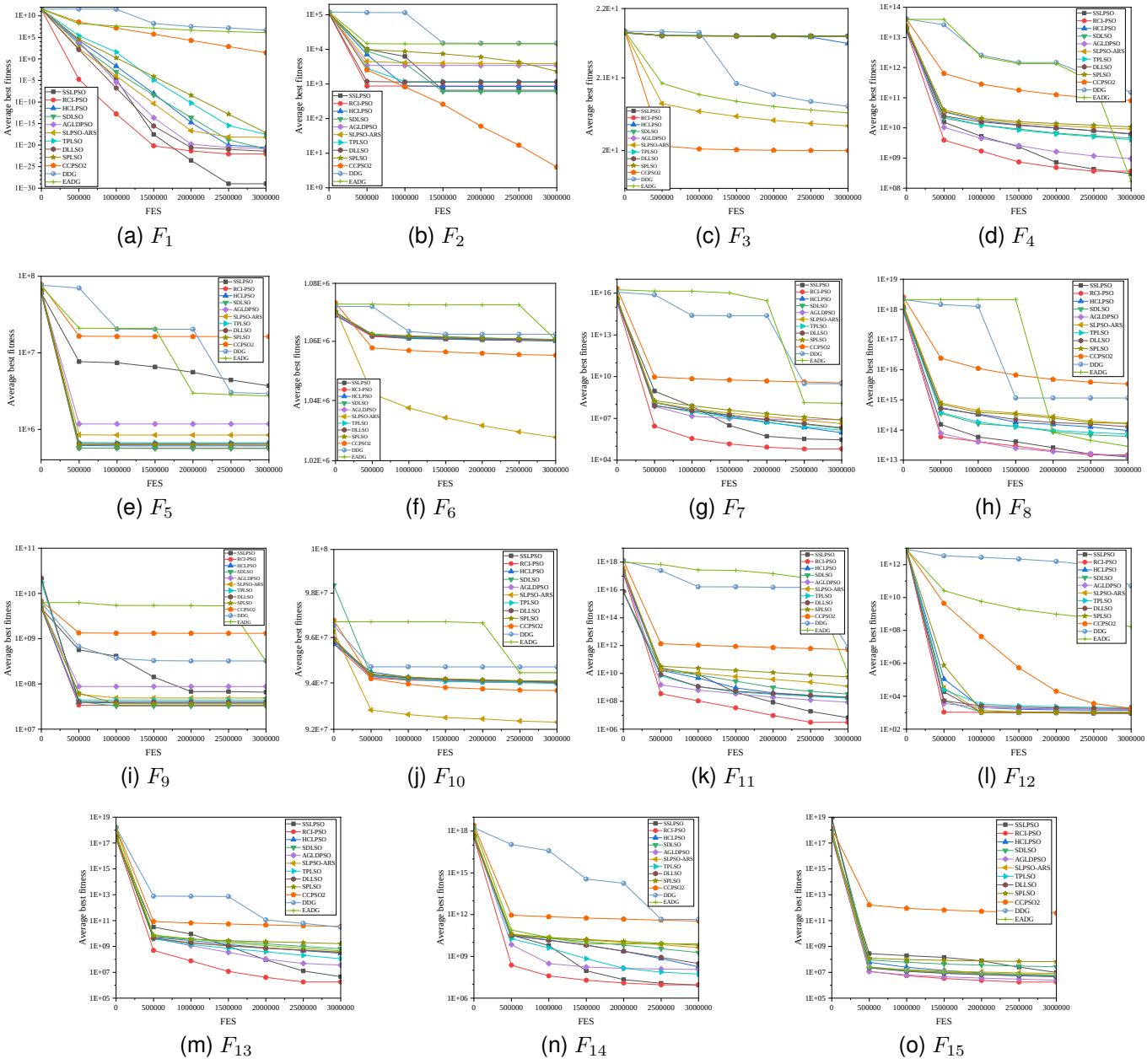


Fig. S.1. Comparison of the convergence behavior of BLPSO with other comparison algorithms on the 1000-dimensional CEC2010 benchmark functions.



(a) F_1 (b) F_2 (c) F_3 (d) F_4

(e) F_5 (f) F_6 (g) F_7 (h) F_8

(i) F_9 (j) F_{10} (k) F_{11} (l) F_{12}

(m) F_{13} (n) F_{14} (o) F_{15}

Fig. S.2. Comparison of the convergence behavior of BLPSO with other comparison algorithms on the 1000-dimensional CEC2013 benchmark functions.