

Calculus(I)

Assignment 2

due date: Wednesday, September 29, 2021, 4pm

Please write down your student ID clearly. There are four problems with *, each of them counts 25 credits. You get full credits of each problem only if your written reasoning is complete and the answer is correct. Problems in the assignment may appear in the exams.

Part I: Calculus, Metric Edition, 9th Edition

§1.5: 7* §1.6: 67* §1.7: 11*

Part II

Problem* 1. Find the following limits:

- (i) $\lim_{x \rightarrow 0} \frac{f(3x)}{x}$, knowing that $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$,
- (ii) $\lim_{x \rightarrow 1} f(x)$, knowing that $\lim_{x \rightarrow 1} f(x)$ exists and $f(x) = x^2 + 3x \lim_{x \rightarrow 1} f(x)$,
- (iii) $\lim_{x \rightarrow -1} \left(\frac{1}{x+1} - \frac{3}{x^3+1} \right)$,
- (iv) $\lim_{x \rightarrow 0} \frac{1}{x} \sin(x^2 \sin(\frac{1}{x}))$. **Hint:** $\lim_{x \rightarrow 0} x^2 \sin(\frac{1}{x}) = 0$ and $\lim_{y \rightarrow 0} \frac{\sin(y)}{y} = 1$.

Part III

Extra Problem 1. (extra 10 + 5 credits) Suppose that your teaching assistant shows $\lim_{x \rightarrow 0} x \sin(\frac{1}{x}) = 0$ by the following argument:

$$\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) = \left(\lim_{x \rightarrow 0} x\right) \cdot \left(\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)\right) = 0 \cdot \left(\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)\right) = 0.$$

- (i) Does $\lim_{x \rightarrow 0} \sin(\frac{1}{x})$ exist? **Hint:** You may consider the following sequences:

$$x_n = \frac{1}{2n\pi + \frac{\pi}{2}}, \quad y_n = \frac{1}{2n\pi - \frac{\pi}{2}}, \quad \text{for } n \in \mathbb{N}.$$

Then $\lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} y_n = 0$. Is $\lim_{n \rightarrow \infty} x_n \sin(\frac{1}{x_n}) = \lim_{n \rightarrow \infty} y_n \sin(\frac{1}{y_n})$?

- (ii) Is your teaching assistant's argument correct? If yes, why? If no, why?