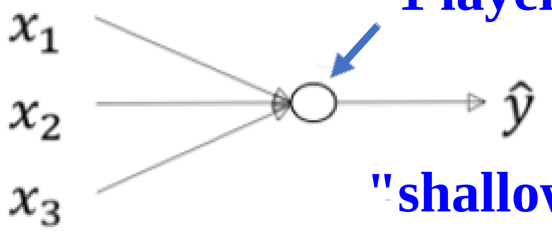
Deep Neural Network

Deep L-layer Neural Network

What is a deep neural network

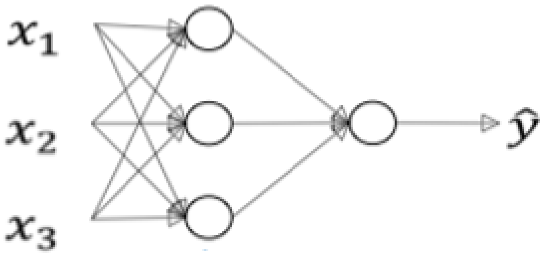
"1 layer NN"



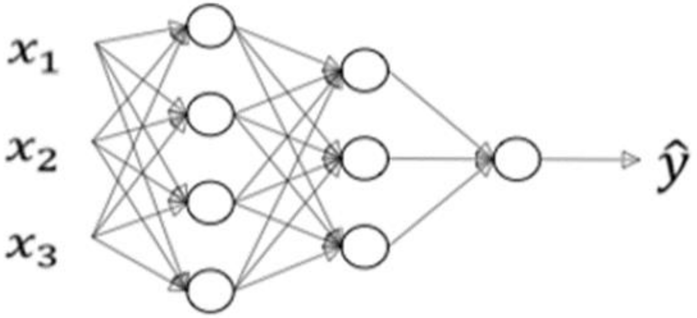
"shallow"

logistic regression

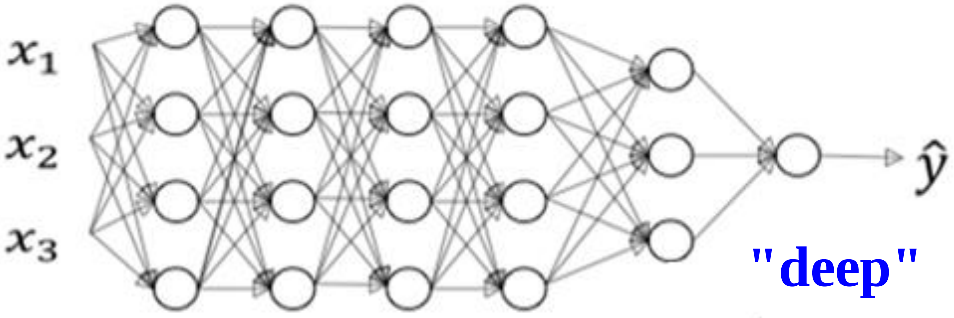
"2 layer NN"



1 hidden layer



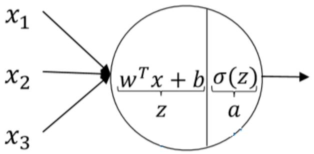
2 hidden layers



"deep"

5 hidden layers

Deep neural network notation



layer "0"

1

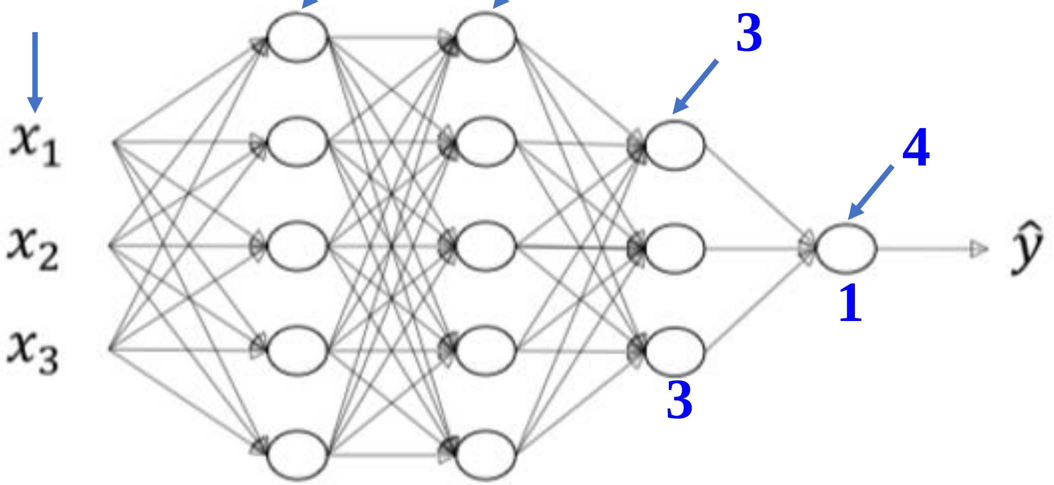
2

3

4

4 layer NN

$x = a^{[0]}$



$\hat{y} = a^{[L]}$

5 5 3 1

L = 4 (#layer)

$n^{[l]}$ = #units in layer l

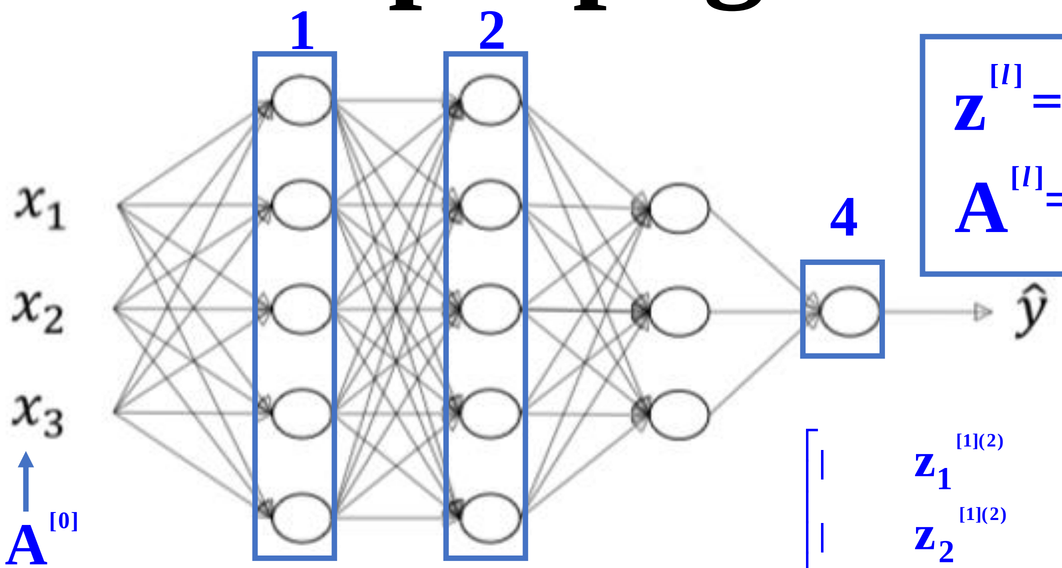
$a^{[l]}$ = activation in layer l

$a^{[l]} = g^{[l]}(z^{[l]})$, $w^{[l]}$, $b^{[l]}$ = weights for $z^{[l]}$

$n^{[1]} = 5,\ n^{[2]} = 5,\ n^{[3]} = 3,\ n^{[4]} = n^{[L]} = 1$

$n^{[0]} = n_x = 3$

Forward propagation in a deep network



x : $A^{[0]}$

$$z^{[l]} = w^{[l]T} A^{[l-1]} + b^{[l]}$$
$$A^{[l]} = g^{[l]}(z^{[l]})$$
$$A^{[0]} = x$$

$$\begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

Vectorized:

除了層外

$$z^{[1]} = w^{[1]T} A^{[0]} + b^{[1]}$$
$$A^{[1]} = g^{[1]}(z^{[1]})$$

$$z^{[2]} = w^{[2]T} A^{[1]} + b^{[2]}$$
$$A^{[2]} = g^{[2]}(z^{[2]})$$

......

$$\hat{Y} = g(z^{[4]}) = A^{[4]}$$

for $l = 1 \ldots 4$

$$\begin{bmatrix} | & z_1^{[1](2)} & & | \\ | & z_2^{[1](2)} & & | \\ z^{[1](1)} & z_3^{[1](2)} & \cdots & z^{[1](m)} \\ | & z_4^{[1](2)} & & | \\ | & z_5^{[1](2)} & & | \end{bmatrix} = \begin{bmatrix} w_{1,1}^{[1]} & w_{1,2}^{[1]} & w_{1,3}^{[1]} & w_{1,4}^{[1]} & w_{1,5}^{[1]} \\ w_{2,1}^{[1]} & w_{2,2}^{[1]} & w_{2,3}^{[1]} & w_{2,4}^{[1]} & w_{2,5}^{[1]} \\ w_{3,1}^{[1]} & w_{3,2}^{[1]} & w_{3,3}^{[1]} & w_{3,4}^{[1]} & w_{n^{[0]},n^{[1]}}^{[1]} \end{bmatrix}^T \begin{bmatrix} A_1^{[0](1)} & A_1^{[0](2)} & \cdots & A_1^{[0](m)} \\ A_2^{[0](1)} & A_2^{[0](2)} & \cdots & A_2^{[0](m)} \\ A_3^{[0](1)} & A_3^{[0](2)} & \cdots & A_3^{[0](m)} \end{bmatrix}$$

$(n^{[1]}, m)$

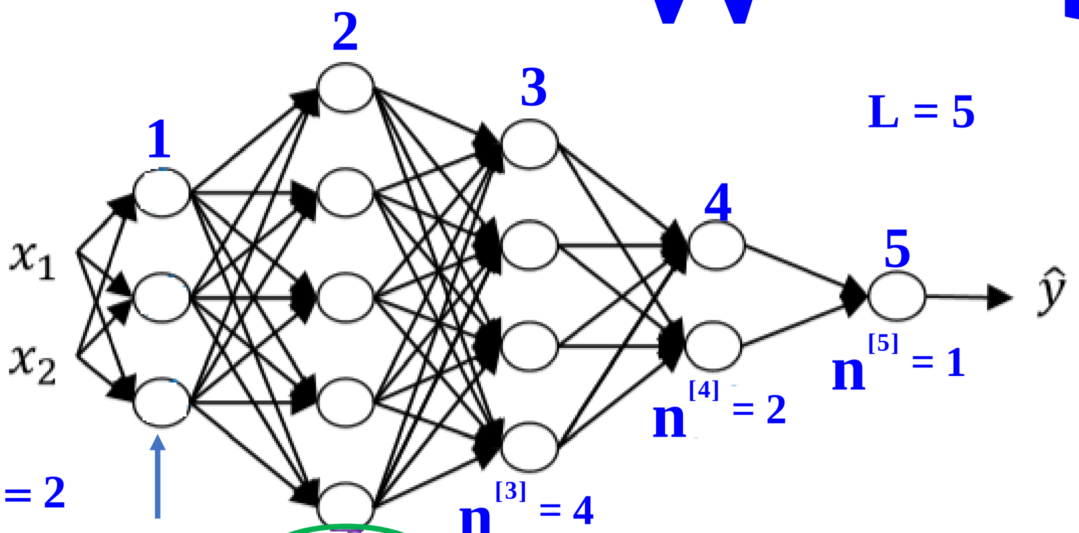
$$= \begin{bmatrix} w_{1,1}^{[1]} & w_{2,1}^{[1]} & w_{3,1}^{[1]} \\ w_{1,2}^{[1]} & w_{2,2}^{[1]} & w_{3,2}^{[1]} \\ w_{1,3}^{[1]} & w_{2,3}^{[1]} & w_{3,3}^{[1]} \\ w_{1,4}^{[1]} & w_{2,4}^{[1]} & w_{3,4}^{[1]} \\ w_{1,5}^{[1]} & w_{2,5}^{[1]} & w_{3,5}^{[1]} \end{bmatrix} \begin{bmatrix} A_1^{[0](1)} & A_1^{[0](2)} & \cdots & A_1^{[0](m)} \\ A_2^{[0](1)} & A_2^{[0](2)} & \cdots & A_2^{[0](m)} \\ A_3^{[0](1)} & A_3^{[0](2)} & \cdots & A_3^{[0](m)} \end{bmatrix} + b^{[1]}$$

$(n^{[1]}, n^{[0]})$ $(n^{[0]}, m)$ $(n^{[1]}, 1)$

broadcasting

Parameters: W^{l} and b^{l}

$w^{[l]T}$:有時簡寫$w^{[l]}$



L = 5

$n^{[0]} = n_x = 2$

$n^{[1]} = 3$

$n^{[2]} = 5$

$n^{[3]} = 4$

$n^{[4]} = 2$

$n^{[5]} = 1$

$$\left\{\begin{array}{l} w^{[l]T} : (n^{[l]}, n^{[l-1]}) \\ b^{[l]} : (n^{[l]}, 1) \\ dw^{[l]T} : (n^{[l]}, n^{[l-1]}) \\ db^{[l]} : (n^{[l]}, 1) \end{array}\right.$$

有d無d,

同矩陣大小(以此記)

$$z^{[1]} = w^{[1]T}x + b^{[1]}$$

(3,1)

$(n^{[1]},1)$

$(3,1) = (3,2)\,(2,1)$

$(n^{[1]},1) = (n^{[1]},n^{[0]})(n^{[0]},1)$

$$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

Assume

1 sample

$w^{[1]T} : (n^{[1]}, n^{[0]})$

$$\underset{(5,1)}{z^{[2]}} = \underset{(5,3)}{w^{[2]T}}\ \underset{(3,1)}{a^{[1]}} + \underset{(5,1)}{b^{[2]}},$$

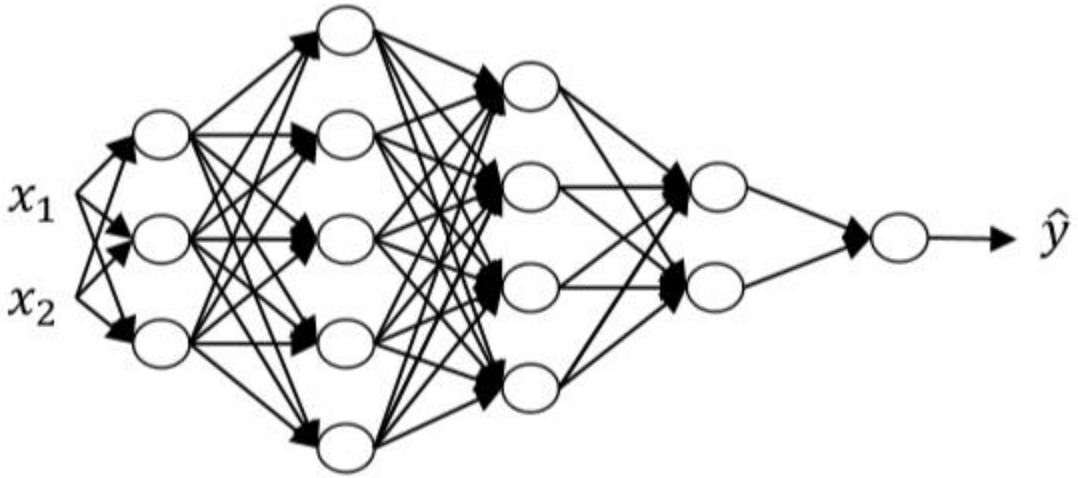
$(n^{[2]},1)$

$w^{[2]T} : (5,3),\ (n^{[2]}, n^{[1]})$

$w^{[3]T} : (4,5)$

$w^{[4]T} : (2,4),\quad w^{[5]T} : (1,2)$

Vectorized implementation



$$\underset{(n^{[1]},1)}{z^{[1]}} = \underset{(n^{[1]},n^{[0]})}{w^{[1]}}\underset{(n^{[0]},1)}{x} + \underset{(n^{[1]},1)}{b^{[1]}}$$

$$\begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$\underset{(n^{[1]},m)}{z^{[1]}} = \underset{(n^{[1]},n^{[0]})}{w^{[1]}}\underset{(n^{[0]},m)}{x} + \underset{(n^{[1]},1)}{b^{[1]}}$$

$$(n^{[1]},1) \downarrow (n^{[1]},m)$$

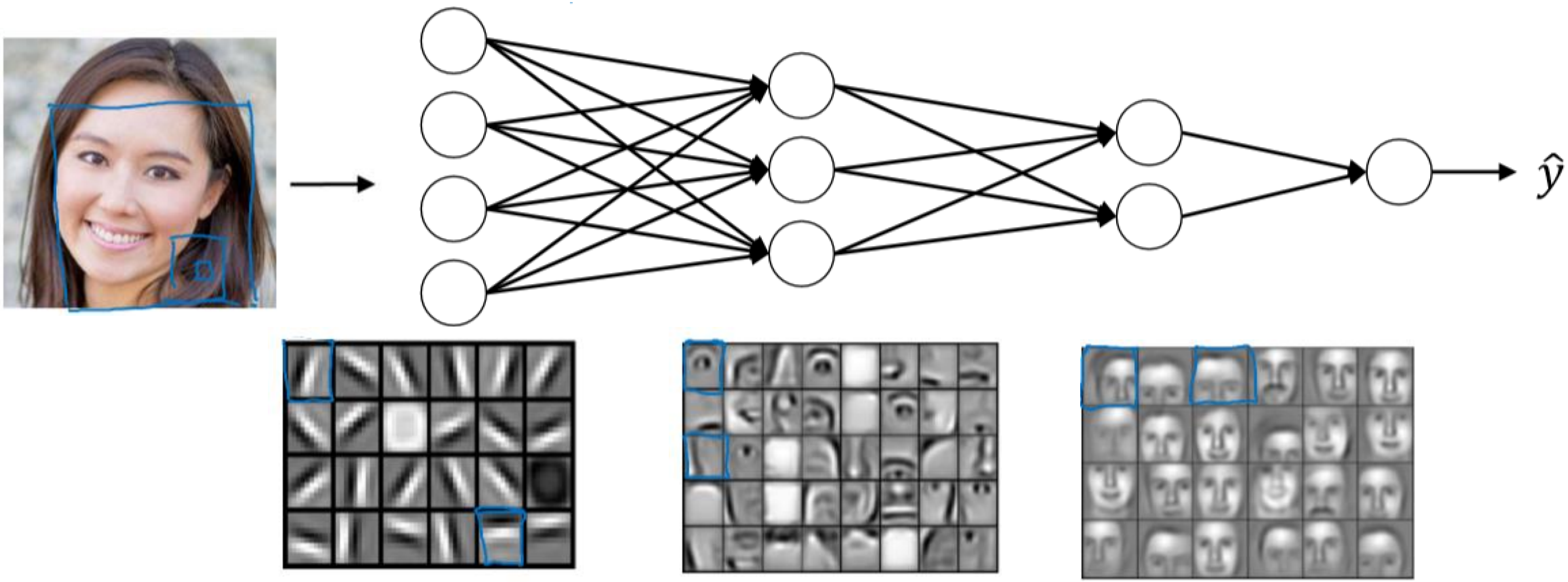
$$z^{[l]}, a^{[l]} : (n^{[l]},1)$$

$$Z^{[l]}, A^{[l]} : (n^{[l]},m)$$

$$l = 0 \quad A^{[0]} = X = (n^{[0]},m)$$

$$dZ^{[l]}, dA^{[l]} : (n^{[l]},m)$$

Intuition about deep representation

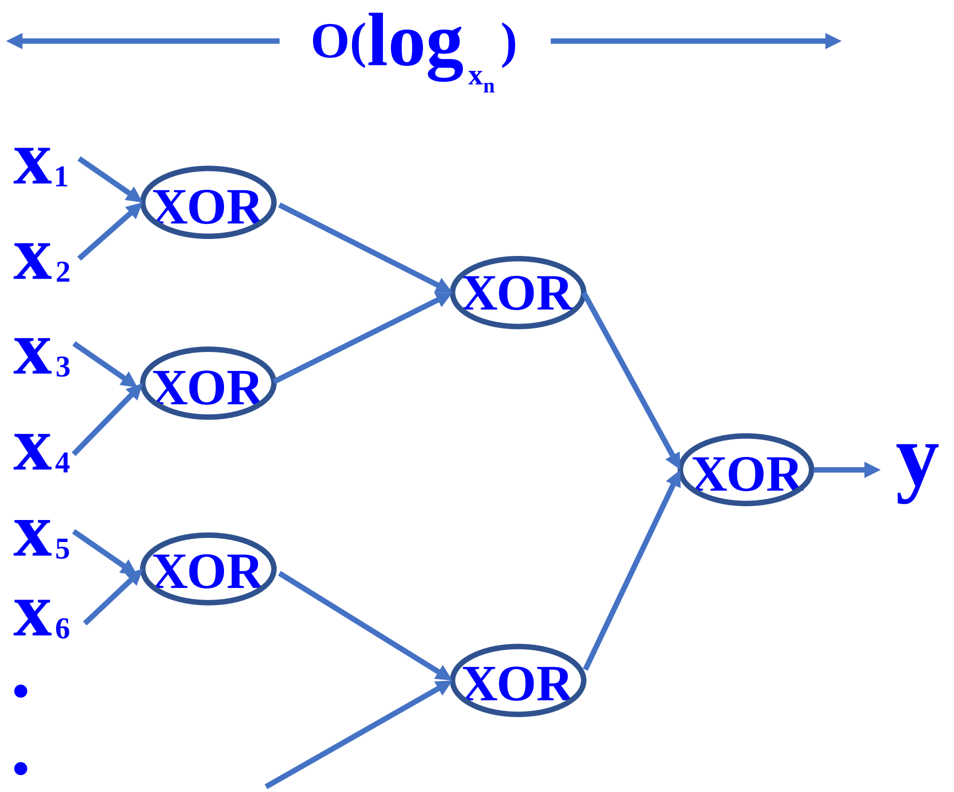


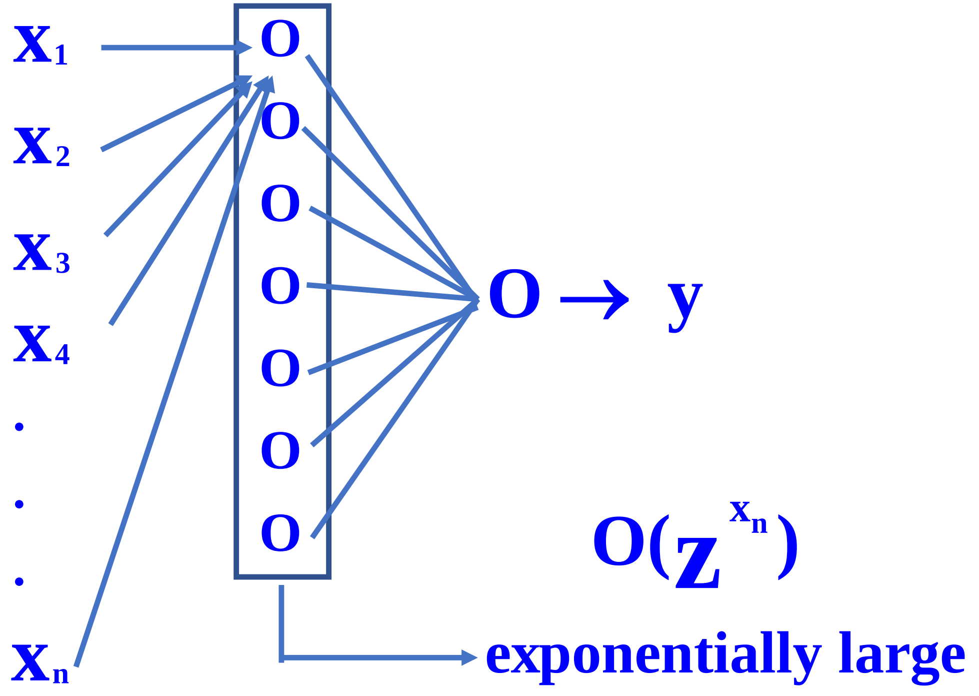
Audio → low level audio waveform → Phonemes C A T → Words → Sentence phrase

Circuit theory and deep learing

Informally: There are functions you can compute with a "*small*" L-layer deep neural network that shallower networks require exponentially more hidden units to compute.

$y = x_1 \text{ XOR } x_2 \text{ XOR } x_3 \text{ XOR } \cdots \text{ XOR } x_6$

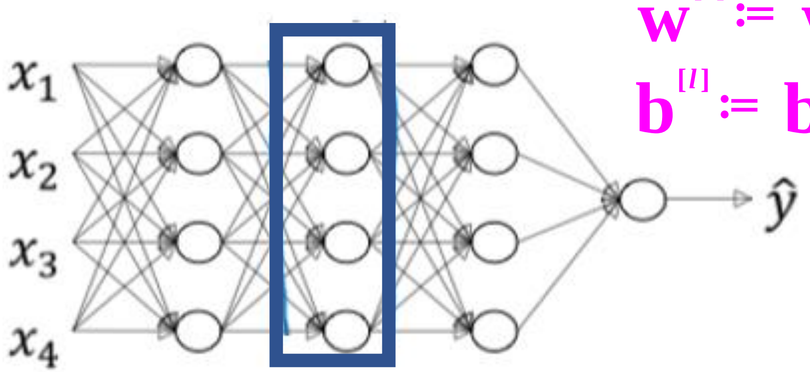




Deep Neural Network

Building blocks of deep neural networks

Forward and backward functions



$w^{[l]} := w^{[l]} - \alpha dw^{[l]}$

$b^{[l]} := b^{[l]} - \alpha db^{[l]}$

layer l : $w^{[l]}, b^{[l]}$

Forward: Input $a^{[l-1]}$, Output $a^{[l]}$

$z^{[l]} = w^{[l]} a^{[l-1]} + b^{[l]}$ cache $z^{[l]}$...

$a^{[l]} = g^{[l]}(z^{[l]})$

Backward: Input $da^{[l]}$, Output $da^{[l-1]}$

cache($z^{[l]}$)... $dw^{[l]}, db^{[l]}$

$dz^{[2]} = dA^{[2]} * g^{[2]'}(z^{(2)})$

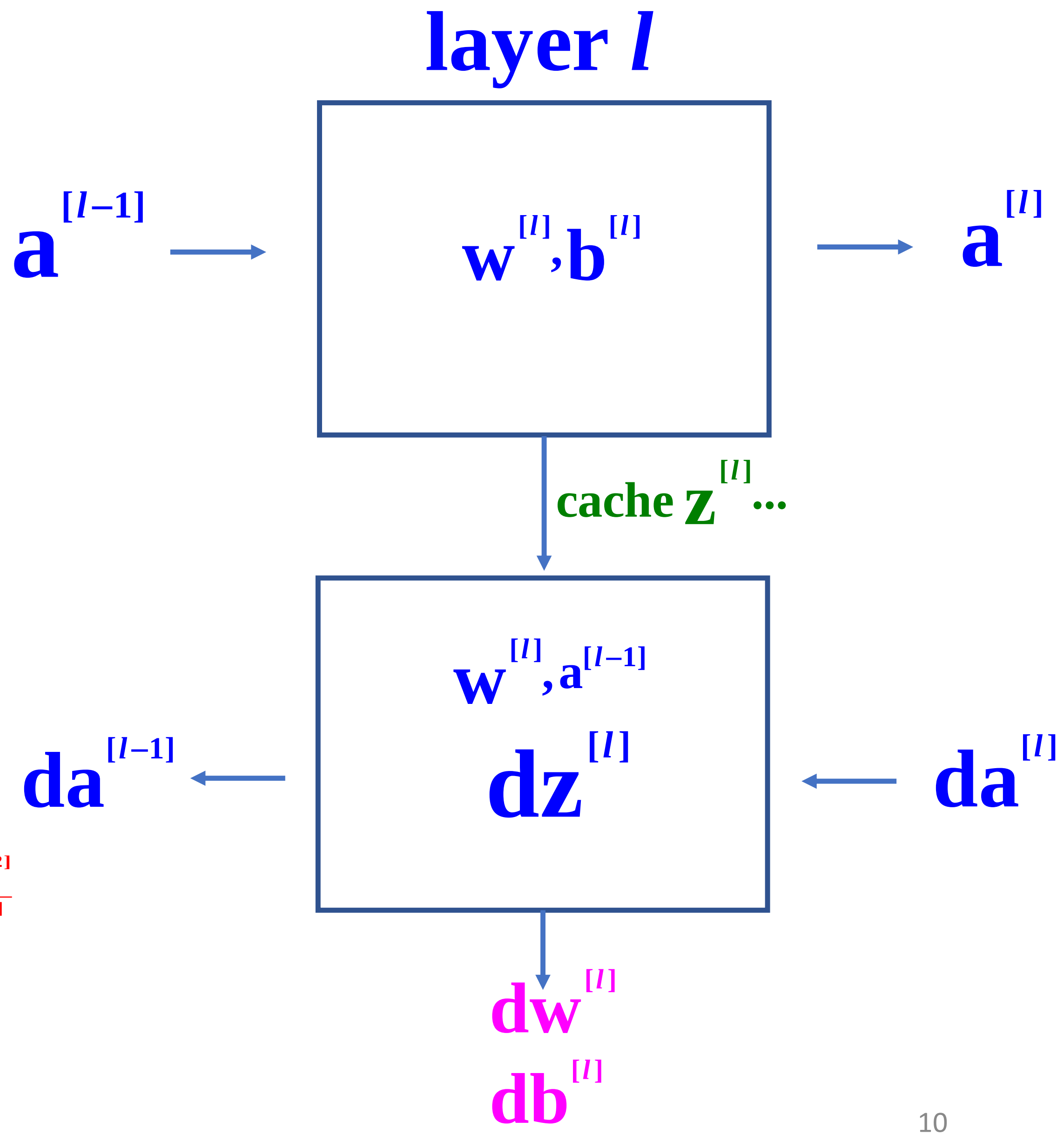
$dw^{[2]} = \frac{1}{m} dz^{[2]} A^{[1]^T}$

$db^{[2]} = \frac{1}{m}$ np.sum($dz^{[2]}$,...)

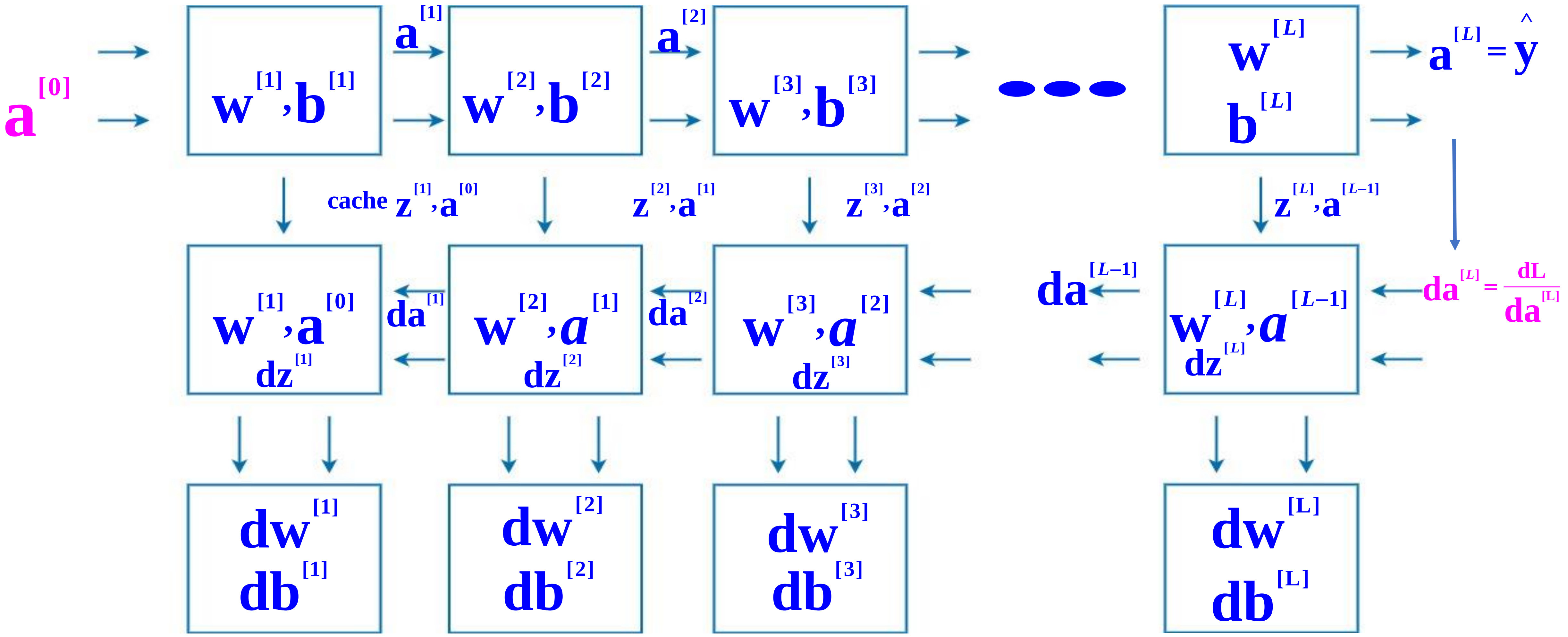
$dA^{[1]} = w^{[2]} dz^{[2]}$

Recall : $\frac{dL}{dz^{[2]}} = dz^{[2]} = \frac{dL}{dA^{[2]}} \frac{dA^{[2]}}{dz^{[2]}}$

$= dA^{[2]} * g^{[2]'}(z^{(2)})$



Forward and backward functions



$$w^{[l]} := w^{[l]} - \alpha dw^{[l]}$$
$$b^{[l]} := b^{[l]} - \alpha db^{[l]}$$

Deep Neural Network

Forward and backward propagation

Backward propagation for layer *l*

Input $da^{[l]}$ cache: $z^{[l]}, a^{[l-1]}$

Output $da^{[l-1]}, dw^{[l]}, db^{[l]}$

1 sample

$$dz^{[l]} = da^{[l]} * g^{[l]'}(z^{[l]})$$

$$dw^{[l]} = a^{[l-1]} dz^{[l]T}$$

$$db^{[l]} = dz^{[l]}$$

$$da^{[l-1]} = w^{[l]} dz^{[l]}$$

$(n^{[l-1]}, n^{[l]})$ $(n^{[l]}, 1)$

m samples

$$dZ^{[l]} = dA^{[l]} * g^{[l]'}(Z^{[l]})$$

$$dw^{[l]} = \frac{1}{m} A^{[l-1]} dZ^{[l]T}$$

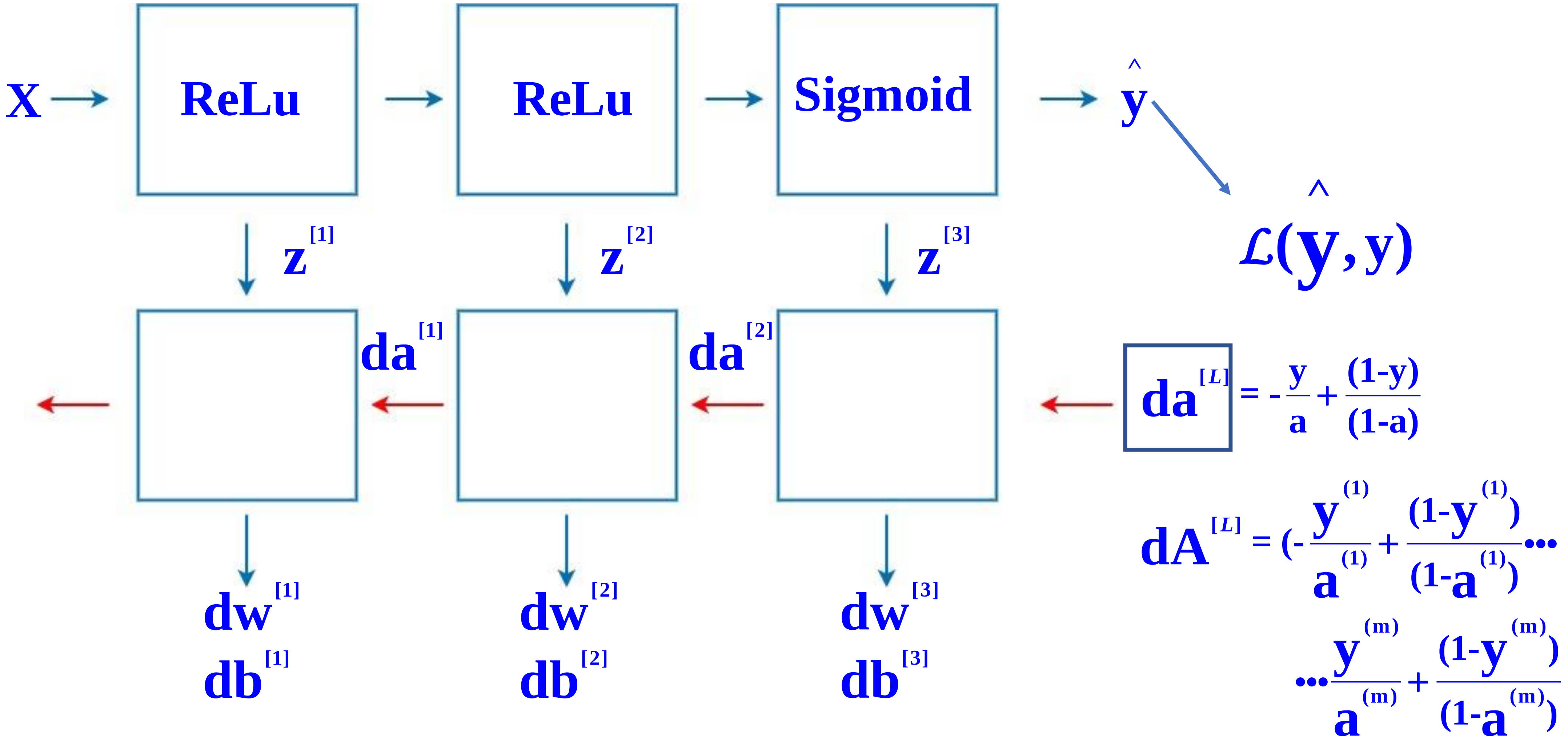
$(n^{[l-1]}, n^{[l]})$ $(n^{[l-1]}, m)$ $(m, n^{[l]})$

$$db^{[l]} = \frac{1}{m} \text{np.sum}(dZ^{[l]}, \text{axis=1}, \text{keepdims=True})$$

$$dA^{[l-1]} = w^{[l]} dZ^{[l]}$$

$(n^{[l-1]}, m)$ $(n^{[l-1]}, n^{[l]})(n^{[l]}, m)$

Summary


X
ReLu
ReLu
Sigmoid
$\hat{y}$
$\mathcal{L}(\hat{y}, y)$
$z^{[1]}$
$z^{[2]}$
$z^{[3]}$
$da^{[1]}$
$da^{[2]}$
$\boxed{da^{[L]}} = -\frac{y}{a} + \frac{(1-y)}{(1-a)}$
$dA^{[L]} = (-\frac{y^{(1)}}{a^{(1)}} + \frac{(1-y^{(1)})}{(1-a^{(1)})} \cdots$
$\cdots \frac{y^{(m)}}{a^{(m)}} + \frac{(1-y^{(m)})}{(1-a^{(m)})})$
$dw^{[1]}$
$db^{[1]}$
$dw^{[2]}$
$db^{[2]}$
$dw^{[3]}$
$db^{[3]}$

Deep Neural Network

Parameters vs Hyperparameters

What are hyperparamerters?

Parameters : $\mathbf{W}^{[1]},\mathbf{b}^{[1]},\mathbf{W}^{[2]},\mathbf{b}^{[2]},\mathbf{W}^{[3]},\mathbf{b}^{[3]}$

Hyperparameters : learning rate α

#iterations

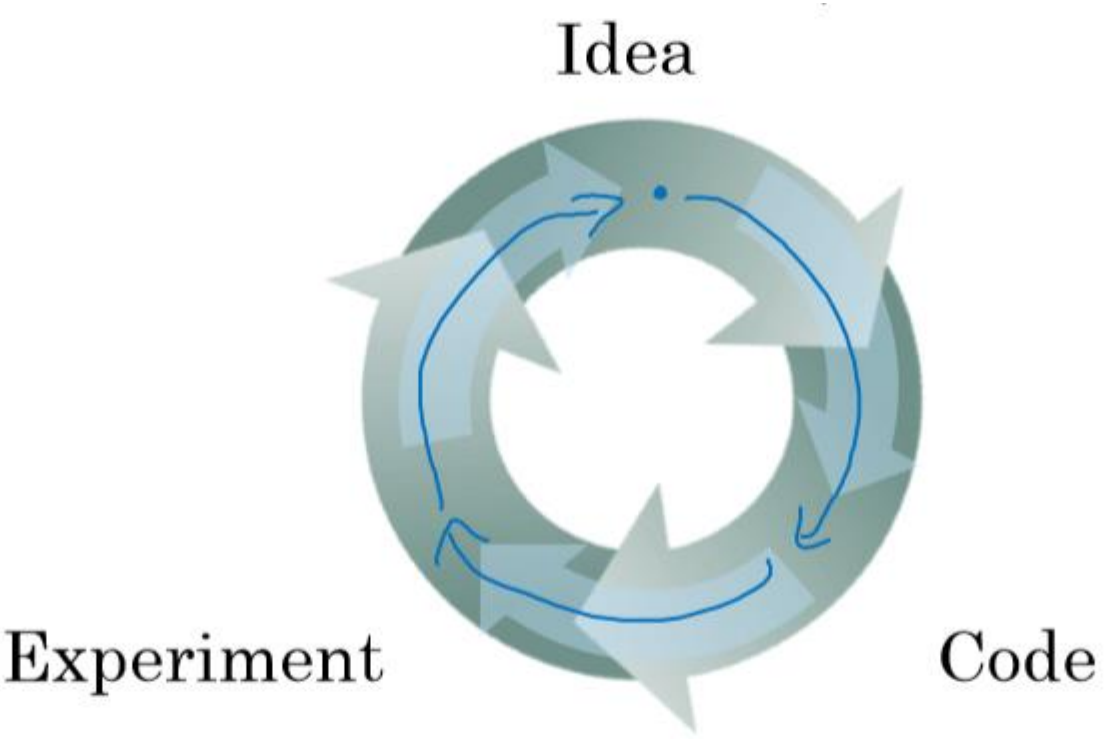
#hidden layers L

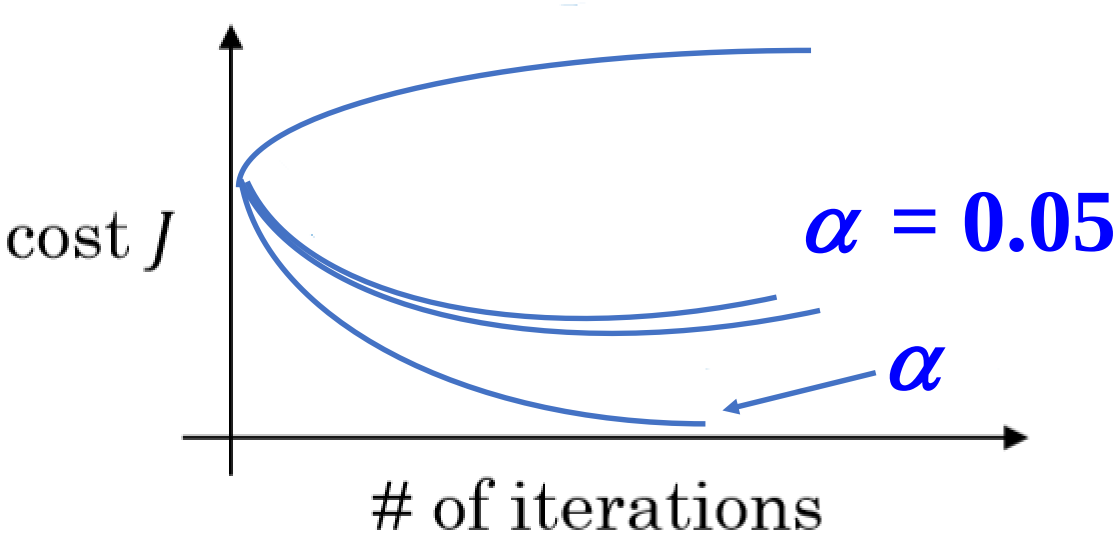
#hidden units $\mathbf{n}^{[1]},\mathbf{n}^{[2]},\cdots$

choice of activation function

Later: momentum,minibatch size,regularzation,•••

Applied deep learning is a very empirical process





Vision, Speech,NLP,Ad,Search,Recommendations

Deep Neural Network

What does this have to do with the brain?

Forward and backward propagation

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$
$$A^{[1]} = g^{[1]}(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = g^{[2]}(Z^{[2]})$$
$$\vdots$$
$$A^{[L]} = g^{[L]}(Z^{[L]}) = \hat{Y}$$

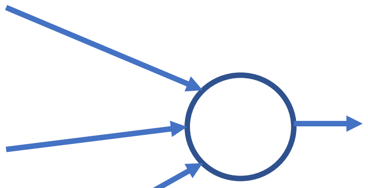
$$dZ^{[L]} = A^{[L]} - Y$$
$$dW^{[L]} = \frac{1}{m} dZ^{[L]} A^{[L]^T}$$
$$db^{[L]} = \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[L]}, axis = 1, keepdims = True)$$
$$dZ^{[L-1]} = dW^{[L]^T} dZ^{[L]} g'^{[L]}(Z^{[L-1]})$$
$$\vdots$$
$$dZ^{[1]} = dW^{[L]^T} dZ^{[2]} g'^{[1]}(Z^{[1]})$$
$$dW^{[1]} = \frac{1}{m} dZ^{[1]} A^{[1]^T}$$
$$db^{[1]} = \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[1]}, axis = 1, keepdims = True)$$

$$\mathbf{dw}^{[l]} = \frac{1}{m} \mathbf{A}^{[l-1]} \mathbf{dz}^{[l]T}$$

看W怎樣定義

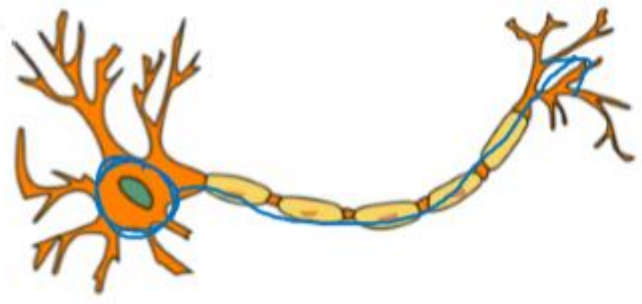
"It's like the brain"

x_1
x_2
x_3



$\hat{y}$

x_1
x_2
x_3



$x \rightarrow y$

3 – layer neural network programming

1. Change alpha
2. Change initial condition to others, ex. All zero, can you make it faster
3. Reduce layer numbers to two, one layer and redraw the distribution plot
4. Change A0 by:

1. Turn in 4/18 before class with code, output, and your comments
2. Code should not use global variables

```
import sklearn; import sklearn.datasets
# load image dataset: blue/red dots in circles
train_X, train_Y = sklearn.datasets.make_circles(n_samples=35, noise=.05)
A0=train_X.T; Y=train_Y.reshape(1,35)
print("np.shape(Y):",np.shape(Y),"np.shape(A0):",np.shape(A0))
for i in range(35):
    if(Y[0][i]<0.5):
        plt.scatter(A0[0][i], A0[1][i],c="blue",marker="o")
    else:
        plt.scatter(A0[0][i], A0[1][i],c="purple",marker="x")
plt.title('x1, x2, y')
plt.show()
```

3 – layer neural network programming

a standard Normal distribution (mean=0, stdev=1).

```
import matplotlib
import time
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
n0=2;n1=3;n2=4;n3=1;m=35
alpha=0.01
```

```
# Prepare input A0 and labelling output Y
x1=[0.4,1.2,1.1,1.9,1.8,1.5,2.6,2.2,3.3,3.0,4.1,4.2,5.6,0.7,0.7
x2=[4.1,1.5,4.8,0.7,2.2,3.8,1.8,4.8,1.0,4.3,1.8,3.1,6.0,5.9,7.0
y0=[0,0,0,0,0,0,0,0,0,0,0,0,0];y1=[1,1,1,1,1,1,1,1,1,1,1,1,1,1,
A0=np.array([x1,x2])
Y= np.concatenate((y0,y1),axis=None).reshape(1,35)
print("np.shape(Y):",np.shape(Y),"np.shape(A0):",np.shape(A0),
      "A0[0][0]:",A0[0][0],"A0[0][1]:",A0[0][1],"Y",Y[0][0:3])
plt.scatter(A0[0][0:13], A0[1][0:13],marker="o")
plt.scatter(A0[0][13:35], A0[1][13:35],marker="x")
plt.title('x1, x2, y')
plt.show()
```

```
# Initialization
np.random.seed(1)
WT1=np.random.randn(n1,n0)*0.01;b1=np.random.randn(n1,1)
WT2=np.random.randn(n2,n1)*0.01;b2=np.random.randn(n2,1)
WT3=np.random.randn(n3,n2)*0.01;b3=np.random.randn(n3,1)
```

3 – layer neural network programming

```python
# Initialization
np.random.seed(1)
WT1=np.random.randn(n1,n0)*0.01;b1=np.random.randn(n1,1)
WT2=np.random.randn(n2,n1)*0.01;b2=np.random.randn(n2,1)
WT3=np.random.randn(n3,n2)*0.01;b3=np.random.randn(n3,1)
```

```python
def forwardfunc():
    global A0,A1,A2,A3,WT1,WT2,WT3,b1,b2,b3
    Z1=np.dot(WT1,A0)+b1;
    A1=(np.exp(Z1)-np.exp(-Z1))/(np.exp(Z1)+np.exp(-Z1))
    Z2=np.dot(WT2,A1)+b2;
    A2=(np.exp(Z2)-np.exp(-Z2))/(np.exp(Z2)+np.exp(-Z2))
    Z3=np.dot(WT3,A2)+b3;
    A3=1/(1+np.exp(-Z3))
```

3 – layer neural network programming

```
def backwardprop():
    global Y,A3,A2,A1,A0,dA2,dA1,dZ3,dZ2,dZ1,
    dWT3,dWT2,dWT1,db3,db2,db1
#     dZ3=A3-Y  #dZ3 comes from dA3 and dA3/dZ3
    dA3=-Y/A3+(1-Y)/(1-A3)
    dZ3=dA3*(A3*(1-A3))
    dWT3=1/m*np.dot(dZ3,A2.T)
    db3=1/m*np.sum(dZ3,axis=1,keepdims=True)
    dA2=np.dot(WT3.T,dZ3)
    dZ2=dA2*(1-A2**2)
    dWT2=1/m*np.dot(dZ2,A1.T)
    db2=1/m*np.sum(dZ2,axis=1,keepdims=True)
    dA1=np.dot(WT2.T,dZ2)
    dZ1=dA1*(1-A1**2)
    dWT1=1/m*np.dot(dZ1,A0.T)
    db1=1/m*np.sum(dZ1,axis=1,keepdims=True)
```

3 – layer neural network programming

```
itera=0
cost = 100
while (cost > 0.05):  #0.05, 0.2
    forwardfunc()
    cost=-1/m*np.sum((Y*np.log(A3)+(1-Y)*np.log(1-A3)),axis=1,keepdims=True)
    if(itera%10000==0):
        print("itera ", itera, "cost ", cost)
    backwardprop()
    WT3=WT3-alpha*dWT3; b3=b3-alpha*db3; WT2=WT2-alpha*dWT2; b2=b2-alpha*db2
    WT1=WT1-alpha*dWT1; b1=b1-alpha*db1
    itera=itera+1
print("np.shape(Y)",np.shape(Y), "np.shape(A3)",np.shape(A3))
```

```
itera  0 cost  [[0.65971979]]
itera  10000 cost  [[0.65971121]]
itera  20000 cost  [[0.65970933]]
itera  30000 cost  [[0.65967928]]
```

3 – layer neural network programming

```python
    # Inference
    A0 = np.zeros((n0,1600))#[xx1, xx2][xy1,xy2]
    for i in range(40):
        for j in range(40):
            A0[0][i*39+j]=i*0.2
            A0[1][i*39+j]=j*0.2
    forwardfunc()
    print("A3 ",A3)
    plt.scatter(A0[0][0:2], A0[1][0:2],c="red",marker=".")
    plt.scatter(A0[0][2], A0[1][2],c="green",marker=".")
    for i in range(40):
        for j in range(40):
            if(A3[0][i*39+j]>0.5):
                plt.scatter(A0[0][i*39+j],
                 A0[1][i*39+j],c="red",marker=".")
            else:
                plt.scatter(A0[0][i*39+j],
                 A0[1][i*39+j],c="green",marker=".")
```

3 – layer neural network programming

```
    A0=np.array([x1,x2])
    forwardfunc()
    for i in range(m):
        if(A3[0][i]<0.5):
            plt.scatter(A0[0][i], A0[1][i],c="blue",marker="o")
        else:
            plt.scatter(A0[0][i], A0[1][i],c="purple",marker="x")
    plt.title('xx, xy, y')
    plt.show()
```