of the snare drum produces a copy of the tuba sound. Since each stroke is noisy and broadband, it acts like a reverberator. The series of strokes acts like several diffusing boundaries and produces the type of echo that can be found in natural landscapes [DMT99, m-tubg5sna].

The convolution can be used to map a rhythm pattern onto a sampled sound. The rhythm pattern can be defined by positioning a unit impulse at each desired time within a signal block. The convolution of the input sound with the time pattern will produce copies of the input signal at each of the unit impulses. If the unit impulse is replaced by a more complex sound, each copy will be modified in its timbre and in its time structure. If a snare drum stroke is used, the attacks will be smeared and some diffusion will be added [m-gendsna]. The convolution has an effect both in the frequency and in the time domain. Take a speech sound with sharp frequency resonances and a rhythm pattern defined by a series of snare-drum strokes. Each word will appear with the rhythm pattern, also the rhythm pattern will be heard in each word with the frequency resonances of the initial speech sound [m-chu5sna].

The convolution as a tool for musical composition has been investigated by composers such as Horacio Vaggione [m-Vag96, Vag98] and Curtis Roads [Roa97]. Because the convolution has a combined effect in the time and frequency domains, some expertise is necessary to foresee the result of the combination of two sounds.

2.3 Equalizers

Introduction and Musical Applications

In contrast to low/highpass and bandpass/reject filters, which attenuate the audio spectrum above or below a cut-off frequency, equalizers shape the audio spectrum by enhancing certain frequency bands while others remain unaffected. They are built by a series connection of first- and second-order shelving and peak filters, which are controlled independently (see Fig. 2.19). Shelving filters boost or cut the low or high frequency bands with the parameter cut-off frequency f_c and gain G. Peak filters boost or cut mid-frequency bands with parameters cut-off frequency f_c, bandwidth f_b and gain G. One often used filter type is the constant Q peak filter. The Q factor is defined by the ratio of the bandwidth to cut-off frequency $Q = \frac{f_b}{f_c}$. The cut-off frequency of peak filters are then tuned, while keeping the Q factor constant. This means that the bandwidth is increased when the cut-off frequency is increased and vice versa. Several proposed digital filter structures for shelving and peak filters can be found in the literature [Whi86, RM87, Dut89a, HB93, Bri94, Orf96, Orf97, Zöl97].

Applications of these parametric filters can be found in parametric equalizers, octave equalizers (f_c=31.25, 62.5, 125, 250, 500, 1000, 2000, 4000, 8000, 16000 Hz) and all kinds of equalization devices in mixing consoles, outboard equipment and foot pedal controlled stomp boxes.

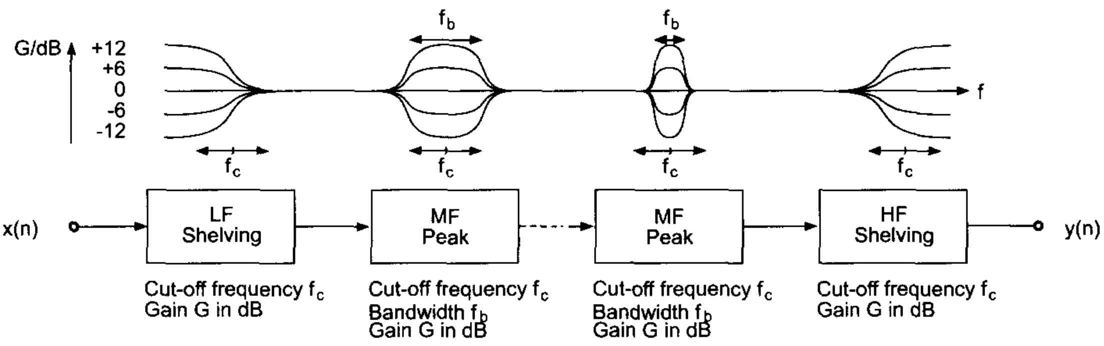

Figure 2.19 Series connection of shelving and peak filters.

2.3.1 Shelving Filters

First-order Design

First-order low/high frequency shelving filters [Zöl97] can be described by the transfer function

$$H(z) = 1 + \frac{H_0}{2}\left[1 + \pm A(z)\right] \quad \text{(LF/HF +/-)} \tag{2.45}$$

with the first-order allpass

$$A(z) = \frac{z^{-1} + a_{B/C}}{1 + a_{B/C}z^{-1}}. \tag{2.46}$$

The block diagram in Fig. 2.20 shows a first-order low/high-frequency shelving

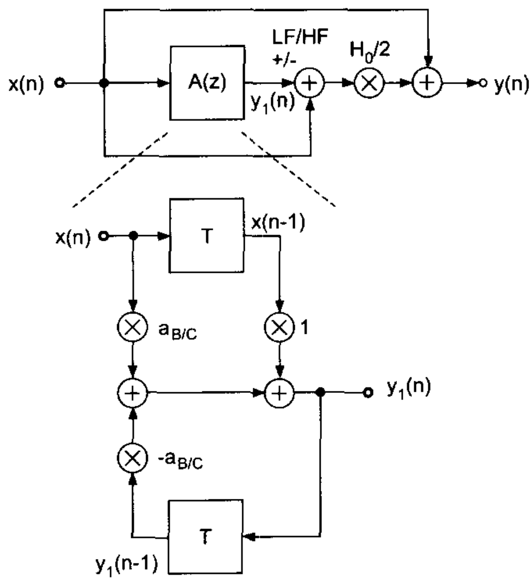

Figure 2.20 First-order low/high-frequency shelving filter.

filter, which leads to the following difference equations:

$$y_1(n) = a_{B/C}x(n) + x(n-1) - a_{B/C}y_1(n-1) \quad (2.47)$$

$$y(n) = \frac{H_0}{2}\left[x(n) \pm y_1(n)\right] + x(n). \quad (2.48)$$

The gain G in dB for low/high frequencies can be adjusted by the parameter

$$H_0 = V_0 - 1, \quad \text{with} \quad V_0 = 10^{G/20}. \quad (2.49)$$

The cut-off frequency parameter a_B for boost and a_C for cut can be calculated as

$$a_B = \frac{\tan((\pi f_c/f_s) - 1}{\tan((\pi f_c/f_s) + 1}, \omega_c = 2\pi f_c \quad (2.50)$$

$$a_C = \frac{\tan((\pi f_c/f_s) - V_0}{\tan((\pi f_c/f_s) + V_0}. \quad (2.51)$$

The cut-off frequency parameters for boost and cut for a first-order high-frequency shelving filter [Zöl97] are calculated by

$$a_B = \frac{\tan(\pi f_c/f_s) - 1}{\tan((\pi f_c/f_s) + 1} \quad (2.52)$$

$$a_C = \frac{V_0\tan((\pi f_c/f_s) - 1}{V_0\tan((\pi f_c/f_s) + 1}. \quad (2.53)$$

Magnitude responses for a low-frequency shelving filter are illustrated in the left part of Fig. 2.21 for several cut-off frequencies and gain factors. The slope of the frequency curves for these first-order filters are with 6 dB per octave.

Second-order Design

For several applications especially in advanced equalizer designs the slope of the shelving filter is further increased by second-order transfer functions. Design formulas for second-order shelving filters are given in Table 2.3 from [Zöl97]. Magnitude responses for second-order low/high frequency shelving filters are illustrated in the right part of Fig. 2.21 for two cut-off frequencies and several gain factors.

2.3.2 Peak Filters

A second-order peak filter [Zöl97] is given by the transfer function

$$H(z) = 1 + \frac{H_0}{2}\left[1 - A_2(z)\right], \quad (2.54)$$

where

$$A_2(z) = \frac{-a_B + (d - da_B)z^{-1} + z^{-2}}{1 + (d - da_B)z^{-1} - a_B z^{-2}} \quad (2.55)$$

Table 2.3 Second-order shelving filter design with $K = \tan(\pi f_c/f_s)$ [Zöl97].

low-frequency shelving (boost $V_0 = 10^{G/20}$)				
b_0	b_1	b_2	a_1	a_2
$\frac{1+\sqrt{2V_0}K+V_0K^2}{1+\sqrt{2}K+K^2}$	$\frac{2(V_0K^2-1)}{1+\sqrt{2}K+K^2}$	$\frac{1-\sqrt{2V_0}K+V_0K^2}{1+\sqrt{2}K+K^2}$	$\frac{2(K^2-1)}{1+\sqrt{2}K+K^2}$	$\frac{1-\sqrt{2}K+K^2}{1+\sqrt{2}K+K^2}$

low-frequency shelving (cut $V_0 = 10^{-G/20}$)				
b_0	b_1	b_2	a_1	a_2
$\frac{1+\sqrt{2}K+K^2}{1+\sqrt{2V_0}K+V_0K^2}$	$\frac{2(K^2-1)}{1+\sqrt{2V_0}K+V_0K^2}$	$\frac{1-\sqrt{2}K+K^2}{1+\sqrt{2V_0}K+V_0K^2}$	$\frac{2(V_0K^2-1)}{1+\sqrt{2V_0}K+V_0K^2}$	$\frac{1-\sqrt{2V_0}K+V_0K^2}{1+\sqrt{2V_0}K+V_0K^2}$

high-frequency shelving (boost $V_0 = 10^{G/20}$)				
b_0	b_1	b_2	a_1	a_2
$\frac{V_0+\sqrt{2V_0}K+K^2}{1+\sqrt{2}K+K^2}$	$\frac{2(K^2-V_0)}{1+\sqrt{2}K+K^2}$	$\frac{V_0-\sqrt{2V_0}K+K^2}{1+\sqrt{2}K+K^2}$	$\frac{2(K^2-1)}{1+\sqrt{2}K+K^2}$	$\frac{1-\sqrt{2}K+K^2}{1+\sqrt{2}K+K^2}$

high-frequency shelving (cut $V_0 = 10^{-G/20}$)				
b_0	b_1	b_2	a_1	a_2
$\frac{1+\sqrt{2}K+K^2}{V_0+\sqrt{2V_0}K+K^2}$	$\frac{2(K^2-1)}{V_0+\sqrt{2V_0}K+K^2}$	$\frac{1-\sqrt{2}K+K^2}{V_0+\sqrt{2V_0}K+K^2}$	$\frac{2(K^2/V_0-1)}{1+\sqrt{2/V_0}K+K^2/V_0}$	$\frac{1-\sqrt{2/V_0}K+K^2/V_0}{1+\sqrt{2/V_0}K+K^2/V_0}$

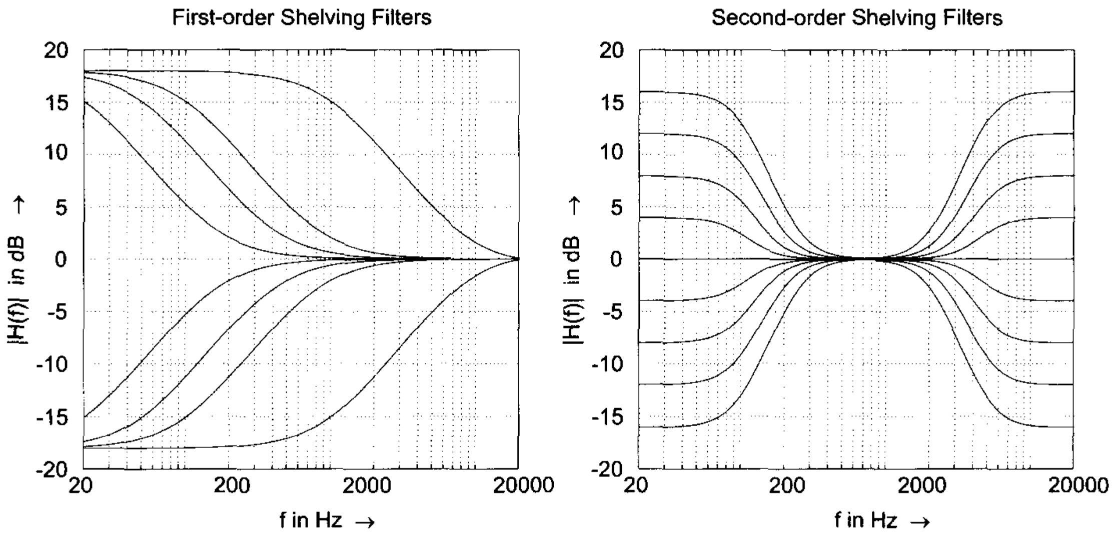

Figure 2.21 Frequency responses for first-order and second-order shelving filters.

is a second-order allpass filter. The block diagram in Fig. 2.22 shows the second-order peak filter, which leads to the following difference equations:

$$
\begin{aligned}
y_1(n) &= -a_{B/C}x(n) + d(1-a_{B/C})x(n-1) + x(n-2) \\
&\quad -d(1-a_{B/C})y_1(n-1) + a_{B/C}y_1(n-2) \qquad (2.56)\\
y(n) &= \frac{H_0}{2}\left[x(n) - y_1(n)\right] + x(n). \qquad (2.57)
\end{aligned}
$$

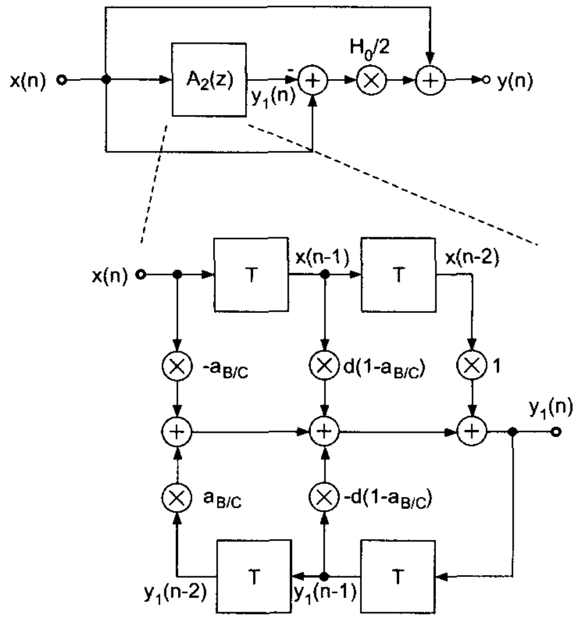

Figure 2.22 Second-order peak filter.

The center/cut-off frequency parameter d and the coefficient H_0 are given by

$$\begin{aligned} d &= -cos(2\pi f_c/f_s) && (2.58)\\ V_0 &= H(e^{j2\pi f_c/f_s}) = 10^{G/20} && (2.59)\\ H_0 &= V_0 - 1. && (2.60) \end{aligned}$$

The bandwidth f_b is adjusted through the parameters a_B and a_C for boost and cut and are given by

$$a_B = \frac{\tan(\pi f_b/f_s) - 1}{\tan(\pi f_b/f_s) + 1} \quad (2.61)$$

$$a_C = \frac{\tan(\pi f_b/f_s) - V_0}{\tan(\pi f_b/f_s) + V_0}. \quad (2.62)$$

This peak filter offers almost independent control of all three musical parameters center/cut-off frequency, bandwidth and gain. Another design approach from [Zöl97] shown in Table 2.4 allows direct computation of the five coefficients for a second-order transfer function as given in the difference equation (2.2).

Frequency responses for several settings of a peak filter are shown in Fig. 2.23. The left part shows a variation of the gain with a fixed center frequency and bandwidth. The right part show for fixed gain and center frequency a variation of the bandwidth or Q factor.

Table 2.4 Peak filter design with $K = \tan(\pi f_c/f_s)$ [Zöl97].

peak (boost $V_0 = 10^{G/20}$)				
b_0	b_1	b_2	a_1	a_2
$\frac{1+\frac{V_0}{Q_\infty}K+K^2}{1+\frac{1}{Q_\infty}K+K^2}$	$\frac{2(K^2-1)}{1+\frac{1}{Q_\infty}K+K^2}$	$\frac{1-\frac{V_0}{Q_\infty}K+K^2}{1+\frac{1}{Q_\infty}K+K^2}$	$\frac{2(K^2-1)}{1+\frac{1}{Q_\infty}K+K^2}$	$\frac{1-\frac{1}{Q_\infty}K+K^2}{1+\frac{1}{Q_\infty}K+K^2}$
peak (cut $V_0 = 10^{-G/20}$)				
b_0	b_1	b_2	a_1	a_2
$\frac{1+\frac{1}{Q_\infty}K+K^2}{1+\frac{V_0}{Q_\infty}K+K^2}$	$\frac{2(K^2-1)}{1+\frac{V_0}{Q_\infty}K+K^2}$	$\frac{1-\frac{1}{Q_\infty}K+K^2}{1+\frac{V_0}{Q_\infty}K+K^2}$	$\frac{2(K^2-1)}{1+\frac{V_0}{Q_\infty}K+K^2}$	$\frac{1-\frac{V_0}{Q_\infty}K+K^2}{1+\frac{V_0}{Q_\infty}K+K^2}$

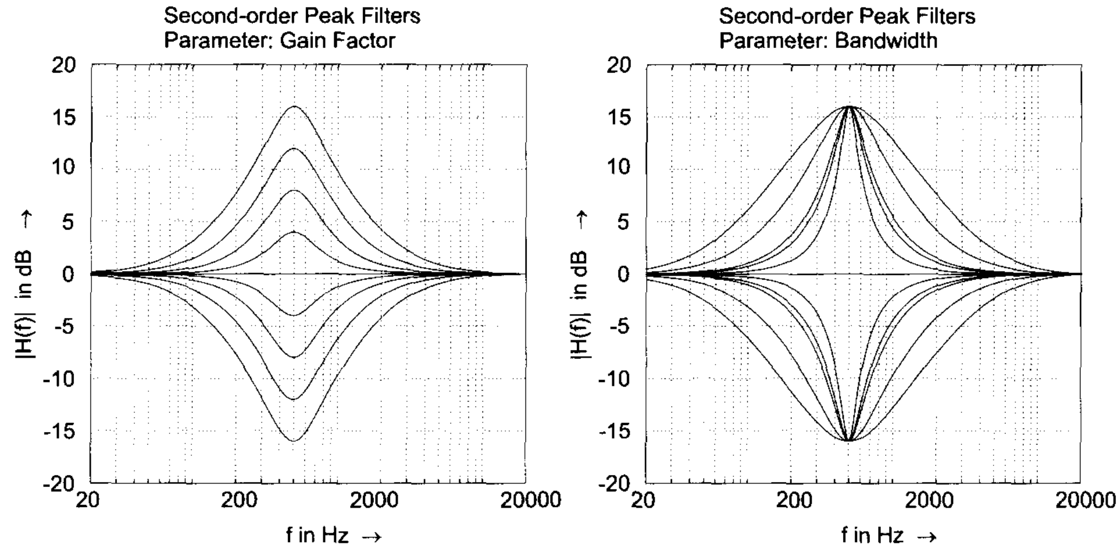

Figure 2.23 Frequency responses second-order peak filters.

2.4 Time-varying Filters

The parametric filters discussed in the previous sections allow the time-varying control of the filter parameters gain, cut-off frequency and bandwidth or Q factor. Special applications of time-varying audio filters will be shown in the following.

2.4.1 Wah-wah Filter

The wah-wah effect is produced mostly by foot-controlled signal processors containing a bandpass filter with variable center/resonant frequency and a small bandwidth. Moving the pedal back and forth changes the bandpass cut-off/center frequency. The "wah-wah" effect is then mixed with the direct signal as shown in Fig. 2.24. This effect leads to a spectrum shaping similar to speech and produces a speech like "wah-wah" sound. If the variation of the center frequency is controlled by the

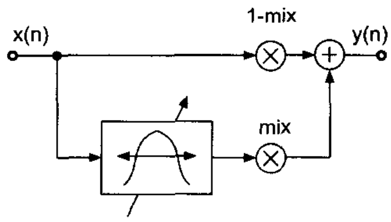

Figure 2.24 Wah-wah: time-varying bandpass filter.

input signal, a low-frequency oscillator is used to change the center frequency. Such an effect is called an auto-wah filter. If the effect is combined with a low-frequency amplitude variation, which produces a tremolo, the effect is denoted a tremolo-wah filter. Replacing the unit delay in the bandpass filter by an M tap delay leads to the M-fold wah-wah filter [Dis99], which is shown in Fig. 2.25. M bandpass filters are spread over the entire spectrum and simultaneously change their center frequency. When a white noise input signal is applied to an M-fold wah-wah filter, a spectrogram of the output signal shown in Fig. 2.26 illustrates the periodic enhancement of the output spectrum. Table 2.5 contains several parameter settings for different effects.

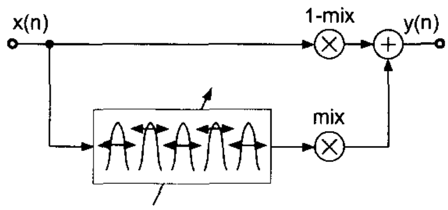

Figure 2.25 M-fold wah-wah filter.

Table 2.5 Effects with M-fold wah-wah filter [Dis99].

	M	Q^{-1}/f_m	$\Delta\omega/2\pi$
Wah-Wah	1	-/3kHz	200Hz
M-fold Wah-Wah	5-20	0.5/-	200-500Hz
Bell effect	100	0.5/-	100Hz

2.4.2 Phaser

The previous effect relies on varying the center frequency of a bandpass filter. Another effect uses notch filters: *phasing*. A set of notch filters, that can be realized as a cascade of second-order IIR sections, is used to process the input signal. The output of the notch filters is then combined with the direct sound. The frequencies of the notches are slowly varied using a low-frequency oscillator (Figure 2.27) [Smi84]. "The strong phase shifts that exist around the notch frequencies combine

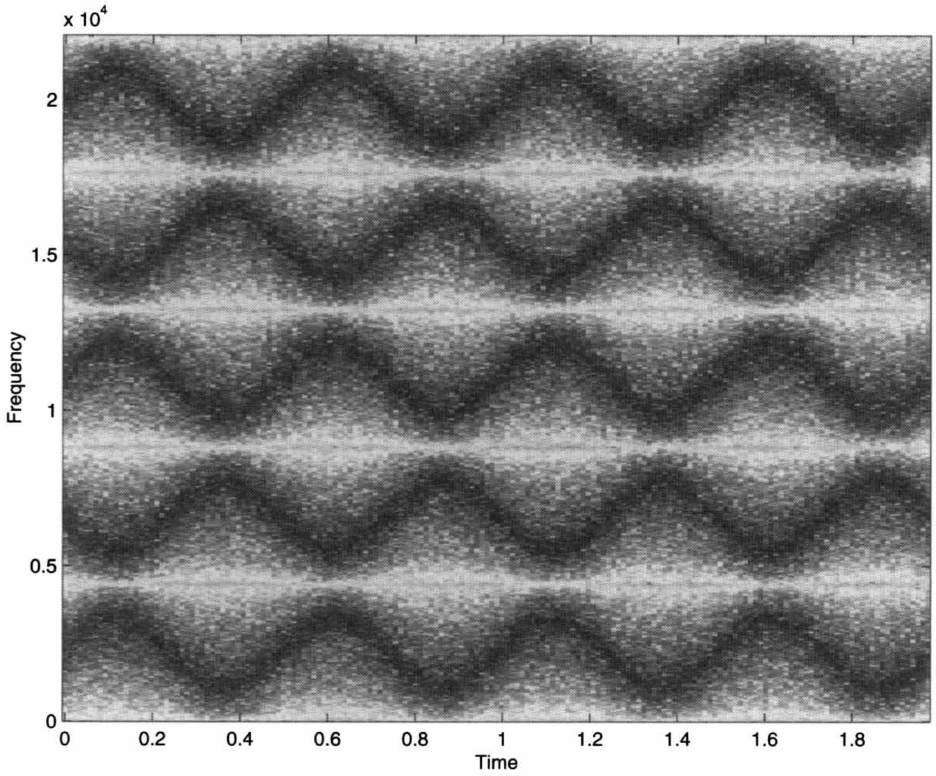

Figure 2.26 Spectrogram of output signal of a time-varying M-fold wah-wah filter [Dis99].

with the phases of the direct signal and cause phase cancellations or enhancements that sweep up and down the frequency axis" [Orf96]. Although this effect does not rely on a delay line, it is often considered to go along with delay-line based effects because the sound effect is similar to that of *flanging*. An extensive discussion on this topic is found in [Str83]. A different phasing approach is shown in Figure 2.28. The notch filters have been replaced by second-order allpass filters with time-varying center frequencies. The cascade of allpass filters produces time-varying phase shifts which lead to cancellations and amplifications of different frequency bands when used in the feedforward and feedback configuration.

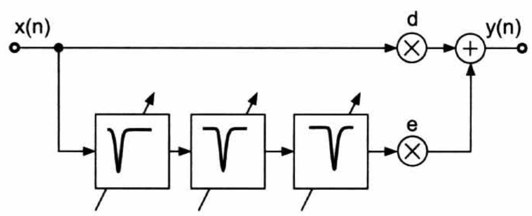

Figure 2.27 Phasing.