Linear Algebra
Assignment 5

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Question 1

(CH4.7 P216 Q16) Find the coordinate matrix of

$$\boldsymbol{x} = (0, -20, 7, 15)$$

in $\mathbb{R}^4$ relative to the basis

$$B' = \{(9, -3, 15, 4), (3, 0, 0, 1), (0, -5, 6, 8), (3, -4, 2, -3)\}.$$

Solution: Let

$$\boldsymbol{x} = x_1(9, -3, 15, 4) + x_2(3, 0, 0, 1) + x_3(0, -5, 6, 8) + x_4(3, -4, 2, -3),$$

then we have

$$\begin{bmatrix} 9 & 3 & 0 & 3 \\ -3 & 0 & -5 & -4 \\ 15 & 0 & 6 & 2 \\ 4 & 1 & 8 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -20 \\ 7 \\ 15 \end{bmatrix}.$$

We can use the Gauss elimination method to solve this system of equations. The result is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 3 \\ 2 \end{bmatrix}.$$

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Question 2

(CH4.7 P216 Q22) Find the transition matrix form B to B'.

$$B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}, \quad B' = \{(1, 3, -1), (2, 7, -4), (2, 9, -7)\}$$

Solution:

$$\left[T\right]_B^{B'} = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

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Question 3

(CH4.7 P217 Q38)

(a) Find the transition matrix from B to B'.

(b) Find the transition matrix from B' to B.

(c) Verity that two transition matrices are inverses of each other.

(d) Find the coordinate matrix $\left[\boldsymbol{x}\right]_{B}$, given the coordinate matrix $\left[\boldsymbol{x}\right]_{B'}$.

$$B = \{(2,-2),(6,3)\}, B' = \{(1,1),(32,31)\},$$

$$\left[\boldsymbol{x}\right]_{B} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Solution:

(a) By

$$(2,-2) = -126 \cdot (1,1) + 4 \cdot (32,31)$$

and

$$(6,3) = -90 \cdot (1,1) + 3 \cdot (32,31)$$

$$\Longrightarrow \left[T\right]_{B}^{B'} = \begin{bmatrix} -126 & -90 \\ 4 & 3 \end{bmatrix}$$

(b) By

$$(1,1) = -\frac{1}{6} \cdot (2,-2) + \frac{2}{9} \cdot (6,3)$$

and

$$(32,31) = -5 \cdot (2,-2) + 7 \cdot (6,3)$$

$$\Longrightarrow \left[T\right]_{B'}^{B} = \begin{bmatrix} -\frac{1}{6} & -5 \\ \frac{2}{9} & 7 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -126 & -90 \\ 4 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{1}{6} & -5 \\ \frac{2}{9} & 7 \end{bmatrix}$$

and

$$\begin{bmatrix} -\frac{1}{6} & -5 \\ \frac{2}{9} & 7 \end{bmatrix}^{-1} = \begin{bmatrix} -126 & -90 \\ 4 & 3 \end{bmatrix}.$$

(d)

$$\left[\boldsymbol{x}\right]_{B} = \left[T\right]_{B'}^{B} \left[\boldsymbol{x}\right]_{B'}$$

$$\Longrightarrow \left[\boldsymbol{x}\right]_{B} = \begin{bmatrix} -\frac{1}{6} & -5 \\ \frac{2}{9} & 7 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\Longrightarrow \left[\boldsymbol{x}\right]_{B} = \begin{bmatrix} \frac{14}{3} \\ -\frac{59}{9} \end{bmatrix}$$

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