Linear Algebra
Assignment 6

110307039 財管四 黃柏維

April 26, 2025

Question 1

(CH5.4 P275 Q20) Find the projection of the vector $\boldsymbol{v}$ onto the subsapce S, where

$$S = \text{span}\left\{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}\right\}, \quad \boldsymbol{v} = \begin{bmatrix}1\\2\\3\\4\end{bmatrix}.$$

Solution: Let $A = \begin{bmatrix}1 & 0 & 0\\1 & 1 & 1\\1 & -1 & 1\\1 & 0 & 0\end{bmatrix}$, then

$$\begin{aligned}\text{proj}_S\boldsymbol{v} &= A\left(A^TA\right)^{-1}A^T\boldsymbol{v}\\ &= \begin{bmatrix}\frac{5}{2}\\2\\3\\\frac{5}{2}\end{bmatrix}\end{aligned}$$

😄

Question 2

(CH5.4 P275 Q23) Find bases for the four fundamental subspaces of the matrix

$$A = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 1\\1 & 1 & 1\\1 & 2 & 2\end{bmatrix}.$$

Solution:

$$\text{RS}(A) = \text{span}\left\{\begin{bmatrix}1 & 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1 & 0\end{bmatrix}, \begin{bmatrix}0 & 0 & 1\end{bmatrix}\right\}$$

$$\text{CS}(A) = \text{span}\left\{\begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\1\\2\end{bmatrix}\right\}$$

$$\text{N}(A) = \text{span}\left\{\begin{bmatrix}0\\-1\\1\end{bmatrix}\right\}$$

$$\text{N}\left(A^T\right) = \text{span}\left\{\begin{bmatrix}-1 & -1 & 1 & 0\end{bmatrix}, \begin{bmatrix}-2 & -2 & 0 & 1\end{bmatrix}\right\}$$

😄

Question 3

(CH5.4 P275 Q28) Find the least squares solution of the system $A\boldsymbol{x} = \boldsymbol{b}$, where

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}.$$

Solution:

$$\hat{\boldsymbol{x}} = (A^T A)^{-1} A^T \boldsymbol{b} = \begin{bmatrix} 0.06 \\ 0.16 \\ 0 \end{bmatrix}$$

😄

Question 4

(CH5.4 P276 Q34) Find the lesat squares regression line for the data points

$$(-2, 0), (-1, 5), \left(0, \frac{7}{2}\right), (1, 2), (2, -1).$$

Solution: Let $y = a + bx \implies \begin{cases} 0 = a - 2b \\ 5 = a - b \\ \frac{7}{2} = a \\ 2 = a + b \\ -1 = a + 2b \end{cases}$.

The problem is equivalent to solve the least squares solution of the system

$$A\boldsymbol{x} = \boldsymbol{b}, \quad A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad \boldsymbol{x} = \begin{bmatrix} a \\ b \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 5 \\ \frac{7}{2} \\ 2 \\ -1 \end{bmatrix}$$

The least squares solution is

$$\hat{\boldsymbol{x}} = (A^T A)^{-1} A^T \boldsymbol{b} = \begin{bmatrix} 1.9 \\ -0.5 \end{bmatrix}$$

😄