p306

8. (a) $T(v) = (\sqrt{3}-2, -2, 4)$

(b) $w = T(2, 0)$

16. $\forall A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \in M_{2\times 2}$

$\forall \alpha, \beta \in \mathbb{R}$

$$T(\alpha A + \beta B) = T\left(\begin{bmatrix} \alpha a + \beta e & \alpha b + \beta f \\ \alpha c + \beta g & \alpha d + \beta h \end{bmatrix}\right)$$

$$= (\alpha a + \beta e) + (\alpha b + \beta f) + (\alpha c + \beta g) + (\alpha d + \beta h)$$

$$= \alpha(a+b+c+d) + \beta(e+f+g+h)$$

$$= \alpha T(A) + \beta T(B)$$

$\therefore$ T is a linear transformation.

18. (Same notation as Q16.)

$$T(\alpha A + \beta B) = (\alpha b + \beta f)^2$$

$$\neq \alpha b^2 + \beta f^2$$

$\therefore$ T is not a linear transformation.

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56. $$T\left(\begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}\right) = 1 \cdot T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) + 3 \cdot T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) - T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right)$$

$$+ 4 \cdot T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 6 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 12 & -4 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -1 \\ 7 & 4 \end{bmatrix}$$

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18. (a)

$$A \overset{r}{\sim} \begin{bmatrix} -1 & 3 & 2 & 1 & 4 \\ 0 & 9 & 9 & 2 & 8 \\ 0 & 0 & 18 & 13 & 16 \end{bmatrix} \triangleq U$$

$$\Rightarrow \ker(T) = \ker(A) = \ker(U) = \operatorname{span}\left\{ \begin{bmatrix} 19 \\ 9 \\ -13 \\ 18 \\ 0 \end{bmatrix}, \begin{bmatrix} 20 \\ 0 \\ -8 \\ 0 \\ 9 \end{bmatrix} \right\}$$

(b)

$$R(T) = R(A) = \operatorname{span}\left\{ \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} \right\}$$

30. (a)

$$\ker(T) = \ker(A) = \operatorname{span}\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

(b) $\text{nullity}(T) = 2$

(c) $\text{range}(T) = \mathbb{R}^2$

(d) $\text{rank}(T) = 2$

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54. $\ker(T) \neq \{\vec{0}\}$, $\therefore$ T is not 1-1

$R(T) = \mathbb{R}^3$, $\therefore$ T is onto

56. (a)(d)(e)(g)

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30. $T(x,y) = T_2 \circ T_1(x,y)$
$= T_2(x,y,y)$
$= (y,y)$

$\Rightarrow A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$T'(x,y,z) = T_1 \circ T_2(x,y,z)$
$= T_1(y,z)$
$= (y,z,z)$

$\Rightarrow A' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

36. $T(x) = Ax$, where $A = \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$\because$ rank$(A) = 4$ $\therefore$ A is invertible

$A^{-1} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow T^{-1}(x) = (x_1 + 2x_2, x_2, x_4, x_3 - x_4)$

38. (a) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \Rightarrow T(v) = (-2,-2)$

(b) $[T]_B^{B'} [v]_B = \begin{bmatrix} 0 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$