Singapore University of Technology & Design
Engineering Product Development
30.114 Advanced Feedback & Control – Fall 2023

Homework #2 Solutions

1. Consider the two systems defined below:
   a. Are they completely state controllable? Why?
   b. Are they completely observable? Why?
   c. Are they completely output controllable? Why?

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-1 & -2 & -2\\ 0 & -1 & 1\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}2\\ 0\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 3 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0 & 1\\ 1 & 0\\ 0 & 1\end{bmatrix}u$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B}\end{bmatrix}=\begin{bmatrix}2 & -4 & 0\\ 0 & 1 & 0\\ 1 & 1 & -5\end{bmatrix}$$

Rank of controllability matrix is 3.
System is completely state controllable

$$\begin{bmatrix}\mathbf{C}^* & \mathbf{A}^*\mathbf{C}^* & (\mathbf{A}^*)^2\mathbf{C}^*\end{bmatrix}=\begin{bmatrix}1 & -1 & 0\\ 1 & -3 & 5\\ 0 & -1 & 0\end{bmatrix}$$

Rank of observability matrix is 3.
System is completely observable

$$\begin{bmatrix}\mathbf{CB} & \mathbf{CAB} & \mathbf{CA}^2\mathbf{B} & \mathbf{D}\end{bmatrix}=\begin{bmatrix}2 & -3 & 0 & 0\end{bmatrix}$$

Rank of the output controllability matrix is 1. Since the requirement is that the matrix be at least rank 1, system is completely output controllable

$$\begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 2 & 0 & 4\\ 1 & 0 & 2 & 0 & 4 & 0\\ 0 & 1 & 3 & 1 & 9 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0 & 2/3 & -2 & 2\\ 0 & 1 & 0 & 2 & 0 & 4\\ 0 & 0 & 1 & -1/3 & 3 & -1\end{bmatrix}$$

Rank of the controllability matrix is 3. There are 3 pivots in the row reduced form. System is completely state controllable.

$$\begin{bmatrix}\mathbf{C}^* & \mathbf{A}^*\mathbf{C}^* & (\mathbf{A}^*)^2\mathbf{C}^*\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 2 & 0 & 4 & 0\\ 0 & 1 & 0 & 2 & 0 & 4\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

Rank of the observability matrix is 2. System is **not** completely observable

$$\begin{bmatrix}\mathbf{CB} & \mathbf{CAB} & \mathbf{CA}^2\mathbf{B} & \mathbf{D}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 2 & 0 & 4 & 0 & 0\\ 1 & 0 & 2 & 0 & 4 & 0 & 0 & 0\end{bmatrix}$$

Rank of the output controllability matrix is 2. Since the requirement is that the matrix be at least rank 2, system is completely output controllable

2. Consider the following matrix **A**. Find the characteristic equation and find the roots.
   a. Find the transformation matrix **S** that will transform **A** to the Jordan form and use that result to find $e^{\mathbf{A}t}$.

b. Evaluate $e^{\mathbf{A}t}$ when t=0.

$$\mathbf{A}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & -3 & 3\end{bmatrix}$$

$$|s\mathbf{I}-\mathbf{A}|=\left\|\begin{bmatrix}s & 0 & 0\\ 0 & s & 0\\ 0 & 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & -3 & 3\end{bmatrix}\right\|=\left\|\begin{bmatrix}s & -1 & 0\\ 0 & s & -1\\ -1 & 3 & s-3\end{bmatrix}\right\|=0$$

Characteristic equation of **A**:

$$s^2(s-3)-1+3s=0$$
$$s^3-3s^2+3s-1=0$$
$$(s-1)(s-1)(s-1)=0$$

Roots are s=1, 1 and 1.

To find **S**, we need to find the generalized eigenvectors.

$$(\mathbf{A}-\lambda\mathbf{I})\mathbf{v}_1=0$$

$$\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 1 & -3 & 2\end{bmatrix}\begin{bmatrix}v_{11}\\ v_{12}\\ v_{13}\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$v_{11}=v_{12} \qquad v_{12}=v_{13} \qquad v_{11}=3v_{12}-2v_{13}$$

$$v_{11}=1 \qquad v_{12}=1 \qquad v_{13}=1$$

$$(\mathbf{A}-\lambda\mathbf{I})\mathbf{v}_2=\mathbf{v}_1$$

$$\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 1 & -3 & 2\end{bmatrix}\begin{bmatrix}v_{21}\\ v_{22}\\ v_{23}\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$$

$$-v_{21}+v_{22}=1 \qquad -v_{22}+v_{23}=1 \qquad v_{21}-3v_{22}+2v_{23}=1$$

$$v_{21}=-2 \qquad v_{22}=-1 \qquad v_{23}=0$$

$$(\mathbf{A}-\lambda\mathbf{I})\mathbf{v}_3=\mathbf{v}_2$$

$$\begin{bmatrix}-1 & 1 & 0\\ 0 & -1 & 1\\ 1 & -3 & 2\end{bmatrix}\begin{bmatrix}v_{31}\\ v_{32}\\ v_{33}\end{bmatrix}=\begin{bmatrix}-2\\ -1\\ 0\end{bmatrix}$$

$$-v_{31}+v_{32}=-2 \qquad -v_{32}+v_{33}=-1 \qquad v_{31}-3v_{32}+2v_{33}=0$$

$$v_{31}=0 \qquad v_{32}=-2 \qquad v_{33}=-3$$

$$\mathbf{S}=[\mathbf{v_1} \quad \mathbf{v_2} \quad \mathbf{v_3}]=\begin{bmatrix}1 & -2 & 0\\ 1 & -1 & -2\\ 1 & 0 & -3\end{bmatrix} \qquad \mathbf{S}^{-1}=\begin{bmatrix}3 & -6 & 4\\ 1 & -3 & 2\\ 1 & -2 & 1\end{bmatrix}$$

Hence you will get the Jordan form using: $\mathbf{J}=\mathbf{S}^{-1}\mathbf{A}\mathbf{S}=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}$

Therefore to find the exponential of **A**, we can use the above expression:

$$e^{\mathbf{A}t}=\mathbf{S}e^{\mathbf{J}t}\mathbf{S}^{-1}=\begin{bmatrix}1&-2&0\\1&-1&-2\\1&0&-3\end{bmatrix}\begin{bmatrix}e^t&te^t&0.5t^2e^t\\0&e^t&te^t\\0&0&e^t\end{bmatrix}\begin{bmatrix}3&-6&4\\1&-3&2\\1&-2&1\end{bmatrix}$$

$$=\begin{bmatrix}e^t&te^t-2e^t&0.5t^2e^t-2te^t\\e^t&te^t-e^t&0.5t^2e^t-te^t-2e^t\\e^t&te^t&0.5t^2e^t-3e^t\end{bmatrix}\begin{bmatrix}3&-6&4\\1&-3&2\\1&-2&1\end{bmatrix}$$

$$=\begin{bmatrix}3e^t+te^t-2e^t+0.5t^2e^t-2te^t&-6e^t-3\left(te^t-2e^t\right)-2\left(0.5t^2e^t-2te^t\right)&4e^t+2\left(te^t-2e^t\right)+0.5t^2e^t-2te^t\\3e^t+te^t-e^t+0.5t^2e^t-te^t-2e^t&-6e^t-3\left(te^t-e^t\right)-2\left(0.5t^2e^t-te^t-2e^t\right)&4e^t+2\left(te^t-e^t\right)+0.5t^2e^t-te^t-2e^t\\3e^t+te^t+0.5t^2e^t-3e^t&-6e^t-3te^t-2\left(0.5t^2e^t-3e^t\right)&4e^t+2te^t+0.5t^2e^t-3e^t\end{bmatrix}$$

$$=e^t\begin{bmatrix}1-t+0.5t^2&t-t^2&0.5t^2\\0.5t^2&1-t-t^2&t+0.5t^2\\t+0.5t^2&-3t-t2&1+2t+0.5t^2\end{bmatrix}$$

$e^{\mathbf{A}(t=0)}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$. The result is the identity matrix and is to be expected.

3. Consider the system:

$$\dot{\mathbf{x}}=\begin{bmatrix}3&0&0\\0&3&0\\0&3&2\end{bmatrix}\mathbf{x}$$
$$y=\begin{bmatrix}1&1&1\end{bmatrix}\mathbf{x}$$

a. Show that the system is not completely observable.

The observability matrix is: $\begin{bmatrix}\mathbf{C}^* & \mathbf{A}^*\mathbf{C}^* & \left(\mathbf{A}^*\right)^2\mathbf{C}^*\end{bmatrix}=\begin{bmatrix}1&3&9\\1&6&24\\1&2&4\end{bmatrix}$. The determinant of this matrix is zero. Hence it is not full rank. System is not completely observable.

b. Show that the system is completely observable if the output is now:

$$\mathbf{y}=\begin{bmatrix}1&1&1\\1&2&3\end{bmatrix}\mathbf{x}$$

The observability matrix is: $\begin{bmatrix}\mathbf{C}^* & \mathbf{A}^*\mathbf{C}^* & \left(\mathbf{A}^*\right)^2\mathbf{C}^*\end{bmatrix}=\begin{bmatrix}1&1&3&3&9&9\\1&2&6&15&24&63\\1&3&2&6&4&12\end{bmatrix}$

Let's row reduce the matrix abovce to obtain $\begin{bmatrix} 1 & 0 & 0 & -9 & -6 & -45 \\ 0 & 1 & 0 & 3 & 0 & 9 \\ 0 & 0 & 1 & 3 & 5 & 15 \end{bmatrix}$. This matrix has 3 pivots, and is rank 3. System is completely observable.

4. For the standard state space system defined: $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$
where **x** = state vector (n-vector), **u** = control vector (r-vector), **A** = $n \times n$ constant matrix, **B** = $n \times r$ constant matrix.

Obtain the response of the system to each of the following inputs:

a. The r components of **u** are impulse functions of various magnitudes.

The general expression for the solution to the nonhomogeneous state equation is:

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{Bu}(\tau)d\tau$$

Let $\mathbf{u}(t) = \delta(t)\mathbf{w}$, where the vector **w** contains the magnitudes of each of the r impulse functions.

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\delta(\tau)\mathbf{w}d\tau$$

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\mathbf{B}\left(\int_0^t \delta(\tau)d\tau\right)\mathbf{w} = e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\mathbf{Bw} \qquad t \geq 0$$

b. The r components of **u** are step functions of various magnitudes.

Let $\mathbf{u}(t) = 1(t)\mathbf{k}$, where the vector **k** contains the magnitudes of each of the r step functions.

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}1(\tau)\mathbf{k}d\tau$$

$$\begin{aligned}
\mathbf{x}(t) &= e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}d\tau\mathbf{Bk} \\
&= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\int_0^t e^{-\mathbf{A}\tau}d\tau\mathbf{Bk} \\
&= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\int_0^t\left(\mathbf{I} - \mathbf{A}\tau + \frac{\mathbf{A}^2\tau^2}{2!} + \cdots\right)d\tau\mathbf{Bk} \\
&= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left[\mathbf{I}t - \frac{\mathbf{A}t^2}{2} + \frac{\mathbf{A}^2t^3}{3!} + \cdots\right]\mathbf{Bk} \\
&= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left(-\mathbf{A}^{-1}\right)\left[-\mathbf{A}t + \frac{\mathbf{A}^2t^2}{2} - \frac{\mathbf{A}^3t^3}{3!} + \cdots\right]\mathbf{Bk} \\
&= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left(-\mathbf{A}^{-1}\right)\left[e^{-\mathbf{A}t} - \mathbf{I}\right]\mathbf{Bk} \\
&= e^{\mathbf{A}t}\mathbf{x}(0) + \mathbf{A}^{-1}\left[e^{\mathbf{A}t} - \mathbf{I}\right]\mathbf{Bk} \qquad t \geq 0
\end{aligned}$$

c. The r components of **u** are ramp functions of various magnitudes (Challenging!).

Let $\mathbf{u}(t) = t\mathbf{v}$, where the vector **v** contains the magnitudes of each of the r ramp functions.

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\tau\mathbf{v}d\tau$$

$$= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\int_0^t e^{-\mathbf{A}\tau}\tau d\tau\mathbf{B}\mathbf{v}$$

$$= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left[\int_0^t \left(\mathbf{I} - \mathbf{A}\tau + \frac{\mathbf{A}^2\tau^2}{2!} + \cdots\right)\tau d\tau\right]\mathbf{B}\mathbf{v}$$

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left(\frac{\mathbf{I}t^2}{2} - \frac{\mathbf{A}t^3}{3} + \frac{\mathbf{A}^2t^4}{2!4} - \cdots\right)\mathbf{B}\mathbf{v}$$

$$= e^{\mathbf{A}t}\mathbf{x}(0) + \left(\mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \cdots\right)\left(\frac{\mathbf{A}^2t^2}{2!} - \frac{2\mathbf{A}^3t^3}{3!} + \frac{3\mathbf{A}^4t^4}{4!} - \cdots\right)\left(\mathbf{A}^{-1}\right)^2\mathbf{B}\mathbf{v}$$

$$= e^{\mathbf{A}t}\mathbf{x}(0) + \left(\frac{\mathbf{A}^2t^2}{2!} + \frac{\mathbf{A}^3t^3}{3!} + \frac{\mathbf{A}^4t^4}{4!} + \cdots\right)\left(\mathbf{A}^{-1}\right)^2\mathbf{B}\mathbf{v}$$

$$= e^{\mathbf{A}t}\mathbf{x}(0) + \left(\mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2t^2}{2!} - \frac{\mathbf{A}^3t^3}{3!} + \frac{\mathbf{A}^4t^4}{4!} + \cdots - \mathbf{I} - \mathbf{A}t\right)\left(\mathbf{A}^{-1}\right)^2\mathbf{B}\mathbf{v}$$

$$= e^{\mathbf{A}t}\mathbf{x}(0) + \left(e^{\mathbf{A}t} - \mathbf{I} - \mathbf{A}t\right)\left(\mathbf{A}^{-1}\right)^2\mathbf{B}\mathbf{v}$$

$$= e^{\mathbf{A}t}\mathbf{x}(0) + \left(\mathbf{A}^{-1}\right)^2\left(e^{\mathbf{A}t} - \mathbf{I} - \mathbf{A}t\right)\mathbf{B}\mathbf{v} \qquad t \geq 0$$

5. You are given a transfer function of a system: $G(s) = \dfrac{10}{(s+1)(s+2)(s+3)}$. Find the differential equation describing this system and express the system in State-Space to design a full state feedback controller such that the closed loop poles are at:

$$s = -2 + j2\sqrt{3}$$
$$s = -2 - j2\sqrt{3}$$
$$s = -10$$

From the transfer function, the differential equation describing the system can be expressed as: $\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 10u$

One possible state space representation is as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}u,\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

The desired characteristic equation of the system is:

$$\left(s + 2 + j2\sqrt{3}\right)\left(s + 2 - j2\sqrt{3}\right)(s + 10) = s^3 + 14s^2 + 56s + 160$$

The gain can be computed using Ackermann's Formula:

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} \end{bmatrix}^{-1}\phi(\mathbf{A})$$

$$\phi(\mathbf{A}) = \begin{bmatrix} 154 & 45 & 8 \\ -48 & 66 & -3 \\ 18 & -15 & 84 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 10 \\ 0 & 10 & -60 \\ 10 & -60 & 250 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 10 \\ 0 & 10 & -60 \\ 10 & -60 & 250 \end{bmatrix}^{-1} \begin{bmatrix} 154 & 45 & 8 \\ -48 & 66 & -3 \\ 18 & -15 & 84 \end{bmatrix} = \begin{bmatrix} 15.4 & 4.5 & 0.8 \end{bmatrix}$$

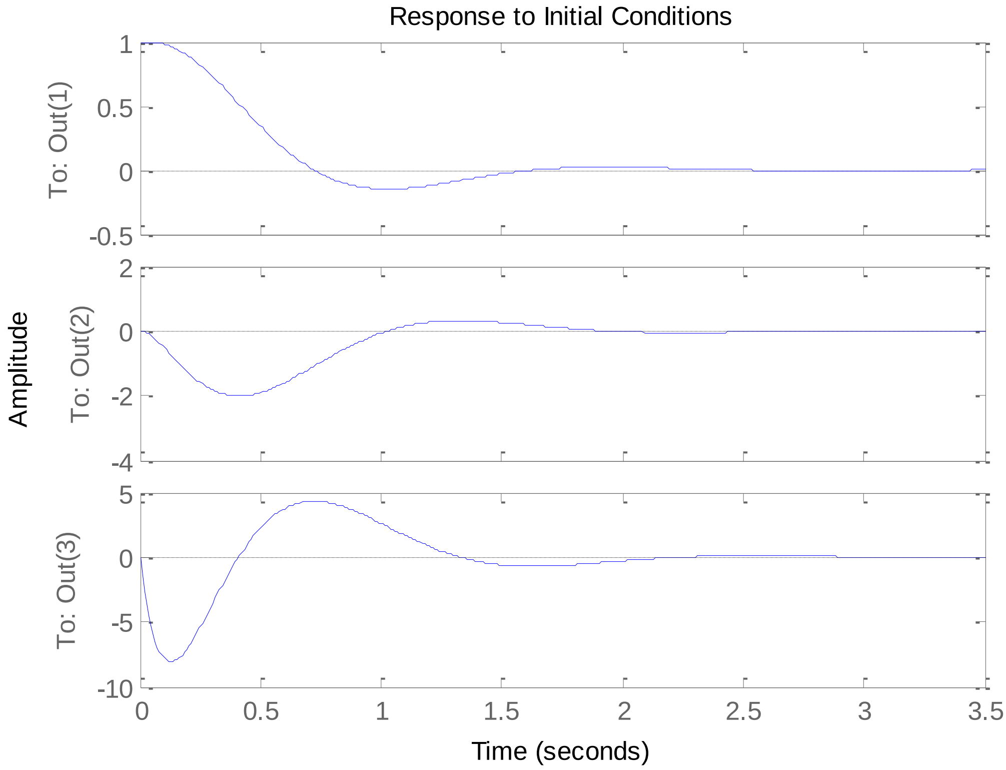
a. Use **MATLAB**’s *acker* and *place* commands to verify your design. Please provide a copy of the MATLAB commands or script used to compute the controller gains.

```
A=[0 1 0; 0 0 1; -6 -11 -6];
B=[0;0;10];
P=[-2+j*2*sqrt(3) -2-j*2*sqrt(3) -10]
K=acker(A,B,P)
K=place(A,B,P)
```

b. Use **MATLAB** to plot the time response of all the states of state feedback controlled system under non-zero initial conditions (e.g. $\mathbf{x}(0)=[1\ 0\ 0]^T$). What are the steady state values for each of the states?

```
sys=ss(A-B*K,[0;0;0],eye(3),0)
initial(sys,[1 0 0])
```

Steady state value for all states are zero.



6. The characteristic equation of a state space system is given below. Find the values of K such that the system is unstable.

$$10 + 9s + Ks^2 + 4s^2 + 2s^3 + s^4 = 0$$

Composing the Routh Array:

$$\begin{array}{llll}
s^4 & 1 & 4+K & 10 \\
s^3 & 2 & 9 & \\
s^2 & \dfrac{2K-1}{2} & 10 & \\
s^1 & \dfrac{18K-49}{2K-1} & & \\
s^0 & 10 & &
\end{array}$$

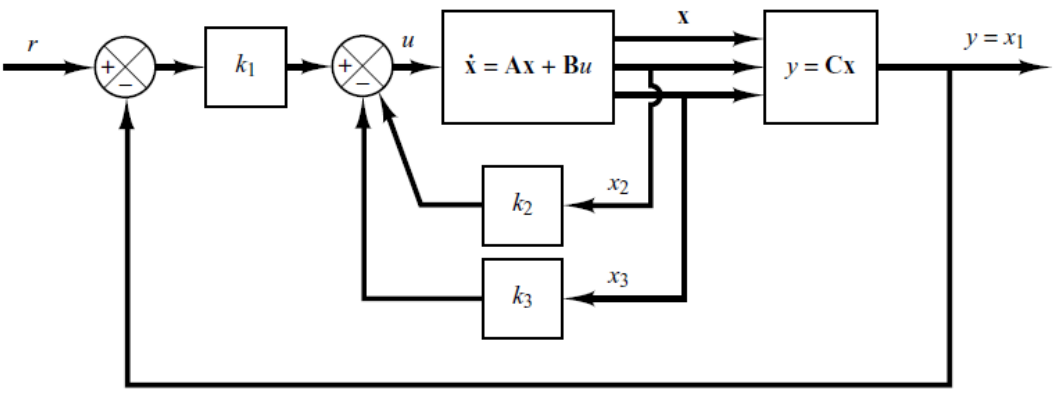
For stability, all coefficients in the first column must be positive.

$$\frac{2K-1}{2} > 0 \quad \Rightarrow K > 0.5$$

$$\frac{18K-49}{2K-1} > 0 \quad \Rightarrow K > \frac{49}{18}$$

Hence the system is unstable if K<49/18

7. Consider a motion system described by the following block diagram.



$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

a. What is the expression of *u(t)* if *r* is a time-varying input *r(t)*?

$$u(t) = -k_1x_1(t) - k_2x_2(t) - k_3x_3(t) + k_1r(t)$$

b. Determine the feedback gains k_1, k_2 and k_3 such that the closed loop poles are located at $s = -2 + j4$, $s = -2 - j4$, $s = -10$. What is the desired characteristic equation?

Checking for complete state controllability:

Controllability matrix: $\begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B}\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$. This matrix is rank 3 (full rank).

System is completely state controllable.

Desired characteristic equation: $(s+2-j4)(s+2+j4)(s+10) = s^3 + 14s^2 + 60s + 200 = 0$

System is already in CCF format. Can directly use method 1:

$$\mathbf{K} = \begin{bmatrix} 200-0 & 60-5 & 14-6 \end{bmatrix} = \begin{bmatrix} 200 & 55 & 8 \end{bmatrix}$$

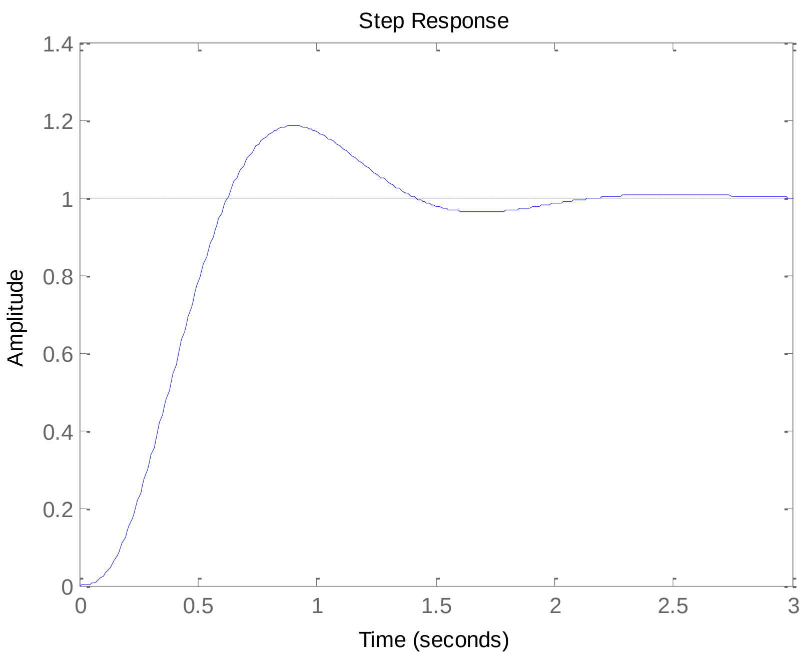
c. Use MATLAB to obtain a unit-step response (*r(t)* is a unit step) and plot the output *y(t)* versus *t* curve. Attach your full MATLAB commands which produced your results and plots.

The system under the control law can be updated to:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}(-\mathbf{Kx} + k_1 r) = (\mathbf{A} - \mathbf{BK})\mathbf{x} + Bk_1 r$$

Using the 'new' A and B matrices, we can simulate the system to see the response to the first state in **x**.

```
A=[ 0 1 0; 0 0 1; 0 -5 -6]
B=[0; 0; 1]
C=[1 0 0]
P=[-2+j*4 -2-j*4 -10]
K=place(A,B,P)
sys_cl=ss(A-B*K,B.*K(1),C,0)
step(sys_cl)
```



d. If you want the system to have zero steady state error, do you need to add an integrator to the system?

No. This is a type 1 system. System already has 1 integrator present in the plant.

8. Consider the system defined by $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad , \quad y = \mathbf{C}\mathbf{x}$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -6 & 0 \end{bmatrix} \ , \ \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ , \ \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \ D = 0$$

a. Use the Ackermann's formula to design a full-order state observer such that the desired poles of the observer are located at $s = -10 \ , \quad s = -15 \ , \ s = -10$

First check for observability of the system:

$$\begin{bmatrix} \mathbf{C}^* & \mathbf{A}^*\mathbf{C}^* & (\mathbf{A}^*)^2\mathbf{C}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The matrix is full rank. System is fully observable.

The desired characteristic equation of the system is:

$$(s+10)(s+10)(s+15) = s^3 + 35s^2 + 400s + 1500$$

Using $\mathbf{K}_e = \phi(\mathbf{A}) \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$,

$$\phi(\mathbf{A}) = \begin{bmatrix} 1495 & 394 & 35 \\ -175 & 1285 & 394 \\ -1970 & -2539 & 1285 \end{bmatrix} \text{ and } \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{K}_e = \phi(\mathbf{A}) \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 35 \\ 394 \\ 1285 \end{bmatrix}$$