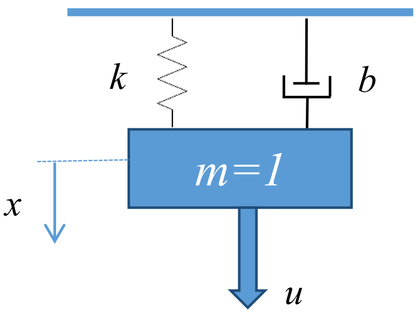
Singapore University of Technology & Design
Engineering Product Development
30.114 Advanced Feedback & Control – Fall 2023

Homework #1

1. Recall the following state space systems that supposedly represented the mass spring damper system (it is slightly different from the example in class). First determine the EOM of the system below and by determining the **Transfer Matrix** for each system, verify that that is the case. Can you also identify the canonical form of each expression, if any? (are they CCF? OCF?)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -b & -k \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \qquad x = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -k \\ 1 & -b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \qquad x = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -b & 1 \\ -k & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2. A car's suspension system can be described with the following Transfer Function. Find the state space representation of the system in modal form and construct the block diagram.

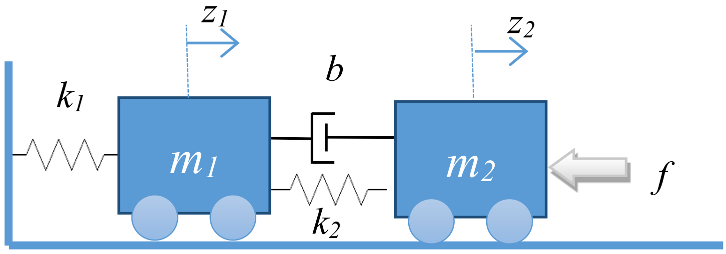
$$G(s) = \frac{s+4}{s^4 + 3s^3 + 2s^2}$$

3. Recall from the second problem of the Quiz. We wish to transform that system to observable canonical form (OCF). Determine the transformation matrix **T** to achieve this and the resultant OCF.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

4. Consider the following mechanical system. The displacements z_1 and z_2 are measured from respective equilibrium positions before the input force f is applied. Derive a State-Space representation of the system. The outputs of the system would be $\mathbf{y} = [z_1 \;\; z_2]^T$.



5. For the following state matrix **A**,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

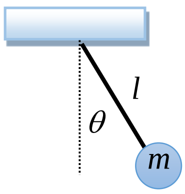
   a. Find the eigenvalues and eigenvectors of **A**.

   b. Find a matrix **P** such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix. What are the numerical values for the diagonal elements of the diagonal matrix?

   c. Verify your answer in b. that $\mathbf{P}^{-1}\mathbf{AP}$ is diagonal.

6. A simple pendulum system is shown below. An input torque u is applied to the pendulum (The torque is defined such that positive u results in negative θ). (gravity is acting downwards)
   a. Derive the non-linear equation of motion.

   b. Linearize the system about the equilibrium position and represent the system in state-space (Both in CCF and OCF).



7. Compute $e^{\mathbf{A}t}$ where $\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

8. Obtain the response *y(t)* of the system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where *u(t)* is the unit step input.