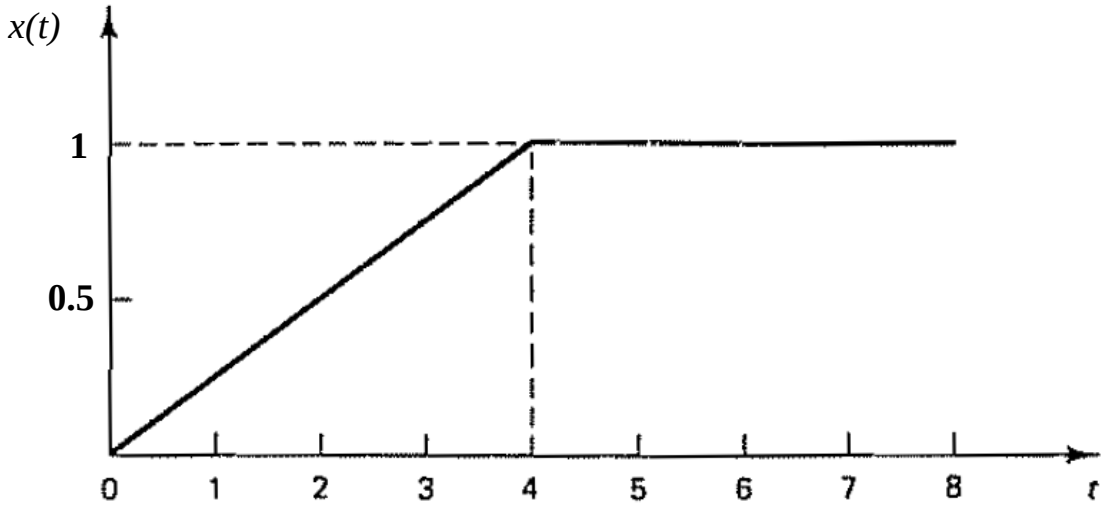
Singapore University of Technology & Design
Engineering Product Development
30.114 Advanced Feedback & Control – Fall 2023

Homework #3

1. What is the range of the quantization error if you want to use a 12-bit ADC to represent a range of 0 to 10 V?

2. Obtain the *z* transform of the curve *x(t)* as shown below. The sampling period is 1 sec. We would like to obtain a closed form expression for *X(z)*.



3. For the following expressions, find poles of the system and use that to determine the **closed form expression** *x(k)* for the inverse *z* transform of *X(z)*:

   a. $X(z)=\dfrac{2z^3+z}{(z-2)^2(z-1)}$. Plot the poles and zeros on the *z* plane.

   b. $X(z)=\dfrac{z+2}{(z-2)z^2}$. Verify with the first 4 terms of *x(k)* using the Direct Division method.

   c. $X(z)=\dfrac{5}{(z-1)(z-2)}$. Use both the Partial Fraction Expansion method and the Inversion Integral method.

   d. $X(z)=\dfrac{0.5z^{-1}(1-2z^{-1})}{(1-0.5z^{-1})(1-0.8z^{-1})^2}$. Is the system stable?

4. Consider the following difference equation:

$$x(k+2)-1.3679x(k+1)+0.3679x(k)=0.3679u(k+1)+0.2642u(k)$$

where *x(k)* is the output and *x(k)*=0 for $k\leq 0$ and where *u(k)* is the input and given by:

$$\begin{aligned} u(k) &= 0, \quad k<0 \\ u(0) &= 1 \\ u(1) &= 0.2142 \\ u(2) &= -0.2142 \\ u(k) &= 0, \quad k=3,4,5,\ldots \end{aligned}$$

Determine the output *x(k)*. (Remember, even though *x(0)*=0, *x(1)* is not necessarily 0).

5. Consider the system described by:

$$y(k)-0.6y(k-1)-0.81y(k-2)+0.67y(k-3)-0.12y(k-4)=x(k)$$

Where *x(k)* is the input and *y(k)* is the output of the system. Determine the stability of the system.

6. For the following characteristic equation, determine whether or not if any roots of the c.e. lie outside the unit circle centred at the origin of the z plane.

$$0.32+1.44z+2.1z^2+z^3=0$$

7. Determine the stability of the following discrete-time system:

$$\frac{Y(z)}{X(z)}=\frac{2z^{-3}}{1+0.4z^{-1}-1.34z^{-2}+0.28z^{-3}}$$