Singapore University of Technology & Design
Engineering Product Development
30.114 Advanced Feedback & Control – Fall 2023

Homework #2

1. Consider the two systems defined below:
   a. Are they completely state controllable? Why?
   b. Are they completely observable? Why?
   c. Are they completely output controllable? Why?

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2. Consider the following matrix **A**. Find the characteristic equation and find the roots.
   a. Find the transformation matrix **S** that will transform **A** to the Jordan form and use that result to find $e^{\mathbf{A}t}$.
   b. Evaluate $e^{\mathbf{A}t}$ when t=0.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

3. Consider the system:

$$\dot{\mathbf{x}} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 3 & 2 \end{bmatrix} \mathbf{x}$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \mathbf{x}$$

   a. Show that the system is not completely observable.

   b. Show that the system is completely observable if the output is now:

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \mathbf{x}$$

4. For the standard state space system defined: $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$
where **x** = state vector (n-vector), **u** = control vector (r-vector), **A** = $n \times n$ constant matrix, **B** = $n \times r$ constant matrix.

Obtain the response of the system to each of the following inputs:
   a. The r components of **u** are impulse functions of various magnitudes.
   b. The r components of **u** are step functions of various magnitudes.
   c. The r components of **u** are ramp functions of various magnitudes (Challenging!).

5. You are given a transfer function of a system: $G(s)=\dfrac{10}{(s+1)(s+2)(s+3)}$. Find the differential equation describing this system and express the system in State-Space to design a full state feedback controller such that the closed loop poles are at:

$$s=-2+j2\sqrt{3},\ s=-2-j2\sqrt{3}.\ s=-10$$

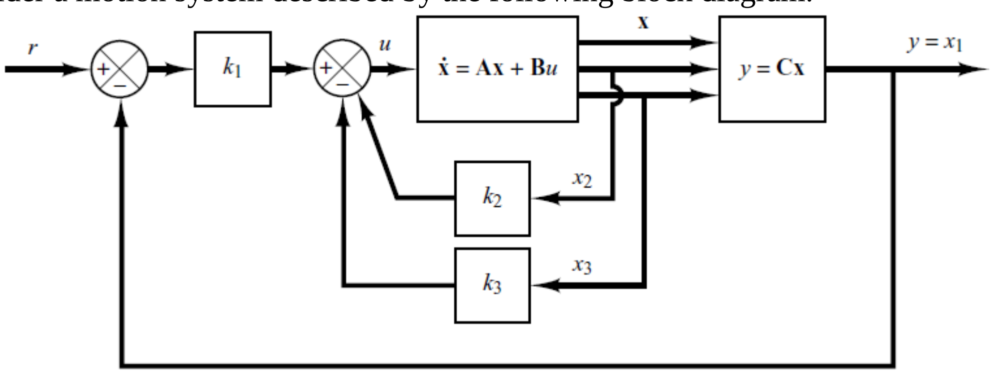
   a. Use **MATLAB**'s *acker* and *place* commands to verify your design. Please provide a copy of the **MATLAB** commands or script used to compute the controller gains.

   b. Use **MATLAB** to plot the time response of all the states of state feedback controlled system under non-zero initial conditions (e.g. $\mathbf{x}(0)=[1\ 0\ 0]^T$). What are the steady state values for each of the states?

6. The characteristic equation of a state space system is given below. Find the values of K such that the system is unstable.

$$10+9s+Ks^2+4s^2+2s^3+s^4=0$$

7. Consider a motion system described by the following block diagram.



$$\mathbf{A}=\begin{bmatrix}0&1&0\\0&0&1\\0&-5&-6\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1&0&0\end{bmatrix}$$

   a. What is the expression of *u(t)* if *r* is a time-varying input *r(t)*?

   b. Determine the feedback gains k_1, k_2 and k_3 such that the closed loop poles are located at $s=-2+j4,\ s=-2-j4,\ s=-10$. What is the desired characteristic equation?

   c. Use **MATLAB** to obtain a unit-step response (*r(t)* is a unit step) and plot the output *y(t)* versus *t* curve. Attach your full **MATLAB** commands which produced your results and plots.

8. Consider the system defined by $\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}u\ ,\ \ y=\mathbf{C}\mathbf{x}$

$$\mathbf{A}=\begin{bmatrix}0&1&0\\0&0&1\\-5&-6&0\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}0\\0\\1\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1&0&0\end{bmatrix},\ D=0$$

   a. Use the Ackermann's formula to design a full-order state observer such that the desired poles of the observer are located at $s=-10\ ,\ \ s=-15\ ,\ s=-10$