

Independence and Bayesian Networks (Part 2)

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Lecture 8

Readings: RN 13.2. PM 8.3.

Outline

Learning Goals

D-Separation

Constructing Bayesian Networks

Revisiting the Learning goals

Learning Goals

By the end of the lecture, you should be able to

- ▶ Determine whether an independence relationship holds by applying d-separation.
- ▶ Given a Bayesian network and an order of the variables, construct a Bayesian network that correctly represents the independence relationships among the variables.

Learning Goals

D-Separation

Constructing Bayesian Networks

Revisiting the Learning goals

D-Separation

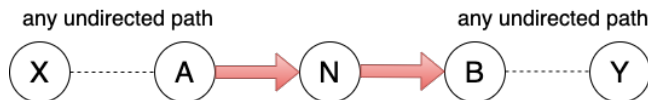
Are two variables X and Y independent given the set of observed variables E ?

Definition (D-Separation)

E d-separates X and Y
iff E blocks every un-directed path between X and Y .

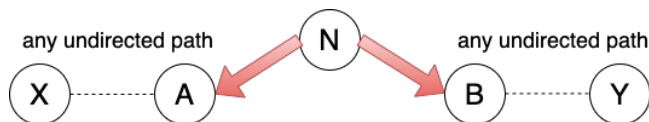
If E d-separates X and Y ,
then X and Y are conditionally independent given E .

Blocked Path - Scenario 1/3



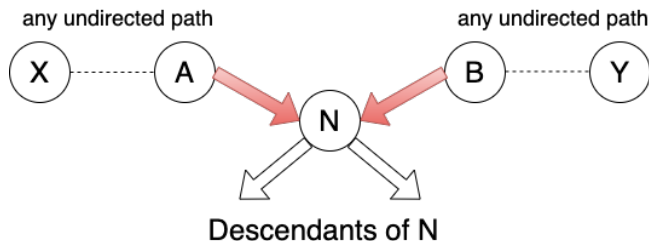
If N is observed,
then it blocks the path between X and Y.

Blocked Path - Scenario 2/3



If N is observed,
then it blocks the path between X and Y.

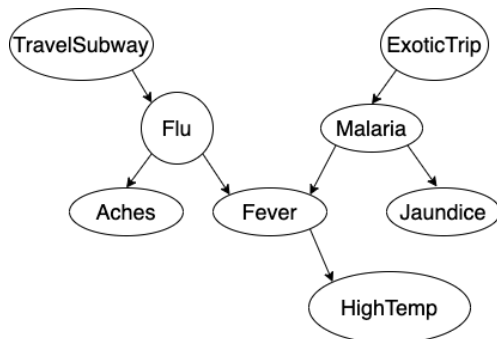
Blocked Path - Scenario 3/3



If N and N's descendants are NOT observed,
then they block the path between X and Y.

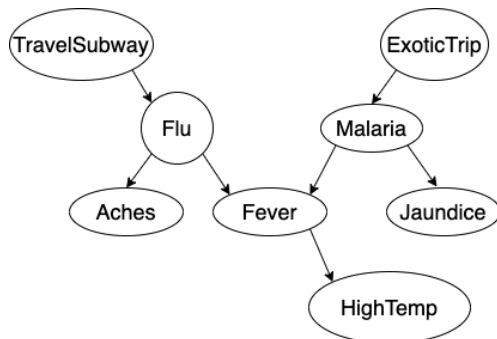
Q: Applying D-separation

Q #1: Are **TravelSubway** and **HighTemp** independent?



Q: Applying D-separation

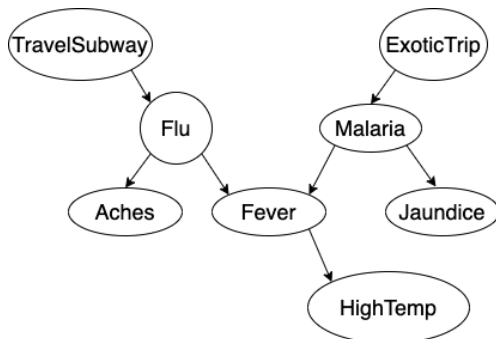
Q #1: Are **TravelSubway** and **HighTemp** independent?



→ No.

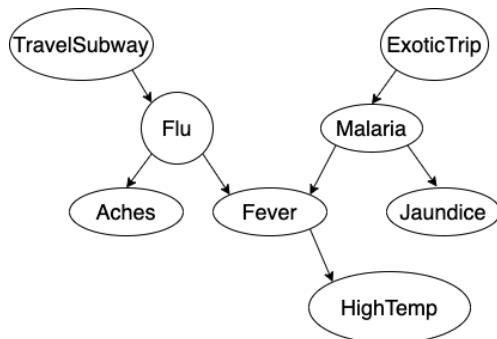
Q: Applying D-separation

Q #2: Are **TravelSubway** and **HighTemp** independent given **Flu**?



Q: Applying D-separation

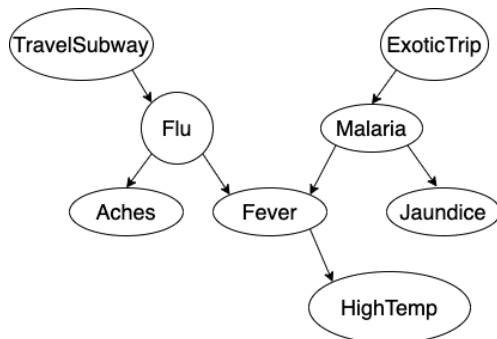
Q #2: Are **TravelSubway** and **HighTemp** independent given **Flu**?



→ Yes.

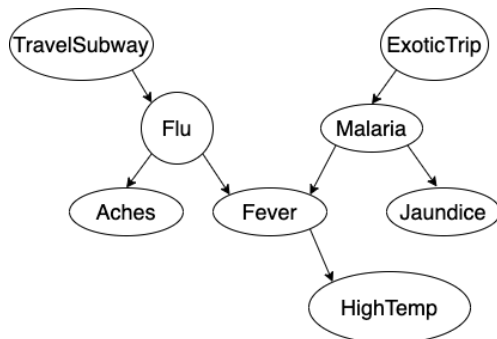
Q: Applying D-separation

Q #3: Are **Aches** and **HighTemp** independent?



Q: Applying D-separation

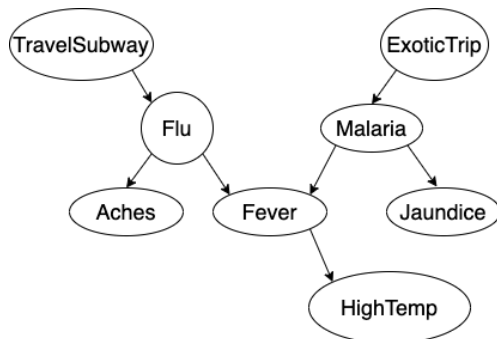
Q #3: Are **Aches** and **HighTemp** independent?



→ No.

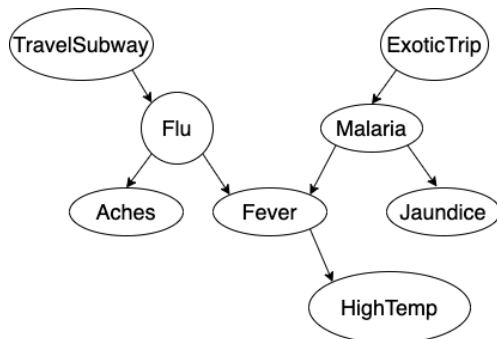
Q: Applying D-separation

Q #4: Are **Aches** and **HighTemp** independent given **Flu**?



Q: Applying D-separation

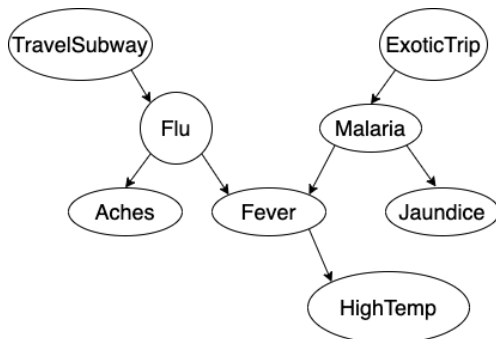
Q #4: Are **Aches** and **HighTemp** independent given **Flu**?



→ Yes.

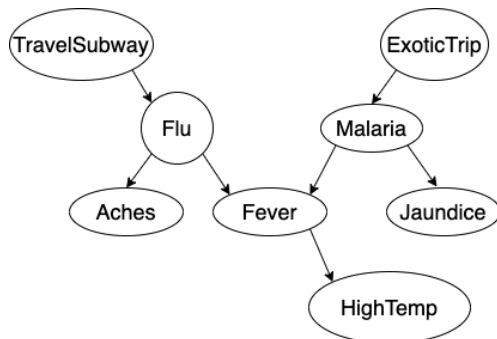
Q: Applying D-separation

Q #5: Are **Flu** and **ExoticTrip** independent?



Q: Applying D-separation

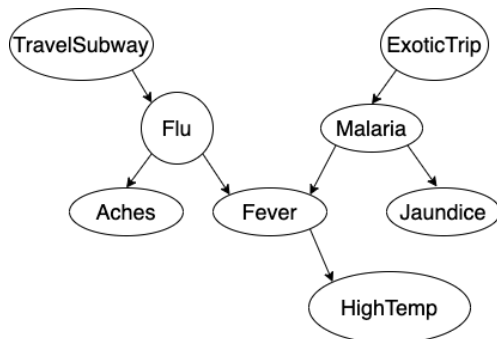
Q #5: Are **Flu** and **ExoticTrip** independent?



→ Yes.

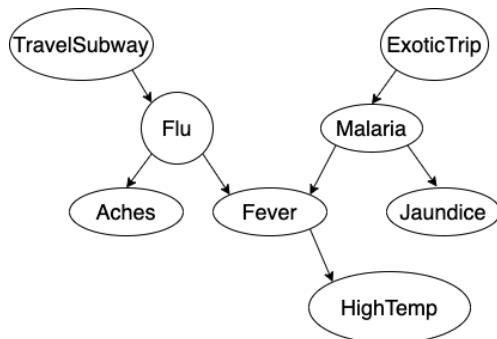
Q: Applying D-separation

Q #6: Are **Flu** and **ExoticTrip** independent given **HighTemp**?



Q: Applying D-separation

Q #6: Are **Flu** and **ExoticTrip** independent given **HighTemp**?



→ No.

Learning Goals

D-Separation

Constructing Bayesian Networks

Revisiting the Learning goals

Constructing Bayesian Networks

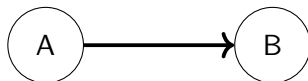
- ▶ For a joint probability distribution, there are many correct Bayesian networks.
- ▶ Given a Bayesian network A, a Bayesian network B is correct if and only if the following is true:

If Bayesian network B requires two variables to satisfy an independence relationship, Bayesian network A must also require the two variables to satisfy the same independence relationship.

- ▶ We prefer a Bayesian network that requires fewer probabilities.

Requiring an Independence Relationship

- ▶ Having an edge between two variables DOES NOT mean that the two variables are DEPENDENT.



- ▶ The absence of an edge between two variables MEANS that the two variables satisfy an INDEPENDENCE relationship.



Constructing a Correct Bayesian Network

1. Order the variables $\{X_1, \dots, X_n\}$.
2. For each variable X_i in the ordering,

2.1 Choose the node's parents:

Choose the smallest set of parents from $\{X_1, \dots, X_{i-1}\}$ such that given $Parents(X_i)$, X_i is independent of all the nodes in $\{X_1, \dots, X_{i-1}\} - Parents(X_i)$. Formally,

$$P(X_i | Parents(X_i)) = P(X_i | X_{i-1} \wedge \dots \wedge X_1).$$

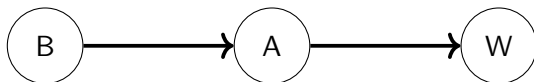
2.2 Create a link from each parent of X_i to the node X_i .

2.3 Write down the conditional probability table

$$P(X_i | Parents(X_i)).$$

Example 1: Construct a Bayes Net

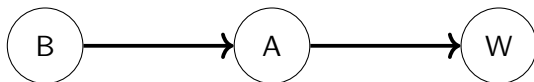
Consider the Bayesian network.



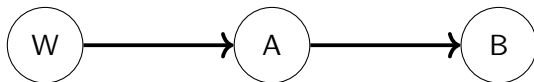
Construct a correct Bayesian network by adding the variables in the order: W, A, and B.

Example 1: Construct a Bayes Net

Consider the Bayesian network.

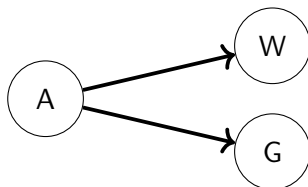


Construct a correct Bayesian network by adding the variables in the order: W, A, and B.



Example 2: Construct a Bayes Net

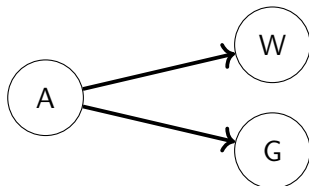
Consider the Bayesian network:



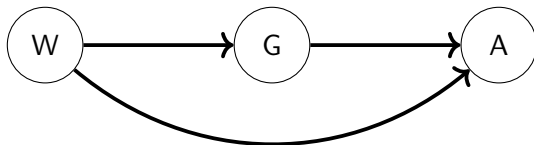
Construct a correct Bayesian network by adding the variables in the order: W, G, and A.

Example 2: Construct a Bayes Net

Consider the Bayesian network:

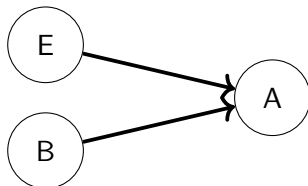


Construct a correct Bayesian network by adding the variables in the order: W, G, and A.



Example 3: Construct a Bayes Net

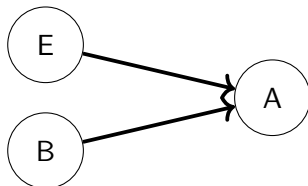
Consider the Bayesian network.



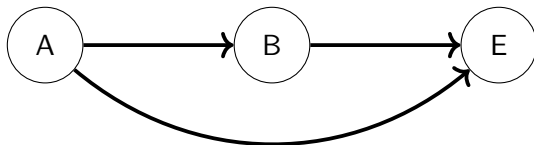
Construct a correct Bayesian network by adding the variables in the order: A, B, and E.

Example 3: Construct a Bayes Net

Consider the Bayesian network.

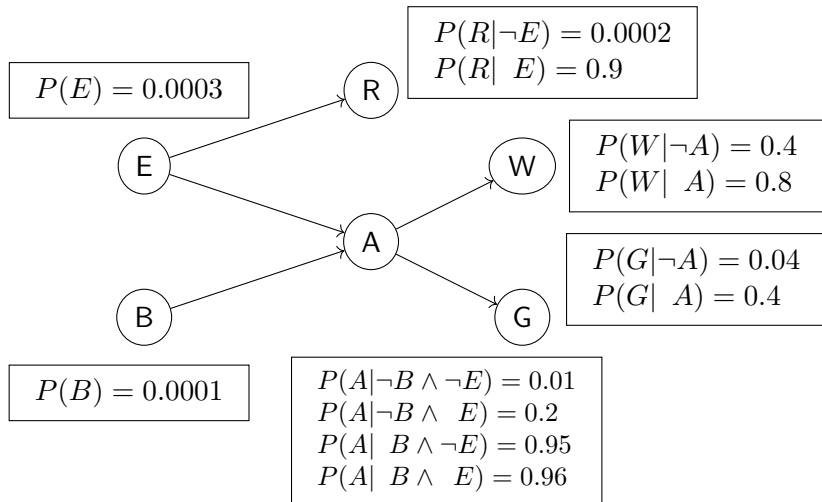


Construct a correct Bayesian network by adding the variables in the order: A, B, and E.



Example 3: Reconstruct the Holmes Scenario Network

Construct a new Bayesian network from the Holmes scenario, using the following order for adding variables: G, W, E, B, A, R .



Constructing a Compact Bayesian Network

- ▶ What does an edge mean?

Does an edge always represent a causal relationship?

→ An edge indicates an associational relationship that is not necessarily causal.

- ▶ How can we construct a Bayesian network with the smallest number of edges?

→ Cause precedes effect. So add causes first, then effects.

Revisiting the Learning Goals

By the end of the lecture, you should be able to

- ▶ Determine whether an independence relationship holds by applying d-separation.
- ▶ Given a Bayesian network and an order of the variables, construct a Bayesian network that correctly represents the independence relationships among the variables.