2a)

j2tham

keystream = 1111000100110101

2b) 1010

2c)

First 8 bits: 10101100

$$\begin{aligned} s_4 &= c_0 s_0 + c_1 s_1 + c_2 s_2 + c_3 s_3 = 1 = c_1 + c_3 \\ s_5 &= c_0 s_1 + c_1 s_2 + c_2 s_3 + c_3 s_4 = 1 = c_2 + c_4 \end{aligned}$$

$$s_6 &= c_0 s_2 + c_1 s_3 + c_2 s_4 + c_3 s_5 = 0 = c_1 + c_3 + c_4$$

$$s_7 &= c_0 s_3 + c_1 s_4 + c_2 s_5 + c_3 s_6 = 0 = c_2 + c_3$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

2d)

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \\
= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \\
= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

2e)

