

Uninformed Search

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Lecture 2

Readings: RN 3.1, 3.2, 3.3, 3.4.1, 3.4.3, 3.4.4

Outline

Learning Goals

Applications of Search

Formulating a Search Problem

Generic Search Algorithm

Uninformed Search Algorithms

- Depth-First Search

- Breadth-First Search

- Iterative-Deepening Search

Revisiting the Learning Goals

Learning goals

By the end of the lecture, you should be able to

- ▶ Formulate a real world problem as a search problem.
- ▶ Trace the execution of and implement uninformed search algorithms (Breadth-first search, Depth-first search, Iterative-deepening search).
- ▶ Given an uninformed search algorithm, explain its space complexity, time complexity, and whether it has any guarantees on the quality of the solution found.
- ▶ Given a scenario, explain whether and why it is appropriate to use an uninformed algorithm.

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Example: Robotics

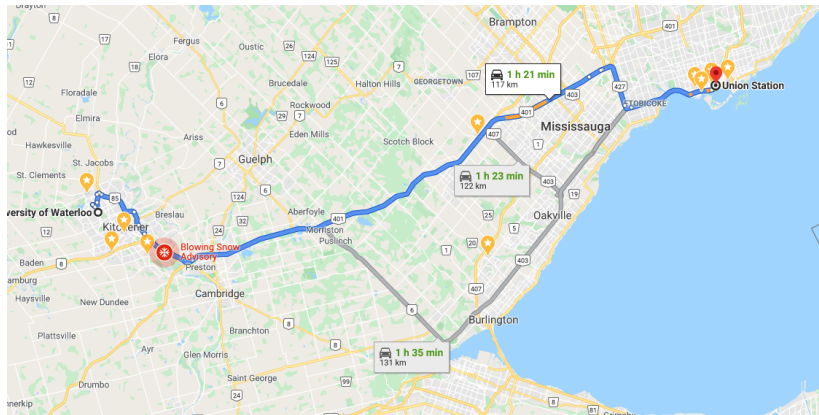


→ Letting Roomba vacuum the house.

Sending a robot to explore the planet Mars.

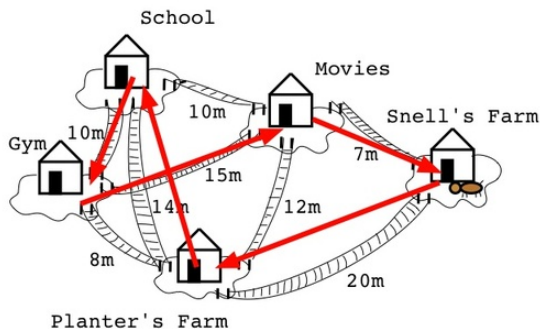
Letting robots handle search and rescue during natural disasters.

Example: Route Planning



Traveling Salesperson Problem

What is the shortest path that starts at city A, visits each city only once, and returns to A?



Applications: DNA sequencing. School bus routes. Parcel pickups.

[See this article for a recent breakthrough.](#)

Propositional Satisfiability

Propositional satisfiability: Given a formula, is there a way to assign true/false to the variables to make the formula true?

$$((((a \wedge b) \vee c) \wedge d) \vee (\neg e))$$

FCC spectrum auction: buy radio spectrums from TV broadcasters and sell them to the telecom companies

Check out [this news article](#) and [this paper](#).

Hua Rong Dao Puzzle



Check out more initial configurations [here](#).

8-puzzle

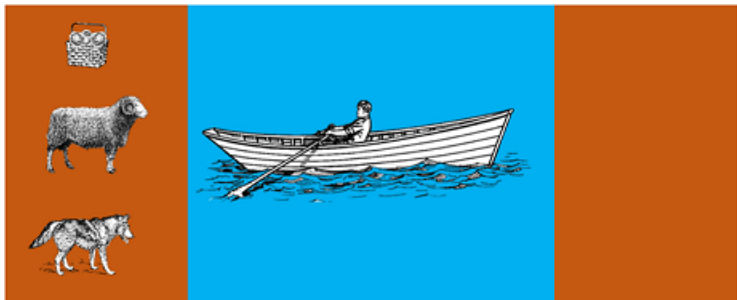
Initial State

5	3	
8	7	6
2	4	1

Goal State

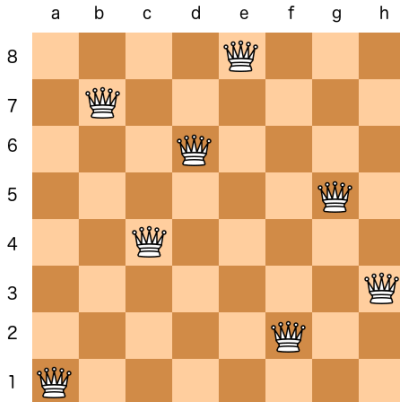
1	2	3
4	5	6
7	8	

A River Crossing Puzzle



N-Queens Problem

The n -queens problem: Place n queens on an $n \times n$ board so that no pair of queens attacks each other.



http://yue-guo.com/wp-content/uploads/2019/02/N_queen.png

Q: Why search?

Q: Why do we use search to solve these problems? Are there efficient algorithms to solve them?

- (A) We use search because there aren't efficient algorithms to solve some of these problems.
- (B) We use search for another reason.
- (C) Search and efficient algorithms are both valid approaches to solve these problems.
- (D) We use search because it works better than efficient algorithms.

Q: Why search?

Q: Why do we use search to solve these problems? Are there efficient algorithms to solve them?

- (A) We use search because there aren't efficient algorithms to solve some of these problems.**
- (B) We use search for another reason.**
- (C) Search and efficient algorithms are both valid approaches to solve these problems.**
- (D) We use search because it works better than efficient algorithms.**

→ All answers are correct!

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Why search?

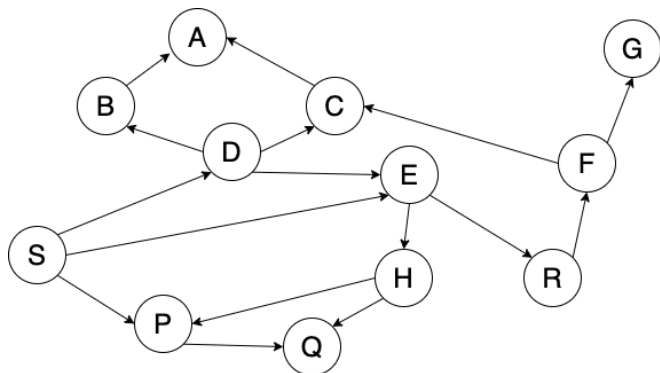
We are facing a difficult problem.

- ▶ Given a description that helps us recognize a solution
- ▶ Not given an algorithm to solve the problem

We have to search for a solution!

→ May remind you of NP-hard problems: Efficient to recognize solutions. No efficient methods for finding solutions. Search is a necessary part of solving these problems.

Graph Searching



A Search Problem

Definition (Search Problem)

A **search problem** is defined by:

- ▶ A set of **states**
- ▶ An **initial state**
- ▶ **Goal states** or a **goal test**
 - ▶ a boolean function which tells us whether a given state is a goal state
- ▶ A **successor (neighbour) function**
 - ▶ an action which takes us from one state to other states
- ▶ (Optionally) a **cost** associated with each action

A solution to this problem is a path from the start state to a goal state (optionally with the smallest total cost).

Example: 8-Puzzle

Initial State

5	3	
8	7	6
2	4	1

Goal State

1	2	3
4	5	6
7	8	

Formulating 8-Puzzle as a Search Problem

- ▶ State:
- ▶ Initial state:
- ▶ Goal states:
- ▶ Successor function:
- ▶ Cost function:

Formulating 8-Puzzle as a Search Problem

- State: $x_{00}x_{01}x_{02}, x_{10}x_{11}x_{12}, x_{20}x_{21}x_{22}$
 x_{ij} is the number in row i and column j . $i, j \in \{0, 1, 2\}$.
 $x_{ij} \in \{0, \dots, 8\}$. $x_{ij} = 0$ denotes the empty square.

Formulating 8-Puzzle as a Search Problem

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- ▶ Initial state: 530, 876, 241.

Formulating 8-Puzzle as a Search Problem

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- ▶ Initial state: 530, 876, 241.
- ▶ Goal states: 123, 456, 780.

Formulating 8-Puzzle as a Search Problem

- ▶ **State:** $x_{00}x_{01}x_{02}, x_{10}x_{11}x_{12}, x_{20}x_{21}x_{22}$
 x_{ij} is the number in row i and column j . $i, j \in \{0, 1, 2\}$.
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- ▶ **Initial state:** 530, 876, 241.
- ▶ **Goal states:** 123, 456, 780.
- ▶ **Successor function:** Consider the empty square as a tile. State B is a successor of state A if and only if we can convert A to B by moving the empty tile up, down, left, or right by one step.

Formulating 8-Puzzle as a Search Problem

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- ▶ **Initial state:** 530, 876, 241.
- ▶ **Goal states:** 123, 456, 780.
- ▶ **Successor function:** Consider the empty square as a tile. State B is a successor of state A if and only if we can convert A to B by moving the empty tile up, down, left, or right by one step.
- ▶ **Cost function:** Each move has a cost of 1.

Q: The successor function

Q: Which of the following is a successor of 530,876,241?

Initial State

- (A) 350,876,241
- (B) 536,870,241
- (C) 537,806,241
- (D) 538,076,241

5	3	
8	7	6
2	4	1

Q: The successor function

Q: Which of the following is a successor of 530,876,241?

Initial State

5	3	
8	7	6
2	4	1

- (A) 350,876,241
- (B) 536,870,241
- (C) 537,806,241
- (D) 538,076,241

→ (B) is correct.

Choosing among multiple formulations

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- ▶ The state definition determines the nodes.
The successor function determines the directed edges.

Choosing among multiple formulations

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- ▶ Ideally, we want to minimize the number of nodes and edges in the graph.

Choosing among multiple formulations

- ▶ The state definition determines the nodes.
The successor function determines the directed edges.
- ▶ Ideally, we want to minimize the number of nodes and edges in the graph.
- ▶ Choosing a state definition may make it easier or harder to implement the successor function.

An alternative state definition for the 8-puzzle:

A state is defined by 8 coordinates.

(x_i, y_i) is the coordinates for tile i where $1 \leq i \leq 8$.

Learning Goals

Applications of Search

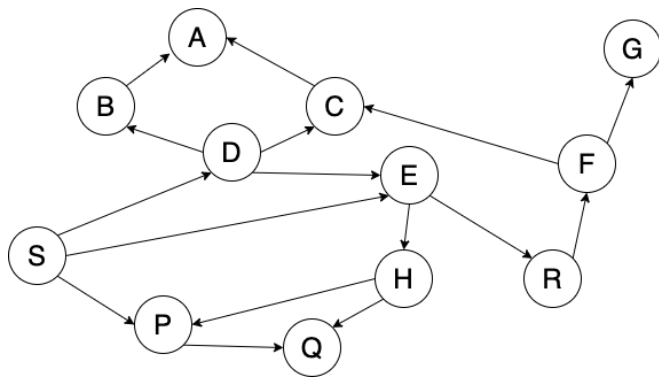
Formulating a Search Problem

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Revisiting the Learning Goals

The Search Graph



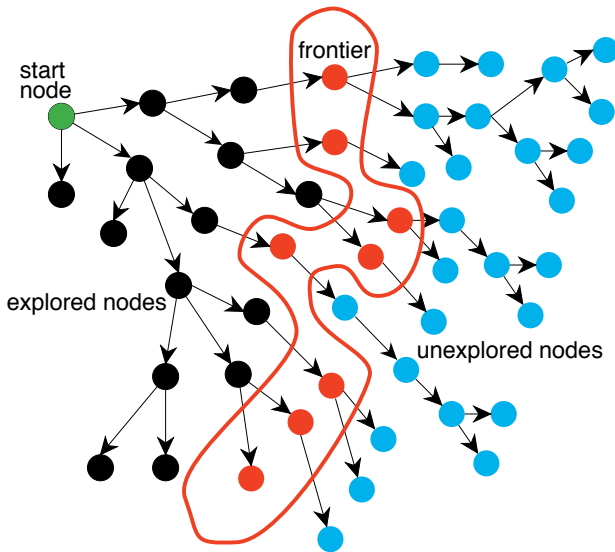
Searching for a Solution

→ A formulation gives us enough info to generate the search graph. However, we often don't generate the search graph explicitly and store it. It may be large or infinite and it is not necessary.

Instead, we will generate a search tree as we explore the search graph.

- ▶ Construct the search tree as we explore paths incrementally from the start node.
- ▶ Maintain a frontier of paths from the start node.
- ▶ Frontier contains all the paths available for expansion.
- ▶ Expanding a path: removing it from the frontier, generating all the neighbours of the last node, and adding the paths ending with each neighbour to the frontier.

The Search Tree



Generic Search Algorithm

Algorithm 1 A Generic Search Algorithm

```
1: procedure SEARCH(Graph, Start node  $s$ , Goal test  $goal(n)$ )
2:   frontier :=  $\{\langle s \rangle\}$ 
3:   while frontier is not empty do
4:     select and remove path  $\langle n_0, \dots, n_k \rangle$  from frontier
5:     if  $goal(n_k)$  then
6:       return  $\langle n_0, \dots, n_k \rangle$ 
7:     for every neighbour  $n$  of  $n_k$  do
8:       add  $\langle n_0, \dots, n_k, n \rangle$  to frontier
9:   return no solution
```

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- Breadth-First Search

- Iterative-Deepening Search

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Depth-First Search

Breadth-First Search

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Depth-First Search

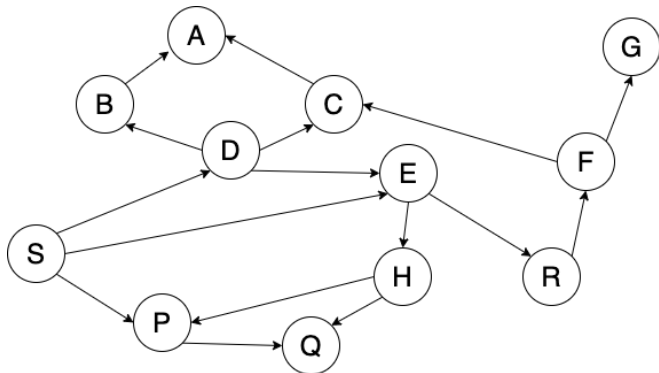
- ▶ Treats the frontier as a stack (LIFO).
- ▶ Expands the last/most recent node added to the frontier.

→ Search one path to completion before trying another path.

Backtracks to another alternative after it has explored all the paths from the first alternative.

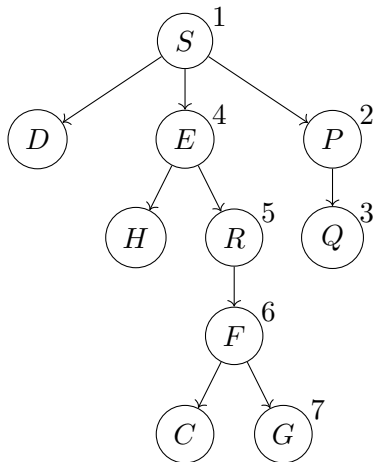
Trace DFS on a Search Graph

- ▶ Trace DFS on the search graph below, with S as the initial state.
- ▶ Add nodes to the frontier in alphabetical order.



Trace DFS on a Search Graph

The final search tree:



Properties of DFS

Properties

- ▶ *Space Complexity*: size of frontier in worst case
- ▶ *Time Complexity*: # nodes visited in worst case
- ▶ *Completeness*: does it find a solution when one exists?
- ▶ *Optimality*: if solution found, is it the one with the least cost?

Useful Quantities

- ▶ b is the branching factor.
- ▶ m is the maximum depth of the search tree.
- ▶ d is the depth of the shallowest goal node.

Properties of DFS - Space Complexity

Space Complexity

Properties of DFS - Space Complexity

Space Complexity

- ▶ $O(bm)$

b is the branching factor.

m is the max depth of the search tree.

- ▶ Linear in m

- ▶ Remembers m nodes on current path and at most b siblings for each node.

Properties of DFS - Time Complexity

Time Complexity

Properties of DFS - Time Complexity

Time Complexity

- ▶ $O(b^m)$
- ▶ Exponential in m .
- ▶ Visit the entire search tree in the worst case.

Properties of DFS - Completeness

Is DFS guaranteed to find a solution if a solution exists?

Properties of DFS - Completeness

Is DFS guaranteed to find a solution if a solution exists?

- ▶ No.
- ▶ Will get stuck in an infinite path.
- ▶ An infinite path may or may not be a cycle.

Properties of DFS - Optimality

Is DFS guaranteed to return an optimal solution if it terminates?

Properties of DFS - Optimality

Is DFS guaranteed to return an optimal solution if it terminates?

- ▶ No.
- ▶ Pays no attention to the costs and makes no guarantee on the solution's quality.

When should we use DFS?

DFS is useful when:

- ▶ Space is restricted.
- ▶ Many solutions exist, perhaps with long paths.

DFS is a poor method when:

- ▶ There are infinite paths.
- ▶ Solutions are shallow.
- ▶ There are multiple paths to a node.

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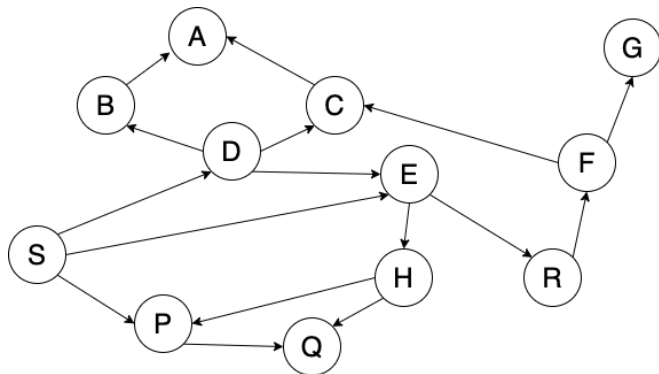
Revisiting the Learning Goals

Breadth-First Search

- ▶ Treats the frontier as a queue (FIFO).
 - ▶ Expands the first/oldest node added to the frontier.
- Select a path with the fewest arcs at each step.

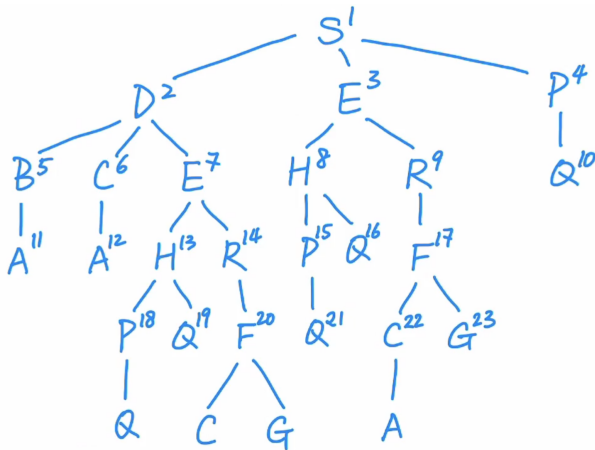
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Properties of BFS - Space Complexity

Space Complexity

Properties of BFS - Space Complexity

Space Complexity

- ▶ $O(b^d)$

b is the branching factor.

d is the depth of the shallowest goal node.

- ▶ Exponential in d .

- ▶ Must visit the top d levels.

Size of frontier is dominated by the size of level d .

Properties of BFS - Time Complexity

Time Complexity

Properties of BFS - Time Complexity

Time Complexity

- ▶ $O(b^d)$
- ▶ Exponential in d .
- ▶ Visit the entire search tree in the worst case.

Properties of BFS - Completeness

Is BFS guaranteed to find a solution if a solution exists?

Properties of BFS - Completeness

Is BFS guaranteed to find a solution if a solution exists?

- ▶ Yes.
- ▶ Explores the tree level by level until it finds a goal.

Properties of BFS - Optimality

Is BFS guaranteed to return an optimal solution if it terminates?

Properties of BFS - Optimality

Is BFS guaranteed to return an optimal solution if it terminates?

- ▶ No.
- ▶ Guaranteed to find the shallowest goal node.

When should we use BFS?

BFS is useful when:

- ▶ Space is not a concern.
- ▶ Want a solution with the fewest arcs.

BFS is a poor method when:

- ▶ All the solutions are deep in the tree.
- ▶ The problem is large and the graph is dynamically generated.

Q: BFS v.s. DFS

Q 1: Suppose that memory is very limited.
Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

Q: BFS v.s. DFS

Q 1: Suppose that memory is very limited.
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- (A) BFS is a better choice.
- (B) **DFS is a better choice.**
- (C) Both are good choices.
- (D) Neither is a good choice.

→ (B) DFS is better.

BFS requires exponential memory.

DFS requires linear memory.

Q: BFS v.s. DFS

Q 2: Suppose that all the solutions are deep in the search tree. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

Q: BFS v.s. DFS

Q 2: Suppose that all the solutions are deep in the search tree. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) **DFS is a better choice.**
- (C) Both are good choices.
- (D) Neither is a good choice.

→ (B) DFS is better.

DFS will explore long paths first and find a solution faster.

BFS will explore shallow nodes first and will likely require more time to find a solution.

Q: BFS v.s. DFS

Q 3: Suppose that the search graph contains cycles. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

Q: BFS v.s. DFS

Q 3: Suppose that the search graph contains cycles.
Which of BFS and DFS would you choose?

- (A) **BFS is a better choice.**
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

→ (A) BFS is better.

DFS may get stuck in cycles.

BFS has no problem with cycles.

Q: BFS v.s. DFS

Q 4: Suppose that the branching factor is large/infinite. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

Q: BFS v.s. DFS

Q 4: Suppose that the branching factor is large/infinite.
Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) **DFS is a better choice.**
- (C) Both are good choices.
- (D) Neither is a good choice.

→ (B) DFS is better.

BFS will be slow for large b and will not terminate with infinite b .
DFS requires linear memory.

Q: BFS v.s. DFS

Q 5: Suppose that we must find the shallowest goal node. Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

Q: BFS v.s. DFS

Q 5: Suppose that we must find the shallowest goal node. Which of BFS and DFS would you choose?

- (A) **BFS is a better choice.**
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

→ (A) BFS is better.

BFS is guaranteed to find the shallowest goal node.

DFS will likely find a deep goal node first.

Q: BFS v.s. DFS

Q 6: Suppose that all the solutions are very shallow.
Which of BFS and DFS would you choose?

- (A) BFS is a better choice.
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

Q: BFS v.s. DFS

Q 6: Suppose that all the solutions are very shallow.
Which of BFS and DFS would you choose?

- (A) **BFS is a better choice.**
- (B) DFS is a better choice.
- (C) Both are good choices.
- (D) Neither is a good choice.

→ (A) BFS is better.

BFS is guaranteed to find the shallowest goal node.

DFS will likely find a deep goal node first.

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Combining The Best of BFS and DFS

Can we create a search algorithm
that combines the best of BFS and DFS?

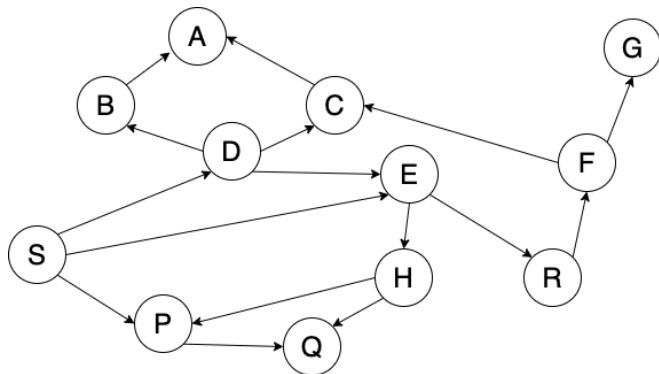
BFS	DFS
$O(b^d)$ exponential space	$O(bm)$ linear space
Guaranteed to find a solution if one exists	May get stuck on infinite paths

Iterative-Deepening Search:

For every depth limit,
perform depth-first search until the depth limit is reached.

Trace IDS on a Search Graph

- ▶ Trace IDS on the search graph below.
- ▶ Add nodes to the frontier in alphabetical order.



Trace IDS on a Search Graph

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Properties of IDS - Space Complexity

Space Complexity

Properties of IDS - Space Complexity

Space Complexity

- ▶ $O(bd)$

b is the branching factor.

d is the depth of the shallowest goal node.

- ▶ Linear in d .
Similar to DFS.

- ▶ Executes DFS for each depth limit.
Guaranteed to terminate at depth d .

Properties of IDS - Time Complexity

Time Complexity

Properties of IDS - Time Complexity

Time Complexity

- ▶ $O(b^d)$
- ▶ Exponential in d .
Same as BFS.
- ▶ Visits all the nodes on the top d levels in the worst case.

Properties of IDS - Completeness

Is IDS guaranteed to find a solution if a solution exists?

Properties of IDS - Completeness

Is IDS guaranteed to find a solution if a solution exists?

- ▶ Yes.
Same as BFS.
- ▶ Explores the tree level by level until it finds a goal.

Properties of IDS - Optimality

Is IDS guaranteed to return an optimal solution if it terminates?

Properties of IDS - Optimality

Is IDS guaranteed to return an optimal solution if it terminates?

- ▶ No.
- ▶ Guaranteed to find the shallowest goal node.
Same as BFS.

A Summary of IDS Properties

- ▶ Space Complexity:

$O(bd)$, linear in d . Similar to DFS.

- ▶ Time Complexity:

$O(b^d)$, exponential in d . Same as BFS.

- ▶ Completeness:

Yes. Same as BFS.

- ▶ Optimality:

No, but guaranteed to find the shallowest goal node.
Same as BFS.

Revisiting the learning goals

By the end of the lecture, you should be able to

- ▶ Formulate a real world problem as a search problem.
- ▶ Trace the execution of and implement uninformed search algorithms (Breadth-first search, Depth-first search, Iterative-deepening search).
- ▶ Given an uninformed search algorithm, explain its space complexity, time complexity, and whether it has any guarantees on the quality of the solution found.
- ▶ Given a scenario, explain whether and why it is appropriate to use an uninformed algorithm.