AST5220 – Milestone I

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1 Theory

1.1 Components of the universe

We consider a flat, expanding universe, goverened by the ΛCDM model. Our universe contains some densities of baryonic matter (ρ_b), cold dark matter (ρ_{CDM}), radiation (ρ_r), and dark energy (ρ_Λ). Let $\Omega_i = \dfrac{\rho_i}{\rho_c}$ be the *relative densities* of each component. Here, ρ_c is the *critical density*, being the total density which would make the universe entirely flat. Since our universe is indeed flat, is follows that

$$\sum_i \rho_i = \rho_c \quad \Rightarrow \quad \sum_i \Omega_i = 1$$

We also define $\rho_{i,o}$ and $\Omega_{i,o}$ to represents the critical and relative densities today.

It can be shown that the density components evolve with time as $\rho_i(a) = \rho_{i,0} a^{-3(1+w_i)}$ where w_i is some constant for each component. For our four compoenents, we have that

$$\begin{aligned}
\rho_b &= \rho_{b,0} a^{-3} \\
\rho_{\text{CDM}} &= \rho_{\text{CDM},0} a^{-3} \\
\rho_r &= \rho_{r,0} a^{-4} \\
\rho_\Lambda &= \rho_{\Lambda,0}
\end{aligned}$$

where a is the *scale factor* of the universe, describing its relative size to today, which is defined to be $a_0 = 1$.

We wish to avoid the densities where possible, and work directly with the relative densities and the scale factor. The relative densities can be rewritten to exclude ρ_i the following way.

$$\Omega_i = \frac{\rho_i}{\rho_c} = \frac{\rho_{i,0} a^{-3(1+w_i)}}{\rho_c} = \frac{\rho_{c,0}\Omega_{i,0} a^{-3(1+w_i)}}{\rho_c}$$

Knowing that $\rho_c = \frac{3H^2}{8\pi G}$, which gives $\rho_{c,0} = \frac{3H_0^2}{8\pi G}$ we get that

$$\Omega_i = \left(\frac{H_0}{H}\right)^2 \Omega_{i,0} a^{-3(1+w_i)}$$

1.2 The Friedmann equation

The evolution of the scale factor is goverened by the (first) Friedmann equation, which for the universe described above reads

$$H(a) = \frac{\dot{a}}{a} = H_0 \sqrt{\Omega_{b,0} a^{-3} + \Omega_{\text{CDM},0} a^{-3} + \Omega_{r,0} a^{-4} + \Omega_{\Lambda,0}} \tag{1}$$

Since the universe takes on scales of many different orders of magnitude, the linear scale factor a is not always well suited for analysis of large time spans. We introduce time(and scale) quantity

$$x = \log a \quad \Rightarrow \quad a = e^x \tag{2}$$

The Friedmann equation now reads

$$H(x) = H_0\sqrt{\Omega_{b,0}e^{-3x} + \Omega_{\mathrm{CDM},0}e^{-3x} + \Omega_{r,0}e^{-4x} + \Omega_{\Lambda,0}} \tag{3}$$

We also introduce the scaled Hubble parameter $\mathcal{H} = aH = \dot{a}$.

1.3 Conformal Time

As well as a, x, and t for measuring time in the universe, we will introduce the conformal time η. η has units of length, and represents the size of the event horizon at any given time. In other words, η is the distance traversed by undisturbed light since the Big Bang.

[Insert definition of Conformal time]

2 Results

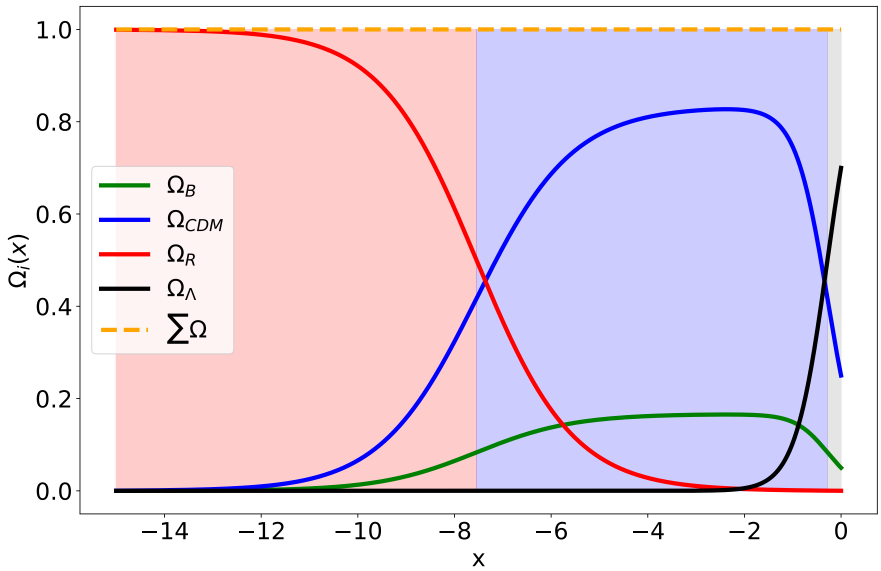

Figure 1:

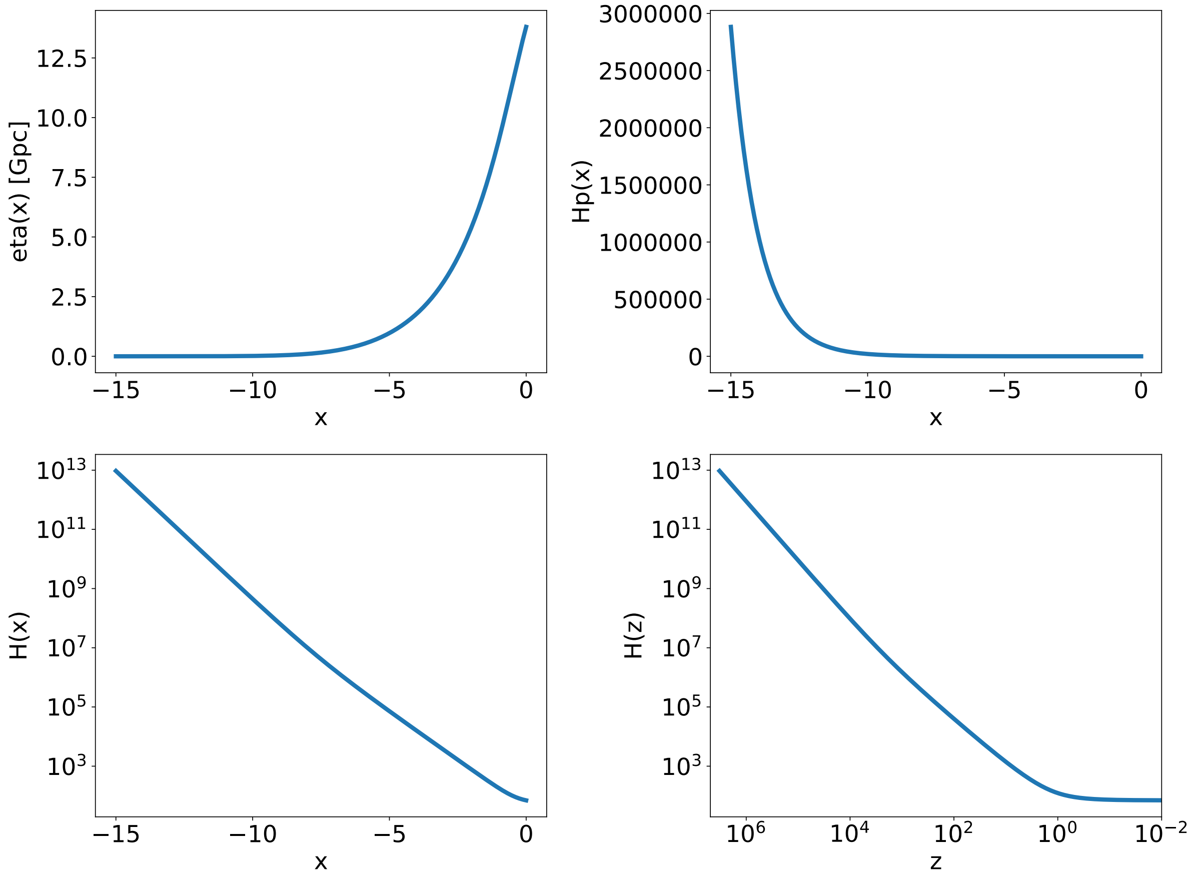

Figure 2: