The Hubble Constant

March 18, 2019

Overview

In this lab, you'll measure the Hubble Constant, H_0, by building a hierarchical model for the distance ladder and ultimately using SNe Ia as standard candles. The data and general method we'll use is described in Riess et al. 2016 (R16) and Riess et al. 2009.

Although we'll use the same data as Riess et al. (and thus had better get a consistent value of H_0), we'll use a hierarchical Bayesian approach instead of the maximum likelihood approach they use. This means less juggling of large covariance matrices, and free quantification of the degeneracies between H_0 and other parameters.

Problem

0) Read the following papers and be prepared to discuss on Monday 4/1/19: Riess et al. 2016 (R16) and Riess et al. 2009.

a) Download the Cepheid data in Table 4 of R16. Discard the data for Cepheids in M31. Why are we not using the M31 data?

b) For each of the remaining 20 galaxies, plot the Cepheid period vs. apparent "Wesenheit" magnitude relation. First you'll need to calculate the Wesenheit magnitude. Explain what the point of Wesenheit magnitudes is. Why don't we just use F160W?

Color the Cepheids in each galaxy by their metallicity. The zeropoints of the relation will be different in each galaxy, since the galaxies are at different distances. Comment on any trends with metallicity.

c) The galaxy NGC 4258 is particularly useful to us because its distance is known very precisely. Explain physically how we know this galaxy's distance. That is, what quantity was measured, and how was this measurement used to calculate the galaxy's distance? We can't we do this for all galaxies?

Select just the Cepheids in NGC 4258. Fit their period–apparent Wesenheit magnitude relation, in the form

$$m_H^W = \mathrm{zp}_{\mathrm{W,NGC\,4258}} + b_W \log P + Z_W \Delta \log(\mathrm{O/H}). \tag{1}$$

Here m_H^W is the apparent Wesenheit magnitude, P is the period in days, and $\Delta \log(\mathrm{O/H})$ is the metallicity with respect to a reference value. You can calculate $\Delta \log(\mathrm{O/H})$ from the data in Table 4 using the Solar oxygen abundance, $\log(\mathrm{O/H}) = 8.66$, as the reference. $\mathrm{zp}_{\mathrm{W,NGC\,4258}}$, b_W, and Z_W are free parameters to be fit. In addition to these, also fit for the intrinsic scatter in the relation.

Later in this problem, you will have to sample from a model in about 25 dimensions. This is sufficiently high dimensionality that `emcee`, the ensemble sampler we've used in the past, will not work well. Achieving convergence with `emcee` would take an impractically large number of steps. More dangerously, samplers like `emcee` are liable to produce trace plots in high-dimensional problems that make it look very much like they have converged, long before they really have.

Instead of using `emcee`, we'll implement and sample models in Stan, which is a powerful language designed for hierarchical statistical modeling. It is implemented in C++ and is very fast. Stan uses a Hamiltonian Monte Carlo sampler (specifically, a "No-U-Turn-Sampler"), which offers several advantages over traditional sampling methods. The easiest way for you to implement a Stan model for this problem is to use the python package pystan.

As a warm up, use Stan to measure the period-apparent magnitude relation for Cepheids in NGC 4258. Show a corner plot of your marginalized constraints and their covariances.

Hint: because Stan uses somewhat different syntax from what you are used to, it is a good idea to first try a toy problem (such as fitting a line) to make sure that you get consistent answers between Stan and emcee.

Another Hint: Because it is intentionally highly parallelized, Stan may try to use all your available memory by running on many cores at once. This is probably fine if you are running on your own computer, but it will cause problems if you are running on Datahub. You can prevent this by forcing Stan to run on a single core.

d) Now build a model to determine the Cepheid period–luminosity relation using data from all galaxies simultaneously, and fit it using Stan. Your model should be hierarchical, based on the following to assumptions: (i) the period – absolute magnitude relation is the same in all galaxies (when the metallicity trend is accounted for), and (ii) while the distances to different galaxies are different, all Cepheids in the same galaxy have the same distance.

Equation 2 of R16 provides the model to be fit. The distance moduli of individual galaxies *relative to that of NGC 4258* should be treated as nuisance parameters.

Make a (very large) corner plot showing the constraints you obtain.

e) Compare the constraints on the 4 free parameters of the period-luminosity relation obtained in (d) to those obtained in (c) for NGC 4258 alone by overplotting the error ellipses on a single corner plot. You should find that the constraint are consistent, but that those from part (d) are significantly tighter.

f) Compare the distance moduli from your fit to those in Table 5 of R16. Also compare the uncertainties on each distance modulus. Are your values consistent with those found by Riess et al? Comment on any inconsistencies.

g) Type 1a SNe are not strictly standard candles, but they are "standardizable". What this means is, roughly, that variations in their intrinsic brightness largely follow well-understood trends with color and light curve morphology. Riess et al. have already carried out standardization for you (you're welcome). Summarize how this standardization is done. The papers by Guy et al. (2005, 2010) provide useful references.

Standardized B-band apparent magnitudes for SNe 1a in galaxies with known Cepheids are provide in Table 5.

Use them, together with the known distance modulus of NGC 4258, to determine the standardized absolute, B-band magnitude of a Ia SN, $M_{B,0}^{\rm Ia}$. Do this by adding $M_{B,0}^{\rm Ia}$ to you hierarchical model as a free parameter, and again fitting the model using Stan. Your should now have a single model that takes the Cepheid and Ia data as inputs and fits them simultaneously.

Your model should use the following assumptions: (i) the Cepheid period – absolute magnitude relation is the same in all galaxies (when the metallicity trend is accounted for), (ii) all Cepheids in the same galaxy have the same distance, (iii) all SNe Ia have exactly the same standardized absolute magnitude, (iv) SNe Ia and Cepheids in the same galaxy have the same distance, and (v) the distance modulus to NGC 4258 is known.

h) Finally, you can calculate H_0 from $M_{B,0}^{\mathrm{Ia}}$ using Equation (4) of R16. Undergrads can take the value of a_B, the intercept of the B-band SN Ia magnitude-redshift relation, from the text. This value, and its uncertainty, are obtained by fitting data from higher-redshift SNe Ia, which are not provided in R16.

Grad students only: rather than assuming the value of a_B reported in R16, download the actual data in Figure 8 (you will need to trace it back to earlier papers), and include it as an ingredient in your hierarchical Stan model. That is, a_B should be a nuisance parameter that gets marginalized over. Compare your constraints on a_B to the value reported in R16.

Compare your final estimate of H_0, and its uncertainty, to the value obtained by Riess et al. using the same data. In particular, overplot your marginalized pdf of H_0 and the constraints from R16, assuming their error bars are Gaussian. Also overplot the constraints from Planck (2016).

i) Reflect on the assumptions you made throughout your analysis and the sources of error accounted for in your estimate of H_0. If there are any differences between your results and those obtained by Riess et al., what might account for them?

The value of H_0 obtained from the local distance ladder (what we did in this lab) is inconsistent with value obtained from fitting the CMB, which is $H_{0,\,\mathrm{Planck}} = (66.93 \pm 0.62)\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, at the 3.4σ level. Based on this lab, do you think this tension is the result of physics beyond the standard ΛCDM cosmological model, underestimated uncertainties, or something else? Explain. If you suspect underestimated uncertainties are to blame, what are the likely culprits?