Ordinary Differential Equations

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                              ( ODEs )
                           /            \
                  ( 1st Order )        ( 2nd Order )
                 /          \         /           \
      ( Separable )    ( Linear )( Homogeneous )   ( Inhomogeneous )
                          |        /        \        /          \
                 ( Constant coeff. ) ( Euler-Cauchy )( Constant coeff. ) ( Factorization )( Variation of
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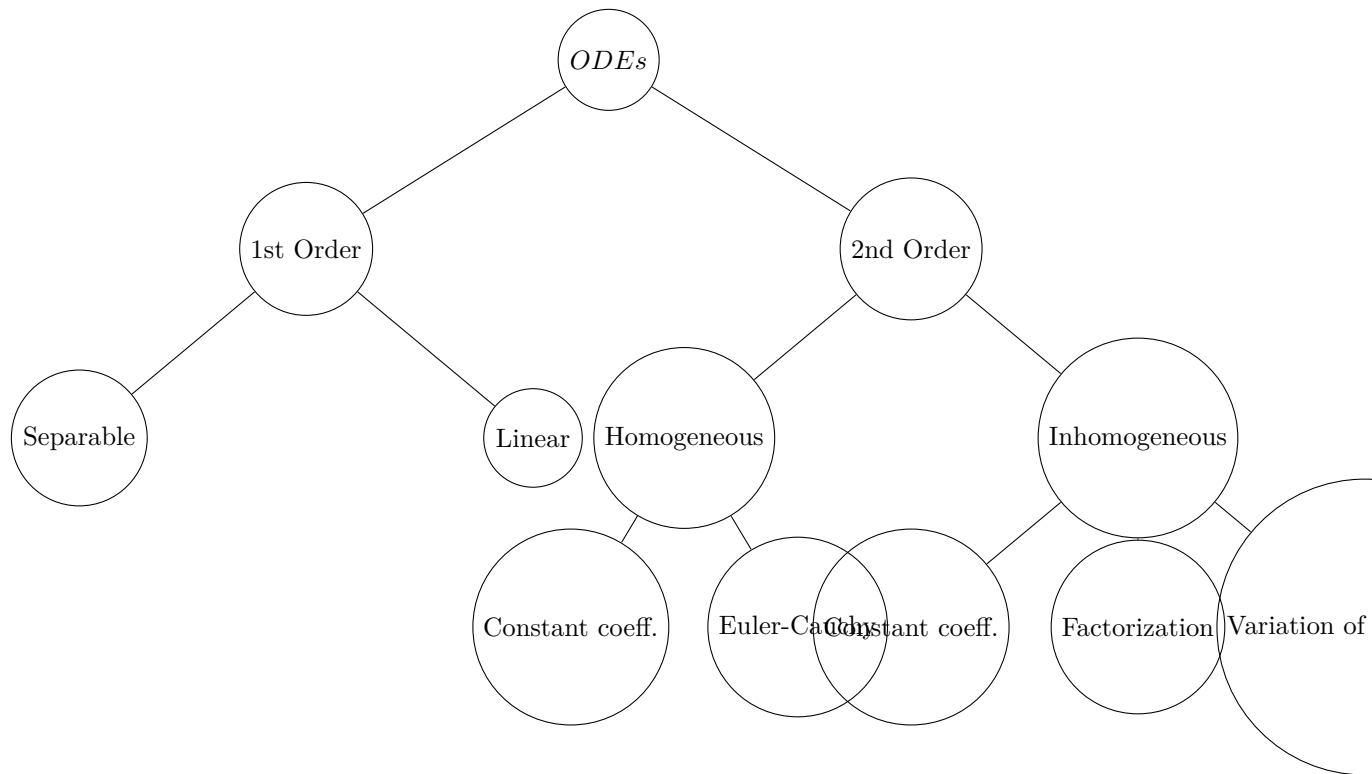
First Order, Linear, ODEs - Integrating Factor

$$y'(x) + P(x)y(x) = Q(x)$$

$$y(x)\mu(x) = \int Q(x)\mu(x)\,\mathrm{d}x + C \qquad \text{with} \qquad \mu(x) = e^{\int P(x)\mathrm{d}x}$$

$$\int (uv') = uv - \int (u'v)$$

Second Order, Homogenous, Linear ODEs, with constant coefficients - Particular Equation

$$y''(x) + ay'(x) + by(x) = 0$$

Solve the particular equation

$$\lambda^2 + a\lambda + b = 0$$

for λ_1 and λ_2.

Two, real roots

$$y(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$$

One, real root

$$y(x) = (C_1 + xC_2)e^{\lambda x}$$

Two, complex roots

$$y(x) = Ae^{\lambda_1 x} + Be^{\lambda_2 x} = e^{-a/2x}\left[Ae^{i\omega x} + Be^{-i\omega x}\right] = e^{-a/2x}\left[\hat{A}\cos\omega x + \hat{B}\sin\omega x\right]$$