OBLIG 2

1a) $F_X(x) = \int_K^x f_X(x)\,dx = \int_K^x \theta K^\theta x^{-\theta-1}\,dx$

$$= \left[\frac{\theta K^\theta x^{-\theta}}{-\theta}\right]_K^x = \left[-K^\theta x^{-\theta}\right]_K^x = -K^\theta x^{-\theta} + K^\theta K^{-\theta}$$

$$= 1 - K^\theta x^{-\theta}$$

• Median er gitt som

$$F_X(x) = \frac{1}{2} = 1 - K^\theta x^{-\theta}$$

$$K^\theta x^{-\theta} = \frac{1}{2}$$

$$x^\theta = 2K^\theta$$

$$\sqrt[\theta]{x^\theta} = \sqrt[\theta]{2K^\theta}$$

$$x = 2^{1/\theta} K$$

b) $E[X] = \int_K^\infty x f_X(x)\,dx = \int_K^\infty \theta K^\theta x^{-\theta}\,dx = \theta K^\theta \left[\frac{x^{-\theta+1}}{-\theta+1}\right]_K^\infty$

$$= \frac{\theta K^\theta}{1-\theta}\left[\infty^{-\theta+1} - K^{-\theta+1}\right] = -\frac{\theta K^\theta K^{-\theta+1}}{1-\theta}$$

$$= \frac{\theta K}{\theta - 1}$$

c) $\theta = 3 \; ; \quad K = 400'000$

$$M[X] = 2^{1/\theta} K = 2^{1/3} \cdot 400'000 \approx 504'000$$

$$E[X] = \frac{\theta K}{\theta - 1} = \frac{3 \cdot 400'000}{2} = 600'000$$

d) $V[X] = E[X^2] - E[X]^2$

$$E[X^2] = \int_K^\infty x^2 f(x)\, dx = \theta K^\theta \int_K^\infty x^{-\theta+1}\, dx = \theta K^\theta \left[\frac{x^{-\theta+2}}{-\theta+2} \right]_K^\infty$$

$$= \frac{\theta K^\theta}{2-\theta} \left[\infty^{-\theta+2} - K^{-\theta+2} \right] = -\frac{\theta K^\theta K^{-\theta+2}}{2-\theta} = \frac{\theta K^2}{\theta - 2}$$

$$V[X] = \frac{\theta K^2}{\theta-2} - \frac{\theta^2 K^2}{(\theta-1)^2} = \frac{(\theta-1)^2 \theta K^2 - \theta^2 K^2 (\theta - 2)}{(\theta-2)(\theta-1)^2}$$

$$= \frac{\theta^3 K^2 - 2\theta^2 K^2 + \theta K^2 - \theta^3 K^2 + 2\theta^2 K^2}{(\theta-2)(\theta-1)^2} = \frac{\theta K^2}{(\theta-2)(\theta-1)^2}$$

$$= \frac{3 \cdot (400\,000)^2}{(3-2)(3-1)^2} = 1.2 \times 10^{11}$$

$$\sigma_x = \sqrt{V[X]} \approx 346'410$$

e) $f_Y(y) = f_X(h(y))\,|h'(y)|$

hvor $h(y)$ er den inverse av $\theta \log(X/K)$

altså:

$$Y = \theta \log(X/K) \Rightarrow \frac{X}{K} = e^{Y/\theta} \Rightarrow X = K e^{Y/\theta}$$

som gir

$$h(Y) = K e^{Y/\theta} \Rightarrow |h'(Y)| = \left|\frac{K}{\theta} e^{Y/\theta}\right|$$

slik at

$$f_Y(y) = \theta K^{\theta} \left[K e^{Y/\theta}\right]^{-\theta-1} \left|\frac{K}{\theta} e^{Y/\theta}\right|$$

$$= \theta K^{\theta} K^{-\theta-1} e^{(Y/\theta)\cdot(-\theta-1)} \cdot \frac{K}{\theta} e^{Y/\theta}$$

$$= e^{-y}.$$

2 a) $1 = \int_0^1 \int_0^x k(x-y)\,dy\,dx = k\int_0^1 \left[xy - \frac{1}{2}y^2\right]_0^x dx = k\int_0^1 \frac{1}{2}x^2\,dx$

$= k\left[\frac{1}{6}x^3\right]_0^1 = \frac{k}{6} = 1 \Rightarrow \underline{\underline{k = 6}}$

b) $P(2Y \leq X) = \int_0^1 \int_0^{x/2} 6(x-y)\,dy\,dx = 6\int_0^1 \left[xy - \frac{1}{2}y^2\right]_0^{x/2} dx$

$= 6\int_0^1 \frac{3}{8}x^2\,dx = 6\left[\frac{3}{8}\frac{x^3}{3}\right]_0^1 = 6\left[\frac{1}{8}\right] = \frac{6}{8}$

c) $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^x 6(x-y)\,dy = \left[6(xy - \frac{1}{2}y^2)\right]_{y=0}^{y=x}$

$= 3x^2$ for $0 \leq x \leq 1$

d) $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_y^1 6(x-y)\,dx = \left[6(\frac{1}{2}x^2 - xy)\right]_{x=y}^{x=1}$

$= \left[(3-6y) - (3y^2 - 6y^2)\right] = 3y^2 - 6y + 3$

e) $f_X(x) \cdot f_Y(y) = 9x^2(1-2y) \neq f(x,y)$

De er <u>ikke</u> uavhengige.

3 a) $P(X \leq x) = P(F^{-1}(U) \leq x) = P(U \leq F(x)) = F(x)$

$$F_X(x) = 1 - K^{\theta} x^{-\theta}$$

$$u = 1 - K^{\theta} F_X^{-1}(u)^{-\theta}$$

$$F_X^{-1}(u)^{-\theta} = \frac{1-u}{K^{\theta}}$$

$$X = F_X^{-1}(u) = \sqrt[\theta]{\frac{K^{\theta}}{1-u}} \qquad u \in \text{uniform } [0,1]$$

$$= \frac{K}{\sqrt[\theta]{1-u}}$$

b) Generer uniforme U-verdier i [0, 1], sett de inn i formelen over.

c) Median = 504'200

Gjennomsnitt = 600'373