University of Toronto at Scarborough

CSC A67/MAT A67 - Discrete Mathematics, Fall 2017

Exercise #3: Predicate Logic

Due: October 1, 2017 at 11:59 p.m. This exercise is worth 3% of your final grade.

Warning: Your electronic submission on MarkUs affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSC A67/MAT A67.

Late exercises will not be accepted.

1. Define the following predicates: [6]

$$R(x) : \text{“}x \text{ is a right angle triangle.”}$$
$$O(x) : \text{“}x \text{ has an obtuse angle.”}$$

   Now consider the following statments:

$$S = \neg\exists x(R(x) \land O(x))$$
$$T = \forall x(R(x) \to \neg O(x))$$

   (a) Write S in ordinary English.

   (b) Write T in ordinary English.

   (c) Prove that $S \iff T$.

2. For each set of sentences, define the domain X, the value of $a \in X$ (for part b), and the predicates $A(x)$ and $B(x)$ such that the last sentence is false and the other sentences are true. [4]

   REQUIREMENT: $|X| \leq 2$ (this means the size of the domain X must be less than or equal to 2).

   (a)

(T)	$\forall x \in X, A(x) \to B(x)$
(F)	$\exists x \in X, A(x) \land B(x)$

   (b)

(T)	$\forall x \in X, A(x) \to B(x)$
(T)	$\neg A(a)$
(F)	$\neg B(a)$

3. You may recall from Calculus the definition of the limit of a sequence a_n. We say that the limit of the sequence a_n as n goes to infinity equals L and write: [5]

$$\lim_{n\to\infty} a_n = L$$

We can write this using quantifiers as follows:

$$\forall \epsilon > 0, \exists N \in \mathbb{Z}, \forall n \in \mathbb{Z}, n > N \to L - \epsilon < a_n < L + \epsilon. \tag{1}$$

Explain in words what the negation of this definition means. Now write the negation of (1). If you have never seen this definition don't worry, you can still answer the question!

4. Prove [5]

$$\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, (a|b \wedge a|c) \to a|(2b - 3c)$$

Note that $x|y$ means that x *divides* y, or equivalently that there exists some n such that $x \cdot n = y$.

5. Prove [5]

$$\forall n \in \mathbb{Z}, n \text{ is odd} \to \exists m \in \mathbb{Z}, n^2 = 8m + 1$$

[Total: 25 marks]