zhan5263. CSCA67 Exercise#3.

Exercise 3.

1.(a) There does not exist x that is both a right angle triangle ~~and~~ or has an obtuse angle.

(b). ~~There~~ For all values of x, if x is a right angle triangle, then x does not has an obtuse angle.

(c) T: $\forall x (R(x) \rightarrow \neg O(x)) \Rightarrow \forall x(\neg R(x) \vee \neg O(x))$

$\Rightarrow \forall x \neg (R(x) \wedge O(x))$.

$= \neg \exists x (R(x) \wedge O(x)) = S$.

2. a) $A(x)$: $-\frac{1}{4} \leq x \leq \frac{1}{4}$

$B(x)$: $-\frac{1}{2} \leq x \leq \frac{1}{2}$

$x \in [-\frac{1}{2}, \frac{1}{2}]$

b) $A(x)$: $-\frac{1}{4} \leq x \leq \frac{1}{4}$

$B(x)$: $-\frac{1}{2} \leq x \leq \frac{1}{2}$

$a = \frac{3}{8}$

$x \in [-\frac{1}{2}, \frac{1}{2}]$

3. Original: $\forall \varepsilon > 0, \exists N \in \mathbb{Z}, \forall n \in \mathbb{Z}, n > N \rightarrow L - \varepsilon < a_n < L + \varepsilon$

Negation: $\exists \varepsilon > 0, \forall N \in \mathbb{Z}, \exists n \in \mathbb{Z}, (n > N) \cap a_n \in (-\infty, L-\varepsilon] \cup [L+\varepsilon, +\infty)$

Explination: There exist ε which is bigger than 0,

for every possible value of N which is integer,

there exist n which is a integer,

Satisfy both n bigger than N

and a_n ~~belong~~ either smaller or equal to $(L-\varepsilon)$

or bigger or equal to $(L+\varepsilon)$

4. Prove: $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, (a|b \wedge a|c) \Rightarrow a|(2b-3c)$

Proof: by defination of $a|b$ & $a|c$,

let $b = an_1$, $c = an_2$ $(n_1, n_2 \in \mathbb{Z})$

$\therefore 2b-3c = 2an_1 - 3an_2 = a(2n_1 - 3n_2)$

as $n_1, n_2 \in \mathbb{Z}$, $\therefore 2n_1 - 3n_2 \in \mathbb{Z}$.

$\therefore a | a(2n_1 - 3n_2)$

$\therefore a | (2b-3c)$

5. Prove: $\forall n \in \mathbb{Z}$, n is odd $\Rightarrow \exists m \in \mathbb{Z}, n^2 = 8m+1$.

Proof: as n is odd,

let $n = 2k+1$ $(k \in \mathbb{Z})$

$\therefore n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 4k(k+1) + 1$.

as k is integer, there must be an even number between k or $k+1$. This means that either

$k = 2p$ $(p \in \mathbb{Z})$

or $k+1 = 2p$ $(p \in \mathbb{Z})$.

if $k = 2p$, then $n^2 = 4 \cdot 2p \cdot (k+1) + 1$

$= 8p(k+1) + 1$

as $p \in \mathbb{Z}$, $k \in \mathbb{Z}$, $\therefore p(k+1) \in \mathbb{Z}$. $\therefore p(k+1) = m$.

$\therefore n^2 = 8m+1$.

if $k+1 = 2p$, then $n^2 = 4 \cdot 2p \cdot k + 1$

$= 8pk + 1$.

as $p \in \mathbb{Z}$, $k \in \mathbb{Z}$, $\therefore pk \in \mathbb{Z}$ $\because m \in \mathbb{Z}$, $\therefore m = pk$.

$\therefore n^2 = 8m+1$.

Therefore for any value of n which is odd integer, there exist m which is integer satisfies that $n^2 = 8m+1$.