CSCA67 Exercise 8.

1. $y_1+y_2+y_3+y_4=7$

$C(7+4-1, 7) = C(10,7)$

$0 \le y_1 \le 4,$

$0 \le y_2 \le 3,$

$0 \le y_3 \le 5$

$0 \le y_4 \le 5.$

if $y_1 \ge 5$, $y_1+y_2+y_3+y_4=7$

$z_1+y_2+y_3+y_4=2$

$C(2+4-1, 2) = C(5,2)$

if $y_2 \ge 4$. $y_1+z_2+y_3+y_4=3$.

$C(3+4-1, 3) = C(6,3)$

if $y_3 \ge 6$, $y_1+y_2+z_3+y_4=1$

$C(1+4-1, 1) = C(4,1)$

if $y_4 \ge 6$, $y_1+y_2+y_3+z_4=1$

$C(1+4-1, 1) = C(4,1)$

$C(10,7) - C(5,2) - C(6,3) - C(4,1) - C(4,1) = 82.$

2. $\binom{72+5-1}{72} = 1282975$

3. $y_1+y_2+y_3+y_4+y_5 = 62$

$\binom{62+5-1}{62} = 720720$

4. a) 16 Samples space:

$S = \{[TTTT], [TTTH], [TTHT], [TTHH]$

$[THTT], [THTH], [THHT], [THHH],$

$[HTTT], [HTTH], [HTHT] [HTHH],$

$[HHTT], [HHTH], [HHHT], [HHHH]\}$

b) There are 6 elements at exactly 2 heads

$$\frac{6}{16} = 0.375$$

There are 11 elements at least 2 tails

$$\frac{11}{16} = 0.6875$$

5. There are 2^{100} tuples in the sample space, so it is infeasible to write it.

{ There are $\binom{100}{2}$ ways of flipping exactly 2 heads

$$P = \frac{\binom{100}{2}}{2^{100}}$$

{ using indirect method:

$$P(0 \text{ tail}) = \frac{1}{2^{100}}$$

$$P(1 \text{ tail}) = \frac{\binom{100}{1}}{2^{100}}$$

$$1 - P(0 \text{ tail}) - P(1 \text{ tail}) = P(\text{at least 2 tails})$$

$$= 1 - \frac{1}{2^{100}} - \frac{\binom{100}{1}}{2^{100}}$$

$$= \frac{2^{100} - 101}{2^{100}}$$