CSCA67 Tutorial, Week 6

1 Pigeon Hole Principle

Recall that the Pigeon Hole Principle (PHP) says that

> *If $k+1$ or more pigeons are distributed among k pigeonholes, then at least one pigeonhole contains two or more pigeons.*

Proving things with the Pigeonhole Principle.

There are four steps involved.

1. Decide what the pigeons are. They will be the things that you would like several of, to have some special property.

2. Set up the pigeonholes. You want to do this so that when you get two pigeons in the same pigeonhole, they have the property you want. To use the Pigeonhole Principle, it is necessary to set things up so that there are fewer pigeonholes than pigeons. Sometimes the way to do this relies on some astute observation.

3. Give a rule for assigning the pigeons to the pigeonholes. It is important to note that the conclusion of the Pigeonhole Principle holds for any assignment of pigeons to pigeonholes, so it holds for any assignment you describe. Pick the rule so that when ?enough? pigeons occupy the same pigeonhole, that collection has the property you want.

4. Apply the Pigeonhole Principle to your setup and get the desired conclusion.

Example.

Prove that if seven distinct numbers are selected from $\{1, 2, ..., 11\}$, then some two of these numbers sum to 12.

Example

Prove that if five points are selected from the interior of a 1×1 square, then there are two points whose distance is less than $2\sqrt{2}$.

Exercises.

1. Prove that if four points are selected from the interior of a unit circle, then there are two points whose distance apart is less than $\sqrt{2}$.

2. How many points must be selected from the interior of an equilateral triangle of side two in order to guarantee that there are two points whose distance apart is less than than one?

Example.

Over a 44 day period, Gary will train for triathlons at least once per day, and a total of 70 times in all. Show that there is a period of consecutive days during which he trains exactly 17 times.