CSCA67.

Exercise 4. zhan5263.

1. Prove: $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, a \nmid bc \Rightarrow a \nmid b$.

a). contradition: $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, a \nmid b \Rightarrow a \nmid bc$.

This means that for all a, b. c which is integer, if a does not divide b then a does not divide bc.

assume $b = ak_1 + m$, $k_1 \in \mathbb{Z}$, $a \nmid m$.

$c = ak_2$ $k_2 \in \mathbb{Z}$.

then $a \nmid b$,

$bc = (ak_1 + m)ak_2 = a(ak_1+m)k_2$,

which is divisible by a. ($(ak_1+m)k_2$ is integer)

Therefore the contradition statement is wrong, the original statement is true.

b). contrapositive: $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, a | b \Rightarrow a | bc$.

This means that for all a. b. c which is integer. if a divide b then a divide b.c.

There are 2 situations. assume $b = ak_1$ ($k_1 \in \mathbb{Z}$)

① c is divisible by a.

assume $c = ak_2$ ($k_2 \in \mathbb{Z}$).

$\therefore bc = ak_1 \cdot ak_2 = a(a \cdot k_1 k_2)$, which is divisible by a.

② c is not divisible by a.

assume $c = ak_3 + m$ ($k_3 \in \mathbb{Z}$, $a \nmid m$)

$\therefore bc = ak_1 \cdot (ak_3 + m)$, which is divisible by a.

In conclusion, bc is divisible by a. when $a | b$

Therefore the contrapositive statement is true, the original statement is true.

2. Prove: $\forall n \in N (n > 3 \land n \text{ is prime}) \Rightarrow \exists q \in N (n = 6q+1 \lor n = 6q+5)$

Proof: ~~as~~ We can divide all natural numbers into 6 parts:

$6q,\ 6q+1,\ 6q+2,\ 6q+3,\ 6q+4,\ 6q+5 \quad (q \in N)$

as
$$6q = 2(3q)$$
$$6q+2 = 2(3q+1)$$
$$6q+3 = 3(2q+1)$$
$$6q+4 = 2(3q+1)$$

these 4 parts are all not prime numbers (except 2 and 3)

So only $(6q+1)$, $(6q+5)$ represents prime numbers (except 2 and 3)

So for all $n \in N$ which is prime and bigger than 3,

there exists $q \in N$ where n can be represented as

$n = 6q+1$ or $n = 6q+5$.

3. Prove: $\forall n \in Z, \forall p (p \text{ is prime}), p \mid n \Rightarrow p \mid (n+1)$

Proof: the contradition of the statement is that

$\forall n \in Z, \forall p (p \text{ is prime}), p \mid (n+1) \Rightarrow p \mid n$.

~~for the contradition. if p is prime, the only factors for p is 1 and p~~

for the ~~State~~ contradition, give a set of example

that $p = 3$, $n = 5$. which satisfiy $p \mid (n+1)$,

but this set does not satisfy $p \mid n$.

So the contradition is false.

Therefore the original statement is True.

4. Prove that $\sqrt[3]{7}$ is irrational.

Proof: suppose that $\sqrt[3]{7}$ is rational.

let $\sqrt[3]{7} = \frac{p}{q}$ (p and q are positive integers and relatively prime)

$7 = \frac{p^3}{q^3}$

$7q^3 = p^3$.

let $p = 7N$ (N is positive integer)

$7q^3 = 7^3N^3$

$q^3 = 7^2N^3$

let $q = 7M$ (M is positive integer)

That's a contradiction which p. q are not relatively prime.

Therefore the assumption is wrong, $\sqrt[3]{7}$ is irrational.

5. Proof: give $n=2$, $n^2+2n-3=5$, 5 is a prime number.

Therefore the statement is true. when $n=2$.

Unique means that this prime number is different from the others.

which cannot satisfy this statement.

Suppose that there exists a number $n>2$, that n^2+2n-3 is a prime.

So $n = k+2$ (k is positive integer)

$n^2+2n-3 = (k+2)^2+2(k+2)-3$

$= k^2+4k+4+2k+4-3$

$= k^2+6k+5$

$\cancel{=(k+3)^2-2^2}$ $=(k+1)(k+5)$

So it's not a prime when $n>2$.

There does not exist any other values for n which n^2+2n-3 is prime

$n=2$ is the only value.