University of Toronto at Scarborough
CSCA67 / MATA67—Discrete Mathematics, Fall 2017

Exercise #5: Simple & Strong Induction

Due: November 5, 2017 at 11:59 p.m.
This assignment is worth 3% of your final grade.

Warning: Your electronic submission on *MarkUs* affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSCA67 / MATA67.

This exercise is due by 11:59 p.m. November 5. **Late exercises will not be accepted.**

[5] 1. A Pythagorean triple consists of three naturals i, j, and k, each greater than 0, such that $i^2 + j^2 = k^2$; one example is $(3, 4, 5)$. It has been proven that there exist infinitely many Pythagorean triples.

Prove or disprove that there exist three naturals i, j, and k, each greater than 0, such that $i^3 + j^3 = k^3$.

Hints: (1) Do some research before attempting this question. **Cite your references.** (2) *Boldly go where no man or woman has gone before*.

For questions #2 - #5, first write the claim in precise mathematical notation, and then prove or disprove the claim using any proof technique you wish.

[5] 2. The sum of the first n natural numbers, starting at 1, is $n(n + 1)(2n + 1)/6$.

[5] 3. The sum of the first n odd natural numbers, starting at 1, is n^2.

[5] 4. Consider the recurrence:

$$a_n = \begin{cases} 2 & n = 1 \\ 6 & n = 2 \\ a_{n-1} + 9a_{n-2} & n \geq 3 \end{cases}$$

For all natural numbers n, starting at 1, $a_n < 3^n$.

[5] 5. Consider the recurrence:

$$t_n = \begin{cases} 1 & n = 1 \\ 1 & n = 2 \\ 1 & n = 3 \\ t_{n-1} + t_{n-2} + t_{n-3} & n \geq 4 \end{cases}$$

For all natural numbers n, starting at 1, $t_n < 2^n$.

[total: 25 marks]