CSC A67/MAT A67 - Discrete Mathematics, Fall 2017

Exercise #2: Logic and Venn Diagrams

Due: September 24, 2017 at 11:59 p.m. This exercise is worth 3% of your final grade.

Warning: Your electronic submission on MarkUs affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSC A67/MAT A67.

Late exercises will not be accepted.

1. (a) Construct a truth table for the following statement: $(p \leftrightarrow q) \rightarrow r$. [10]

p	q	r	$(p \leftrightarrow q) \rightarrow r$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

   (b) Derive from this truth table an equivalent formula to $(p \leftrightarrow q) \rightarrow r$ that only uses connectives from $\{\wedge, \vee, \neg\}$ **and** has exactly 6 propositional variables in it (for example, $(p \wedge q) \vee (\neg q \wedge r)$ has exactly 4 propositional variables).

   (c) Explain how you derived your formula.

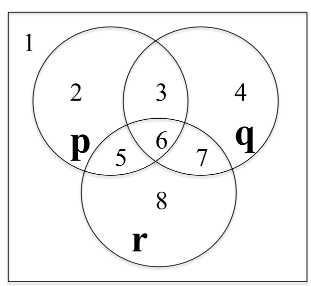
   (d) Prove that $(p \leftrightarrow q) \rightarrow r$ is logically equivalent to $(\neg p \wedge q) \vee (\neg q \wedge p) \vee r$ using the logical equivalence laws given in class. You may assume the meaning of $\leftrightarrow$ that we learned in class, namely that

$$A \leftrightarrow B \Leftrightarrow (A \rightarrow B) \wedge (B \rightarrow A)$$

   and refer to this as the definition of $\leftrightarrow$ or definition of biconditional.

2. List the regions of the Venn diagram where each of the following statements is true: [10]

   (a) $(p \rightarrow \neg q) \rightarrow \neg r$
   (b) $p \rightarrow (\neg q \rightarrow \neg r)$
   (c) $(p \leftrightarrow q) \leftrightarrow r$
   (d) $p \leftrightarrow (q \leftrightarrow r)$
   (e) $(p \wedge r) \rightarrow (q \vee \neg r)$



3. What do you observe from your solutions to 2(a)-(d) above about $\rightarrow$ and $\leftrightarrow$? [2]

4. Disprove $(\neg r \rightarrow \neg(p \vee q)) \iff (p \rightarrow q) \rightarrow r$. How can one disprove an equivalence without providing the *entire* truth table? [3]

[Total: 25 marks]

Logical Equivalences

Commutative	$p \wedge q \iff q \wedge p$	$p \vee q \iff q \vee p$
Associative	$(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$	$(p \vee q) \vee r \iff p \vee (q \vee r)$
Distributive	$p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$
Identity	$p \wedge T \iff p$	$p \vee F \iff p$
Negation	$p \vee \neg p \iff T$	$p \wedge \neg p \iff F$
Double Negative	$\neg(\neg p) \iff p$	
Idempotent	$p \wedge p \iff p$	$p \vee p \iff p$
Universal Bound	$p \vee T \iff T$	$p \wedge F \iff F$
De Morgan's	$\neg(p \wedge q) \iff (\neg p) \vee (\neg q)$	$\neg(p \vee q) \iff (\neg p) \wedge (\neg q)$
Absorption	$p \vee (p \wedge q) \iff p$	$p \wedge (p \vee q) \iff p$
Conditional or ($\rightarrow$) Law	$(p \rightarrow q) \iff (\neg p \vee q)$	$\neg(p \rightarrow q) \iff (p \wedge \neg q)$
Biconditional	$p \leftrightarrow q \iff (p \rightarrow q) \wedge (q \rightarrow p)$	