CSCA67 Assignment 2 Zhan5263

1. a) the size of the sample space is 365^{200}

Description: The sample space for $n=200$ is the set of all possible outcomes of the experiment, corresponding to the Cartesian product of the set of 365 possible birth dates, with itself 200 times as there are 200 people

This will produce 365^{200} ordered pairs.

b) P(event happen) + P(event doesn't happen) = 1.

P(two people share birthday) + P(not two people share birthday) = 1.

So, P(two people share birthday) = 1 − P(not two people share birthday)

To calculate P(not two person share birthday) for 200 people is

$$\left[\frac{365-1}{365}\right]\cdot\left[\frac{365-2}{365}\right]\cdot\left[\frac{365-3}{365}\right]\cdot \cdots \cdot\left[\frac{365-199}{365}\right]$$

$$= \frac{365\times(365-1)\cdots(365-199)}{365^{200}} = \frac{365!}{365^{200}(365-200)!}$$

Then P(at least two person share birthday) $= 1 - \frac{365!}{365^{200}(365-200!)}$

$$= \frac{365^{200}165! - 365!}{365^{200}165!}$$

c). sample size for $n=200$: 365^{200}

number of tuples that at least 2 have same Birthday:

Total − no same

$$= 365^{200} - \frac{365!}{200!}$$

Seconds: $\frac{365^{200}}{33.86\times10^{15}} = \frac{2.87\times10^{513}}{0.3386\times10^{17}} = 8.47\times10^{496}$ (s).

$$\approx \cancel{\frac{1\times10^{400}\cdot3.65^{200}}{33.86\times10^{15}} = 2.95\times10^{383}\times3.65^{200}}$$

days: $\frac{8.47\times10^{496}}{60\times60\times24} = 9.8\times10^{491}$.

years: $\frac{9.8\times10^{491}}{365} = 2.7\times10^{489}$ years

2. E denoted the event that everyone has a distinctive birthday.
Here is to find $P(\bar{E})$ for $n=3$.
Given the birthday of any person can fall on any day in the 365 days, the exhaustive number of n person is 365^n.
The number of faverable cases are

$$365(365-1)(365-2)\cdots[365-(n-1)]$$

$$P(\bar{E}) = \frac{365(365-1)(365-2)\cdots[365-(n-1)]}{365^n}$$

For $n=3$, $P(\bar{E}) = \frac{365(365-1)(365-2)}{365^3} \approx 0.9918$ $1-P(\bar{E}) = 0.0082$

$P(E)$ means that 3 person has at least 2 in common birthday.
Among them 3, 2 or more have same BD on a specific day is:
$\frac{1}{365} + \frac{1}{365} - \frac{1}{365} \times \frac{1}{365}$, the second & third person have the same BD but not the day before is $\frac{364}{365} \times \frac{364}{365} \times \frac{1}{364}$.

$$P(E) = \frac{1}{365} + \frac{1}{365} - \frac{1}{365} \times \frac{1}{365} + \frac{364}{365} \times \frac{364}{365} \times \frac{1}{364} = \frac{2}{365} + \frac{3}{365^2} \approx 1 - \frac{364 \times 363}{365^2},$$

which means that $P(E) = 1 - P(\bar{E})$)

3. For $n=4$, $P(\bar{E}) = \frac{365(365-1)(365-2)(365-3)}{365^4}$)

$$P(E) = 1 - P(\bar{E}) = \frac{365^3 - 364 \times 363 \times 362}{365^3}$$

Using counting principle to count $P(E)$,
$P(E) =$

But this would be more & more difficult as n increase, as the people needs to be considered & the set of people having the same birthday is larger & larger. The increasing is not like one plus one but increase by multiple times.

4. P(at least 3 person have same BD) = 1 - P(less than 3 have same BD)

$$\Downarrow$$

$$P(>=3 \text{ same BD}) = 1 - P(<3 \text{ same BD})$$

$$\Downarrow$$

$$P(<3 \text{ same BD}) = P(\text{all different BD}) + P(2 \text{ same BD})$$

$$P(\text{all different BD}) = \frac{365(365-1)\cdots(365-n+1)}{365^n} = \frac{365!}{365^n(365-n)!}$$

$$\Downarrow$$

$$P(2 \text{ same BD}) = {}_nC_2 \cdot \frac{365}{365} \cdot \frac{364}{365} \cdots \frac{365-n+2}{365}$$

$$= \frac{n(n-1)}{2} \cdot \frac{365!}{365^n \cdot (365-n+1)!}$$

$$\Downarrow$$

$$P(<3 \text{ same BD}) = \frac{365!}{365^n(365-n)!} + \frac{365!}{365^n \cdot (365-n+1)!} \cdot \frac{n(n-1)}{2}$$

$$= \frac{365!}{365^n(365-n)!} \cdot \frac{n(n-1)+1}{2(365-n+1)}$$

$$\Downarrow$$

$$P(>=3 \text{ same BD}) = 1 - P(<3 \text{ same BD})$$

$$= 1 - \frac{365!}{365^n(365-n)!} \cdot \frac{n(n-1)+1}{2 \cdot (365-n+1)}$$

5.

$$\begin{array}{ccccccccc} &&&&1&&&& \\ &&&1&&1&&& \\ &&1&&2&&1&& \\ &1&&3&&3&&1& \\ 1&&4&&6&&4&&1 \end{array}$$

0th row

1st row

2nd row

3rd row

4th row

↑ ↑ ↑ ↑ ↑

0th 1th 2th 3th 4th

a) I.H. $P(n)$: rth entry of nth row is ${}^nC_r$

Base Case: $P(1)$ is true since ${}^1C_0 = {}^1C_1 = 1$

I.S. Suppose that $P(m)$ is true.

r^{th} entry in m^{th} row

$= {}^mC_r + {}^mC_{r-1}$ (By construction of Pascal triangle)

$= {}^{m+1}C_r$

Which is true for r in $\{0, 1, \cdots (m+1)\}$

Therefore $P(n)$ is true by induction laws.

b) Sum of n^{th} row:

I.H. $S(n)$: the sum of the n^{th} row, ${}^nC_0 + {}^nC_1 + \cdots + {}^nC_n = 2^n$.

Base case: $n=1$, $S(1) = 2 = 2^1$ $S(1)$ is true.

I.S. Suppose that $S(k)$ is true.

for $n = k+1$, $S(n) = {}^{k+1}C_0 + {}^{k+1}C_1 + \cdots + {}^{k+1}C_{k+1}$

$= 2^{k+1}$.

Therefore $S(n)$ is true by induction rules.

6. Let A = "0 is sent"
B = "1 is sent"
C = "2 is sent"
X = "1 is received"

According to given information, $P(A) = 0.3$. $P(X/A) = 0.2$
$P(B) = 0.4$
$P(C) = 0.3$. $P(X/C) = 0.1$

We've know that $0 \to 1$ with probability 0.2.
$1 \to 2$ with probability 0.1.

$$P(X/B) = P(B) \cdot P(\overline{1 \text{ has changed to } 0}) + P(B) \cdot P(\overline{1 \text{ has changed to } 2})$$
$$= 0.4 \times 0.8 + 0.4 \times 0.9$$
$$= 0.32 + 0.36$$
$$= 0.68$$

So $P(B/X) = \dfrac{P(X/B)P(B)}{P(X/B)P(B) + P(X/A)P(A) + P(X/C)P(C)}$

$$= \frac{0.68 \times 0.4}{0.68 \times 0.4 + 0.3 \times 0.2 + 0.3 \times 0.1}$$

$$= \frac{0.272}{0.272 + 0.09}$$

$$= \frac{0.272}{0.362}$$

$$\approx 0.75$$