University of Toronto at Scarborough
CSC A67/MAT A67 - Discrete Mathematics, Fall 2017

Exercise #4: Direct and Indirect Proofs

Due: October 15, 2017 at 11:59 p.m.
This exercise is worth 3% of your final grade.

Warning: Your electronic submission on MarkUs affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSC A67/MAT A67.

This exercise is due by 11:59 p.m. October 15. **Late exercises will not be accepted**.

1. Prove [5]
$$\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \forall c \in \mathbb{Z}, a \nmid bc \rightarrow a \nmid b$$
   (a) Using proof by contradiction.
   (b) Using proof by contraposition.

   Note that $x \nmid y$ means "*x does not divide y*".

2. Prove [5]
$$\forall n \in \mathbb{N}, (n > 3 \wedge n \text{ is prime}) \rightarrow \exists q \in \mathbb{N}, (n = 6q + 1 \vee n = 6q + 5).$$

3. Prove that for any integer n and prime number p, if $p \mid n$ then $p \nmid (n+1)$. Hint: Try an indirect proof method. [5]

4. Prove that $\sqrt[3]{7}$ is irrational. [5]

5. Prove that there exists a *unique* prime number of the form $n^2 + 2n - 3$ where n is a positive integer. What does *unique* mean? [5]

[Total: 25 marks]