STAC51 TUT02

Week 3: Sept 24, 2020

Binomial Theorem

Theorem 3.2.2 (Binomial Theorem) *For any real numbers x and y and integer* $n \geq 0$,

(3.2.4) $$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}.$$

$$\sum_{i=0}^{n} \binom{n}{i} =$$

Mgf of X ~ binomial(n,p)

$M_X(t)$=

Multinomial Theorem

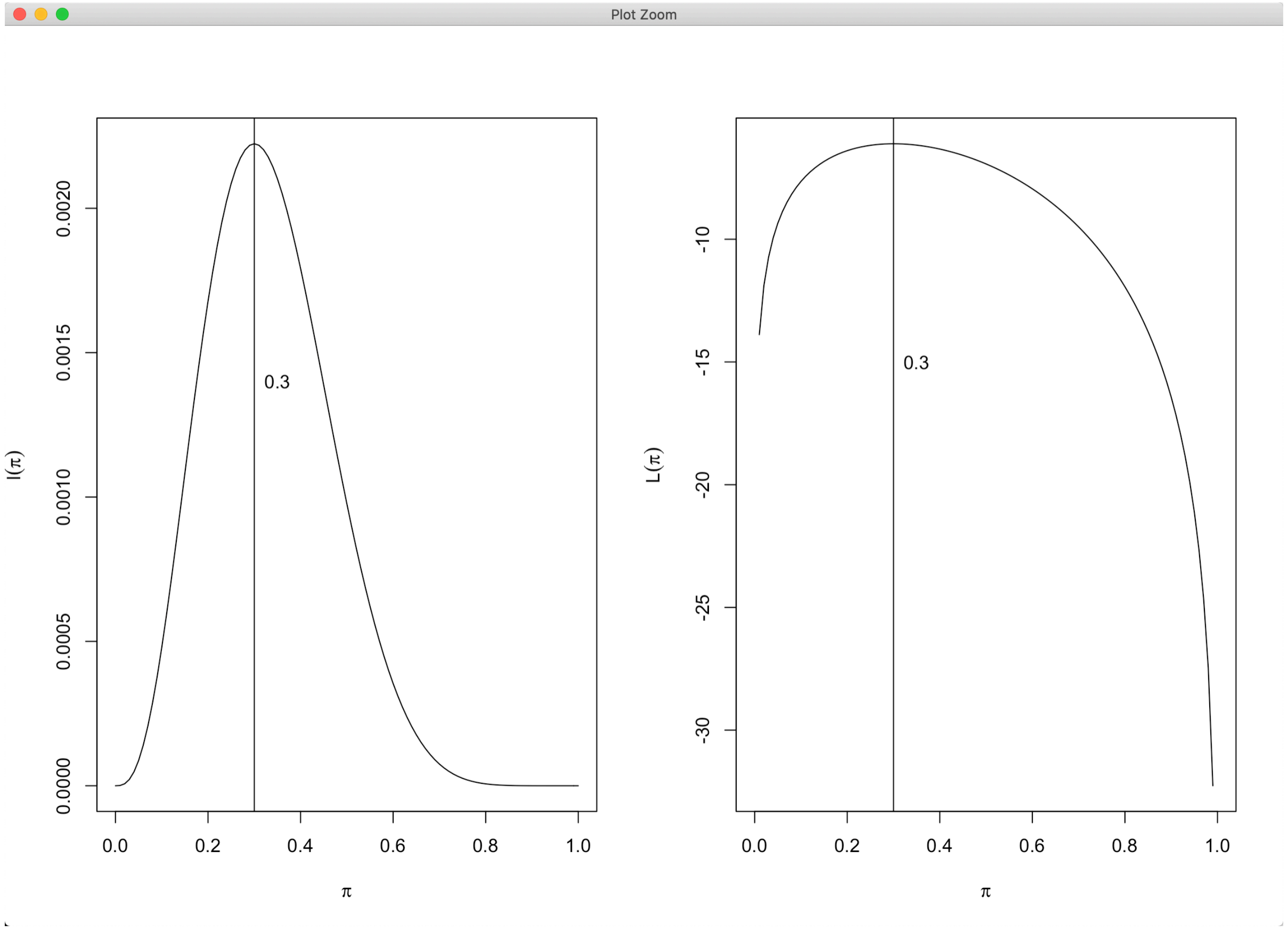
Theorem 4.6.4 (Multinomial Theorem) *Let m and n be positive integers. Let $\mathcal{A}$ be the set of vectors $\mathbf{x} = (x_1, \ldots, x_n)$ such that each x_i is a nonnegative integer and $\sum_{i=1}^{n} x_i = m$. Then, for any real numbers $p_1, \ldots, p_n$,*

$$(p_1 + \cdots + p_n)^m = \sum_{\mathbf{x} \in \mathcal{A}} \frac{m!}{x_1! \cdot \cdots \cdot x_n!} p_1^{x_1} \cdot \cdots \cdot p_n^{x_n}.$$

Binomial Example

- Let's assume that we want to test that a coin is fair
- We toss the coin 10 (n) times and we got 3 (Y) heads
- $\ell(\pi) = \pi^3(1-\pi)^7$ that is $L(\pi) = 3\log(\pi) + 7\log(1-\pi)$. Thus $\hat{\pi} = 3/10$
- Plot the likelihood function and find the value of π that maximizes $\ell(\pi)$.

R Tutorial: The likelihood plots



Example

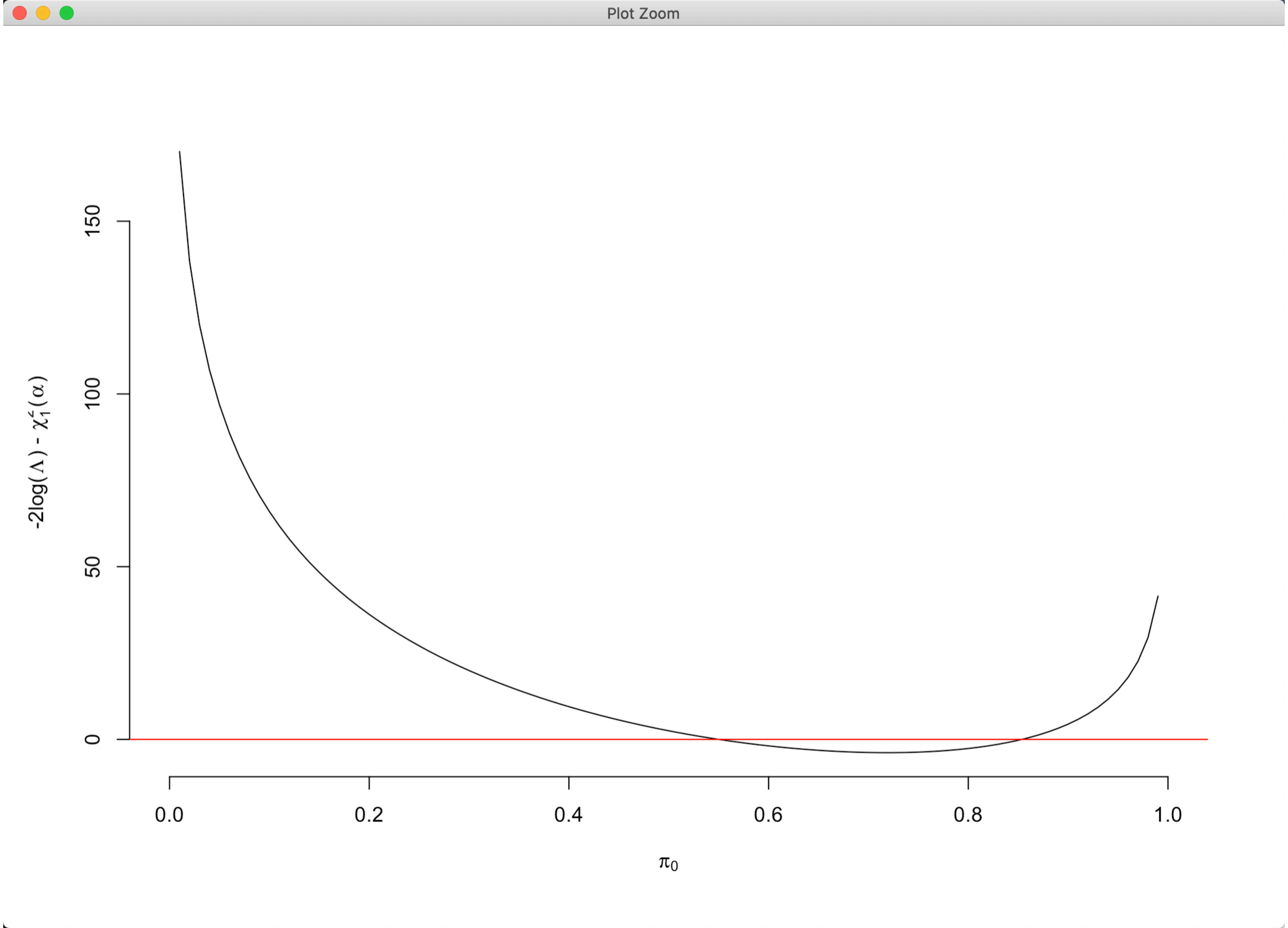
A coin was tossed 32 times and observed 23 heads. Use the likelihood ratio test to test $H_0 : \pi = 0.5$ and $H_a : \pi \neq 0.5$

R Tutorial: LR based Confidence Interval

- Likelihood based confidence interval for π is the set of values of π_0 for which, $-2\log(\Lambda) < \chi^2_1(\alpha)$
- We can find the boundaries of the interval by solving the equation $-2(L_0 - L_1) = \chi^2_1(\alpha)$
- The solution may not be obtainable analytically but we can use numerical methods
- For the previous example we need to solve the following equation,

$$2\left[23\log\left(\frac{23}{32\pi_0}\right) + (32-23)\log\left(\frac{32-23}{32-32\pi_0}\right)\right] - 3.84 = 0$$

- We will solve it with ‘`R`’ in the following slide


Plot Zoom
-2log(Λ) - χ²₁(α)
0
50
100
150
0.0
0.2
0.4
0.6
0.8
1.0
π₀

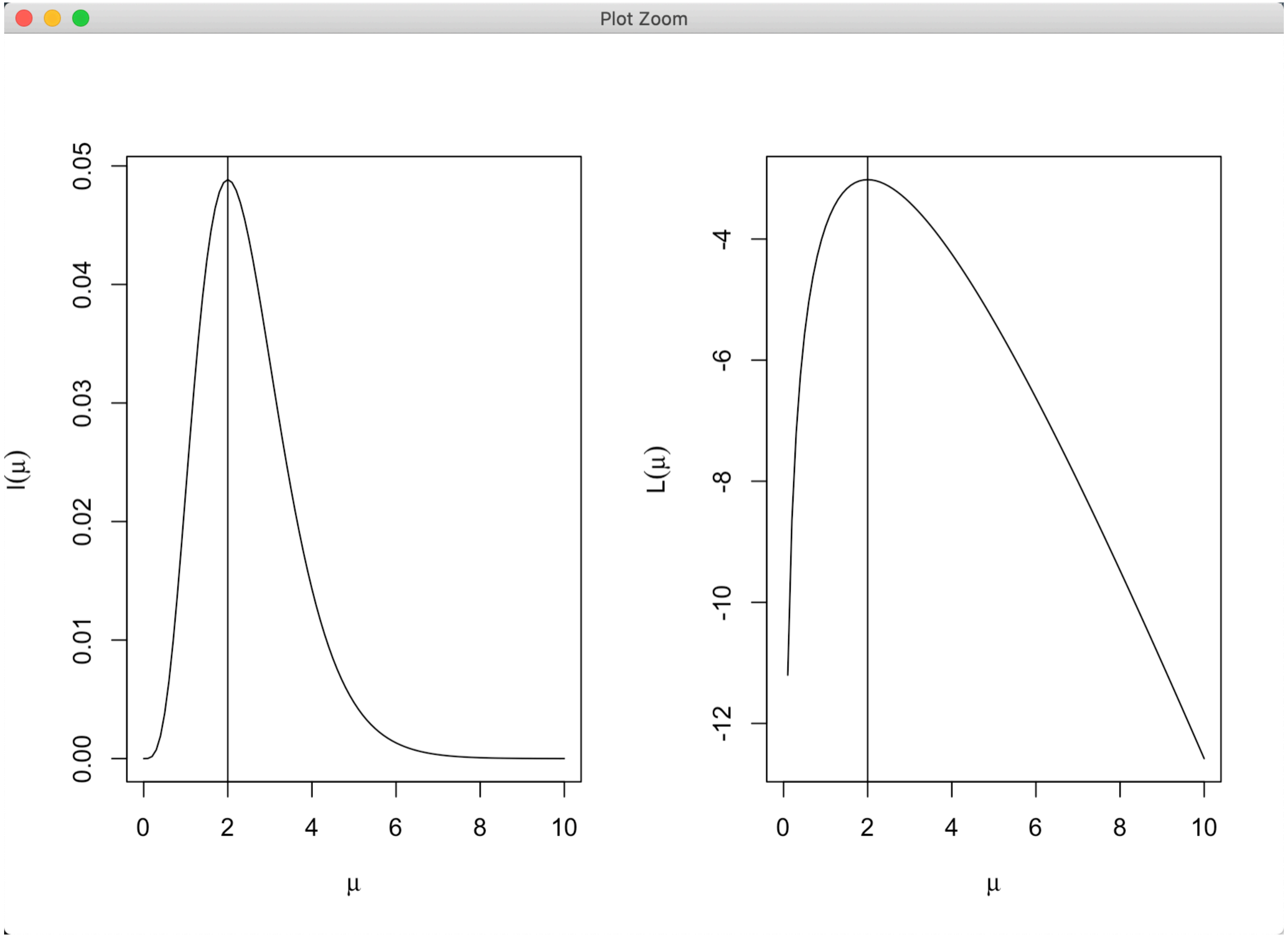
Assume that the number of cases of tetanus reported in the United States during a single month in 2005 has a Poisson distribution with parameter μ. The number of cases reported in January and February are 1 and 3 respectively.

(a) Find and plot the likelihood function over the space of potential values for μ.

(b) What is the maximum likelihood estimate (MLE) of μ?

(c) Give an estimate of the probability that there is no case of tetanus reported for a given month.

R: plot Poisson likelihood



If Y_1 and Y_2 are independent Poisson random variable with parameters μ_1 and μ_2 respectively. Then find the conditional distribution of Y_1 given $Y_1 + Y_2 = n$. That is calculate $P(Y_1 = k|Y_1 + Y_2 = n)$. [5 Marks]

If $Y_1 \sim \text{Bin}(n_1, \pi)$ and $Y_2 \sim \text{Bin}(n_2, \pi)$ are independent. Then find the conditional distribution of Y_1 given $Y_1 + Y_2 = m$. That is calculate $P(Y_1 = k | Y_1 + Y_2 = m)$. [5 Marks]