PID Tuning for Optimal Closed-Loop Performance With Specified Gain and Phase Margins

Keyu Li

***Abstract*— A new robust proportional-integrative-derivative (PID) tuning method based on nonlinear optimization is developed. The closed-loop bandwidth is maximized for specified gain and phase margins (GPM) with constraint on overshoot ratio, so that criteria of closed-loop performance and robustness are both satisfied. The equations of open-loop amplitude ratio and phase change are derived based on frequency analysis for the first-order plus time delay system with a PID controller in parallel form. The equation of closed-loop amplitude ratio is also explicitly given. The formulated optimization problem from the tuning method based on GPM with constraint on closed-loop amplitude ratio is further given. The method is demonstrated in simulation examples and compared with previous work on this topic.**

***Index Terms*— Gain margin, nonlinear optimization, phase margin, process control, proportional-integrative-derivative (PID) tuning, robustness.**

I. Introduction

DESPITE the widespread application of the proportional-integrative-derivative (PID) control in industry, it can still be a challenge to find a general and effective PID tuning method. Large amount of research work has been done on this topic and many tuning formulas have been derived [1]–[3].

Most modern PID tuning methods are model-based and there is always model mismatch. Thus, robustness plays an important role in PID design. In practice, gain and phase margins (GPMs) serve as important indicators of system robustness. Intensive research has been done on designing PID to meet GPM specifications [4]–[8]. In general, the PI design process usually leads to four equations and four unknowns ($\omega_g, \omega_p, K_c, \tau_i$). Numerical methods can be used to directly solve the PI design problem, or approximation on *arctan* function can be adopted to simplify the problem and get the analytical solution [9], [10]. For PID, there are five unknowns for total four equations [10], [14], [15]. Extra approximations on derivative term are made to reduce the number of unknowns in [10] and [15]. The extra degree of freedom can also be used to achieve another objective, such as the minimization of the integral square error (ISE) [14]. The internal model control (IMC) tuning algorithm can also be combined with GPM, which significantly simplifies the problem since there is only one tuning parameter for IMC [11], [12], [16]. The analytical solution can be easily obtained and suitable for online application, such as adaptive control. However, the down side is that there would be only three unknowns for four equations, thus GPM are not independent in such case and the robustness criterion cannot be exactly met.

Besides robustness, good closed-loop controller performance is also required in PID tuning, and usually there is a tradeoff between performance and robustness. The maximum value of closed-loop amplitude ratio M_T and bandwidth ω_b is important closed-loop performance criteria. A larger M_T leads to a faster response, but also implies a larger overshoot and a lower robustness level, so it should be properly bounded. The bandwidth ω_b, however, should be as large as possible, since larger bandwidth values result in faster closed-loop responses and smaller settling time. However, there is no report yet on directly applying performance criteria M_T and ω_b with robustness criteria GPM in PID tuning. Thus, the motivation of this work is to develop a novel robust PID tuning method by using M_T and ω_b as performance criteria combined with the robustness criteria GPM. In this work, the constraint on M_T bounds the overshoot and constraints on GPM guarantee the robustness. Meanwhile, the maximized bandwidth makes the response as fast as possible so that the settling time is minimized. The main contribution of this work is to formulate the PID tuning into a nonlinear optimization problem to maximize the bandwidth with constraints on both GPM and M_T for the first time, so that criteria on robustness and closed-loop performance are both satisfied simultaneously. The whole idea here is that we want the closed-loop response as fast as possible (minimized settling time) for given robustness criteria and bound on the overshoot ratio.

This paper is organized as follows. In the next section, the open-loop amplitude ratio and the phase equations for first-order plus time-delay (FOPTD) model and PID in parallel form are explicitly given. Furthermore, the closed-loop amplitude ratio equation is also derived to calculate the bandwidth and the overshoot ratio. In the following section, the new tuning method to meet both robustness and performance criteria is described and the resulting optimization problem is formulated. The solution technique to the nonlinear optimization problem is also discussed. In the end of this paper, four typical examples in process control are illustrated and results are compared with other methods in previous work.

II. System Frequency Analysis

A. Open-Loop Frequency Analysis

The transfer function of a FOPTD process is given by

$$G_p(s) = \frac{K_p}{\tau s + 1} e^{-\theta s} \tag{1}$$

Manuscript received September 20, 2011; accepted April 19, 2012. Manuscript received in final form May 2, 2012. Date of publication May 30, 2012; date of current version April 17, 2013. This work was supported in part by the National Basic Research Program of China 973 Program under Grant 2012CB720500 and the Science Foundation of China University of Petroleum, Beijing, under Grant YJRC-2011-16. Recommended by Associate Editor Q. Wang.

The author is with the Department of Thermal Energy Engineering, China University of Petroleum, Beijing 102249, China (e-mail: likeyu@cup.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCST.2012.2198479

and the transfer function of the PID controller in parallel form is given by

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right). \tag{2}$$

Then the open-loop transfer function is given by

$$G_{ol}(s) = G_c(s)G_p(s) \tag{3}$$
$$= \frac{K_c K_p(\tau_I\tau_D s^2 + \tau_I s + 1)e^{-\theta s}}{\tau_I s(\tau s + 1)}. \tag{4}$$

With frequency analysis on each term in (4), the amplitude ratio AR_{ol} and phase change ϕ_{ol} are explicitly given by

$$AR_{ol} = \frac{K_c K_p\sqrt{(1-\omega^2\tau_I\tau_D)^2 + (\omega\tau_I)^2}}{\omega\sqrt{\omega^2\tau^2+1}\tau_I} \tag{5}$$

$$\phi_{ol} = \begin{cases} \Lambda(\omega) - \omega\theta - \tan^{-1}(\omega\tau) - \frac{\pi}{2}, & \text{if } \Lambda(\omega) \geq 0 \\ \Lambda(\omega) - \omega\theta - \tan^{-1}(\omega\tau) + \frac{\pi}{2}, & \text{if } \Lambda(\omega) < 0 \end{cases} \tag{6}$$

where

$$\Lambda(\omega) = \tan^{-1}\left(\frac{\omega\tau_I}{1-\omega^2\tau_I\tau_D}\right). \tag{7}$$

B. Closed-Loop Frequency Analysis

For open-loop system G_{ol}, the closed-loop transfer function is given by

$$G_{cl} = \frac{G_{ol}}{1+G_{ol}}. \tag{8}$$

With relatively simple mathematical manipulation, the amplitude ratio of closed-loop system can be calculated by

$$AR_{cl} = \frac{1}{\sqrt{\left(\frac{1}{AR_{ol}} + \cos\phi_{ol}\right)^2 + \sin^2\phi_{ol}}}. \tag{9}$$

Thus, the amplitude ratio of the closed-loop system AR_{cl} can be calculated directly from the open-loop amplitude ratio AR_{ol} and phase change ϕ_{ol}. The bandwidth ω_b is then can be calculated by solving the equation

$$AR_{cl}(\omega) = 0.707. \tag{10}$$

The maximum closed-loop amplitude ratio M_T can be obtained by calculating

$$M_T = \max(AR_{cl}(\omega)) \qquad \forall\ \omega. \tag{11}$$

III. Optimal PID Design Based on GPMs

From definition, the gain margin A and phase margin ϕ can be calculated from the following equations:

$$A = \frac{1}{|G_{ol}(j\omega_p)|} \tag{12}$$

$$\phi = \angle G_{ol}(j\omega_g) + \pi \tag{13}$$

where

$$|G_{ol}(j\omega_g)| = 1 \tag{14}$$

$$\angle G_{ol}(j\omega_p) = -\pi. \tag{15}$$

Substituting (5) and (6) into (12)–(15), we have

$$A = \frac{\omega_p\tau_I}{K_c K_p}\sqrt{\frac{\omega_p^2\tau^2+1}{(1-\omega_p^2\tau_I\tau_D)^2+\omega_p^2\tau_I^2}} \tag{16}$$

$$\phi = \begin{cases} \Lambda(\omega_g) - \omega_g\theta - \tan^{-1}(\omega_g\tau) + \frac{\pi}{2}, & \text{if } \Lambda(\omega_g) \geq 0 \\ \Lambda(\omega_g) - \omega_g\theta - \tan^{-1}(\omega_g\tau) + \frac{3}{2}\pi, & \text{if } \Lambda(\omega_g) < 0 \end{cases} \tag{17}$$

and

$$\frac{K_c K_p}{\omega_g\tau_I}\sqrt{\frac{(1-\omega_g^2\tau_I\tau_D)^2+\omega_g^2\tau_I^2}{\omega_g^2\tau^2+1}} = 1 \tag{18}$$

$$\begin{cases} \Lambda(\omega_p) - \omega_p\theta - \tan^{-1}(\omega_p\tau) + \frac{\pi}{2} = 0, & \text{if } \Lambda(\omega_p) \geq 0 \\ \Lambda(\omega_p) - \omega_p\theta - \tan^{-1}(\omega_p\tau) + \frac{3}{2}\pi = 0, & \text{if } \Lambda(\omega_p) < 0. \end{cases} \tag{19}$$

For given gain margin A and phase margin ϕ, there are five unknowns ω_g, ω_p, K_c, τ_i, and τ_D in four equations (16)–(19), so we cannot solve them directly. However, the extra degree-of-freedom can be used to maximize the closed-loop bandwidth. The optimization problem with constraints on gain margin, phase margin, and maximum closed-loop amplitude ratio can be formulated as

$$\max_{\omega_g,\omega_p,K_c,\tau_i,\tau_D} \omega_b \tag{20}$$

s.t

$$AR_{cl}(\omega_b) = 0.707 \tag{21}$$
$$A \geq A^* \tag{22}$$
$$\phi \geq \Phi^* \tag{23}$$
$$M_T \leq O^* \tag{24}$$

where A^* and Φ^* are given GPM criterions, respectively, O^* is the upper bound of the maximum amplitude ratio.

A. Discussions

1) M_T is used as the constraint on overshoot, and the corresponding percentage overshoot (PO) for unit step response is given by

$$PO = (M_T - 1) \times 100\%.$$

2) Note that M_T is related to the GPM bounds by [13]

$$A \geq 1 + \frac{1}{M_T} \qquad \phi \geq 2\sin^{-1}\left(\frac{1}{2M_T}\right).$$

If O^* is small, then the actual value of A and ϕ may larger than A^* and ϕ^*, which means a more robust and less aggressive PID tuning, and vice versa.

3) We need to find the maximum of $AR_{cl}(\omega)$ to get M_T in the entire frequency range $(0, \infty)$, but it is hard to do that due to the nonlinearity of function $AR_{cl}(\omega)$. Luckily, The corresponding frequency for M_T is actually in the range $(0, \omega_b)$ in this problem. Since ω_b is unknown, an extra parameter $\omega_{\max}$ is adopted in solving the optimization problem, and AR_{cl} is actually evaluated in a limited range $(0, \omega_{\max}]$. We just need to make sure

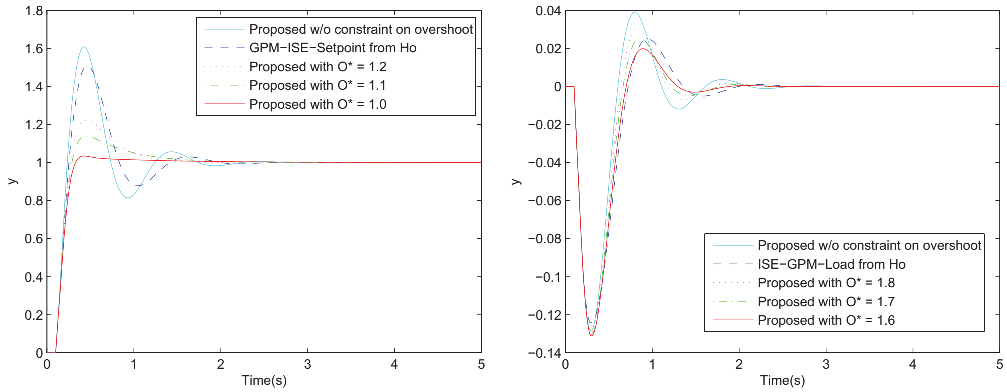


Fig. 1. Step set-point responses and load disturbance responses for Example 1. (a) Step responses. (b) Load disturbance responses.

TABLE I
SIMULATION RESULTS OF EXAMPLE 1 FOR STEP SET-POINT RESPONSE

Parameter	K_c	τ_I	τ_D	A	ϕ	T_S	ISE
Proposed w/o O^*	6.2144	0.1842	0.0347	3	30	1.6500	0.2400
Proposed with $O^* = 1.2$	6.2139	0.4383	0.0270	3	52.1735	1.2300	0.1639
Proposed with $O^* = 1.1$	6.2139	0.5927	0.0256	3	57.2442	1.4400	0.1559
Proposed with $O^* = 1.0$	6.2139	1.1018	0.0239	3	63.3348	0.6800	0.1504
ISE-GPM-setpoint	5.7474	0.2082	0.0382	3	30	1.7900	0.2215

TABLE II
SIMULATION RESULTS OF EXAMPLE 1 FOR STEP LOAD RESPONSE

Parameter	K_c	τ_I	τ_D	A	ϕ	ISE
Proposed w/o O^*	6.2144	0.1842	0.0347	3	30	0.0042
Proposed with $O^* = 1.8$	6.2140	0.2061	0.0333	3	33.4149	0.0043
Proposed with $O^* = 1.7$	6.2140	0.2217	0.0324	3	35.5635	0.0044
Proposed with $O^* = 1.6$	6.2138	0.2415	0.0315	3	38.0100	0.0046
ISE-GPM-load	5.8789	0.2082	0.0382	3	30	0.0045

that the tuning parameter $\omega_{\max}$ is large enough such that $\omega_{\max} \gg \omega_b$.

4) The constrained nonlinear optimization problem for proposed method is solved by *fmincon* function from MATLAB optimization toolbox. To evaluate the constraint functions (21)–(24) in the optimization solver, the gain margin A, phase margin ϕ, and bandwidth ω_b are calculated by solving (16)–(19) and (21) with *fzero* function in MATLAB.

IV. SIMULATION EXAMPLES

Simulation examples of FOPTD system as well as other typical systems for process control are illustrated in this section. Closed-loop responses to step change at time 0 in both set-point and load disturbance are analyzed and compared with other methods in previous work.

A. Example 1: FOPTD System

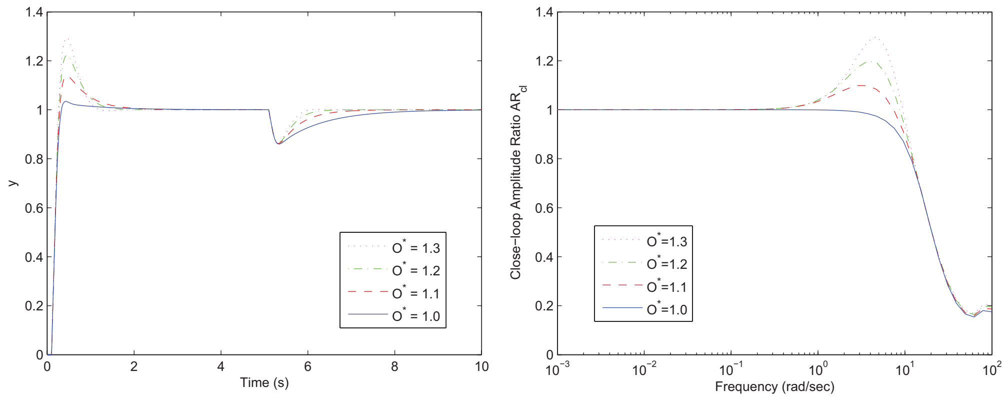
In [14], Ho *et al.* use ISE as the objective function in the optimization problem with constraints on GPM, which results in two sets of tuning formula: ISE-GPM-load and ISE-GPM-setpoint. The same example is illustrated here and results are compared for both load disturbance response and step set-point response.

Consider the plant model given by

$$G_1(s) = \frac{1}{s+1}e^{-0.1s}. \tag{25}$$

For required gain margin of $A^* = 3$ and phase margin $\phi^* = 30°$, different PID tuning methods are used and results are compared. For proposed method, parameter $\omega_{\max}$ is set to 100 for the optimization problem, and results of optimization with and without the constraint on M_T are both illustrated. Closed-loop responses to unit step change on set-point for proposed method with different O^* values and ISE-GPM-setpoint are shown in Fig. 1(a). The corresponding PID controller parameters and key simulation results, such as ISE, settling time T_S, actual gain A, and phase margin ϕ, are compared in Table I.

We can see that the proposed method without constraint on overshoot ratio gives the worst closed-loop performance on step set-point change in terms of ISE. The proposed method with an overshoot constraint $O^* = 1.0$ gives the best performance in both settling time and ISE.



(a) (b)

Fig. 2. Time responses and frequency responses for Example 1 with different O^* values. (a) Time responses. (b) Frequency responses.

The unit disturbance responses for proposed method and ISE-GPM-load from Ho are also shown in Fig. 1(b). The corresponding PID controller parameters and results on closed-loop performance are given in Table II. In this case, the proposed method without overshoot constraint gives the smallest ISE value but more oscillations. A smaller O^* leads to a smoother response but larger ISE.

From results above, we can see that tighter overshoot constraints (smaller O^*) lead to better set-point responses but worse load disturbance rejection in terms of ISE in this example, so the constraint on overshoot ratio basically set a balance between set-point tracking and disturbance rejection. Typical closed-loop responses to both set-point change and load disturbance with different constraints on overshoot are shown in Fig. 2(a). The corresponding closed-loop frequency responses are shown in Fig. 2(b). We can see that the closed-loop amplitude ratios are properly bounded by specified O^* values. Thus, the uniqueness of the proposed method is that the required closed-loop response to set-point or load disturbance can be obtained by simply tuning the parameter O^*. Simulation results of the bandwidth ω_b and the phase margin ϕ with different O^* are shown in Fig. 3. We can clearly see that a smaller O^* results in a smaller closed-loop bandwidth and a larger phase margin, which means a more robust and less aggressive controller.

B. Example 2: FOPTD System With Long Time Delay

The second example is a first-order with a long time-delay system, which is from [17]

$$G_2 = \frac{1}{20s+1}e^{-20s}. \tag{26}$$

An iterative feedback tuning (IFT) method is used and results are compared with other classical tuning methods in [17]. Here, the proposed method is also applied to the same system and the results are compared with IMC and IFT methods. The IMC principle is from [3] with an optimized $T_f = 14$. For proposed method, we use a gain margin $A^* = 2.5$ and a phase margin $\phi^* = 30°$.

The step set-point and load disturbance responses from different methods are shown in Fig. 4(a) and (b). The PID

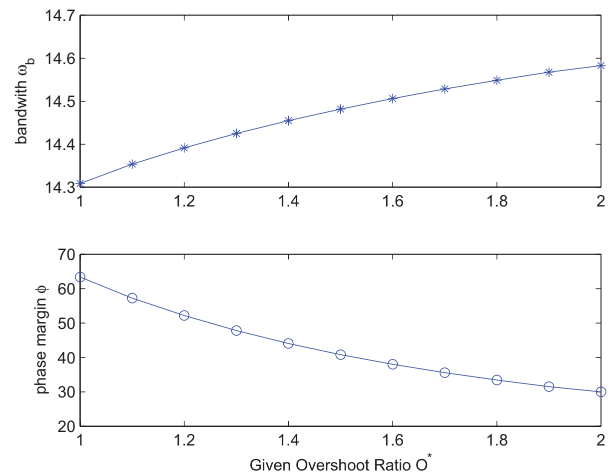


Fig. 3. The bandwidth and phase margin results with different O^* values for Example 1.

parameters and simulation results are shown in Table III, where ISE_{SP} and $\mathrm{ISE}_{\mathrm{load}}$ are the integrated square error for set-point response and load disturbance response, respectively.

When O^* is set to 1.0, the performance of proposed method is very close to IMC and IFT for both step set-point and load disturbance responses, but if we look into the Table III, we can see that the settling time T_s, ISE_{SP}, and $\mathrm{ISE}_{\mathrm{load}}$ are actually slightly better than IMC and IFT.

Set-point and load responses for different O^* values are shown in Fig. 5(a) and (b). When O^* becomes larger, the load disturbance response gets better too, while the set-point response becomes worse. Compared to IMC and IFT, another advantage of the proposed method is the flexibility brought by the constraint on overshoot. The PID controller can be easily tuned for best set-point response or best load disturbance response by simply changing the O^* value. Generally, a smaller O^* is good for set-point response and a larger O^* is good for load disturbance, so that the user could choose appropriate value for O^* for different tuning purposes.

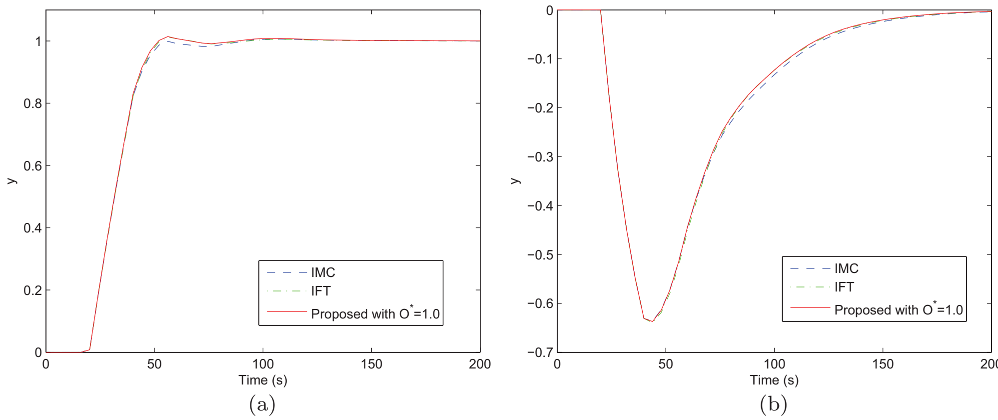


Fig. 4. Step set-point responses and load disturbance responses for Example 2. (a) Step responses. (b) Load disturbance responses.

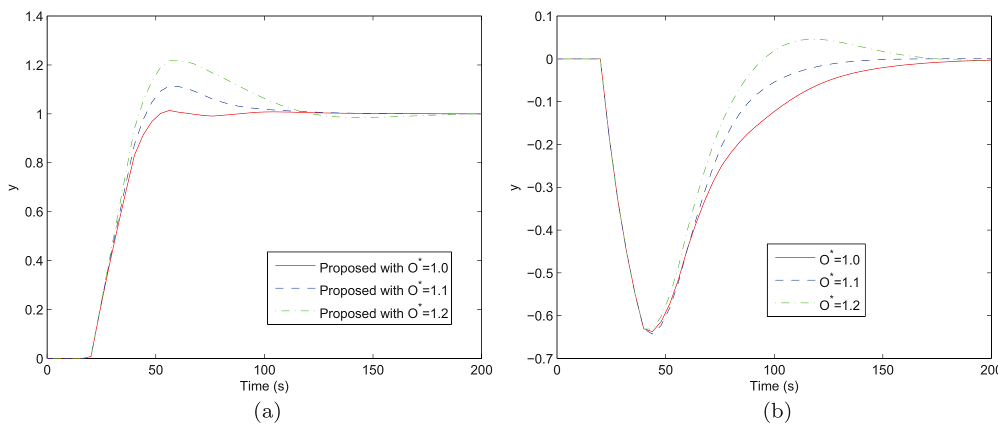


Fig. 5. Step set-point responses and load disturbance responses for Example 2 with different O^* values. (a) Step responses. (b) Load disturbance responses.

TABLE III
PID PARAMETERS AND SIMULATION RESULTS OF EXAMPLE 2

Parameter	K_c	τ_I	τ_D	T_s	ISE_{SP}	ISE_{load}
IMC	0.9351	30.5400	6.4797	52.1972	25.9107	13.1159
IFT	0.9303	30.0593	6.0553	52.2190	25.9160	13.1685
Proposed with $O^* = 1.0$	0.9407	30.0420	6.3021	52.1720	25.8290	12.9573
Proposed with $O^* = 1.1$	0.9515	26.2210	5.3864	100.1200	25.8060	12.2643
Proposed with $O^* = 1.2$	0.9470	20.8807	7.0175	116.1443	26.7173	11.1026

C. Example 3: Higher Order System With Repeated Poles

The third example is also from [17], and the process system is given by

$$G_3 = \frac{1}{(10s+1)^8}. \tag{27}$$

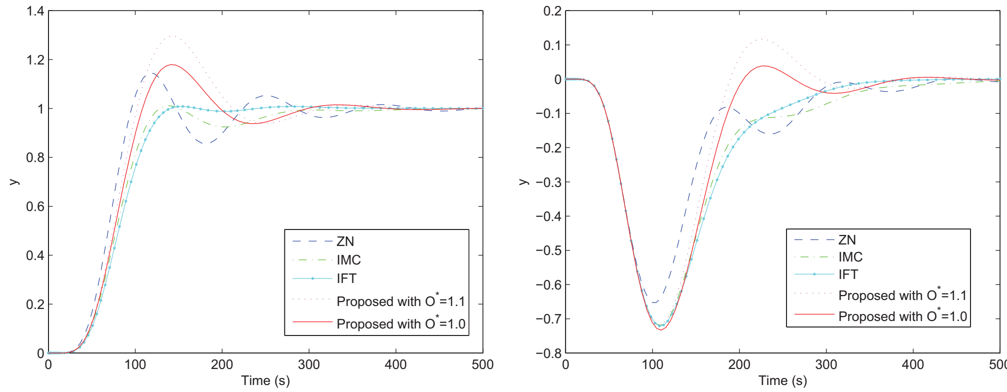
The proposed method is also applied to the same system and the results are compared. Based on [17] and [3, Sec. 2.9], the system G_3 can be approximated by

$$G_p = \frac{1}{43.1505s+1}e^{-43.0691s} \tag{28}$$

so that the IMC and proposed method can be applied. The IMC algorithm is from [3] with an optimized $T_f = 42$. The specified GPM are 3° and 30°, respectively.

The step set-point responses from different methods are shown in Fig. 6(a). The PID parameters and simulation results are shown in Table IV. The classical ZN gives the smallest ISE value but very oscillated closed-loop response with longest settling time. The IFT method performs the best in terms of overshoot and settling time, but with larger ISE and longest rising time. The proposed method leads to a balanced response between ZN and IFT, with smaller ISE and rising time but larger settling time than IMC and IFT. Although we set overshoot constraint to 1.0 for proposed method, the actual overshoot is almost 20%, which caused by the approximation of the model.

The load disturbance responses are shown in Fig. 6(b). The ZN method leads to the smallest ISE_{load} value but with very oscillated behavior. The IMC and IFT methods both give very sluggish responses and large ISE_{load} values. The proposed method with $O^* = 1.0$ performs better than IMC and IFT with a faster response and a smaller ISE_{load}. As overshoot constraint



(a) (b)

Fig. 6. Step set-point responses and load disturbance responses for Example 3. (a) Step responses. (b) Load disturbance responses.

TABLE IV
PID PARAMETERS AND SIMULATION RESULTS OF EXAMPLE 3

Parameter	K_c	τ_I	τ_D	T_s	ISE_{SP}	$\mathrm{ISE}_{\mathrm{load}}$
ZN	1.1	75.9	18.975	345.25	57.144	26.8151
IMC	0.7604	64.685	14.366	266.07	62.734	37.7329
IFT	0.6641	53.979	18.214	132.4	64.812	38.6650
Proposed with $O* = 1.1$	0.7796	43.694	11.798	298.48	64.164	34.0279
Proposed with $O* = 1.0$	0.7674	52.196	9.3868	282.52	62.224	36.3860

TABLE V
PID TUNING RESULTS FOR EXAMPLE 2 WITH DIFFERENT METHODS

Parameter	K_c	τ_I	τ_D	A	ϕ	ω_b	ISE
Proposed with $O* \geq 1.1$	0.5763	1.8778	0.5348	3	60	0.6771	5.2448
Proposed with $O* = 1.0$	0.5685	1.9527	0.4845	3	61.9	0.6629	5.2895
GPM-PID from Ho	0.33	1	1	3	60	N/A	6.4722

$O*$ increases, such as $O* = 1.1$, the response goes faster and $\mathrm{ISE}_{\mathrm{load}}$ becomes smaller but with a bigger overshoot.

The proposed method is not the best for set-point response due to the process-model mismatch, but a better load disturbance response and a more balanced overall closed-loop response than other methods can be obtained by properly setting overshoot constraint $O*$.

D. Example 4: Non-Minimum Phase System

The third-order system with an unstable zero in [10] is used, and the process is given by

$$G_4(s) = \frac{1-s}{(1+s)^3}. \tag{29}$$

By relay auto-tuning, the process can be approximated by a FOPTD model

$$G_{p1}(s) = \frac{1}{1+1.45s}e^{-2.22s} \tag{30}$$

and a second-order plus time-delay (SOPTD) model [10]

$$G_{p2}(s) = \frac{1}{(1+1.00s)^2}e^{-1.58s}. \tag{31}$$

In [10], a GPM-PID tuning method is developed by using the SOPTD model G_{p2}. The proposed method is applied to the same process and the FOPTD model G_{p1} is used.

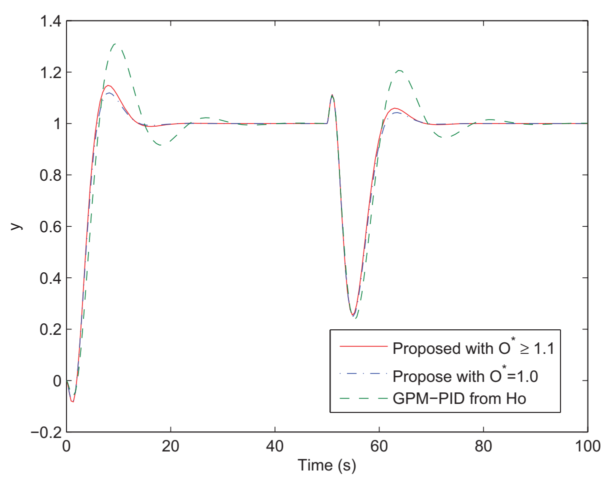


Fig. 7. Responses for two PID tuning methods.

The specified gain margin $A^* = 3$ and phase margin is $\phi^* = 60°$. The response to step set-point change and load disturbance is shown in Fig. 7. The calculated PID parameters and results from GPM-PID and from proposed methods are given in Table V. We can see that the proposed method performs much better than GPM-PID method in this case. Note that when we set the overshoot constraint $O^* = 1.0$, the PID setting becomes more conservative and the actual phase margin is relaxed to 61.9°. If we put $O^* = 1.1$ or larger, the tuning result would be the same, because the PID setting is not possible to be more aggressive to obtain an overshoot larger than 1.1 with specified GPMs. In another words, the constraints on GPM would be the active inequality constraints for the nonlinear optimization problem, and the overshoot constraint would be inactive in that case.

V. Conclusion

In this work, the PID tuning problem is formulated as a nonlinear optimization problem to satisfy both robustness and closed-loop performance criteria simultaneously. The bandwidth and maximum amplitude ratio serve as closed-loop performance criteria, while GPM serve as robustness criteria. The bandwidth was maximized with constraints on gain margin, phase margin, and maximum closed-loop amplitude ratio. The proposed method was demonstrated in four different type examples: FOPTD, first-order with long time-delay, high-order system, and non-minimum phase system. Simulation results showed that the proposed method performs better than existing GPM-based method, IMC, and IFT for first-order systems. With process-model mismatch for high-order systems, the proposed method still leads to comparable performance to IMC and non-model-based method IFT for set-point change response, and better performance for load disturbance response. In non-minimum phase case, the proposed method is significantly better than existing GPM-based method GPM-PID. Moreover, a unique advantage of proposed method is the flexibility brought by the constraint on maximum closed-loop amplitude ratio M_T. By tuning the bound of M_T, we can easily obtain PID parameters for the best set-point change or the best load disturbance response, or a response balanced for both. The proposed idea of directly applying M_T to constraint the overshoot can also be used in other optimization-based methods, such as minimizing ISE or IAE, which has been explored extensively in the literature.

VI. Future Work

The current work is based on nonlinear optimization and the problem is solved numerically, which limits its online application, so an important part of future work is to look into the optimization problem and try to get more theoretical results and the possible analytical solution, probably with reasonable assumptions or approximations.

Another part of future work is to extend the proposed method to second-order and higher order systems to obtain better performance and more general applications.

References

[1] K. J. Åström, C. C. Hang, P. Persson, and W. K. Ho, "Toward intelligent PID control," *Automatica*, vol. 28, no. 1, pp. 1–9, Jan. 1992.

[2] K. J. Åström, T. Hügglund, C. C. Hang, and W. K. Ho, "Automatic tuning and adaptation for PID controllers–a survey," *Control Eng. Pract.*, vol. 1, no. 4, pp. 699–714, Aug. 1993.

[3] K. J. Åström and T. Hügglund, *PID Controllers: Theory, Design and Tuning*. Charlotte, NC: Instrum. Soc. Amer., 1995.

[4] W. K. Ho, O. P. Gan, E. B. Tay, and E. L. Ang, "Performance and gain and phase margins of well-known PID tuning formulas," *IEEE Trans. Control Syst. Technol.*, vol. 4, no. 4, pp. 473–477, Jul. 1996.

[5] W. K. Ho, C. C. Hang, and J. H. Zhou, "Self-tuning PID control of a plant with under-damped response with specifications on gain and phase margins," *IEEE Trans. Control Syst. Technol.*, vol. 5, no. 4, pp. 446–452, Jul. 1997.

[6] O. Yaniv and M. Nagurka, "Design of PID controllers satisfying gain margin and sensitivity constraints on a set of plants," *Automatica*, vol. 40, no. 1, pp. 111–116, Jan. 2004.

[7] C. H. Lee, "A survey of PID controller design based on gain and phase margins (invited paper)," *Int. J. Comput. Cognit.*, vol. 2, no. 1, pp. 63–100, Sep. 2004.

[8] Q. Wang, H. Fung, and Y. Zhang, "PID tuning with exact gain and phase margins," *ISA Trans.*, vol. 38, no. 3, pp. 243–249, Jul. 1999.

[9] W. K. Ho, C. C. Hang, and J. H. Zhou, "Performance and gain and phase margins of well-known PI tuning formulas," *IEEE Trans. Control Syst. Technol.*, vol. 3, no. 2, pp. 245–248, Jun. 1995.

[10] W. K. Ho, C. C. Hang, and L. S. Cao, "Tuning of PID based on gain and phase margin specifications," *Automatica*, vol. 31, no. 2, pp. 497–502, 1995.

[11] W. K. Ho, T. H. Lee, T. P. Han, and Y. Hong, "Self-tuning IMC-PID control with interval gain and phase margins assignment," *IEEE Trans. Control Syst. Technol.*, vol. 9, no. 3, pp. 535–541, May 2001.

[12] I. Kaya, "Tuning PI controllers for stable processes with specifications on gain and phase margins," *ISA Trans.*, vol. 43, no. 2, pp. 297–304, Apr. 2004.

[13] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Design: Analysis and Design*. New York: Wiley, 1996.

[14] W. K. Ho, K. W. Lim, and W. Xu, "Optimal gain and phase margin tuning for PID controllers," *Automatica*, vol. 34, no. 8, pp. 1009–1014, 1998.

[15] W. K. Ho, K. W. Lim, C. C. Hang, and L. Y. Ni, "Geting more phase margin and performance out of PID controllers," *Automatica*, vol. 35, no. 9, pp. 1579–1585, 1999.

[16] C. Chu, B. E. Ydstie, and N. V. Sahinidis, "Optimization of IMC-PID tuning parameters for adaptive control: Part 1," in *Proc. 21st Eur. Symp. Comput. Aided Process Eng.*, vol. 29. Jun. 2011, pp. 758–762.

[17] O. Lequin, M. Gevers, M. Mossberg, E. Bosmans, and L. Triest, "Iterative feedback tuning of PID parameters: Comparison with classical tuning rules," *Control Eng. Pract.*, vol. 11, no. 9, pp. 1023–1033, 2003.