

PMSM Hamiltonian Energy Shaping Control with Parameters Self-tuning PID Control

Qiu Jun, Hu Chao, Yang Sainv

School of Information Science and Engineering, Ningbo Institute of Technology, Zhejiang University, Ningbo 315100

E-mail: qiujuunb@zju.edu.cn, huchao@nit.zju.edu.cn, sainvyang@163.com

Abstract: Aimed at permanent magnet synchronous motor (PMSM) servo control system control requirements, based on Hamiltonian feedback dissipative control strategy, this paper proposed a PID Hamiltonian parameters self-tuning method. Basing on the view of system energy balance point, via port-controlled dissipative Hamiltonian realization, this method designed a permanent magnet synchronous motor speed controller with parameters PID self-tuning to improve the control system speed tracking performance and simplify the system design process. Comparing with fixed gain control parameters, simulation results shown that the proposed method can ensure the system asymptotically stable performance, simplify system parameter setting and improve the system's transient response performance.

Key Words: PMSM, Hamiltonian System, Energy Shaping, Self-tuning

1 Introduction

Hamiltonian systems are widely used in the area such as network, ecological, chemical and other fields. The concept of energy dissipation was introduced into the framework of generalized Hamiltonian system with the development of port-controlled Hamiltonian dissipative systems theory[1-3]. In the field of science and engineering, the system can be considered as a dynamic energy conversion device[4]. When the system is operating in a dynamic equilibrium state, the system energy state is steady, as the system output energy equal to input energy. So we can correct the input energy and output energy to control the system running at the desired equilibrium point, which is named as energy-shaping method. A passivity-based control frame work for port-controlled Hamiltonian system is established in recent series of papers[5-8]. A controller design methodology is brought out which can achieves stabilization via system passivity. Hamilton feedback dissipative control method is such an energy-shaping methods.

In this paper, based on the Hamiltonian feedback dissipative control method, the appropriate function is selected as a closed-loop system's Hamilton Lyapunov function. With Lyapunov stability principle and the global asymptotic stability of system design principles, a PMSM decoupled speed controller is achieved with the coordinate transformation of vector control. The system parameter PID self-tuning method is proposed in this paper which can simplify the design process and improve the transient response performance of the system.

2 Port-controlled dissipative Hamiltonian realization theory

Conventional Hamiltonian system is defined as

$$\dot{x} = F(x)\nabla H(x) \quad (1)$$

Where $x \in R^n$ are state variables, $F(x) \in R^{n \times n}$ is structure matrix, $H(x)$ is Hamiltonian function. If the structure matrix $F(x)$ satisfies

$$F(x) + F^T(x) \leq 0 \quad (2)$$

the system (1) can be said is a dissipative Hamiltonian system, where $F(x)$ can be decomposed as

$$F(x) = J(x) - R(x) \quad (3)$$

Where $J(x) = -J^T(x)$, $J(x)$ is an antisymmetric matrix, $R(x) = R^T(x) \geq 0$, $R(x)$ is a Semi-definite symmetric matrix.

For a system as the following form:

$$\begin{aligned} \dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x) \frac{\partial H}{\partial x}(x) \end{aligned} \quad (4)$$

where $J(x) = -J^T(x)$, $R(x) \geq 0$. Matrix $J(x)$ reflects system's interconnect structure, Matrix $R(x)$ reflects the additional resistive structure on system port.

Proposition 1[4]. Given $J(x), R(x), H(x), g(x)$ and desired equilibrium x^* to be stabilized, if we can find the function $u = \beta(x), R_a(x), J_a(x)$ which can satisfy

$$\begin{aligned} J(x) + J_a(x) &= -[J(x) + J_a(x)]^T \\ R(x) + R_a(x) &= [R(x) + R_a(x)]^T \geq 0 \end{aligned} \quad (5)$$

and a vector function $K(x)$ which can satisfy

$$\begin{aligned} [J(x) + J_a(x) - (R(x) + R_a(x))]K(x) \\ = -[J_a(x) - R_a(x)] \frac{\partial H}{\partial x}(x) + g(x)\beta(x) \end{aligned} \quad (6)$$

and such that

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$$\begin{aligned}
(i) \quad & \frac{\partial \mathbf{K}_i}{\partial x_j}(x) = \frac{\partial \mathbf{K}_i}{\partial x_i}(x), i, j = 1, 2, \dots, n \\
(ii) \quad & \mathbf{K}(x^*) = -\frac{\partial H}{\partial x}(x^*) \\
(iii) \quad & \frac{\partial \mathbf{K}}{\partial x}(x^*) > -\frac{\partial^2 H}{\partial x^2}(x^*)
\end{aligned} \quad (7)$$

Where $\partial \mathbf{K} / \partial x$ is $n \times n$ matrix, the i -th column vector is $\partial \mathbf{K}_i(x) / \partial x$, $\partial^2 H(x^*) / \partial x^2$ is Hessian matrix of $H(x)$ at desired equilibrium x^* . Condition (i) is the gradient of a scalar function which indicates the designed system meeting the integrability conditions. Condition (ii) shows that the closed-loop system $H_d(x)$ has a minimum balance at the system equilibrium point x^* . Condition (iii) indicates that the Jacobian matrix are bounded equilibrium conditions at the system equilibrium point x^* , and the system equilibrium point x^* is an isolated equilibrium point.

Under these conditions, the closed-loop system $\mathbf{u} = \beta(x)$ will be a PCH system with dissipation of the form (8)

$$\dot{x} = [\mathbf{J}_d(x) - \mathbf{R}_d(x)] \frac{\partial H_d}{\partial x}(x) \quad (8)$$

where

$$\begin{aligned}
H_d(x) &= H(x) + H_a(x) \\
\mathbf{J}_d(x) &= \mathbf{J}(x) + \mathbf{J}_a(x) \\
\mathbf{R}_d(x) &= \mathbf{R}(x) + \mathbf{R}_a(x)
\end{aligned} \quad (9)$$

And the new energy function $H_d(x)$ has a strict local minimum at the desired equilibrium x^* , and control law $\mathbf{u} = \beta(x)$ can be obtained through the solution of the formula (6), and the close system (8) will be asymptotically stable if, in addition, the largest invariant set under the closed-loop dynamics contained in

$$\{x \in \mathbf{R}^n \mid (\nabla H_d)^T \mathbf{R}_d(x) \nabla H_d = 0\} \quad (10)$$

equals $\{x^*\}$. An estimate of its domain of attraction is given by the largest bounded level set $\{x \in \mathbf{R}^n \mid H_d(x) \leq c\}$.

By the above method, we can realize the port controlled Hamiltonian system dissipative control based on passive interconnection and damping assignment. Using feedback control, we realize the system damping injection and add dissipative into real system that original system resistive structure $\mathbf{R}(x)$ by injecting $\mathbf{R}_a(x)$ into the system become $\mathbf{R}_d(x)$, and the system Hamiltonian function $H(x)$ is shaped into $H_d(x)$.

3 PMSM Energy Shaping Control Realize

3.1 PMSM's PCH Mode Structure

The electrical and mechanical equations of the PMSM are as follows in this paper [9], which ignore the hysteresis loss and the friction coefficient:

$$\begin{cases} L_d \frac{di_d}{dt} = -R_s i_d + n_p L_q i_q \omega + u_d \\ L_q \frac{di_q}{dt} = -R_s i_q + n_p L_d i_d \omega - n_p \phi \omega + u_q \\ J \frac{d\omega}{dt} = \tau_{em} - \tau_L \\ \quad \quad \quad = n_p [(L_d - L_q) i_d i_q + \phi i_q] - \tau_L \end{cases} \quad (11)$$

Where i_d, i_q are the currents in $d-q$ axis, R_s is the stator resistance per phase, L_d, L_q are d axes and q axes stator inductances; ω is electrical angular speed of rotor, ϕ is the rotor flux linking the stator, n_p is the number of pole pairs, J is the moment of inertia, τ_{em} and τ_L are the electromagnetic and load torque. For salient pole permanent magnet synchronous motors, there are $L_d = L_q$.

Define system's state x , input variable u and output variable y as follows,

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} L_d i_d \\ L_q i_q \\ J \omega \end{bmatrix} \\ &= \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & J \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} = \mathbf{D} \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} \\ \mathbf{u} &= \begin{bmatrix} u_d \\ u_q \\ -\tau_L \end{bmatrix}, \mathbf{y} = \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} \end{aligned} \quad (12)$$

Permanent magnet synchronous motor control system's energy can be considered as the sum of electricity and mechanical energy, which means the permanent magnet synchronous motor Hamiltonian function can be expressed as

$$\begin{aligned} H(x) &= \frac{1}{2} \mathbf{x}^T \mathbf{D}^{-1} \mathbf{x} \\ &= \frac{1}{2} \left[\frac{1}{L_d} x_1^2 + \frac{1}{L_q} x_2^2 + \frac{1}{J} x_3^2 \right] \end{aligned} \quad (13)$$

According to the ports controlled dissipation system requirements of equation (4), combined equations (11) and (12), permanent magnet synchronous motor's port controlled Hamiltonian dissipative system model can be described as

$$\begin{cases} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{cases} = [\mathbf{J}(x) - \mathbf{R}] \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix} + \mathbf{g} \begin{bmatrix} u_d \\ u_q \\ -\tau_L \end{bmatrix} \quad (14)$$

$$\mathbf{y} = \mathbf{g}^T \frac{\partial H}{\partial \mathbf{x}}(x) = [i_d \quad i_q \quad \omega]$$

where $\mathbf{J}(x)$, \mathbf{R} are as follows:

$$\mathbf{J}(x) = \begin{bmatrix} 0 & 0 & n_p x_2 \\ 0 & 0 & -n_p(x_1 + \phi) \\ -n_p x_2 & n_p(x_1 + \phi) & 0 \end{bmatrix}, \quad (15)$$

$$\mathbf{R} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{g} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

3.2 PMSM's Stable Equilibrium Point and Speed Controller Design

In order to make the (12) asymptotically stable at equilibrium \mathbf{x}^* , the closed-loop desired energy function $H_d(x)$ is need to be built, which will get minimum value at point \mathbf{x}^* . Closed-loop energy function is as follows

$$H_d(x) = \frac{1}{2}(\mathbf{x} - \mathbf{x}^*)^T \mathbf{D}^{-1}(\mathbf{x} - \mathbf{x}^*) \quad (17)$$

where $H_d(x)$ will get minimum value at point \mathbf{x}^* , which means in a field of \mathbf{x}^* to any $\mathbf{x} \neq \mathbf{x}^*$, there exists $H_d(x) > H_d(x^*)$

The control object is the asymptotic regulation of the speed ω to the desire speed ω^* , accord to the MTPA (maximum torque per ampere) [10, 11] control theory, at equilibrium point \mathbf{x}^* . At the same time, we should find feedback control law $\mathbf{u} = \boldsymbol{\beta}(x)$ to transform the closed-loop system form as

$$\dot{\mathbf{x}} = [\mathbf{J}_d(x) - \mathbf{R}_d] \frac{\partial H_d}{\partial \mathbf{x}}(x) \quad (18)$$

when the permanent magnet synchronous motor stable operating at the desired speed, there exists $\dot{\mathbf{x}}_3 = 0$, which means

$$\dot{\mathbf{x}}_3 = n_p[(L_d - L_q)i_d^* i_q^* + \phi i_q^*] - \tau_L = 0 \quad (19)$$

For salient pole permanent magnet synchronous motors, at the equilibrium \mathbf{x}^* and formula (19), we have $\tau = \tau^* = \tau_L^*$, $i_d^* = 0$, $i_q^* = \tau_L^* / (n_p \phi)$, which means at the equilibrium point, we have

$$\mathbf{x}^* = [x_1^* \quad x_2^* \quad x_3^*]^T = \left[0 \quad \frac{L_q \tau^*}{n_p \phi} \quad J \omega^* \right]^T \quad (20)$$

Without loss of generality, we choose

$$\mathbf{J}_a(x) = \begin{bmatrix} 0 & -J_{12} & J_{13} \\ J_{12} & 0 & -J_{23} \\ -J_{13} & J_{23} & 0 \end{bmatrix}, \mathbf{R}_a = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (21)$$

where J_{12} , J_{13} , J_{23} , r_1 and r_2 are the interconnection and damping parameters to be designed. From (7), we can obtain

$$\frac{\partial H}{\partial \mathbf{x}}(x) = \mathbf{D}^{-1} \mathbf{x} \quad (22)$$

$$\mathbf{K}(x) = \frac{\partial H_a}{\partial \mathbf{x}}(x) = \frac{\partial H_d}{\partial \mathbf{x}}(x) - \frac{\partial H}{\partial \mathbf{x}}(x) = -\mathbf{D}^{-1} \mathbf{x}^* \quad (23)$$

Substituting (22), (23) into (6), we can get

$$\begin{aligned} & [\mathbf{J}(x) + \mathbf{J}_a(x) - (\mathbf{R}(x) + \mathbf{R}_a(x))] \mathbf{K}(x) \\ &= -[\mathbf{J}_a(x) - \mathbf{R}_a(x)] \frac{\partial H}{\partial \mathbf{x}} + \mathbf{g}(x) \boldsymbol{\beta}(x) \end{aligned} \quad (24)$$

From (15) to (24), we can get

$$\begin{cases} u_d = \frac{-r_1}{L_d} x_1 - \frac{J_{12}}{L_q} x_2 + \frac{J_{13}}{J} x_3 + \frac{(R_s + r_1)}{L_d} x_1^* \\ \quad + \frac{J_{12}}{L_q} x_2^* - \frac{J_{13} + n_p x_2}{J} x_3^* \\ u_q = \frac{J_{12}}{L_d} x_1 - \frac{r_2}{L_q} x_2 - \frac{J_{23}}{J} x_3 - \frac{J_{12}}{L_d} x_1^* \\ \quad + \frac{(R_s + r_2)}{L_q} x_2^* + \frac{(J_{23} + n_p(x_1 + \phi))}{J} x_3^* \\ -\tau_L = \frac{-J_{13}}{L_d} x_1 + \frac{J_{23}}{L_q} x_2 + \frac{(n_p x_2 + J_{13})}{L_d} x_1^* \\ \quad - \frac{(n_p(x_1 + \phi) + J_{23})}{L_q} x_2^* \end{cases} \quad (25)$$

Substituting (20) into (25), we get

$$-\tau_L = -x_1 \left(\frac{J_{13}}{L_d} + \frac{\tau_L^*}{\phi} \right) + J_{23} \left(\frac{x_2}{L_q} - \frac{\tau_L^*}{n_p \phi} \right) - \tau_L^* \quad (26)$$

With the MTPA control theory, when system is operating at the equilibrium point, we have $\tau = \tau^* = \tau_L^*$ where

$$J_{12} = 0, J_{13} = -\frac{L_d n_p x_2^*}{L_q}, J_{23} = 0 \quad (27)$$

we get

$$\begin{cases} u_d = \frac{-r_1}{L_d} x_1 - \frac{L_d \tau_L^*}{J \phi} x_3 + \frac{L_d \tau_L^*}{\phi} \omega^* + n_p x_2 \omega^* \\ u_q = \frac{-r_2}{L_q} x_2 + \frac{R_s + r_2}{n_p \phi} \tau_L^* + n_p(x_1 + \phi) \omega^* \end{cases} \quad (28)$$

3.3 Stability Analysis

By (17) and (23), we get

$$\begin{aligned}\frac{\partial H_d}{\partial x}(x) &= \mathbf{D}^{-1}(\mathbf{x} - \mathbf{x}_0), \\ \frac{\partial^2 H_d}{\partial x^2}(x) &= \mathbf{D}^{-1}, \\ \frac{\partial \mathbf{K}}{\partial x}(x) &= 0\end{aligned}\quad (29)$$

when $\mathbf{x} = \mathbf{x}^*$, there is $\partial H_d / \partial x(x) = 0$ and

$\partial^2 H_d / \partial x^2(x) = \mathbf{D}^{-1} > 0$, Thus the stability determination of Proposition 1 is satisfied, and the system is asymptotically stable. The system control block diagram is as below:

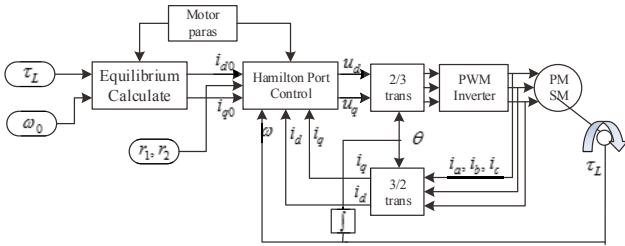


Fig1 System Control Block Diagram with Parameters Fixed

4 Self-tuning PID Parameters Realize

The u_d, u_q control law obtained from (28) usually choose a fixed gain parameters r_1, r_2 [12, 13], or choose a mixed gain control strategy[14]. Such gain parameters selection methods have tentatively and if situation changed, the parameter is not suitable to the system any more. In this paper, a self-tuning PID gain parameter method is proposed according to the speed error system, a PID gain parameters is introduced according to the system speed error to achieve parameters automatic control system. From (21), \mathbf{R}_a is a Semi-definite symmetric matrix, so the parameters r_1, r_2 value range are chosen as (0..200). The system control block diagram becomes:

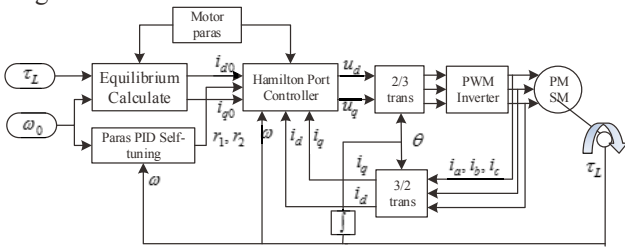


Fig 2 System Control Block Diagram with Parameters PID Self-tuning

5 Simulation Result

We do the system simulation test in simulink environment of matlab 2013a. PMSM parameters are chosen as DC bus voltage is 300V, the number of pole pairs n_p is 4, the moment of inertia J is $0.0021 \text{ kg} \cdot \text{m}^2$, d-axes and q-axes stator inductances $L_d = L_q = 0.153 \text{ H}$, the rotor flux

$\phi = 0.171 \text{ Wb}$, the stator resistance per phase

$R_s = 0.455 \Omega$.

Suppose the system initial rotate speed is 0 rad/s , after 0.2 s, set desire rotate speed $\omega^* = 70 \text{ rad/s}$, at time 1s, the desire rotate speed changes to $\omega^* = 120 \text{ rad/s}$, at time 2s, the desire rotate speed changes to $\omega^* = 80 \text{ rad/s}$, the system load τ^* is $0 \text{ N} \cdot \text{m}$.

The fig3 is the speed response curve in different r_1 and r_2 . From fig3, we can see in the different r_1 and r_2 , the system's dynamic response is different, and choosing a proper parameter r_1 and r_2 is not an easy work.

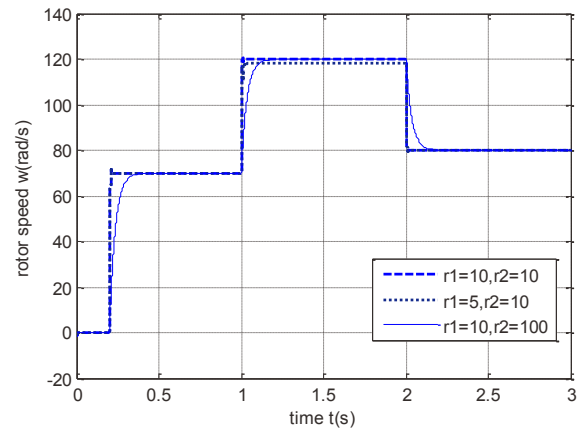


Fig 3 rotor speed for different parameters response figure

The fig 4 is the speed response curve with PID self-tuning parameters. The parameters initial value is 0. From fig 4.a, we can obtain nice speed response, and from fig 4.b, we can see the parameters r_1 and r_2 are changed according to the system speed error.

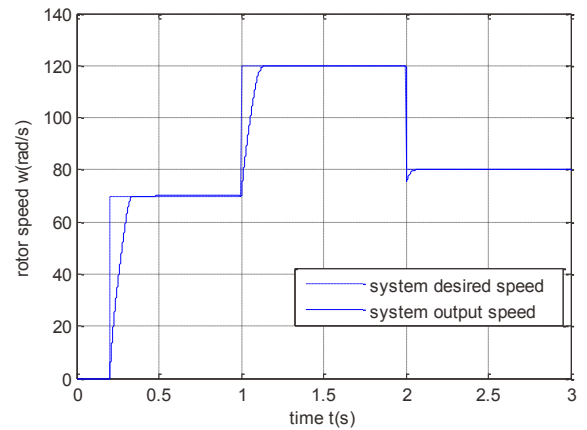


Fig4.a rotor speed response figure

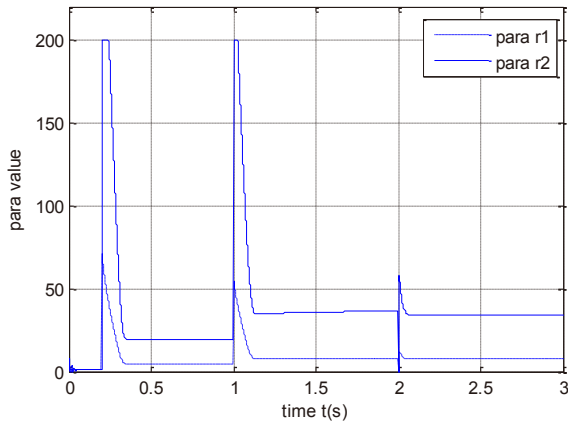


Fig 4.b parameters for PID Self-tuning
Fig 4 simulation result for parameters Self-tuning

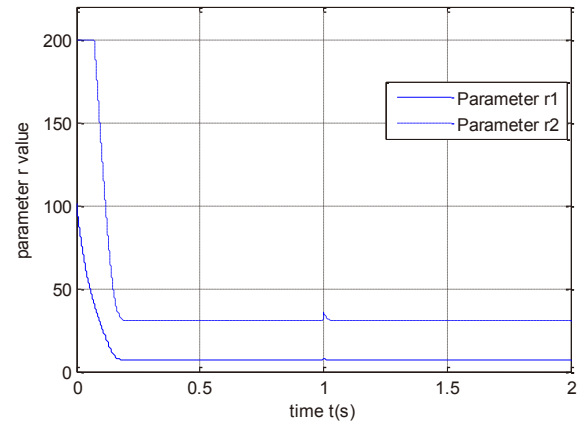


Fig 5.c parameters response figure
Fig 5 simulation result for load torque change

Fig. 5 is the system speed response when load torque changed from $0N \cdot m$ to $5N \cdot m$ at $t = 1s$, and we can see the system gets stable quickly, and system parameters change as the system speed output.

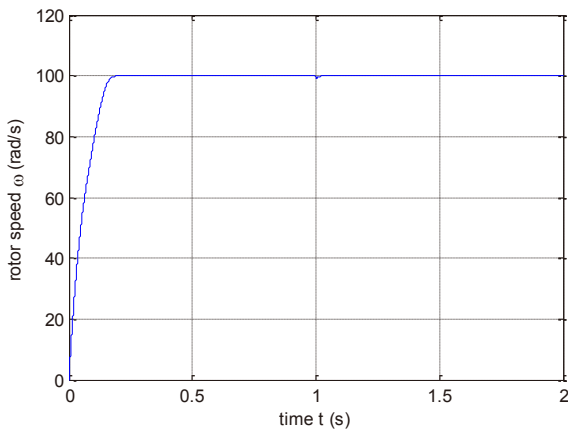


Fig 5.a rotor speed response figure

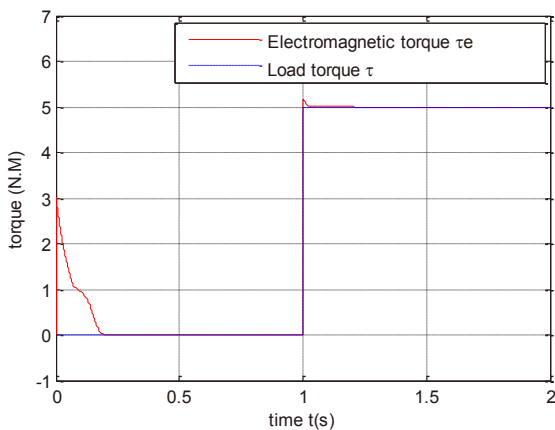


Fig 5.b torque response figure

6 Simulation Result

In this paper, we proposed an energy-based approach to PMSM control with parameters self-tuning PID control which achieves good speed tracking motion by keeping the system's total energy to the desired value. The port-controlled Hamiltonian structure of PMSM system was presented. The speed controller was designed with the MTPA theory. The parameters self-tuning PID control method was introduced in system. The simulation results show that the PMSM have good speed response and well robustness using the proposed method, and the self-tuning parameters have good performance and more easy to design comparing with the fix value parameters. The designed is simple and can be implemented facilely, it is applicable. the further information will be studied as future works.

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