

# A Method for Auto-tuning of PID Control Parameters\*

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*A method for tuning PID parameters in a digitalized controller introduces a concept of characteristic areas of the step response for process parameter estimation and a new type of performance index for controller parameter optimization.*

**Key Words**—PID process control; direct digital control; process parameter estimation; weighted ISE; control parameter optimization; man-machine interactive method; adaptive control.

**Abstract**—A method for automatic tuning of the PID process control parameters, usually called ‘auto-tuning’, is developed. The procedure of applying the method consists of (1) sampling a process response to a test input signal, (2) processing the sampled data for estimating characteristic values of the process, and (3) calculating the optimal values of the PID control parameters. For the optimization, a new type of performance index, i.e. a weighted integral of squared error is introduced. The procedure is implemented on a digital controller using microprocessors and applied to some real processes, yielding satisfactory results.

## 1. INTRODUCTION

IN THE last few years, various kinds of direct digital control (DDC) systems have been developed in Japan offering some new aspects in the process control technique. Most of these systems are utilizing microprocessors as basic components. In this paper, a new method for automatic tuning of PID control parameters in a DDC system is proposed, and some results of field tests obtained by application of the method to real plants are also reported.

In the PID process control, the following tuning methods are typically used; one is the method proposed by Ziegler, Nichols and Rochester (1942) and the other is the method by Chien, Hrones and Reswick (1952). The former needs the ultimate gain and the period of the ultimate oscillation at the stability limit. But it is difficult to determine their exact values experimentally in real processes, because the oscillatory operation must be avoided.

The latter requires an exact form of the process expressed by a transfer function, but many of real processes fail to reveal their transfer functions.

For the automatic tuning based on process parameters estimation, Åström and Wittenmark (1973) proposed the self-tuning controller and Isermann (1978) has been developing the self-adaptive controller. These controllers rely upon the modern control theory, and contain many control parameters. Therefore field engineers and plant operators are often embarrassed in attempting to understand the conceptual structure of the control theory and the relationships between parameter settings and control system actions.

We consider that, in practical designs and operations, the continuity of understanding of the object system among design engineers, field engineers, plant operators, etc. is a crucial point for success. Therefore our automatic tuning method is developed for improving the PID control technique which is still dominant in practical use and is also traditionally familiar to the many classes of engineers.

Our method of automatic tuning is called ‘auto-tuning’ for brevity. The procedure of applying the method consists of (1) sampling a process response signal to a test input signal, (2) processing the sampled data for estimating characteristic values of the process, and (3) calculating the optimal values of PI or PID control parameters.

The operator starts this procedure only when he requires the initial PID settings or to modify them to adapt to a changing situation. By an indication of the operator, the test signal is generated and then the optimal settings are obtained automatically. Although periodical repetition of the procedure is possible for a kind of adaptive control, it is undesirable because excessive test signals are not allowable. Therefore, a man-machine interactive scheme based on the operator’s judgement is better

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than the so-called 'fully automatic' procedure. In this viewpoint, our tuning method differs from the self-tuning controller and self-adaptive controller which continually change the estimated values of the process parameters and adjust the settings.

In order to estimate the process parameters, a concept of characteristic areas of the step response is introduced. The use of these areas is more effective than the use of the maximum slope of the response curve, for reducing the estimation error in somewhat noisy processes.

We also introduce a concept of the weighted integral of squared error (weighted ISE) as a performance index for obtaining the optimal settings of the PID parameters. The weighting factor is an exponential function of the time. This index can be readily calculated in the  $s$  domain, and then the optimal solution is searched in a PID parameter domain by using the conjugate gradient method. It is expected that, from the form of the index, the settings for a desired damping ratio of closed loop response is obtained by prescribing only a scalar parameter in the weighting factor.

The paper is organized as follows. Section 2 describes the process control system under our study. Section 3 presents a method for estimating process parameters, and Section 4, the principal part of the paper, develops procedures for optimizing PID parameters with use of the weighted ISE. Some results of field applications are summarized in Section 5. Section 6, the last section, gives some concluding remarks.

## 2. DESCRIPTION OF THE PROCESS CONTROL SYSTEM

A tuning procedure for a standard PID control system with a single input and a single output, as shown in Fig. 1 is considered. We treat both the self-regulating process and the non-self-regulating process expressed by transfer functions

$$G_p(s) = \frac{K_p e^{-Ls}}{\pi_{j=1}^n (1 + T_j s)} \quad (1)$$

and

$$G_p(s) = \frac{e^{-Ls}}{Ts [\pi_{j=1}^n (1 + T_j s)]} \quad (2)$$

respectively. In (2), the gain constant is defined by  $1/T$  where  $T$  has the dimension of time. This gives the simplest expression for the non-self-regulating process. As space is limited, theoretical development here is confined only to the self-regulating process.

The transfer function of the digital PID controller is given in one of the following forms according to the parallel or the series connection of a differentiation element:

for parallel connection

$$\begin{aligned} G_{c1}(s) &= K_c \left( 1 + \frac{1}{T_i s} + T_d s \right) \\ G_{c2}(s) &= 1 \end{aligned} \quad (3)$$

for series connection

$$\begin{aligned} G_{c1}(s) &= K_c \left( 1 + \frac{1}{T_i s} \right) \\ G_{c2}(s) &= \frac{1 + T_d s}{1 + T_i s}. \end{aligned} \quad (4)$$

As shown in Fig. 1 and (4), the series connection of the PI action and the D action is utilized in actual control systems, and it is adopted also in our DDC system for avoiding an excessively large output of the D action for an abrupt change of the reference input. However, the parallel connection expressed by (3) is used in this paper for convenience of the theoretical investigation, because the actions of P, I and D have no interaction.

## 3. METHOD FOR ESTIMATION OF THE PROCESS PARAMETERS

As mentioned in Section 1, the self-tuning controller was first proposed by Åström and Wittenmark (1973) and the self-adaptive controller was studied by Isermann (1978). These controllers

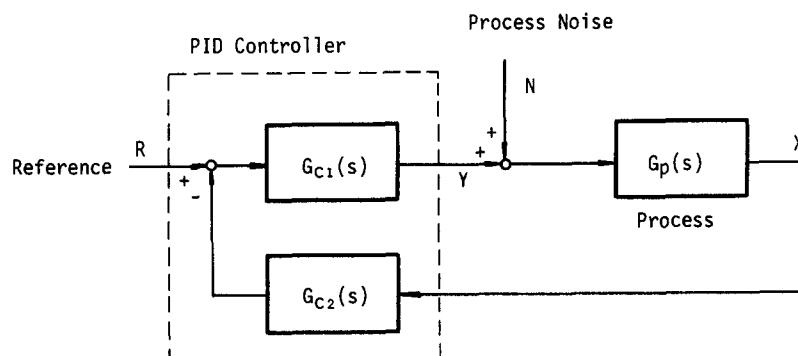


FIG. 1. Block diagram of PID control system.

estimate the process parameters by using transient responses of both the process and the controller to disturbances generated spontaneously in a normal process operation.

In contrast with these, the present procedure applies a kind of intentional disturbance to the process for estimating its parameters. In general, use of an intentional disturbance is not desirable. However, we need only a small pulse signal which does not disturb the normal operation significantly. In practice, the control parameters are kept constant unless the process parameter varies to some extent. Therefore, the tuning procedure should be applied only when a plant operator desires to apply it.

Among various tuning methods proposed previously, the method by Ziegler, Nichols and Rochester (1942) is quite simple and well known. However, exact values of the ultimate gain and the period of the ultimate oscillation of a real process can rarely be obtained, because the ultimate oscillation is not permitted for fear of unstable operation. Therefore, the value of ultimate gain is usually calculated from the Bode diagram for a process assumed in the following form:

$$G_p(s) = \frac{K_p e^{-Ls}}{1 + Ts} \quad (5)$$

where the deadtime  $L$  and the time constant  $T$  are estimated by drawing the maximum slope line along the step response curve of the process.

In this paper, the concept of characteristic areas of the step response is introduced and the relationship between the characteristic areas and the values of process parameters is derived. The use of area which is obtained by an integral calculation is more effective than use of the slope, to reduce the estimation error induced by a slight process noise. The truncation error of the numerical integration with an infinite interval is negligible as compared with the error caused by process disturbances. In this section, both the open-loop and the closed-loop procedures are discussed.

### 3.1. Open-loop procedure

We have previously proposed a method of open-loop estimation based on the idea of characteristic areas (Ohta and co-workers, 1980). The procedure is as follows:

1. The open-loop response of the process output to a pulse input signal is sampled. Use of the pulse input is enough to obtain necessary data for the consequent calculation, avoiding continuous disturbance to the process. The amplitude  $A$  and the width  $T_c$  of the test pulse are automatically determined in a computer, in order to obtain the response suitable for estimating the process parameters. The sampling

interval is determined so that the whole sequence of the sampled data can be stored in a prepared region of the computer memory.

2. The response with or without self-regulation is distinguished on the basis of offset after the test pulse is deleted.

3. By assuming linearity of the process, the step response  $x(t)$  is obtained from the pulse response  $z(t)$  as

$$x(t) = \begin{cases} z(t), & 0 \leq t \leq T_c \\ z(t) + x(t - T_c), & t > T_c \end{cases} \quad (6)$$

4. The characteristic areas are calculated, and then the process parameters are estimated.

As an example, we consider the self-regulating process whose step response is shown in Fig. 2. In this case, the characteristic areas  $S_T$  and  $S(t)$  are defined, respectively, by

$$S_T \triangleq \int_0^\infty [x(\infty) - x(\tau)] d\tau \quad (7)$$

$$S(t) \triangleq \int_0^t x(\tau) d\tau. \quad (8)$$

The process gain  $K_p$  is obtained from the final value  $B$  of the step response as  $K_p = B/A$ . The characteristic area  $S_T$  gives the sum of the time constants and the deadtime, i.e.

$$T_T \triangleq \sum_{j=1}^n T_j + L = \frac{S_T}{B}. \quad (9)$$

We also define the following quantities  $\sigma$  and  $\sigma'$  with aid of (8)

$$\sigma \triangleq \frac{S(T_T)}{B T_T} \quad (10)$$

$$\sigma' \triangleq \frac{S(\kappa T_T)}{B T_T} \quad (11)$$

where  $\kappa$  is a constant between 0 and 1. The value of  $\sigma$  becomes large as the order  $n$  or the deadtime  $L$  decreases in the process (1). When  $n = 1$  and  $L = 0$

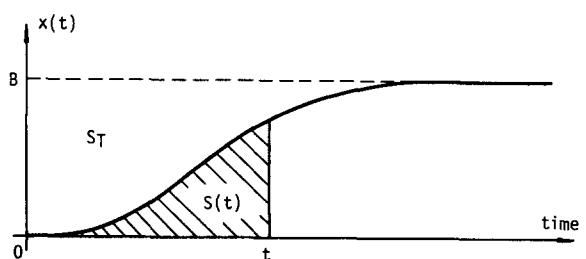


FIG. 2. Characteristic areas of the step response for a self-regulating process.

in (1), it takes the maximum  $\sigma_{\max} = e^{-1} = 0.368$ , which is independent of the time constant  $T_1$ . Therefore,  $\sigma$  is considered to be a measure representing the delay of the process.

If the process is expressed by (1) with  $n = 1$ , the corresponding process parameters  $T_1$  and  $L$  are determined in terms of  $T_T$  and  $\sigma$  as

$$T_1 = e\sigma T_T \quad (12)$$

$$L = T_T - T_1. \quad (13)$$

For a more precise estimation, additional use of  $\sigma'$ , a partial characteristic area, is necessary. If the process is expressed by (1) with  $n = 2$ , that is, the parameters  $T_1$ ,  $T_2$ , and  $L$  are obtained in terms of  $T_T$ ,  $\sigma$ , and  $\sigma'$  from the following equations

$$T_T = L + T_1 + T_2 \quad (14)$$

$$\begin{aligned} T_T \sigma &= \frac{T_1^2}{T_1 - T_2} \exp [-(T_T - L)/T_1] \\ &\quad + \frac{T_2^2}{T_2 - T_1} \exp [-(T_T - L)/T_2] \end{aligned} \quad (15)$$

$$\begin{aligned} T_T \sigma' &= (\kappa - 1)T_T + \frac{T_1^2}{T_1 - T_2} \exp [-(\kappa T_T - L)/T_1] \\ &\quad + \frac{T_2^2}{T_2 - T_1} \exp [-(\kappa T_T - L)/T_2]. \end{aligned} \quad (16)$$

Furthermore, an arbitrarily precise approximation is possible if a necessary number of different partial characteristic areas can be calculated. However, with the increase of  $n$ , the differences among the partial characteristic areas become smaller and the errors of  $T_j$  obtained become larger.

On the other hand, in order to get the optimal PID settings, it is sufficient practically to approximate a self-regulating process by (1) with  $n = 1$  or 2.

In addition, the transformation from the characteristic areas to time constants of (1) by using (12)–(16) is not necessary, if the optimal settings can

be given directly from the characteristic areas. Thus, for the self-regulating process, the gain  $K_p$ , the sum of time constants  $T_T$  and two characteristic areas  $\sigma$ ,  $\sigma'$  are regarded as the parameters to be estimated. As for the non-self-regulating process, see Ohta and co-workers (1980).

### 3.2. Closed-loop procedure

The closed-loop procedure for estimating the process parameters has an advantage to avoid special operation of opening the loop. However, the closed-loop procedure is generally more complicated. In this section, we propose a relatively simple procedure of the closed-loop estimation.

Figure 3 shows a block diagram for this procedure. Either of the following disturbance signals is applied to the control system to sample the closed-loop response of the process, i.e. (a) the pulse input  $R'$  or (b) the step input  $N'$ . In case of (a), the response obtained is converted to the corresponding step response. In either method, the characteristic areas are defined by the following:

$$S_x \triangleq \int_0^\infty [x(\infty) - x(t)] dt \quad (17)$$

$$S_y \triangleq \int_0^\infty [y(\infty) - y(t)] dt \quad (18)$$

$$SA_x(\alpha) \triangleq \int_0^\infty e^{-\alpha t} [x(\infty) - x(t)] dt \quad (19)$$

$$SA_y(\alpha) \triangleq \int_0^\infty e^{-\alpha t} [y(\infty) - y(t)] dt \quad (20)$$

where  $x(\infty)$  and  $y(\infty)$  are the final values of  $x(t)$  and  $y(t)$ , respectively.

Examples of  $S_x$  and  $S_y$  are shown by the hatched areas in Fig. 4, for a closed-loop response to the step input  $R'$ . The characteristic areas  $SA_x$  and  $SA_y$  are the integrals of the responses  $x$  and  $y$ , respectively, weighted by the function  $e^{-\alpha t}$ . The parameter  $\alpha$  is a positive number chosen appropriately from the

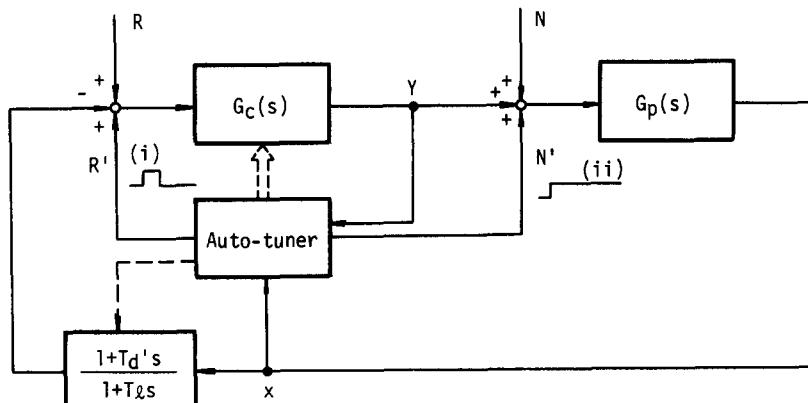


FIG. 3. Block diagram of closed-loop auto-tuning.

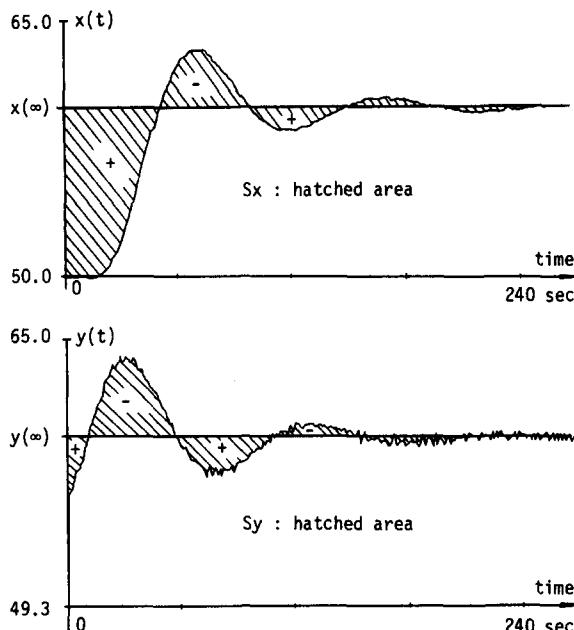


FIG. 4. Example of closed-loop response to step change of the reference.

estimated value of  $T_T$ . If the value of  $\alpha$  is too small the differences between  $SA$  and  $S$  values cannot be detected; on the contrary, if the value of  $\alpha$  is too large  $SA$  values become too small.

These characteristic areas can be calculated numerically by using the trapezoidal-rule formula in terms of the sampled data of  $x(t)$  and  $y(t)$ . The final values  $x(\infty)$ ,  $y(\infty)$ , and the characteristic areas are related to the parameters of the process and the controller when the system structure is given. Those relations can be obtained by applying the final-value theorem of Laplace transformation to the closed loop transfer function. For example, in case of applying the unit step input  $R'$  to the series type PID controller for self-regulating process, they are

$$x(\infty) = 1 \quad (21)$$

$$y(\infty) = \frac{1}{K_p} \quad (22)$$

$$S_x = \frac{T_i + (T_d - T_i)K_p K_c}{K_p K_c} \quad (23)$$

$$S_y = \frac{1}{K_p} \left[ \frac{T_i}{K_p K_c} + T_d - T_i - \left( \sum_{j=1}^n T_j + L \right) \right] \quad (24)$$

$$SA_x(\alpha) = \frac{\frac{1}{\alpha} \left( \frac{(T_d - T_i)\alpha}{1 + T_i\alpha} K_c \left( 1 + \frac{1}{T_i} \right) K_p F_\alpha \right)}{\frac{1 + T_d\alpha}{1 + T_i\alpha} K_c \left( 1 + \frac{1}{T_i\alpha} \right) K_p F_\alpha} \quad (25)$$

$$SA_y(\alpha) = \frac{\frac{1}{\alpha} \left[ \frac{1}{K_p} - \frac{K_c \left( 1 + \frac{1}{T_i\alpha} \right)}{1 + \frac{1 + T_d\alpha}{1 + T_i\alpha} K_c \left( 1 + \frac{1}{T_i\alpha} \right) K_p F_\alpha} \right]}{1 + \frac{1 + T_d\alpha}{1 + T_i\alpha} K_c \left( 1 + \frac{1}{T_i\alpha} \right) K_p F_\alpha} \quad (26)$$

where

$$F_\alpha = \frac{e^{-L\alpha}}{\pi_{j=1}^n (1 + T_j\alpha)}. \quad (27)$$

In an actual procedure, the pulse input is applied instead of the step input due to the same reason as in the open-loop procedure. By using these equations, the parameters  $K_p$ ,  $T_j$ , and  $L$  can be estimated in various ways. For example, the process gain  $K_p$  is obtained from either (22) or (23). Consequently, propriety of the values obtained can be examined by a cross-checking. Because of space limitation, a more detailed description is omitted here.

#### 4. METHOD FOR OPTIMIZATION OF THE PID CONTROL PARAMETERS

There are many propositions on the optimal PID setting. Among them, the method of Ziegler, Nichols and Rochester has found acceptance with most field engineers because the method intends to make efficient use of their common sense, that is, **25% damping**. This means that the latter amplitude of the successive peaks in the oscillation of the process output should be less than 25% of the former amplitude.

However, in real operation of the process, a little more damping than 25% damping is usually preferred for guaranteeing stability even in case where plenty of noise exist, and a much more damping is especially desired in cases where the reference is changed frequently. Further, Ziegler, Nichols and Rochester's method has been found poor for the process with dominant deadtime.

Instead of their method, here we propose another method of realizing a transient response of 'desired' damping. For this purpose, a new performance index is introduced for the optimal PID settings. The index is the weighted integral of squared error (ISE) where the weighting factor is a function increasing exponentially with the time.

##### 4.1. Calculation of ISE by use of the Hurwitz determinant

First, we summarize the procedure for minimizing ISE with respect to the PID parameters. We call it ISE method. The error of the process output is

$$\Delta x(t) \triangleq r(t) - x(t). \quad (28)$$

We want to minimize ISE

$$J = \int_0^\infty [\Delta x(t)]^2 dt. \quad (29)$$

The integral of (29) is evaluated in the  $s$  domain by using Parseval's theorem [as for the procedure, see for instance James, Nichols and Phillips (1947) and Fukawa (1974)].

If the Laplace transform of  $\Delta x$  is given by

$$\mathcal{L}[\Delta x(t)] = \frac{Q(s)}{M(s)} \quad (30)$$

where

$$\begin{aligned} M(s) &= a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n \\ Q(s)Q(-s) &= b_0 s^{2n-2} + b_1 s^{2n-4} + \cdots \\ &\quad + b_{n-2} s^2 + b_{n-1} \end{aligned} \quad (31)$$

then,  $J$  is obtained by the ratio of the two determinants as follows:

$$J = (-1)^{n-1} \frac{B_n}{2a_0 H_n}. \quad (32)$$

In (32)  $H_n$  is the **Hurwitz determinant** of  $M(s)$  and  $B_n$  is obtained from  $H_n$  by replacing the first row by  $(b_0, b_1, \dots, b_{n-2}, b_{n-1})$ .

In the control system of Fig. 1,  $Q(s)$  and  $M(s)$  of (30) are easily formulated for the step change of the reference  $R$  or the process noise  $N$ . For convenience, these two procedures are called the reference change method (RCM) and the process noise method (PNM), respectively. Since  $Q(s)$  and  $M(s)$  contain PID parameters, we minimize the performance index  $J$  by adjusting these parameters. The minimum point is searched by the **conjugate gradient method without the derivative of  $J$**  proposed by Zangwill (1967).

Since the transfer function of the process must be a rational form in order to use (30), the **deadtime element is expressed, by the second-order approximation of Padé**, as

$$e^{-Ls} = \frac{12 - 6Ls + L^2 s^2}{12 + 6Ls + L^2 s^2}. \quad (33)$$

#### 4.2. Optimal PID settings by ISE method

Let us examine the results of control action for the PID setting obtained by ISE method. As an example, for the self-regulating process with the transfer function

$$G_p(s) = 1/[(1 + s)(1 + \mu/3s)^3] \quad (34)$$

the optimal settings of parallel PID control are given as those in Table 1. For comparison, the setting

TABLE 1. OPTIMAL PARAMETERS OF PARALLEL PID CONTROL FOR THE PROCESS (34) WITH  $\mu = 0.5$

| Method | K <sub>c</sub> | T <sub>i</sub> | T <sub>d</sub> |
|--------|----------------|----------------|----------------|
| PNM    | 12.80          | 0.146          | 0.533          |
| RCM    | 4.27           | 0.630          | 0.774          |
| ZNM    | 4.56           | 0.761          | 0.190          |

obtained by the Ziegler-Nichols method (ZNM) is also shown in the table.

Figure 5 shows the process outputs to the process noise and the reference change for the three sets of PID parameters in Table 1. At a glance of these responses, the optimal setting by ZNM seems to give a better result than those by PNM and RCM. Especially, in the case of PNM, we find that the small oscillation remains in the response to the process noise. The effect of the slow damping of the oscillation by PNM is more clearly shown in the response to the reference change. It means that the settings by PNM give undesirable oscillation. In addition, PNM has such a high feedback gain that the margin of the stability is very little and the sensitivity of the process parameters is very high. Hence, these settings are not applicable to the control of real processes.

#### 4.3. Weighted ISE

To overcome the shortcomings of the settings by the ISE method, we introduce a new weighted ISE of the form

$$J(\beta) = \int_0^\infty [\Delta x(t) e^{\beta t}]^2 dt \quad (35)$$

where  $\beta$  is a positive number. The value of  $J(\beta)$  is easily calculated by use of the relation

$$\mathcal{L}[\Delta x(t) e^{\beta t}] = Q(s - \beta)/M(s - \beta). \quad (36)$$

That is to say, we obtain, from  $M(s)$  and  $Q(s)$ , the coefficients of  $M(s - \beta)$  and  $Q(s - \beta)$  and construct the Hurwitz determinants from them. This is the exceeding merit of the form of (35), as compared with the case of integral of time multiplied by the absolute error (ITAE).

The weighted ISE becomes unbounded unless the error  $\Delta x(t)$  converges to zero with a larger damping than  $\beta$ . Therefore, the optimal setting which minimizes the index (35) with the parameter  $\beta$  chosen appropriately yields fast damping characteristics of the oscillation. Further, we can obtain the optimal settings which satisfy a desired damping, if we can predict the period of oscillation of the closed-loop response.

Here we provide the following proposition:

**Proposition.** The period  $P_c$  of the oscillation of the closed-loop response whose PID control parameters are set by a tuning strategy is proportional to the period  $P_u$  of the ultimate oscillation of the loop.

The applicability of this proposition is confirmed by several case studies. An example is shown in Table 2 for various tuning strategy applied to the process (34).

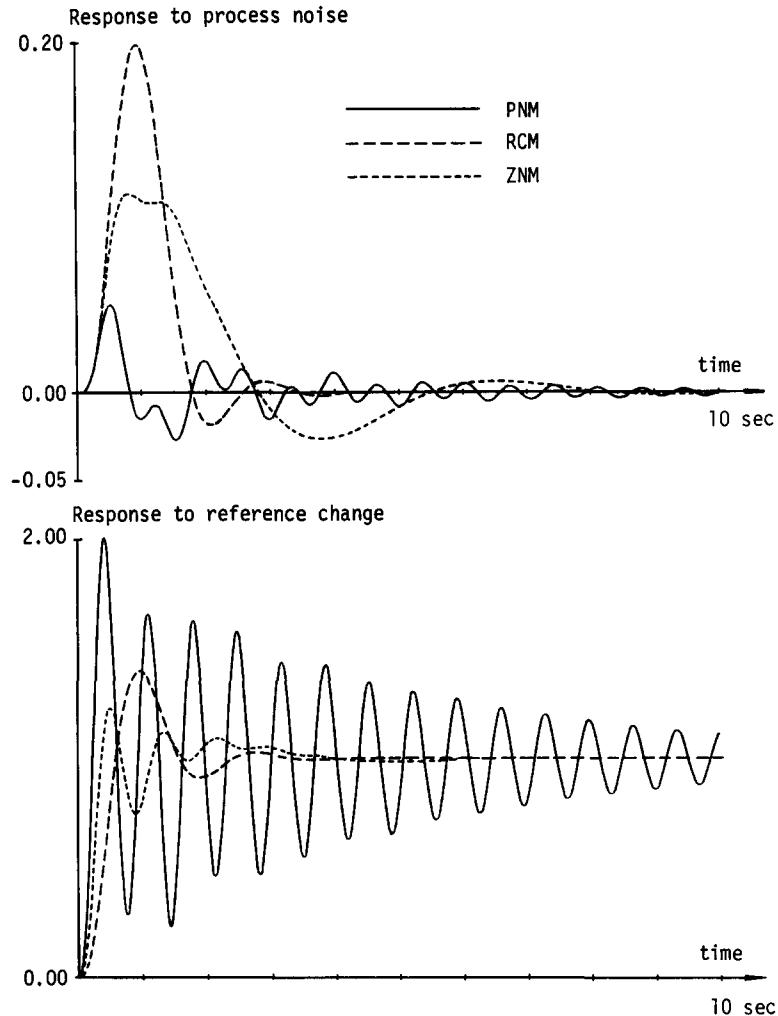


FIG. 5. Results obtained by the settings of Table 1.

According to this proposition, we can set  $\beta$  by

$$\beta = \gamma/P_u \quad (37)$$

where  $P_u$  is determined corresponding to an identified process. The parameter  $\gamma$  is assigned skillfully so as to get the desired damping ratio of the closed-loop response. As we expect, the larger value of  $\gamma$  gives the larger damping factor and the more stable closed-loop response. For various value of  $\gamma$ , the optimal settings of PI and PID parameters of the process expressed by (34) with  $\mu = 1.0$  are listed in Tables 3 and 4, respectively. The corresponding closed-loop responses are shown in Fig. 6. The values of  $\gamma$  are selected from 0 to 1.0 for PI control and from 0 to 2.0 for PID, because a shorter period of the oscillation can be expected for PID than PI.

TABLE 2. RATIO  $P_c/P_u$  FOR VARIOUS  $\mu$  IN (34)

| $\mu$    | 0.5  | 1.0  | 2.0  |
|----------|------|------|------|
| ISE, PNM | 1.20 | 1.19 | 1.20 |
| ISE, RCM | 1.41 | 1.37 | 1.35 |
| ZNM      | 1.40 | 1.35 | 1.31 |

TABLE 3. OPTIMAL PARAMETERS OF PI CONTROL

| $\gamma$ | 0.0            | 0.25 | 0.5  | 1.0  |
|----------|----------------|------|------|------|
| PNM      | K <sub>c</sub> | 3.20 | 2.62 | 2.12 |
|          | T <sub>i</sub> | 2.67 | 2.23 | 1.92 |
| RCM      | K <sub>c</sub> | 2.07 | 1.76 | 1.49 |
|          | T <sub>i</sub> | 2.92 | 2.36 | 1.99 |

TABLE 4. OPTIMAL PARAMETERS OF PID CONTROL

| $\gamma$ | 0.0            | 0.5   | 1.0   | 2.0   |
|----------|----------------|-------|-------|-------|
| PNM      | K <sub>c</sub> | 8.00  | 7.63  | 6.73  |
|          | T <sub>i</sub> | 0.267 | 0.403 | 0.576 |
|          | T <sub>d</sub> | 1.00  | 0.781 | 0.640 |
| RCM      | K <sub>c</sub> | 2.87  | 3.27  | 3.28  |
|          | T <sub>i</sub> | 0.804 | 1.21  | 1.52  |
|          | T <sub>d</sub> | 1.36  | 0.937 | 0.715 |

With the same values of  $\gamma$ , similar responses are obtained also in the case of non-self-regulation process. It is generally recognized that the optimal settings with larger value of  $\gamma$  have smaller sensitivity to errors in the process parameter estimation. In other words, when the optimal

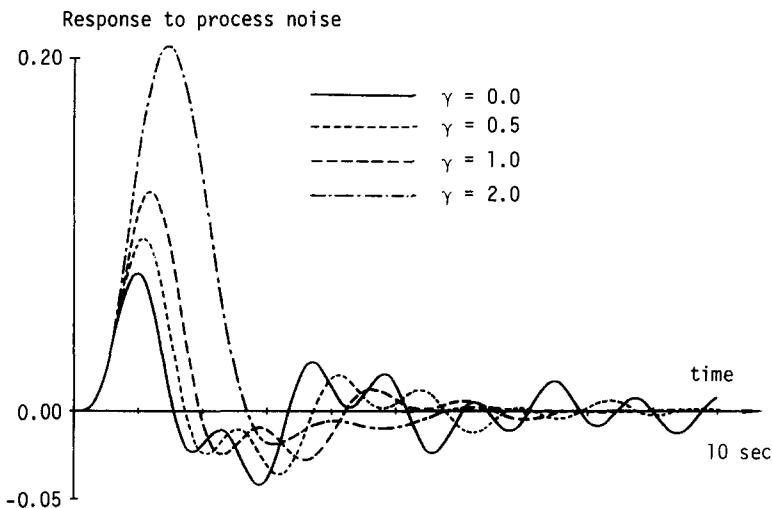


FIG. 6. Results obtained by the optimal PID control (PNM).

parameters are set for a large value of  $\gamma$ , there are only quite small differences among the closed-loop responses, and then instability due to the estimation error can be avoided. As the results of various case studies, we recommend  $\gamma = 0.8$  for the PI control and  $\gamma = 1.5$  for the PID control.

#### 4.4. Optimal PI control of the process with dominant deadtime

It has been thought that the process with a large deadtime cannot be controlled satisfactorily by PI actions. However, the procedure described in Section 4.3 is well applicable to the process expressed by the transfer function (5) with large  $L$ .

Figure 7 shows the optimal PI settings obtained by the proposed method with  $\gamma = 0.8$  as well as by

other conventional tuning methods. In Table 5 and Fig. 8, the control parameters and the corresponding results are also compared for the process (5) with  $K_p = 1.0$  and  $L = T = 0.5$  s. It is observed that the proposed method offers a better result than the other two methods cited. Moreover, it is noted that the present method gives good tunings to the process even with  $L/T > 1$ . The results have enough margin when the process parameters, such as the gain  $K_p$ , or the deadtime  $L$ , change more than 40%.

#### 5. APPLICATIONS OF THE PROPOSED METHOD TO REAL PROCESSES

In order to investigate validity and effectiveness of the present procedure, some field tests have been executed. The proposed algorithm was first

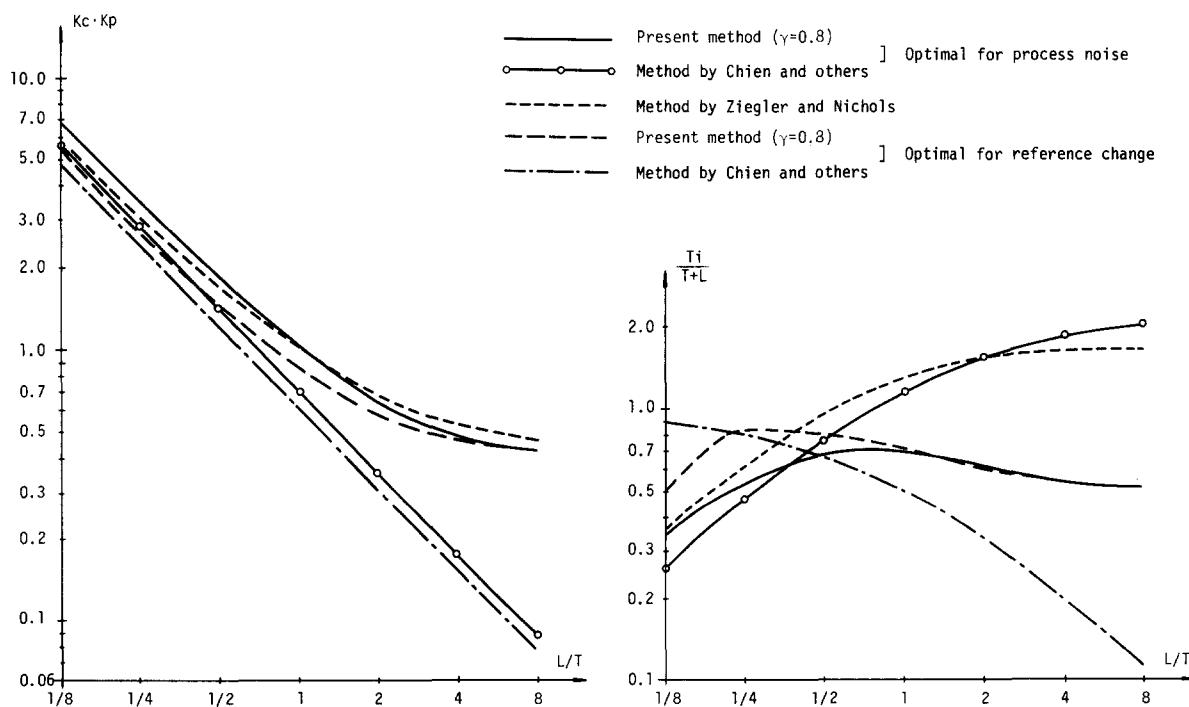


FIG. 7. Optimal parameters of PI control for the process with deadtime.

TABLE 5. OPTIMAL PARAMETERS OF PI CONTROL FOR PROCESS NOISE OBTAINED BY THE VARIOUS METHODS

| Method of tuning              | $K_c$ | $T_i$ |
|-------------------------------|-------|-------|
| Present method                | 1.018 | 0.695 |
| Method by Ziegler and Nichols | 1.018 | 1.29  |
| Method by Chien and others    | 0.700 | 1.15  |

implemented in one card of a microcomputer DDC system, called FUJI MICREX, and presently more complicated DDC systems with hierarchical computer structures are also available.

In the total procedure of the auto-tuning, the calculation of the optimal settings described in Section 4 is too complicated to implement on a small scale computer. Therefore, the calculation procedure is partly modified as follows. First, many kinds of transfer functions (1), (2) and combinations of time constants are chosen for getting various values of characteristic areas. Secondly, the optimal settings of the PID parameters are obtained for all those processes chosen. These calculations are performed by the aid of a large-scale off-line computer. Finally, the relationship between the characteristic areas and the optimal settings is approximated by a set of simple polynomials. It is noted that enough stability margin should be kept with consideration of the influence of approximation errors, even if the result yields an overdamping.

For example, the approximate PID parameters for the self-regulating process are given by

$$\begin{aligned} K_c &= f_1(\sigma, \sigma')/K_p \\ T_i &= f_2(\sigma, \sigma')T_T \\ T_d &= f_3(\sigma, \sigma')T_T \end{aligned} \quad (38)$$

where  $f_i(\sigma, \sigma')$  are the third-order polynomials of  $\sigma$  and  $\sigma'$ . The set of polynomials includes all the combinations of controller, PI or PID; process, the self-regulating or the non-self-regulating; and policy, for the process noise or the reference change. Then we have 8 ( $= 2 \times 2 \times 2$ ) sets of polynomials,

which can be easily memorized in small size on-line computers. The detailed expression of these polynomials are omitted here.

The system for this auto-tuning has worked satisfactorily in several applications to real processes. Typical results are summarized as follows.

**Case 1.** A clinker-feed process of a cement plant as shown in Fig. 9(a) is one of the self-regulating processes. To this process, the auto-tuning was applied in the set up period of the control system, without any prior information for PID settings. A pulse test signal was generated in the form of Fig. 9(b) and the corresponding process response was obtained as shown also in the figure. Then, the response was sampled and converted into the step response by using (6). From the pulse response, this process was regarded as a self-regulation process, and from the converted step response,  $K_p$ ,  $T_T$ ,  $\sigma$ , and  $\sigma'$  were calculated in the computer. Finally the PID parameters were obtained for Test I as

$$K_c = 0.689, \quad T_i = 31.9 \text{ s}, \quad T_d = 7.9 \text{ s}$$

By the same procedure, we had for Test II

$$K_c = 0.632, \quad T_i = 30.0 \text{ s}, \quad T_d = 7.5 \text{ s.}$$

In order to reduce effect of the process noise, the time constant  $T_i$  of a filter had been previously set to 10.0 s for both the test cases. Figure 9(c) shows a result of the closed-loop response obtained by the settings of the auto-tuning.

**Case 2.** Figure 10(a) shows schematically a heat exchanger (self-regulating process) in a power plant of a chemical process. In this plant, tar is used for fuel after it is warmed by steam. The controller TIC regulates the tar temperature at a fixed set point. The auto-tuning was executed in this control loop. Figure 10(b) shows an example of the test signal for the position of the valve together with the corresponding temperature response. As shown in this figure, disturbance to the process is small. From

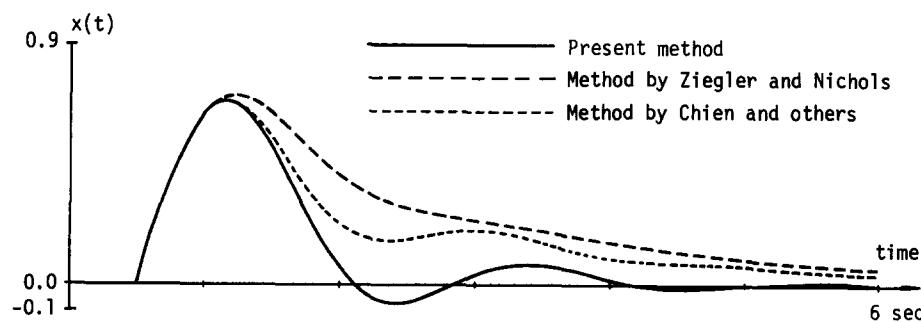


FIG. 8. Responses to the step process noise.

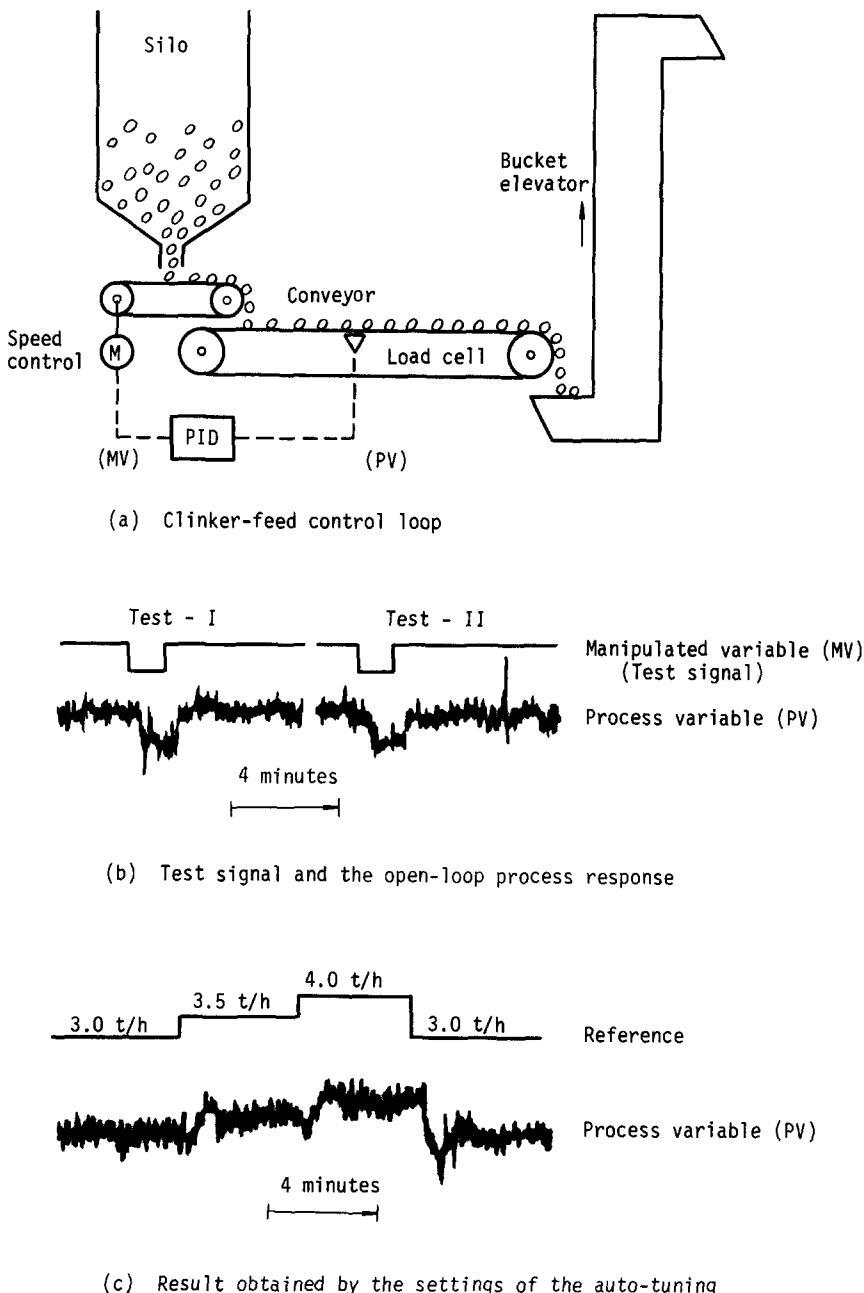


FIG. 9. Application to a cement plant.

this small response, the auto-tuning calculated the settings as

$$K_c = 1.32, \quad T_i = 74 \text{ s}, \quad T_d = 18 \text{ s}, \quad T_l = 2 \text{ s}$$

which gave the stable closed-loop response as shown in Fig. 10(c).

We assumed the transfer function of the process by (5) and sought the values of the parameters  $K_p$ ,  $T$  and  $L$  by both the methods described in Sections 3.1 and 3.2. The open-loop procedure gave

$$K_p = 0.353, \quad T = 85 \text{ s}, \quad L = 87 \text{ s}$$

from the data shown by Fig. 10(b). On the other hand, the closed-loop procedure gave

$$K_p = 0.290, \quad T = 90 \text{ s}, \quad L = 76 \text{ s}$$

from the data shown by Fig. 10(c).

*Case 3.* A tank-level control of Fig. 11(a) is an example of the non-self-regulating process. Up to present, this process has been operated with a very small feedback gain. The auto-tuning based on the data of Fig. 11(b) suggests much larger values. Even with those large values of the feedback gain, the

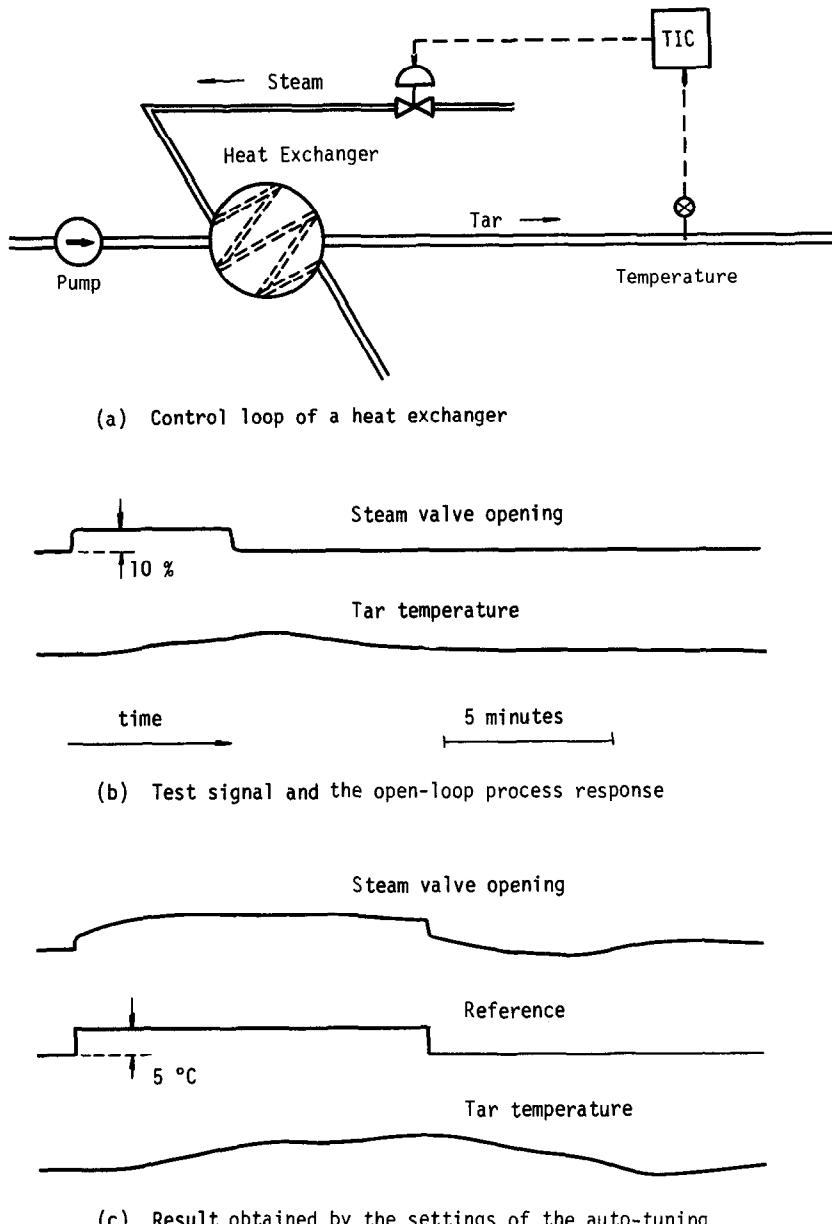


FIG. 10. Application to temperature control of a heat exchanger.

resultant closed-loop response is stable enough as shown in Fig. 11(c).

#### 6. CONCLUSIONS

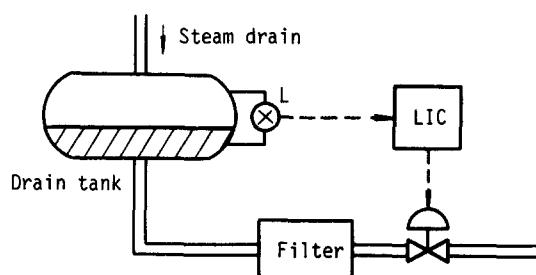
A new idea for automatic tuning of the PID control parameters, auto-tuning, has been proposed. It gives a relatively simple but practically effective algorithm. This algorithm is easily implemented on a small size digital processor and is applicable to the various kinds of the processes such as the self-regulating process, the non-self-regulating process and the process with a long deadtime.

In order to estimate the process parameters, an intentional signal is applied. This is a disadvantage

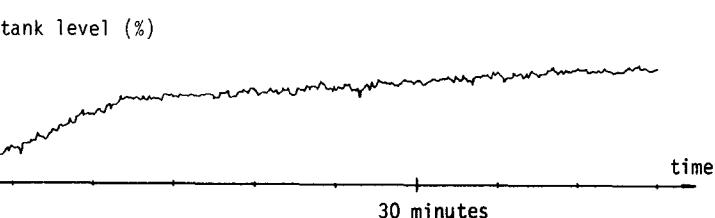
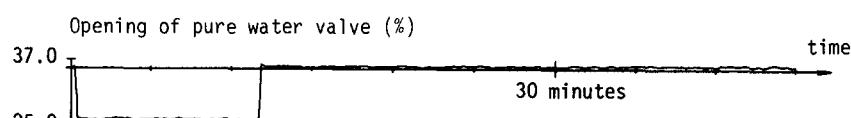
but the signal disturbs the plant operation so little because of a pulse form.

The PID control parameters are determined so as to minimize the weighted ISE. In most cases, the resulting responses are similar, more or less, to those obtained by the conventional 25% damping standard. Therefore, the field engineers can understand readily the feature of the optimal settings.

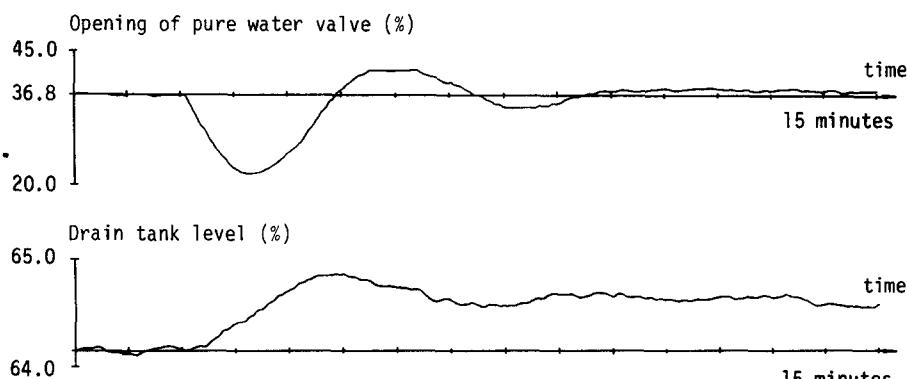
As shown in the results of applying the proposed method to real processes, we can obtain sufficiently good settings of the PID parameters without knowing the exact form of the process transfer function. At least, the settings obtained can be recommended to the field engineers as the initial (or reference) values for the tuning operation.



(a) Control loop of a tank level



(b) Test signal and the open-loop process response



(c) Result obtained by the settings of the auto-tuning

FIG. 11. Application to control of a tank level.

As already mentioned in the introductory section, the proposed procedure is a kind of man-machine interactive one in the sense that revision of the process parameters is initiated by a direction of the operator. In other words, the procedure is not fully automated as in the conventional adaptive techniques. We might assert that, through our experience, this kind of man-machine cooperative procedure is better accepted by engineers and operators in many cases than one which is fully automated.

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