

# A New Method For Independently Tuning PID Parameters

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## Abstract

In this paper, based on specified phase and amplitude margins(SPAM), a new method for tuning of PID regulators is proposed, by which all parameters of PID regulators are tuned independently without using much more knowledge about process dynamics. Simulation results show that SPAM method has excellent performance.

**Key words:** Adaptive control; describing function; PID control; SPAM method; relay control.

## 1 Introduction

The majority of the regulators used in industry are of the PID type structure. In recent years, a number of tuning techniques for PID regulators have been presented in literature. The Ziegler-Nichols rules[1] are one of the more popular schemes, where the plant is required to operate near the critical state as to measure the ultimate gain and period, which thus easily generates oscillation of increasing amplitude and furthermore destroys the physical instrument. To avoid this shortcoming, a method which incorporates a characterized ideal relay into the closed-loop system as to generate a relay oscillation for measuring the ultimate gain and period, was proposed in[2]. A improvement of the Ziegler-Nichols rules was presented in[3], which is, however, not suitable for unstable process. For those methods presented in[4,5], where the closed-loop system would be expected to have a specified phase margin, the objective of tuning PID regulators is to move the crossing point of the Nyquist curve and the ideal relay or the relay with hysteresis to the specified phase on the unit circle of the complex plane, and a linear experimental relation has to be adopted for lack of tuning conditions so that the control performance is not satisfactory. For the same purpose, a new approach was proposed in[6], where a suitable point satisfying the pre-specified condition on process frequency response curve needs to be found by plugging a variable time delay into

control loop, which, however, causes a longer tuning time and inconvenience in practice.

In general, the method based on specified phase margin can not ensure the closed-loop system to have specified amplitude margin. Hence, it is expected that using both phase and amplitude margins to tune parameters of PID regulators should supply a good performances of control and robustness. For the purpose, an analytical method was reported in[7,8], which is, however, also not suitable for unstable process.

In this paper, based on specified phase and amplitude margins(SPAM), a new method for tuning of PID regulators is proposed, by which all parameters of PID regulators are tuned independently for both stable and unstable processes without using much more knowledge about process dynamics. Simulation results show that SPAM method has excellent performance.

## 2 Designing Idea of the SPAM Method

The block diagram of the SPAM method is shown in Fig.1. The negative reciprocal of describing function of the relay with hysteresis is given by

$$-\frac{1}{N(A)} = -\frac{\pi}{4d} \sqrt{A^2 - \varepsilon^2} - j \frac{\pi\varepsilon}{4d} \quad (1)$$

where  $d$  is the relay amplitude,  $\varepsilon$  is the hysteresis width and  $-1/N(A)$  is a straight line parallel to the negative real axis on the complex plane. As shown in Fig.2 and Fig.3, by choosing  $\varepsilon$  and  $d$ , the relay with hysteresis can determine the point Q with a specified imaginary part on Nyquist curve of the controlled process. The SPAM method proposed in this paper will move the point Q to the point P with a specified phase  $\pi + \phi_m$  ( $\phi_m$  is the phase margin) on the unit circle as to make the closed loop system to have the specified phase margin by the expected proportional, integral and derivative actions of the PID regulator, where the imaginary part of the point Q is equal to that of the point P. From the open loop frequency response and the geometrical relation as

shown in Fig.2(a) and Fig.3(a), the proportional factor  $K_p$  and a relation between integral constant  $T_I$  and derivative constant  $T_D$  can be obtained. However, for most methods[4,5], when a relation between  $T_I$  and  $T_D$  was

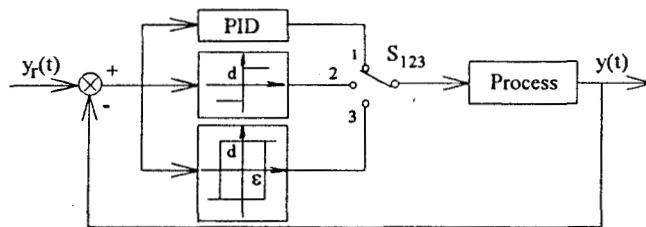


Fig.1. Block diagram of the SPAM method

obtained the following linear experimental relation has to be adopted for lack of tuning conditions

$$T_I = \xi \cdot T_D \quad (2)$$

where  $\xi$  is usually selected in region (1.5,4), therefore,  $T_I$  and  $T_D$  can be tuned respectively. But for the SPAM method proposed in this paper,  $T_I$  and  $T_D$  will be tuned independently by incorporating the amplitude margin  $K_g$  into tuning of PID regulators. To ensure the closed loop system to have the specified amplitude margin, the amplitude at crossover frequency  $\omega_g$  (phase equals  $-180^\circ$ ) on the Nyquist curve of the closed loop system must be  $1/K_g$ . In engineering, it is required that slope at cutting frequency  $\omega_c$  (gain equals 1) on Bode diagram of a closed loop system with specified phase and amplitude margins should be -20db/dec[9]. Define that slope at  $\omega_g$  on Bode diagram of the closed loop system is also equal to -20db/dec then a relation among  $\omega_g$ ,  $\omega_c$  and  $K_g$  can be found, where  $\omega_c$  is equal to the oscillation frequency of the point Q. Furthermore,  $\omega_g$  can be easily calculated by  $\omega_c$  and  $K_g$ , which, in general, is more than the crossover frequency  $\omega_g$  on Nyquist curve of the controlled process. As shown in Fig.2(b) and Fig.3(b), the point S with  $\omega_g$  on Nyquist curve of the controlled process can be not obtained. However, by expected actions of PID regulator, the point S is moved to the point M with the amplitude  $1/K_g$  on negative real axis. From the open loop frequency response and the geometrical relation as shown in Fig.2(b) and Fig.3(b), other relation between  $T_I$  and  $T_D$  can be obtained via introducing a

dimensionless number  $\beta$  ( $\beta = \tan \phi$ ) so that  $T_I$  and  $T_D$  can be calculated independently. This is main idea of the SPAM method proposed in this paper, from which, it is found that almost no a priori knowledge about process dynamics is required.

### 3 SPAM Tuning for PID Regulator

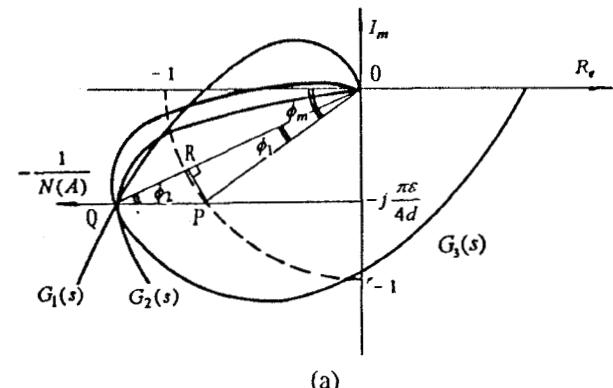
When the switch  $S_{123}$  turns to the point 1 as shown in Fig.1, the open loop transfer function of the system is written as follows

$$G_K(s) = K_p \cdot (1 + sT_D + \frac{1}{sT_I}) \cdot G(s) \quad (3)$$

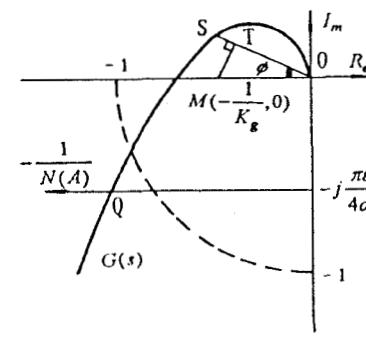
and the open loop frequency response is expressed by

$$G_K(j\omega) = K_p \cdot G(j\omega) + jK_p \cdot G(j\omega)(\omega T_D - \frac{1}{\omega T_I}) \quad (4)$$

#### 3.1 The crossing point of the Nyquist curve and the relay with hysteresis is out of the unit circle



(a)



(b)

Fig.2 SPAM tuning when the point Q is out of the unit circle

As shown in Fig.2(a), for the controlled processes  $G_1(s)$  and  $G_2(s)$  whose Nyquist curves intersect with the negative real axis, define the coordinate of the point P as

$(-\cos\phi_m, -j\sin\phi_m)$  and the coordinate of the point Q as  $(-\chi_0, -j\sin\phi_m)$ , then from (1)  $\mathcal{E}$  can be derived by

$$\varepsilon = \frac{4d}{\pi} \sin\phi_m \quad (5)$$

When the switch  $S_{123}$  turns to the point 3, the relay with hysteresis will force the system to oscillate as to obtain the oscillation frequency  $\omega_c$  and the oscillation amplitude  $A$ .

Calculating the real part  $\chi_0$  of the oscillation point Q from (1) leads to

$$\chi_0 = \frac{\pi}{4d} \sqrt{A^2 - \varepsilon^2} \quad (6)$$

As shown in Fig.2(a), according to the analysis in Section 2 and (4), the first relation between  $T_I$  and  $T_D$  is now described by

$$\tan\phi_1 = \omega_c T_D - \frac{1}{\omega_c T_I} = \alpha \quad (7)$$

The amplitude  $|OR|$  is obtained by acting  $K_p$  on the amplitude  $|G(j\omega_c)|$  of the point Q so  $K_p$  can be derived by

$$K_p = \frac{|OR|}{|OQ|} = \frac{|OP|\cos\phi_1}{|G(j\omega_c)|} = \frac{1}{\sqrt{1+\alpha^2} \cdot \sqrt{\chi_0^2 + \sin^2\phi_m}} \quad (8)$$

The proportional factor tuned by (8) is suitable for  $G_1(s)$ , but not suitable for  $G_3(s)$  whose starting point of Nyquist curve lies in the positive real axis because the crossing point of the negative real axis and the Nyquist curve is too closely to the original. Therefore, for  $G_3(s)$ ,  $K_p$  is adjusted by

$$K_1 = \alpha_1 \cdot K_p \quad (9)$$

where  $\alpha_1$  is selected as 0.5. The dimensionless number  $\alpha$  in (8) is determined by the specified phase margin  $\phi_m$  and the intrinsic property of the controlled process. In the triangle OQP as shown in Fig.2(a), the following relation can be obtained

$$\frac{\sin\phi_2}{|OP|} = \frac{\sin\phi_1}{|QP|} \quad (10)$$

and furthermore,

$$\frac{\alpha}{\sqrt{1+\alpha^2}} = \frac{\sin\phi_m}{\sqrt{\chi_0^2 + \sin^2\phi_m}} (\chi_0 - \cos\phi_m) \quad (11)$$

It is found from (11) that  $\alpha$  is uniquely determined when  $\phi_m$  is given and  $\chi_0$  is calculated from (6). According to the analysis in Section 2, the slopes at  $\omega_c$  and  $\omega_g$  on Bode diagram of the closed loop system would be chosen as -20db/dec to stabilize the system, and the relation among  $\omega_g$ ,  $\omega_c$  and  $K_g$  can be obtained approximately

$$\frac{L_{K_g}}{\log\omega_g - \log\omega_c} = 20 \quad (12)$$

where  $L_{K_g} = 20\log K_g$ . Calculating  $\omega_g$  from (12) leads to

$$\omega_g = K_g \cdot \omega_c \quad (13)$$

As shown in Fig.2(b), according to the analysis in Section 2 and (4), the second relation between  $T_I$  and  $T_D$  is now expressed as

$$\tan\phi = \omega_g T_D - \frac{1}{\omega_g T_I} = \beta \quad (14)$$

where  $\beta$  is a dimensionless number, which will be discussed in sequel. From (7) and (14),  $T_I$  and  $T_D$  are calculated independently by

$$T_I = \frac{K_g^2 - 1}{K_g \omega_c (\beta - \alpha K_g)} \quad (15)$$

$$T_D = \frac{\beta K_g - \alpha}{\omega_c (K_g^2 - 1)} \quad (16)$$

From (15) and (16), it is known that  $\beta$  must satisfy the following

$$\beta > \alpha K_g \quad (17)$$

A lot of simulation studies have indicated that control performance is satisfactory when  $\beta$  is chosen in region  $(\alpha \cdot K_g + 1.0, \alpha \cdot K_g + 1.2)$ . Thus, the parameters of PID regulators are tuned by (8), (15) and (16) in this case, respectively.

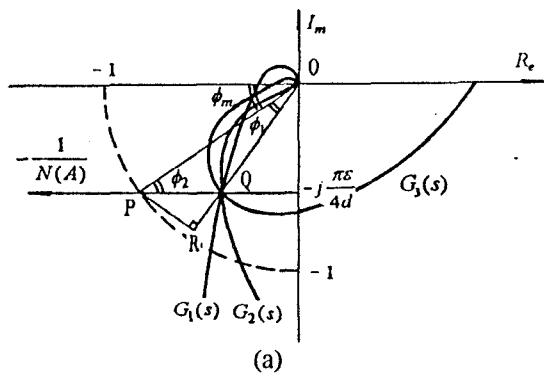
### 3.2 The crossing point of the Nyquist curve and the relay with hysteresis is in the unit circle

As shown in Fig.3(a), for the controlled processes  $G_1(s)$  and  $G_3(s)$  whose Nyquist curves intersect with the negative real axis, define the coordinate of the point P as  $(-\cos\phi_m, -j\sin\phi_m)$  and the coordinate of the point Q as  $(-\chi_0, -j\sin\phi_m)$ , where  $\chi_0$  is calculated by (6) when  $\phi_m$  is given and  $A$  is measured. When the switch  $S_{123}$  turns to the point 3 as shown in Fig.1, the relay with hysteresis will force the system to oscillate as to obtain the oscillation frequency  $\omega_c$  and the oscillation amplitude  $A$ . As shown in Fig.3(a), according to the analysis in Section 2 and (4), the first relation between  $T_I$  and  $T_D$  is now described by

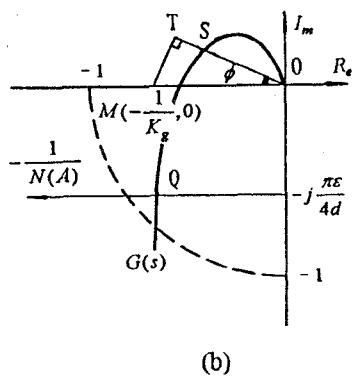
$$\tan\phi_1 = -\omega_c T_D + \frac{1}{\omega_c T_I} = \alpha \quad (18)$$

where  $\alpha$  is also determined by  $\phi_m$  and  $\chi_0$ , which satisfies

$$\frac{\alpha}{\sqrt{1+\alpha^2}} = \frac{\sin\phi_m}{\sqrt{\chi_0^2 + \sin^2\phi_m}} (\cos\phi_m - \chi_0) \quad (19)$$



(a)



(b)

Fig.3 SPAM tuning when the point Q is in of the unit circle

As shown in Fig.3(a),  $K_p$  expressed by (8) can be also obtained by the same analysis adopted in 3.1, and furthermore, for  $G_3(s)$ , it is still adjusted by (9). According to the analysis in Section 2 and (4), the relation (13) among  $\omega_e$ ,  $\omega_c$  and  $K_g$  and the relation (14) between  $T_I$  and  $T_D$  can be also obtained in this case. From (18) and (14),  $T_I$  and  $T_D$  can be calculated independently by

$$T_I = \frac{K_g^2 - 1}{K_g \omega_c (\beta + \alpha K_g)} \quad (20)$$

$$T_D = \frac{\beta K_g + \alpha}{\omega_c (K_g^2 - 1)} \quad (21)$$

It is known from (20) and (21) that  $\beta$  determined by (14) needs to be non-negative only. However, in order to obtain a good control effect, it should be given a better choice. When the switch  $S_{123}$  turns to the point 2 as shown in Fig.1, the ideal relay will force the system to oscillate and  $\omega_g'$  is the oscillation frequency. Define  $n = (\omega_g - \omega_g')/\omega_g'$  and then choose  $\beta \in (n - 0.4, n + 0.4)$ . A lot of simulation studies have indicated that control performance is satisfactory in this way. Thus, the parameters of PID regulators are tuned by (8), (20) and (21) in this case, respectively.

As shown in Fig.2(a) and Fig.3(a), for the controlled process  $G_2(s)$  whose Nyquist curve doesn't intersect with the negative real axis, the intrinsic amplitude margin tends to infinite so it is not required that the closed-loop system has a specified amplitude margin. In this case, the linear experimental relation (2) is used to tune parameters of PID regulators. In two different cases mentioned above, substituting (2) into (7) or (18) can obtain the parameters of PID regulators.

#### 4 Simulation Results

With  $T_S = 0.2s$ ,  $\phi_m = 30^\circ$ ,  $L_{K_g} = 10db$ , the parameters of PID regulators shown in Table 1 are obtained for different processes.

Table 1. Parameters of PID regulators

Process model	$\beta$	$K_p$	$T_I$	$T_D$
1. $\frac{e^{-0.6s}}{s+1}$	0.6	0.7033	0.9068	0.1252
2. $\frac{e^{-2s}}{s+1}$	1.0	0.6518	2.0774	0.4529
3. $\frac{e^{-0.4s}}{(s+1)^2}$	0.6	0.747	1.476	0.306
4. $\frac{1-0.8s}{(s+1)^3}$	1.3	0.649	2.425	0.793
5. $\frac{(1-0.5s)e^{-0.4s}}{s(s+1)^3}$	1.9	0.353	10.00	1.76

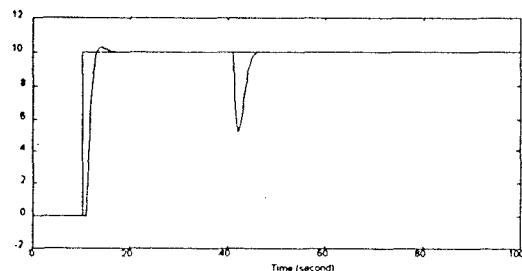


Fig.4(a) for the process model 1

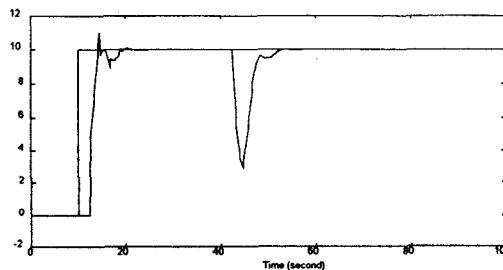


Fig.4(b) for the process model 2

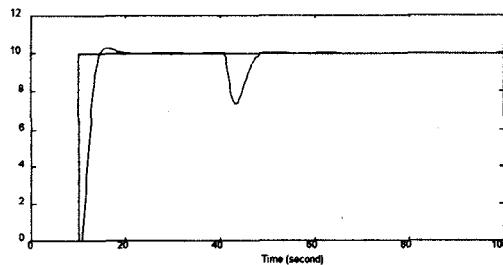


Fig.4(c) for the process model 3

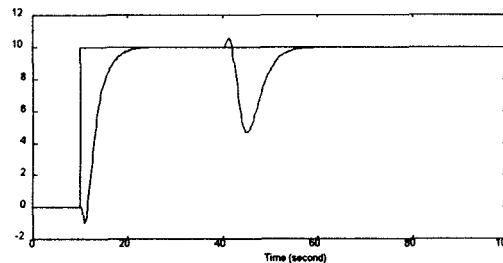


Fig.4(d) for the process model 4

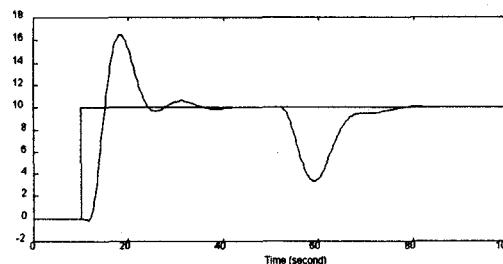


Fig.4(e) for the process model 5

Fig.4 Set-point and load disturbance responses of SPAM tuning

It is observed from the Fig.4 that performance of the SPAM method is excellent.

### 5 Specification on Load Disturbance Rejection

The PID regulator designed by mentioned above can ensure the closed loop system to have certain stable margins, but can not make it have strongest capability of rejecting load

disturbance. By the strong capability of rejecting load disturbance it is meant that the integration of absolute value of tracking error must be minimum when the load disturbance occurs, and from which, the SPAM tuning of rejecting load disturbance can be obtained.

Defining  $t_1$  as the time when a system runs into steady state, then

$$u(t_1) = \frac{K_p}{T_I} \int_0^{t_1} e(s) ds \quad (22)$$

If the step load disturbance with amplitude  $D$  is incorporated into the closed loop, the process input is  $u(t_1) + D$ . Defining  $t_2$  as the time when the system runs into steady state again, then

$$u(t_2) + D = \frac{K_p}{T_I} \int_0^{t_2} e(s) ds + D = u(t_1) \quad (23)$$

and furthermore

$$D = -\frac{K_p}{T_I} \int_{t_1}^{t_2} e(s) ds \quad (24)$$

so

$$\left| \int_{t_1}^{t_2} e(s) ds \right| = \frac{|D| T_I}{K_p} \quad (25)$$

From the inequality  $\int |e(s)| ds \geq \left| \int e(s) ds \right|$ , we can obtain

$$\int_{t_1}^{t_2} |e(s)| ds \geq \frac{|D| T_I}{K_p} \quad (26)$$

It is observed From (26) that integratiion of absolute value of tracking error is propotional to  $T_I$ , and inversely to  $K_p$ .

Therefore, the system has strongest capability of rejecting load disturbance when  $T_I$  is equal to the minimum, since  $K_p$  determined by (8) is not changed. In the case discussed in 3.1, when  $\beta$  is equal to  $\alpha K_g + 1.2$ ,  $T_I$  is selected as the minimum, furthermore, substituting  $\beta$  into (15) and (16) obtains the SPAM tuning of rejecting load disturbance. In the case discussed in 3.2, when  $\beta$  is equal to  $n+0.4$ ,  $T_I$  is selected as the minimum, then, substituting  $\beta$  into (20) and (21) leads to the SPAM tuning of rejecting load disturbance.

### 6 Comparison Results

The PID parameters obtained by the normal SPAM tuning and the SPAM tuning of rejecting load disturbance, and the comparison results for  $G(s) = e^{-0.4s} / (s+1)$  are shown in Table 2 and Fig.5.

Table 2. Parameters of PID regulators

<i>Specified</i>	$T_s$	0.1
	$L_{K_t}$	10db
	$\phi_m$	20°
<i>Normal SPAM tuning</i>	$\beta$	0.655
	$K_p$	0.646
	$T_I$	1.2126
	$T_D$	0.1654
<i>SPAM tuning of rejecting load disturbance</i>	$\beta$	1.455
	$K_p$	0.646
	$T_I$	0.7898
	$T_D$	0.3443

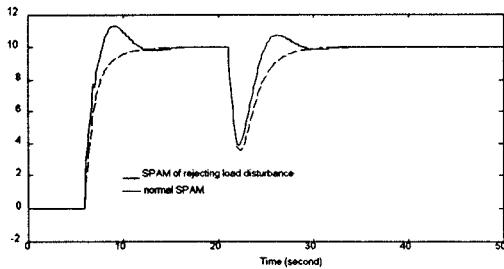


Fig.5 Comparison Results

It is observed from the Fig.5 that the SPAM tuning of rejecting load disturbance has distinct better performance of rejecting load disturbance than the normal SPAM tuning.

## 7 Conclusion

With the new method, PID autotuning is made more accurate over a much larger range of process dynamics. The new method for tuning of PID regulators has several merits. Firstly, it requires only a small set-point change in closed-loop to measure the critical gain and frequency. Secondly, the tuning effect of the new method is excellent. Finally, for unstable process, a stable response can be obtained by the new method.

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