

PID Controller Optimal Tuning

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Abstract – A controller is the most important part of an automatic control system. Its main purpose is to receive the signals from a transmitter and to compare it with a reference value. Depending on the error the controller sends a control signal to an actuator, so that the controlled variable to be equal with the reference value. The most used algorithm by a controller is the PID algorithm. In this paper, two methods of determining the optimum values of the tuning parameters for the PID controller will be presented, using case studies. The optimization criteria are the integral criteria, the square of the error between reference and controlled value being taken into consideration. The results consisted in the optimal values of the PID tuning parameters (gain and reset) for the analyzed cases associated to a temperature control system.

Keywords-optimal tuning; integral performance criteria; PID controller; optimization function

I. INTRODUCTION

The design of an automatic control system (ACS) requires the choice and sizing of its elements, their interconnection and the controller tuning so that the performances imposed to the controlled system are met [1]. The design of ACS implies four stages. First, the objectives associated to the system have to be defined. The second phase is to define the means and methods available for the proposed objective achievement. The third stage is to find technical solutions for the implementation of the control system and the final stage is the actual implementation of the system by connecting all chosen and dimensioned elements and verifying the control system performance.

Controller tuning is the process of determination of the best tuning parameters. This tuning can be done based on theoretical, numerical or experimental methods. Among the experimental methods, Skogestad [2] presents some simple analytic rules for PID controller tuning that ensure a good closed-loop behavior. Padula and Visoli [3] developed tuning rules for standard and fractional-order PID controllers that have been devised in order to minimize the integrated absolute error with a constraint on the maximum sensitivity. In this case, the controller performance can be evaluated using performance indices. Åström and Hägglund [4] present simple tuning rules which help achieve robust performance for monotone response processes. Graphical methods are also used for PID controller tuning using dominant pole placement approach with guaranteed gain margin (GM) and phase margin (PM) [5].

In the following the numerical optimal tuning methods are approached. In literature, a series of studies regarding optimal tuning are presented. Hernández-Riveros et al. made such a study by using MAGO (Multidynamics Algorithm for Global Optimization) algorithm to optimize the tuning parameters of a PID controller for second order systems with dead time [6].

Another optimal tuning method is proposed by Arun et al. [7]. Their research is focused on using Particle Swarm Optimization (PSO) algorithm, which iteratively calculates the optimal solution of a problem, trying to improve a possible solution depending on the chosen optimization criterion [7]. To demonstrate the efficiency of the proposed method the obtained results were compared with the values obtained using conventional optimization methods [8]. Similar research which use the same algorithm were performed also by Lakshmi et al. [9], or Taeib and Chaari [10].

Artificial intelligence is another approach for controllers' optimal tuning. Kim developed a hybrid optimal tuning system composed of genetic algorithms and Bacterial Foraging (BF) optimization algorithm [11]. In the same category as BF algorithm is the Ant Colony Optimization (ACO) algorithm. Researches related to the use of ACO together with fuzzy logic were performed with good results by Neyoy et al. [12]. ACO algorithm is a probabilistic algorithm used for solving optimization problems which can be reduced to finding optimal paths with the help of graph theory [13]. This algorithm simulates ants' natural behavior in search of the shorter path (optimal path) from the colony to the food source [14].

In this paper, the authors will present, using MATLAB software, a method of controllers' optimal tuning based on minimizing integral performance indices.

II. INTEGRAL INDICES USED FOR ACS PERFORMANCE DETERMINATION

The dynamic performances of an automatic control system are analyzed based on integral criteria [1]. These are performance functions and have the following expressions:

$$I_1 = \int_0^{\infty} (y^i - y(t)) dt = \int_0^{\infty} \varepsilon(t) dt, \quad (1)$$

$$I_2 = \int_0^{\infty} |\varepsilon| dt, \quad (2)$$

$$I_3 = \int_0^{\infty} \varepsilon^2 dt, \quad (3)$$

$$I_4 = \int_0^{\infty} (\varepsilon^2 + T^2 \dot{\varepsilon}^2) dt. \quad (4)$$

The meanings of the used notations are:

- y^i – set point of the control system;
- $y(t)$ – control system response to step input;
- ε – control system error;
- T – weight (choosing its value is subjective).

The four criteria have the following names:

- I_1 – integral of error;
- I_2 – integral of absolute value of error;
- I_3 – integral of squared error;
- I_4 – generalized integral of squared error.

III. OPTIMAL CONTROLLER TUNING. CASE STUDY I

A. Structure and model of the control system

The case studies from this paper have as aim the optimal tuning of a temperature control system from a tube furnace, Fig. 1 [15].

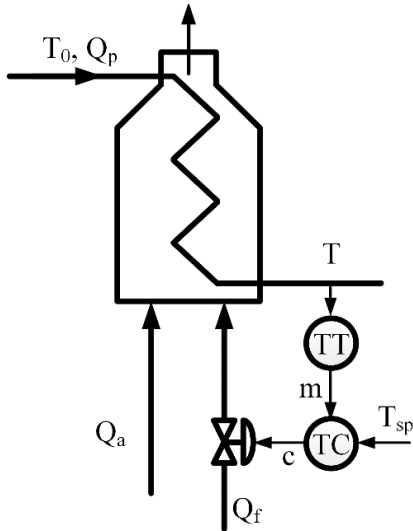


Figure 1. Temperature control system

The block diagram of the temperature control system is presented in Fig. 2.

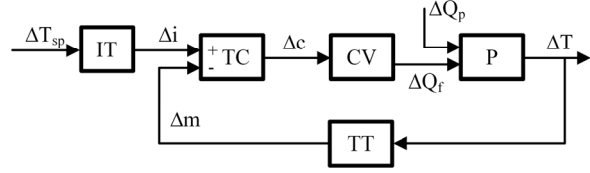


Figure 2. Block diagram of the automatic control system

The notations used in Fig. 2 signify: P – process; TT – temperature transducer; TC – temperature controller; IT – input transducer; CV – control valve.

In this first case study a proportional controller was used. The mathematical models associated to the elements of the control system are:

- Process

$$2\Delta\ddot{T} + 4\Delta\dot{T} + \Delta T = 0.2\Delta Q_f - 0.1\Delta Q_p. \quad (5)$$

- Temperature transducer

$$0.5\Delta\dot{m} + \Delta m = 0.1\Delta T. \quad (6)$$

- Temperature controller

$$\Delta c = k_p (\Delta i - \Delta m). \quad (7)$$

- Control valve

$$\Delta Q_f = 50\Delta c. \quad (8)$$

- Input transducer

$$\Delta i = 0.1\Delta T_{sp}. \quad (9)$$

In order to implement in MATLAB the models from the previous relations, their transfer functions are required, these functions being presented in the following:

$$G_P(s) = \frac{0.2}{2s^2 + 4s + 1}, \quad (10)$$

$$G_{P-d}(s) = \frac{-0.1}{2s^2 + 4s + 1}, \quad (11)$$

$$G_{TT}(s) = \frac{0.1}{0.5s + 1}, \quad (12)$$

$$G_{TC}(s) = k_p, \quad (13)$$

$$G_{CV}(s) = 50, \quad (14)$$

$$G_{IT}(s) = 0.1. \quad (15)$$

The Routh-Hurwitz stability criterion applied to the control system described by transfer functions (10)-(14) leads to the limit value $k_p^{lim} = 17$.

B. Objective function and optimization

The objective function used for the optimal tuning of the temperature controller is

$$f_{ob}(k_p) = \int_0^{t_{\max}} \varepsilon(t)^2 dt. \quad (16)$$

The value of the objective function has to be minimum. The necessary integration time must be comparable with the control system transient time.

For the minimization of the function from (16) was used MATLAB environment. The mathematical model included in MATLAB contains the transfer functions from relations (10) - (15) implemented using MATLAB function *tf* (e.g. the process transfer function is written $G_P=tf(0.2,[2 \ 4 \ 1])$). After the implementation of the individual transfer functions, the transfer function of the whole system is formed according to the block diagram from Fig. 2. The controller, the control valve and the process are in series, so their transfer functions are multiplied. This product is used together with TT transfer function in the MATLAB function *feedback* in order to implement the feedback of the control system. The transfer function obtained in this manner is then multiplied by the IT transfer function to complete the system transfer function.

The optimization algorithm is implemented using MATLAB function *fminbnd*. Because the stability limit of the studied control system is reached for $k_p = 17$, the optimum value of the controller gain was searched within the interval [1...16].

C. Results and Discussions

Fig. 3 presents the objective function (16) related to the gain values. It can be observed that the objective function presents a large range in which its value is quite small.

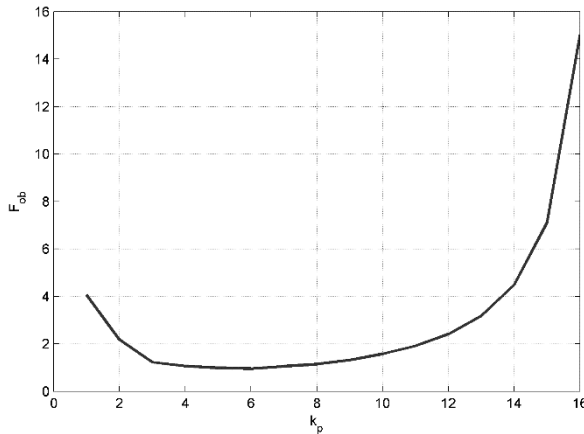


Figure 3. Representation of the objective function (16)

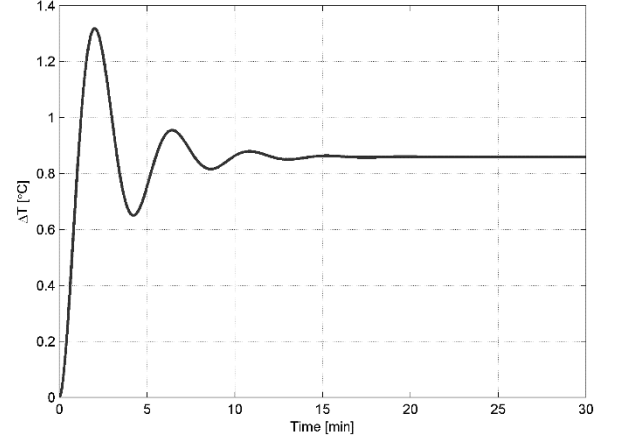


Figure 4. Control system response for optimal k_p

The minimization of the objective function led to the optimal value $k_p^{opt} = 6.0948$. The response of the control system from Fig. 4 is obtained for this value of the gain.

From this figure it can be observed that the system response has overshoot, which is a deficiency. At the same time, the proportional controller cannot eliminate the steady state error.

IV. OPTIMAL CONTROLLER TUNING. CASE STUDY II

A. Structure and model of the control system

In this case, a similar study with the one from section 3 was performed for the same control system from Fig. 1, with the difference that the controller was proportional-integral (PI). The purpose of this case study is to determine the optimal values of gain k_p and reset T_i .

As in the first case, the transfer functions of the control system are required. Compared to the previous case, only the transfer function of the controller has a new expression

$$G_{TC}(s) = k_p \cdot \left(1 + \frac{1}{T_i s}\right), \quad (17)$$

B. The objective function

In this case is required the minimization of the objective function

$$f_{ob}(k_p, T_i) = \int_0^{t_{\max}} \varepsilon(t)^2 dt. \quad (18)$$

The optimization algorithm is implemented in this case using the MATLAB function *fminsearch*. This function requires an initial set of values for the function variables, (k_p, T_i) , necessary to initiate the optimization calculations. In this study, the initial values were $(k_p, T_i) = (1, 1)$.

C. Results and discussions

The minimization of the objective function (18) led to the optimal values $(k_p^{opt}, T_i^{opt}) = (4.43, 7.98)$.

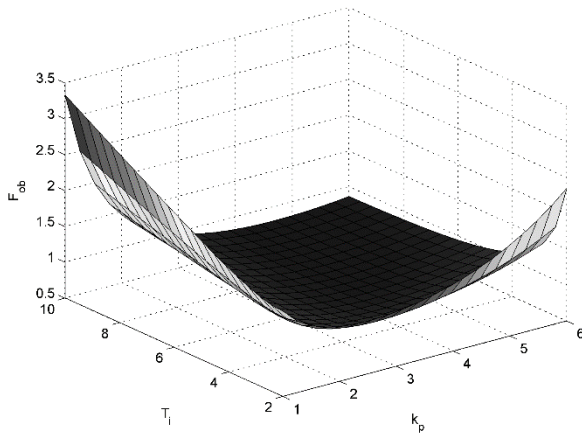


Figure 5. Representation of the objective function (18)

Fig. 5 presents the objective function (18) representation. It can be observed that the objective function has small values on a large portion of the interval.

The system response for the optimal values (k_p^{opt} , T_i^{opt}) is presented in Fig. 6 where can be observed that the overshoot is present.

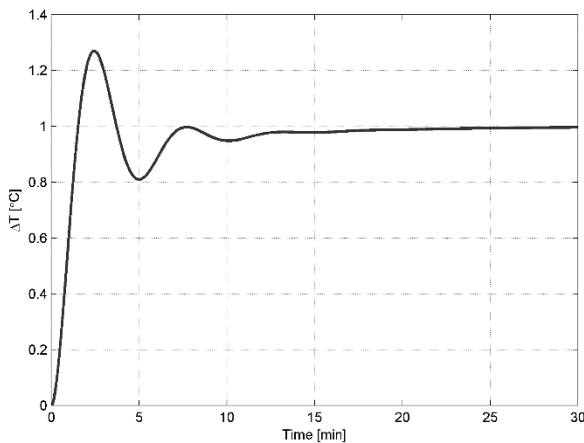


Figure 6. Control system response for optimal values of PI tuning parameters

V. CONCLUSIONS

In this paper two case studies were performed regarding the optimal tuning of a controller from a furnace temperature control system. The first case study considered a proportional controller and the second a proportional-integral controller. Having as objective function the integral of the squared error, the optimal values for the tuning parameters of the two controllers were determined.

In the first case, the optimal value of the controller gain, k_p^{opt} , was 6.0948, for which the system response presented overshoot and the steady state error was not eliminated.

In the second study, the optimal values for the controller gain and reset were determined, with (k_p^{opt} ,

T_i^{opt}) = (4.43, 7.98). In this case the steady state error is eliminated due to the presence of the integral component.

The research from this study is a first step of the process which will allow the real time optimal control of industrial controllers.

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