Self-tuning Gains of PI Controllers for Current Control in a PMSM

Shin-Hung Chang, *Member*, *IEEE*
Mechanical and Systems Research Laboratories,
Industrial Technology Research Institute
Hsinchu, Taiwan
e-mail: vernon.sh.chang@itri.org.tw

Pin-Yung Chen
Mechanical and Systems Research Laboratories,
Industrial Technology Research Institute
Hsinchu, Taiwan
e-mail : kelvin_chen@itri.org.tw

***Abstract*—In this paper, a self-tuning proportional plus integral (PI) controller is proposed for current control in a permanent magnet synchronous motor system (PMSM). Based on the Lyapunov stability theorem, the self-tuning laws are derived and the coefficients of the PI controllers can be suitably adjusted to deal with uncertainties and external disturbances. Thus, the on-line tuned coefficients can force the tracking error to approach zero. The system robustness, as well as stability is proven. In addition, the proposed self-tuning laws are very simple and can be effectively implemented. Numerical simulation demonstrates the effectiveness of the proposed control algorithm.**

Keywords- self-tuning, proportional plus integral (PI), permanent magnet synchronous motor (PMSM), adaptive, Lyapunov.

I. INTRODUCTION

The permanent magnet synchronous motor (PMSM) has been used widely in industrial applications and has advantages such as its high torque to inertia ratio, superior power density, and high efficiency [1]. A popular algorithm called field-orientation control is usually applied for a PMSM with nonlinear characteristics and an inherent coupling problem. In the field-orientation control, this technique make a PMSM be like as torque control performance to a DC motor [2]. Thus, to obtain a high performance of current control for a PMSM is necessary since a current controller influences directly the drive performance.

In past years, a proportional plus integral (PI) control technique based on the field-orientation control is widely used for PMSMs current control in many industrial fields. However, using fixed PI coefficients has to be based on a precise mathematical model for guaranteeing control performance [3]. Note that it is unavoidable that the uncertainties caused by parametric variations and unstructured dynamics in a practical PMSM. Especially the parametric variations are more serious at high speed operation since the parametric variations in flux linkage and stator inductance are proportional to the operating speed. In other words, the fixed coefficients of a conventional PI controller based on the field-orientation control can not sure the system robustness.

For observing the system uncertainties and providing a compensator to the controller, there were some researches in disturbances and uncertainties estimation of PMSMs [4]-[8]. In [4]-[7], robust current control schemes were proposed for PMSMs, system uncertainties and disturbances can be observed through an adaptive internal model and observers, respectively. In order to ensure system robustness, complicated computation is necessary in these observers. [3] and [8] had proposed a real-time tuning PI control and a fuzzy self-adapting PID control to achieve a high performance PMSM system. Note that these algorithms are needed to base on experts' knowledge.

In summary, this paper proposes PI controllers with self-tuning gains for current control in a PMSM. There are no complicated self-tuning laws and many tuned parameters. The proposed method guarantees the system robustness, as well as stability. The paper is organized as follows. In Section II, a dynamic model of PMSM is illustrated. Section III describes the proposed self-tuning PI control algorithm in detail. Section IV is the analysis of system stability. Numerical simulation is shown to validate the effectiveness of the proposed algorithm in Section V. Finally, Section VI gives conclusions.

II. DYNAMIC MODEL OF PMSM

The $d-q$ dynamic model of a PMSM in the synchronously reference frame can be described as

$$\frac{d}{dt}\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} = \begin{bmatrix} -\dfrac{R_s}{L_s} & \omega_e \\ \omega_e & -\dfrac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} + \frac{1}{L_s}\begin{bmatrix} v_{ds}^e \\ v_{qs}^e - \omega_e \phi_F^{'} \end{bmatrix}, \tag{1}$$

$$J_m p \omega_{rm} + B_m \omega_{rm} + T_L = T_e \,, \tag{2}$$

$$T_e = \frac{3P}{4}\phi_F^{'} i_{qs}^e \,, \tag{3}$$

where ω_e is the electrical angular velocity, R_s is the stator resistance for per-phase, L_s is the stator inductance for per-phase, $\phi_F^{'}$ is the permanent flux linkage, i_{ds}^e and i_{qs}^e are the $d-q$ axes stator currents, respectively, v_{ds}^e and v_{qs}^e are the $d-q$ axes stator voltages, respectively, J_m is the moment of inertia of the motor, B_m is the viscous coefficient of the motor,

T_L is the load torque, T_e is the motor torque, and ω_{rm} is the mechanical rotor speed and equals to $\omega_e * 2/P$, where P is the number of the poles. In order to eliminate the coupled term in (1), let new control voltage be

$$v_{ds}^{'} = v_{ds}^{e} + \omega_e i_{qs}^{e} L_s , \tag{4}$$

$$v_{qs}^{'} = v_{qs}^{e} - \omega_e i_{ds}^{e} L_s - \omega_e \phi_F^{'} . \tag{5}$$

Then, the $d-q$ axes linear model of a PMSM can be obtained as

$$\frac{d}{dt}\begin{bmatrix} i_{ds}^{e} \\ i_{qs}^{e} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_s}{L_s} & 0 \\ 0 & -\dfrac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_{ds}^{e} \\ i_{qs}^{e} \end{bmatrix} + \frac{1}{L_s}\begin{bmatrix} v_{ds}^{'} \\ v_{qs}^{'} \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}, \tag{6}$$

where d_1 and d_2 are external disturbances. Assume that $R_s = R_{so} + \Delta R$ and $L_s = L_{so} + \Delta L$, where R_{so} and L_{so} are the nominal parts and ΔR_s and ΔL_s are the parametric variations of R_s and L_s , respectively. Since the $d-q$ axes dynamic equations are independent each other, the control input can be designed separately. From (3), the motor torque is direct proportion to the q axis stator current. For getting a biggest torque-current ratio, all stator current vectors have to be set on the q axis, i.e., there is no stator current on the d axis. In other words, the motor torque can vary linearly to the q axis. The control method is similar to drive a DC motor.

The block diagram of a PMSM control is shown in Fig. 1. There are two PI controllers are employed in current loops of d and q axes, respectively. In Fig. 1, the symbol * represents the reference signal. Fixed coefficients k_p and k_i of PI controllers can be chosen based on the mathematical model (6) according to the poles assignment technique. Through the decouple equations (4) and (5), the command voltage, v_{qs}^{e} and v_{ds}^{e} are computed from feedback signals, ω_e , i_{qs}^{e} , i_{ds}^{e} , $v_{qs}^{'}$, and $v_{ds}^{'}$.

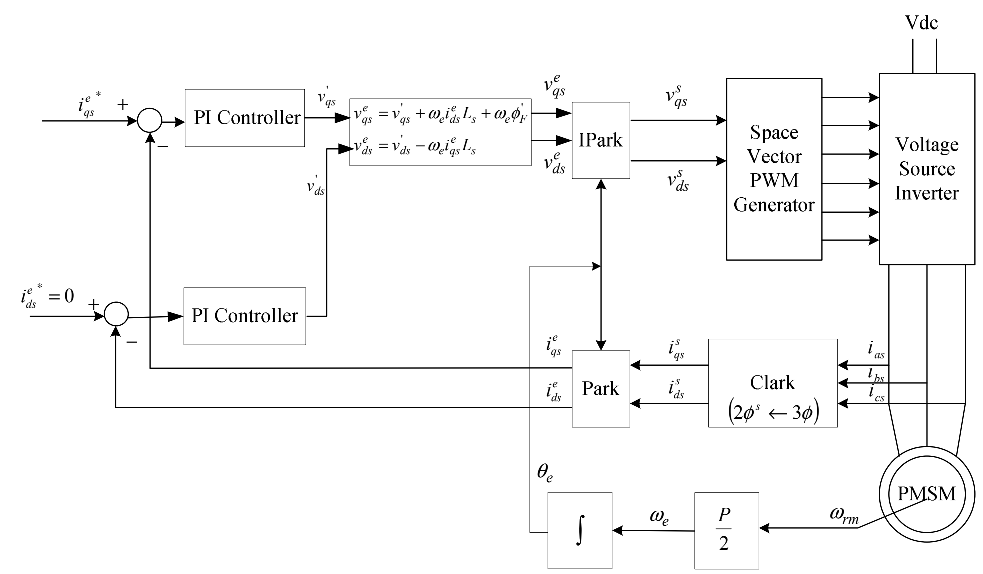

Figure 1. Block diagram of the PMSM control system.

III. SELF-TUNING GAINS OF THE PI CONTROLLER DESIGN

In a precise $d-q$ axes linear model as (6), there is no doubt that the fixed proportional and integral coefficients can be designed suitably for high control performance. The system stability is proven as long as the poles of the characteristic polynomial are negative. Although the PMSM in current control is approximately regarded as a first order system through the vector control algorithm, some unknown load disturbances and parametric variations under different temperature or operating conditions make the PMSM model be inaccuracy. For this reason, Adaptive laws to adjust the proportional and integral coefficients are necessary in difference operating environments. In this paper, a self-tuning PI controller is defined in current control of q axis as

$$v_{qs}^{'} = \hat{k}_p e + \hat{k}_i \int e , \tag{7}$$

where $e = i_{qs}^{e*} - i_{qs}^{e}$. Let the self-tuning laws be

$$\dot{\hat{k}}_p = \eta_p e^2 \,\mathrm{sgn}\left(\frac{\partial i_{qs}^e}{\partial v_{qs}'}\right), \tag{8}$$

$$\dot{\hat{k}}_i = \eta_i e\left(\int e\right)\mathrm{sgn}\left(\frac{\partial i_{qs}^e}{\partial v_{qs}'}\right), \tag{9}$$

where η_p and η_i are positive constants and to decide tuning rates.

Proof:

The validity of the control strategy (7)–(9) can be investigated using the Lyapunov stability method. Choose a Lyapunov function candidate, such as

$$V = \frac{1}{2}e^2 \;. \tag{10}$$

By direct computation,

$$\begin{aligned}
\frac{\partial V}{\partial t} &= \frac{\partial V}{\partial e}\frac{\partial e}{\partial i_{qs}^e}\frac{\partial i_{qs}^e}{\partial v_{qs}'}\frac{\partial v_{qs}'}{\partial t} \\
&= \frac{\partial V}{\partial e}\frac{\partial e}{\partial i_{qs}^e}\frac{\partial i_{qs}^e}{\partial v_{qs}'}\left(\frac{\partial v_{qs}'}{\partial k_p}\frac{\partial \hat{k}_p}{\partial t} + \frac{\partial v_{qs}'}{\partial k_i}\frac{\partial \hat{k}_i}{\partial t}\right) \\
&= -e\frac{\partial i_{qs}^e}{\partial v_{qs}'}\left(\frac{\partial v_{qs}'}{\partial k_p}\frac{\partial \hat{k}_p}{\partial t} + \frac{\partial v_{qs}'}{\partial k_i}\frac{\partial \hat{k}_i}{\partial t}\right) \\
&= -e^2\frac{\partial i_{qs}^e}{\partial v_{qs}'}\dot{\hat{k}}_p - e\frac{\partial i_{qs}^e}{\partial v_{qs}'}\left(\int e\right)\dot{\hat{k}}_i \,.
\end{aligned} \tag{11}$$

By substituting the self-tuning laws (8) and (9) into (11), $\dot{V} < 0$ while $V > 0$ can be obtained. According to the Lyapunov stability theorem, the adjustable coefficients, $\hat{k}_p$ and $\hat{k}_i$ can force the tracking error to approach zero. As the tracking error achieves zero, the self-tuning laws ,(8) and (9) also equal zero, i.e., the tuning motion is stop. Therefore, the convergence of the coefficients, $\hat{k}_p$ and $\hat{k}_i$ are also guaranteed.

In (8) and (9), the self-tuning laws are continuous. However, it is necessary that control algorithms are usually implemented on digital microprocessors. Therefore, the self-tuning laws in a discrete system can be described as follows.

$$\hat{k}_p(k+1) = \hat{k}_p(k) + \eta_p e^2(k)\mathrm{sgn}\left(\frac{i_{qs}^e(k) - i_{qs}^e(k-1)}{v_{qs}'(k) - v_{qs}'(k-1)}\right)\Delta t \,, \tag{12}$$

$$\hat{k}_i(k+1) = \hat{k}_i(k) + \eta_i e(k) * r(k) * \mathrm{sgn}\left(\frac{i_{qs}^e(k) - i_{qs}^e(k-1)}{v_{qs}'(k) - v_{qs}'(k-1)}\right)\Delta t \,, \tag{13}$$

where $r(k) = r(k-1) + e(k) * \Delta t$ and Δt is the sampling time. As long as the sampling time is small enough, the discrete laws (12) and (13) can replace the continuous laws (8) and (9) and be implemented in a discrete system. Herein, figure 2 shows the structure of the self-tuning PI controller in current control of q axis. Similarly, the self-tuning algorithm in current control of d axis can also be easily derived.

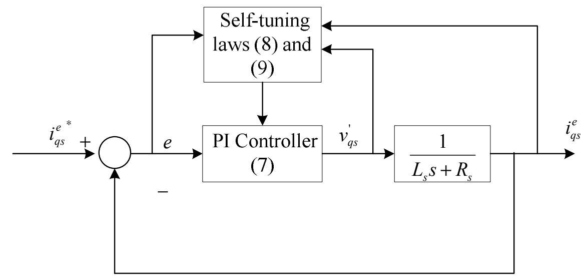

Figure 2. Structure of the self-tuning PI controller.

IV. Analysis of System Stablity

Consider the system model of q axis as

$$G_p(s) = \frac{I_{qs}^e(s)}{V_{qs}'(s)} = \frac{K_a}{s\tau + 1} \tag{14}$$

where $K_a = 1/R_s$ is the system gain and $\tau = L_s / R_s$ is the time constant. In the frequency domain, the self-tuning PI controller is

$$G_c(s) = \hat{k}_p + \frac{\hat{k}_i}{s} \,. \tag{15}$$

Theorem 1: Consider the system (14). Given the control law (15) and the self-tuning laws (8) and (9), then the stability of the closed-loop system of q axis will be stabilized and the tracking error will be eliminated as long as the coefficients $\hat{k}_p$ and $\hat{k}_i$ keep the conditions of the system stability, $\hat{k}_p > -1/K_a$ and $\hat{k}_i > 0$, respectively.

Proof:

In the closed-loop system of current control of q axis, the characteristic polynomial can be represented as

$$1 + G_p(s)G_c(s) = 0 \,. \tag{16}$$

By direction computation,

$$s^2 + \frac{1 + \hat{k}_p K_a}{\tau}s + \frac{\hat{k}_i K_a}{\tau} = 0 \,. \tag{17}$$

In order to satisfy the system stability, the eigenvalues of (17) must be negative. In other words, the conditions have to be guaranteed as follows.

$$\frac{1+\hat{k}_p K_a}{\tau} > 0 \text{ and } \frac{\hat{k}_i K_a}{\tau} > 0 . \tag{18}$$

By direction computation, the conditions $\hat{k}_p > -1/K_a$ and $\hat{k}_i > 0$ guarantee the stability of the system.

V. Numerical Simulation

Consider the dynamic equation of a PMSM as (6). The parametric values of PMSM are as follows: $R_{so} = 0.0146\Omega$, $L_{so} = 0.0000219H$. Assume that the uncertainties are as follows: $\Delta R_s = 0.002\Omega$ and $\Delta L_s = 0.000005H$. The desired currents of d and q axes are ${i_{ds}^e}^* = 0$ and ${i_{qs}^e}^* = 30$, respectively. The external disturbances are random noises. The bias and magnitude of the random noises are 10 and 5, respectively, as shown in Fig. 7. For the current control of q axis, the control laws (7) and the self-tuning laws (8) and (9) are applied. The tuning gains η_p and η_i are chosen to be 0.2 and 20, respectively. For the current control of d axis, the control laws are similar to (7)-(9). The tuning gains, η_p and η_i are chosen to be 10 and 100, respectively. The initial coefficients of the PI controller are set as $\hat{k}_p(0) = 0.01$ and $\hat{k}_i(0) = 1$.

Figure 3 shows the current responses of $d-q$ axes when the uncertainties and external disturbances exist. Figure 4 shows the corresponding command $d-q$ voltages. It is obvious that the tracking errors approach zero in 0.15 sec roughly. The current responses and the control voltages are robust and smooth. The tuned coefficients of PI controllers for $d-q$ axes are shown in Figs. 5 and 6, respectively. Finally, the tuned parameters will hold on constants as long as the tracking errors approach zero.

VI. Conclusions

This work proposes a self-tuning PI controller for current control in a PMSM under parametric variations and external disturbances. The major contributions of this work are: 1) the proposed control algorithm guarantees the PMSM system stability and robustness; 2) based on the Lyapunov theorem, the self-tuning laws can be derived to suitably adjust the coefficients of PI controllers to overcome parametric variations and external disturbances; 3) numerical simulations demonstrate that the proposed control method is very effective; 4) the proposed methodology in this work can be extended to other industry control systems.

Acknowledgment

The authors would like to thank the Bureau of Energy, Ministry of Economic Affairs of Taiwan for financial support under the project No. 8453DD1210.

References

[1] H. Nakai, H. Ohtani., E. Satoh, and Y. Inaguma, “Development and testing of the torque control for the permanent-magnet synchronous motor,” *IEEE Trans. Ind. Electron.*, vol. 52, no. 3, pp. 800-806, Jun. 2005.

[2] T. Jahns, “Motion control with permanent-magnet ac machines,” *Proc. IEEE*, vol. 82, no. 8, pp. 1241-1252, Aug. 1994.

[3] M. Tursini, F. Parasiliti, and D. Zhang, “Real-time gain tuning of PI controllers for high-performance PMSM drives,” *IEEE Trans. Ind. Appl.*, vol. 38, no. 4, pp. 1018-1026, Jul./Aug. 2002.

[4] Y. A.-R. I. Mohamed, “Design and implementation of a robust current-control scheme for a PMSM vector drive with a simple adaptive disturbance observer,” *IEEE Trans. Ind. Electron.*, vol. 54, no. 4, pp. 1981-1988, Aug. 2007.

[5] Y. A.-R. I. Mohamed and E. F. EI-Saadany, “A current scheme with an adaptive internal model for torque ripple minimization and robust current regulation in PMSM drive system,” *IEEE Trans. Energy Convers.*, vol. 23, no. 1, pp. 92-100, Mar. 2008.

[6] K. H. Kim, I. C. Baik, G. W. Moon, and M. J. Youm, “A current control for a permanent magnet synchronous motor with a simple disturbance estimation scheme,” *IEEE Trans. Control. Syst. Technol.*, vol. 7, no. 5, pp. 630-633, Sept. 1999.

[7] K. H. Kim and M. J. Youn, “A simple and robust digital current control technique of a PM synchronous motor using time delay control approach,” *IEEE Trans. Power Electron.*, vol. 16, no. 1, pp. 72-82, Jan. 2001.

[8] E. Chiricozzi, F. Parasiliti, M. Tursini, and D. Q. Zhang, “Fuzzy self-tuning PI control of PM synchronous motor drives,” *Int. J. Electron.*, vol. 80, no. 2, pp. 211-221, 1996.

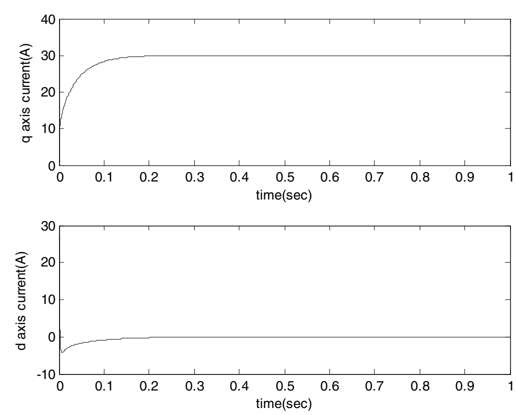

Figure 3. $d-q$ axes current, i_{ds}^e and i_{qs}^e .

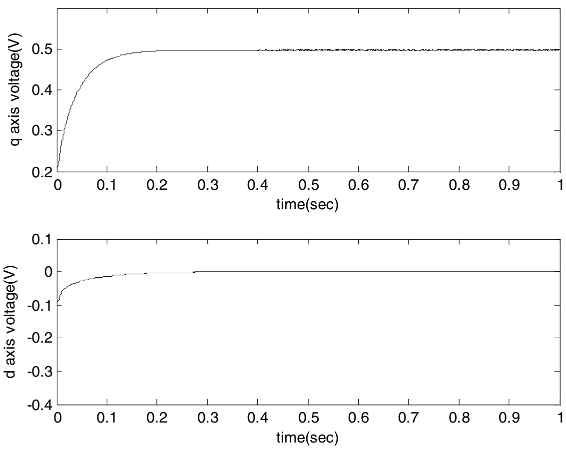

Figure 4. $d-q$ axes voltages, v'_{ds} and v'_{qs}.

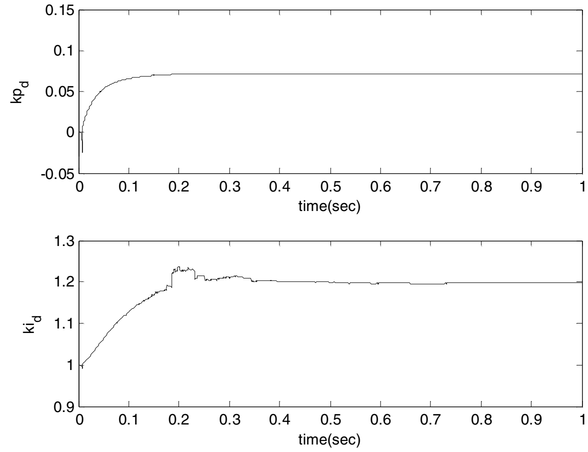

Figure 6. $\hat{k}_p$ and $\hat{k}_i$ for d axis.

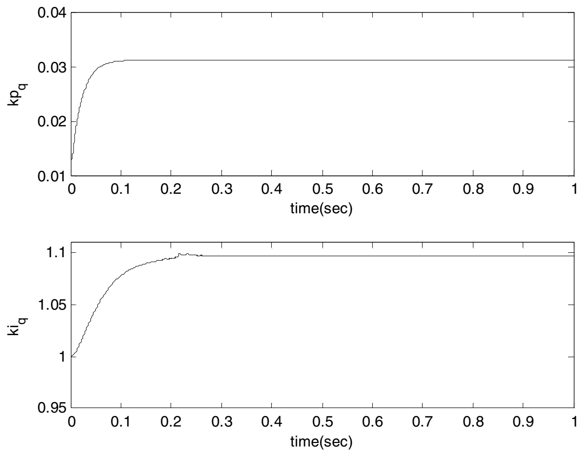

Figure 5. $\hat{k}_p$ and $\hat{k}_i$ for q axis.

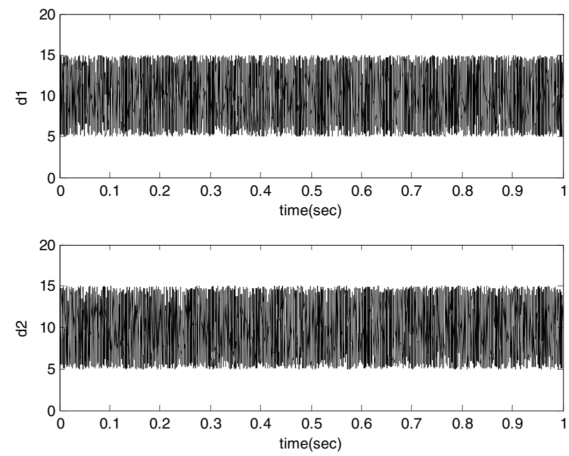

Figure 7. External disturbances, d_1 and d_2.