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A FREQUENCY RESPONSE APPROACH TO AUTOTUNING OF MULTIVARIABLE CONTROLLERS

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The relay auto-tuning technique for PID controllers is here extended to tune multivariable controllers. In the case of significant interaction, a fully cross-coupled multivariable controller rather than a decentralized controller should be employed. In this paper, an autotuning method for multivariable controllers from sequential relay feedback is proposed. The frequency response a $m \times m$ multivariable process is identified from limit cycles using a FFT-based method, and a multivariable controller is computed with linear least squares frequency response fitting. Various typical examples are included for illustration of the effectiveness of the method.

Keywords: autotuning; multivariable system; frequency response; FFT; relay feedback; PID control

INTRODUCTION

PID control has been the most commonly-used technique in the process industries for over fifty years. The classical method of tuning PID controllers is that of Ziegler-Nichols[1]. The Ziegler-Nichols rule is based on the ultimate gain and ultimate frequency at which the process becomes oscillatory. Astrom and Hagglund[2,3] proposed a method to determine the approximate values of the ultimate gain and the ultimate frequency by introducing a relay feedback. One of the advantages of this method is that it is easy to control the amplitude of the limit cycle by an appropriate choice of the relay amplitude. In addition, it is a closed-loop method. The relay feedback autotuning has been proven to be very successful in industrial applications and has led to a number of commercial autotuners[3,4]. It is here being extended to model-based advance regulators[5,6,7] and a multivariable control system[8].

PID-type controllers are routinely used in SISO applications with satisfactory results for simple processes, but the extension of the single-loop design procedure for PID-type controllers to multivariable processes is not straightforward unless the latter is transformed into a decoupled system. The task of developing a satisfactory design procedure for multivariable PID-type process controllers remains a difficult problem[9]. Multivariable processes are difficult to control due to the presence of interactions. If the interaction is modest, one may consider using multiloop SISO controllers for the multivariable system. Luyben[10] presents a tuning procedure for multiloop PID controllers, where the stability of the whole system can only be guaranteed by introducing appropriate detuning factors on the PI/PID parameters, and Economou and Morari[11] describe the design procedure for the IMC multiloop controllers. If the interaction is significant, a decoupler should be used to reduce interaction in conjunction with decentralized controllers or a full multivariable control technique that inherently compensates for interaction should be employed. Koivo and Tanttu[12] give a recent survey of MIMO PI/PID techniques. These controllers are mainly aimed at decoupling the plant at certain frequencies. Davison[13] sets the proportional and integral terms as a fraction of the inverse of the steady-state gain matrix i.e. $G^{-1}(0)$. Maciejowski[14] proposes to use the proportional term as a fraction of $G^{-1}(j\omega_b)$ where ω_b is the desired bandwidth while the integral term is set as in Davison[13]. Recently, Lieslehto[14] developed a heuristic tuning method for these controllers based on the determination of tuning parameters for each element in the plant transfer function matrix using the IMC design approach. However, these methods of tuning the controllers are both manual and time consuming in nature. In addition, most of them require the transfer function matrix to be known and the control performance of the closed loop system is usually not so satisfactory, especially in the case of large interaction and long dead time.

The frequency response method is a very useful tool for control system analysis and design. A controller may be designed for simple processes using only one point on the process Nyquist curve[3]. However, for complex processes, advanced designs such as dominant pole assignment[3], gain and phase margin[15], Smith predictors[5,6] and finite spectrum assignment[7] are necessary to enhance performance and they make use of two or more process frequency response points. It is more so in the MIMO case. In the literature, only one point information has been used to tune the multiloop controllers[8,10,16] but one point information is usually not enough to tune the full multivariable controller due to the significant interaction.

In this paper, an autotuning method of multivariable controllers based on the frequency responses of the process is proposed. To identify the process frequency responses, several sequential relay tests are performed on the process, and the FFT based frequency response identification technique is developed to obtain the process frequency responses. This frequency identification requires little prior knowledge of the process and can yield as many frequency response points as desired. For a $m \times m$ multivariable process, this multiple-point identification is achieved with only m relay tests and the result is accurate since no approximation is made. On the basis of these frequency responses, a design method for fully cross-coupled PID type multivariable controllers is proposed. The proposed method can shape the loop frequency response to optimally match the desired curve over a large range of frequencies so that the closed-loop performance is more guaranteed than one or two points methods. An additional advantage of this method is that it deals with a linear problem and no iteration is needed. The stability of closed-loop system can be analysed from the Gershgorin bands of the designed open loop, and the performance of the designed system can be estimated by comparing the Nyquist array and Gershgorin bands of the actual open-loop transfer matrix with those of the desired open-loop transfer matrix. Extensive simulations have shown that the proposed method gives very satisfactory results for most processes. In some special cases of large interaction, more than one stage of compensators can be added to the PID controller to enhance the control performance. Various typical examples are included to illustrate the method.

The paper is organized as follows. In the next section, the autotuning procedure is presented followed by a section dealing with the frequency response identification from the autotuning test and then the design method for a multivariable controller is proposed. Examples are then given, and next the large dead time process is considered, followed by the conclusion.

AUTOTUNING PROCEDURE

The automatic tuning of controllers consists of a process dynamics identification scheme and a design procedure that computes the controller parameters. An excitation of the process is needed to extract useful information on process dynamics. There are several kinds of excitation or tuning test. The classical one is an open-loop transient response experiment, where a step or a pulse is injected at the process input, and the response is measured. To perform such an experiment, the process must be stable. If a pulse test is used, the process may include an integrator. It is necessary that the process be in equilibrium when the experiment is begun. The main advantage of the method is that it requires little prior knowledge. However, it is sensitive to disturbances and it is an open loop method. In the proposed tuning procedure, the preferred method is to use the relay feedback so that there would be a limit cycle oscillation in the loop. With an ideal relay, the method gives an input signal to the process whose period is close to the crossover frequency of the open-loop system[2,3]. As it is a closed-loop method, the process will not drift away from the setpoint. It is easy to control the amplitude of the oscillation and the test time is short[3].

For a MIMO system, there are three ways to perform the relay test.

(1) Independent Single Relay Tests. One channel loop is subjected to relay feedback control while all other loops are kept open. The main advantage of this method is that the structure is simple and the computation is little. However, it is much more sensitive to disturbance compared to other methods due to the opening of control loops, and it also results in more disturbances to the process.

(2) Sequential Relay Test. The main idea of sequential relay test is to perform the relay test loop by loop, and to close a loop with a simple controller once a relay test has been done, until all the loops are performed. The method results in less disturbance to the process than the first one, and is not so sensitive to disturbance as the first method. A little drawback is that it need a little more computation.

(3) Decentralized Relay Test. All loops are placed on relay feedback simultaneously. Unlike the previous two tuning methods, all of the loops are closed during the tuning process. The method is least sensitive to the disturbance, and results in the least disturbance to the process. However, the resulting limit cycles are often complicated and the modelling from the relay test is difficult.

On the basis of the above analysis, the sequential relay test as shown in Figure 1 is preferred in the current tuning procedure. The sequence of loop closing is important since it affects the amount of interaction entering all the previously tuned loops, and also the tuning time. It is advisable to tune the fastest loop first since it is unlikely to be affected by interactions and the tuning time is shortest.

Some kind of process model is necessary for most multivariable controller designs. Linear multivariable process dynamics can be completely described by state space model, transfer function matrices, impulse response matrices and frequency response matrices. In this design, the frequency response is employed for the reason that control performance specifications are best formulated in frequency domain, and the synthesis method of multivariable controller is frequency response based. The full frequency responses of process can be obtained from the tuning test by the frequency response identification technology (see the next section). The PID controller is the most commonly used in the process industries and it has very good robustness. The fully cross-coupled PID-type controller is employed in the current method to compensate the interaction and achieve the control objective, and more than one stage of lead-lag compensators can be added to the PID controller to enhance the control performance in the case of large interaction. The parameters of the controller are determined with the least squares frequency fitting method, as described in the design section.

The above development can be summarized as the following autotuning procedure.

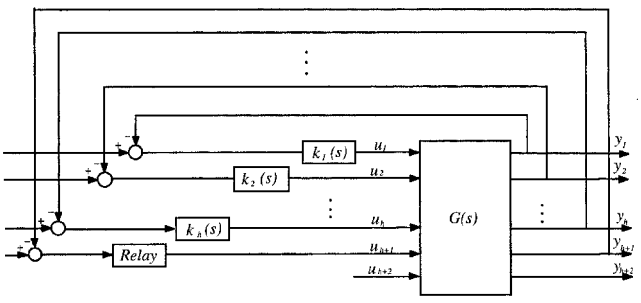

Figure 1. Sequential relay test.

Tuning Procedure

Step 1: Perform the sequential relay test to the process.
Step 2: Estimate frequency responses of the process from the relay test.
Step 3: Design the fully cross-coupled multivariable controllers.
Step 4: Commission the controllers.

PROCESS FREQUENCY RESPONSE ESTIMATION

It is known that the Fourier Transform technique can serve as a bridge between the continuous time domain and the frequency domain. With discrete time data generated from computer control, the discrete Fourier transform (DFT) should be used. The Fast Fourier Transform (FFT) is an efficient and reliable method for computing the DFT. In order to obtain correct results from the FFT, the signal to be transformed should be absolutely integrable. However, the input and output transients $u(t)$ and $y(t)$ resulting from a relay, step or ramp test cannot be directly transferred to frequency response meaningfully using FFT. A decay exponential $e^{-\alpha t}$ is here introduced to form

$$\tilde{u}(t) = u(t)e^{-\alpha t}, \tag{1}$$

and

$$\tilde{y}(t) = y(t)e^{-\alpha t} \tag{2}$$

so that $\tilde{u}(t)$ and $\tilde{y}(t)$ are decayed to zero exponentially as t approaches infinity. As $\tilde{u}(t)$ and $\tilde{y}(t)$ decay to zero quickly, the frequency response transform, which is an infinite integral, can be approximately computed at a finite integral region using FFT. The shifted process frequency response $g(j\omega + \alpha)$ can subsequently be obtained as

$$g(j\omega + \alpha) = \frac{Y(j\omega + \alpha)}{U(j\omega + \alpha)} = \frac{\tilde{Y}(j\omega)}{\tilde{U}(j\omega)}, \tag{3}$$

where $\tilde{Y}(j\omega)$ and $\tilde{U}(j\omega)$ can be computed at discrete frequencies ω_i with the standard FFT technique from $\tilde{y}(t)$ and $\tilde{u}(t)$ (Bergland[18] and Ramirez[19]). Suppose that $u(kT)$, $y(kT)$, $k = 0, 1, 2, \ldots, N-1$, are samples of $u(t)$ and $y(t)$, where T is the sampling interval. N is chosen such that $u((N-1)T)$ and $y((N-1)T)$ must have reached a stationary oscillation. $\tilde{u}(kT)$ and $\tilde{y}(kT)$ are formed from (1) and (2) respectively, and we then have

$$\tilde{U}(j\omega_i) = FFT(\tilde{u}(kT)) = T\sum_{k=0}^{N-1}\tilde{u}(kT)e^{-j\omega_i kT}, \quad i = 1, 2, \ldots, n, \tag{4}$$

$$\tilde{Y}(j\omega_i) = FFT(\tilde{y}(kT)) = T\sum_{k=0}^{N-1}\tilde{y}(kT)e^{-j\omega_i kT}, \quad i = 1, 2, \ldots, n. \tag{5}$$

where $n = N/2$ and $\omega_i = 2\pi i/(NT)$.

To find $g(j\omega)$ from $g(j\omega + \alpha)$, taking the inverse FFT of $g(j\omega_i + \alpha)$ yields

$$\tilde{g}(kT) := FFT^{-1}(g(j\omega_i + \alpha)) = g(kT)e^{-\alpha kT}, \tag{6}$$

and applying the FFT again to $g(kT)$ results in the process frequency response:

$$g(j\omega_i) = FFT(g(kT)). \tag{7}$$

For an $m \times m$ multivariable process, its frequency response matrix can be described as

$$\begin{bmatrix} Y_1(j\omega) \\ \vdots \\ Y_m(j\omega) \end{bmatrix} = \begin{bmatrix} g_{11}(j\omega) & \cdots & g_{1m}(j\omega) \\ \vdots & \ddots & \vdots \\ g_{m1}(j\omega) & \cdots & g_{mm}(j\omega) \end{bmatrix} \begin{bmatrix} U_1(j\omega) \\ \vdots \\ U_m(j\omega) \end{bmatrix}. \tag{8}$$

To estimate the frequency responses of the process, put the first loop into a relay feedback while all other loops are open, record the resultant process input $u_1(t)$ and output $y_i(t)$, $i = 1, 2, \ldots, m$, until the system reaches a stationary oscillation. Then, applying the above method would yield first column of the shifted frequency response matrix $g_{i1}(j\omega + \alpha)$, $i = 1, 2, \ldots, m$, as

$$\begin{bmatrix} g_{11}(j\omega + \alpha) \\ \vdots \\ g_{m1}(j\omega + \alpha) \end{bmatrix} = \begin{bmatrix} \tilde{Y}_1(j\omega)/\tilde{U}_1(j\omega) \\ \vdots \\ \tilde{Y}_m(j\omega)/\tilde{U}_1(j\omega) \end{bmatrix}, \tag{9}$$

from which the frequency response can be computed. Based on the frequency response $g_{11}(j\omega)$, a PID controller $k_1(s)$ can be designed for the loop 1. According to closed-loop specifications, the desired closed-loop transfer function can be chosen as

$$h_1(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}, \tag{10}$$

where the damping rate ζ is usually valued as 0.707, and desired closed-loop nature frequency ω_0 is chosen as 0.6–0.8 times of the process critical frequency which is approximately estimated from the ultimate period of limit cycle in loop 1. It follows that the desired open-loop transfer function is

$$q_1(s) = h_1(s)(1 - h_1(s))^{-1} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s}. \tag{11}$$

The actual open-loop transfer function is $g_{11}k_1$. Matching g_1k_1 to q_1 yields

$$g_{11}(s)k_1(s) = g_{11}(S)\begin{bmatrix} 1 & \frac{1}{s} & s \end{bmatrix}\begin{bmatrix} k_{p1} \\ k_{I1} \\ k_{D1} \end{bmatrix} = q_1(s). \tag{12}$$

The equation is linear in controller parameter. Let $s = j\omega_k$, $k = 1, 2, \ldots, n$, then the parameters (k_{p1}, k_{I1}, k_{D1}) can be determined by solving equation (12) with the least squares method. For SISO system, the frequency ω_k is chosen in the range from the low frequency to the critical frequency of process and the design gives very good results for most cases.

As a demonstration, consider the following SISO example

$$G_p(s) = \frac{1}{(s^2 + s + 1)(s + 1)}e^{-0.2s}.$$

From the relay test, the critical point of $G_p(s)$ is estimated as $(K_u, \omega_u) = (2.16, 1.24)$. Choosing $\zeta = 0.707$, $\omega_0 = 0.75$, ω_k in the range from 0.1 to 1.2 with 0.1 increment, and solving equation (12) yields

$$k(s) = 0.613 + 0.530\frac{1}{s} + 0.655s.$$

For comparison, the modified Ziegler-Nichols method is also presented here. If γ_s for ψ_s are specified as 0.5 and $\pi/4$

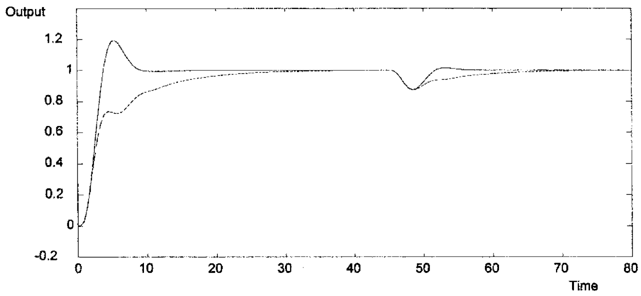

Figure 2. Responses of set-point and load change for a SISO case. —— proposed method; - - - - modified Z–N method.

respectively, the PID controller is computed[3] as $(K_c, T_I, T_D) = (0.764, 3.892, 0.973)$. A set-point change and load disturbance responses for the loop closed with these two controllers are given in Figure 2. It is known that the proposed method gives better results than the modified Z–N method.

Suppose that the first h sequential relay tests have been done, and their loops have been closed with the decentralized PID controllers which are tuned with the proposed method. Then put the $(h+1)$th loop into relay feedback, record all the controllers and relay outputs $u_j(t)$, $j = 1, \ldots, h, h+1$ and the process outputs $y_i(t)$, $i = 1, \ldots, m$, until the system reaches a stationary oscillation. It follows from (3) that

$$\begin{bmatrix} g_{1,h+1}(j\omega + \alpha) \\ \vdots \\ g_{m,h+1}(j\omega + \alpha) \end{bmatrix} = \begin{bmatrix} \left(\tilde{Y}_1(j\omega) - \sum_{j=1}^{h} g_{1,j}(j\omega + \alpha)\tilde{U}_j(j\omega)\right) / \tilde{U}_{h+1}(j\omega) \\ \vdots \\ \left(\tilde{Y}_m(j\omega) - \sum_{j=I}^{h} g_{m,j}(j\omega + \alpha)\tilde{U}_j(j\omega)\right) / \tilde{U}_{h+1}(j\omega) \end{bmatrix}. \tag{13}$$

In the same way, the shifted frequency responses of multivariable process are determined after m relay tests, and so are the frequency responses.

For the decay coefficient α, its value should be such that the input $\tilde{u}(t)$ and output $\tilde{y}(t)$ decay approximately to zero when the time approaches the test time span T_f, regardless of nonzero $u(t)$ and $y(t)$.

It should be noted that this method is not confined to the relay feedback case. It can also be applied to many other input excitative signals such as steps and ramps. In the proposed method, no iterative calculation is performed, and no prior knowledge of the process is required.

MULTIVARIABLE CONTROLLER DESIGN

Consider a multivariable control system as shown in Figure 3, where $G(s)$ is a multivariable process, and $K(s)$ is a fully cross-coupled multivariable controller. For a start, a multivariable PID-type controller which can be described as

$$K(s) = \{k_{ij}(s)\}, \quad k_{ij}(s) = k_{Pij} + k_{Iij}\frac{1}{s} + k_{Dij}s,$$

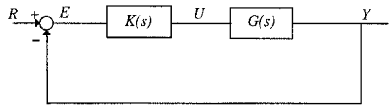

Figure 3. Multivariable control system.

or

$$K(s) = K_P + K_I\frac{1}{s} + K_D s = \begin{bmatrix} 1 & \frac{I}{s} & s \end{bmatrix} \begin{bmatrix} K_P \\ K_I \\ K_D \end{bmatrix} \tag{14}$$

is employed. According to closed-loop specifications and assuming that the process dead time is small, the desired dominant closed-loop transfer matrix can be chosen as

$$H(s) = \{h_{ij}(s)\} = diag\left\{\frac{\omega_{0i}^2}{s^2 + 2\zeta\omega_{0i}s + \omega_{0i}^2}\right\}, \tag{15}$$

where the damping rate ζ is usually valued as 0.707, and the nature frequency ω_{0i} is chosen as 0.5–1.0 times the minimum loop ultimate frequency $\omega_{u\,\min}$ which can be estimated from the relay test. The faster the loop i is, the larger the ω_{0i} can be. It follows that the desired open-loop transfer matrix is

$$Q(s) = H(s)[I - H(s)]^{-1} = diag\left\{\frac{\omega_{0i}^2}{s^2 + 2\zeta\omega_{0i}s}\right\}. \tag{16}$$

The actual open-loop transfer matrix is GK. Matching GK to Q yields

$$G(s)K(s) = G(S)\begin{bmatrix} I & \frac{I}{s} & sI \end{bmatrix} \begin{bmatrix} K_P \\ K_I \\ K_D \end{bmatrix} = Q(s). \tag{17}$$

Let $s = j\omega_k$, $k = 1, 2, \ldots, n$, it follows

$$G(j\omega_k)\begin{bmatrix} I & \frac{I}{j\omega_k} & j\omega_k I \end{bmatrix} \begin{bmatrix} K_P \\ K_I \\ K_D \end{bmatrix} = Q(j\omega_k), \quad k = 1, 2, \ldots, n \tag{18}$$

Equation (18) is a system of linear complex equations as $AX = B$, where

$$A = \begin{bmatrix} G(j\omega_1) & \frac{G(j\omega_1)}{j\omega_1} & j\omega_1 G(j\omega_1) \\ \vdots & \vdots & \vdots \\ G(j\omega_n) & \frac{G(j\omega_n)}{j\omega_n} & j\omega_n G(j\omega_n) \end{bmatrix}, \tag{19}$$

$$X = \begin{bmatrix} K_P \\ K_I \\ K_D \end{bmatrix}, \quad B = \begin{bmatrix} Q(j\omega_1) \\ \vdots \\ Q(j\omega_n) \end{bmatrix}. \tag{20}$$

Define

$$A' = \begin{bmatrix} Re(A) \\ Im(A) \end{bmatrix}, \quad B' = \begin{bmatrix} Re(B) \\ Im(B) \end{bmatrix}, \tag{21}$$

a system of linear real equations is obtained:

$$A'X = B' \tag{22}$$

Then the parameters (K_P, K_I, K_D) can be determined by solving equation (22) with the least squares method. The frequencies ω_k are usually chosen in the range from the low frequency to the maximum loop ultimate frequency $\omega_{u\,max}$.

It is noted that the change of the desired ith closed-loop nature frequency ω_{0i} will only affect ith column elements of the PID controller matrix in our design, and the increase of ω_{0i} will make the closed-loop set-point and load responses less sluggish, but it will cause the interaction of loops more significant and the system less stable.

The stability of the designed system can be analysed from the Gershgorin bands of the designed open-loop transfer matrix GK, which can be plotted from the identified process frequency responses and controller frequency responses. By Gershgorin's theorem[20] it is known that the union of the Gershgorin bands 'traps' the union of the characteristic loci. If all the Gershgorin bands exclude the point $(-1, 0)$, then $(I + GK)$ is diagonally dominant, and the closed-loop stability can be assessed by counting the encirclements of $(-1, 0)$ by the Gershgorin bands. It follows from the Generalized Nyquist's stability theorem[20] that the stability of a closed-loop system is guaranteed if the Gershgorin bands of the desired open-loop transfer matrix GK unencircle the point $(-1, 0)$ assuming the open-loop system is stable. The stability margin can also be estimated from the Gershgorin bands. The closer the Gershgorin bands approaches the $(-1, 0)$ point, the closer the multivariable system is to closed-loop instability, and the worse the robustness of the system.

The performance of the designed system can be estimated by comparing the Nyquist array and Gershgorin bands of the actual open-loop transfer matrix GK with those of desired open-loop transfer matrix Q. The narrower the Gershgorin bands is, the more closely does the actual GK resemble m non-interacting SISO transfer functions, so does the actual closed-loop resemble m non-interacting single loops, which is desired in our closed-loop specifications. The closer the Nyquist array of the actual open-loop transfer matrix GK to that of desired open-loop transfer matrix Q, the better the actual system achieves the desired closed-loop specifications.

Another way to assess the fitting of actual system to the desired one is the achieved least squares error matrix of (22), which is specified by

$$J(K) = (B' - A'X^*)^T(B' - A'X^*) \tag{23}$$

where X^* is the least squares optimal solution of (22). In fact, the matrix is related to ω_{0i} and ω_k. The larger ω_{0i} is, the greater the corresponding ith diagonal element of $J(K)$. The more number m of frequency points ω_k is, the greater the elements of $J(K)$. A scalar index x can be defined as the maximum element of $J(K)$. The less the x is, the better the actual open-loop transfer matrix fits the desired one. Based on the extensive simulations, it has been observed and could be concluded that the performance of the designed system is satisfied if x satisfies the criterion

$$\frac{x}{n\omega_{u\,min}} \le \varepsilon \tag{24}$$

where $\omega_{u\,min}$ is the minimum loop ultimate frequency, and ε is the specified threshold and usually takes the value of $0.05 \sim 0.1$.

In some special cases, there are large interactions in the process, or the frequency characteristic of loops are significantly different. To enhance the control performance, one more stage lead-lag compensator can be added to the PID controller as

$$K(s) = \{k_{ij}(s)\},$$

$$k_{ij}(s) = \left(k_{Pij} + k_{Iij}\frac{1}{s} + k_{Dij}s\right)\frac{s + b_{ij}}{s + a}$$

$$= \frac{k_{1ij} + k_{2ij}\frac{1}{s} + k_{3ij}s + k_{4ij}s^2}{s + a}$$

or

$$K(s) = \frac{K_1 + K_2\frac{1}{s} + K_3 s + K_4 s^2}{s + a}$$

$$= \frac{1}{s + a}\begin{bmatrix} I & \frac{I}{s} & sI & s^2 I \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix}$$

where the a is usually valued in the range $0.2 \sim 0.8$ times of $2\zeta\omega_{u\,min}$, and the parameters (K_1, K_2, K_3, K_4) can be determined by solving the following equation

$$G(s)K(s) = G(s)\frac{1}{s + a}\begin{bmatrix} I & \frac{I}{s} & sI & s^2 I \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} = Q(s) \tag{26}$$

with least squares method for $s = j\omega_k$, $k = 1, 2, \ldots, n$.

The employment of this compensator will reduce least squares method error $J(K)$. More than one stage of compensators can be added until the error is small enough to satisfy the criterion (24) and the control specifications are met.

The above development can be summarized as the following design procedure.

Design Procedure

Step 1: Solve (22) for appropriate $\omega_{0i} \ge 0.5\omega_{u\,min}$ and ω_k. If $\omega_{0i} \le 0.5\omega_{u\,min}$, go to step 4.
Step 2: Check the stability. If it is not satisfied, reduce ω_{0i}, return to step 1.
Step 3: Verify the criterion (24). If it is satisfied, stop. Otherwise, reduce ω_{0i}, return to step 1.
Step 4: Employ one more compensator to the controller and solve new frequency response fitting problem as shown in (26). Verify the criterion (24). If it is satisfied, then stop. Otherwise, return to step 4.

EXAMPLES

Four typical examples are given to illustrate the proposed method.

Example 1

Consider the binary distillation column plant in Luyben[10]:

$$G(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{1 + 16.7s} & \dfrac{-18.9e^{-3s}}{1 + 21s} \\ \dfrac{6.6e^{-7s}}{1 + 10.9s} & \dfrac{-19.4e^{-3s}}{1 + 14.4s} \end{bmatrix}$$

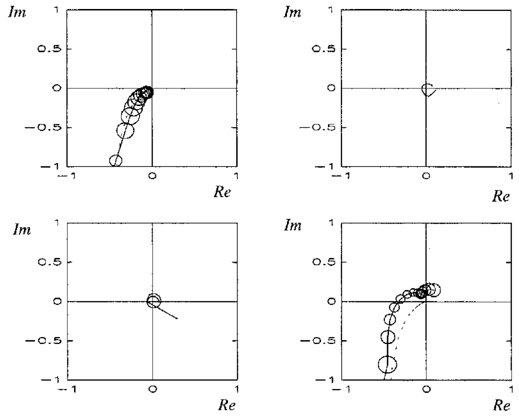

Figure 4. Open-loop Nyquist array with Gershgorin bands for Example 1. - - - - desired; —— designed.

For this method, loop 1 is first put into relay feedback while loop 2 is open. The frequency responses of first column elements are then estimated using the proposed method. Based on the frequency response $g_{11}(j\omega)$, the parameters of the first PID controller $k_1(s)$ for loop 1 are determined by solving (12). Choosing $\zeta = 0.707$, $\omega_0 = 1.0$, ω_k in the range from 0.1 to 1.6 with 0.1 increment, and solving (12) yields

$$k_1(s) = 0.911 + 0.055\frac{1}{s} + 0.290s.$$

Loop 1 is closed with this controller at $t = 15$ and then relay test is performed for the loop 2 at $t = 35$. The frequency responses of second column elements are then estimated using the proposed method.

Based on the frequency responses, the multivariable controllers are determined by solving (22) with the least squares method. Choosing $\zeta = 0.707, \omega_{01} = 0.35, \omega_{02} = 0.3$, ω_k in the range from 0.1 to 1.6 with 0.1 increment, and solving (22) yields

$$K(s) = \begin{bmatrix} 0.156 + 0.053\frac{1}{s} + 0.065s & -0.029 - 0.032\frac{1}{s} + 0.046s \\ -0.021 + 0.024\frac{1}{s} + 0.028s & -0.103 - 0.024\frac{1}{s} - 0.094s \end{bmatrix}.$$

The fully cross-coupled PID controller is commissioned at $t = 81$. The Nyquist array and Gershgorin bands of the $G(j\omega)K(j\omega)$ and the desired open-loop $Q(j\omega)$ are given in Figure 4. The whole tuning transients are shown in Figure 5, where the set-point change of loop 1 and loop 2 occur at $t = 141.5$ an $t = 301.5$, respectively. For comparison, the BLT tuning method of Luyben[10] is also considered. For this multivariable process, the method gives the parameters of multiloop PI controllers as $K_c = (0.375, -0.075)$ and $T_i = (8.29, 23.6)$. The set-point change responses are also shown in Figure 5. The results indicate that the proposed method gives significant improvement both in the decoupling and the performances of the control system.

Example 2

This example is adopted from Palmor *et al.*[16] as

$$G(s) = \begin{bmatrix} \dfrac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \dfrac{-1}{(0.1s+1)(0.2s+1)^2} \\ \dfrac{1}{(0.1s+1)(0.2s+1)^2} & \dfrac{2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \end{bmatrix}.$$

There exist large interactions in this process. If the decentralized PID controllers are tuned via the Ziegler-Nichols method ignoring interaction, i.e. one loop at a time while the other is open, the resulting closed loop system will be unstable. The sequential relay tests are performed for this process and the frequency responses of the elements are

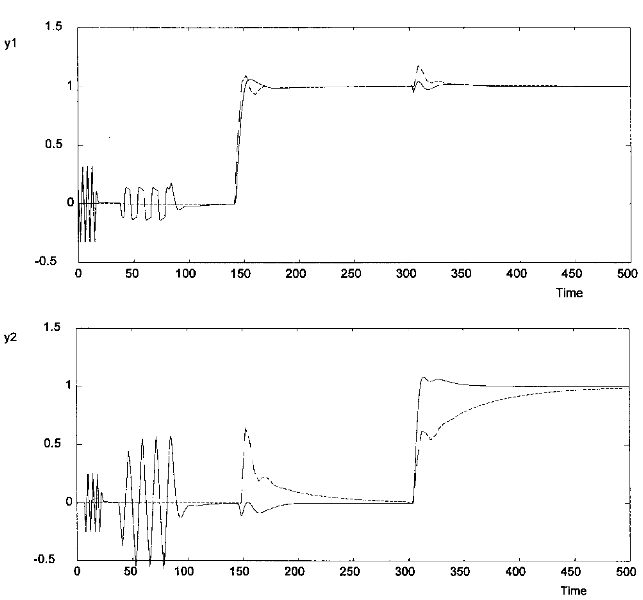

Figure 5. Autotuning process for Example 1. —— proposed method; - - - - BLT method.

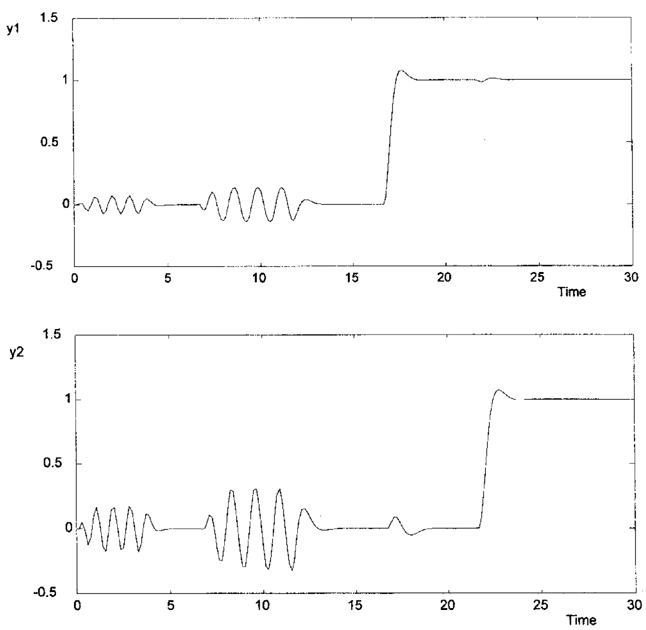

Figure 6. Autotuning process for Example 2.

obtained. Choosing $\zeta = 0.707$, $\omega_{01} = 4.5$, $\omega_{02} = 4.0$, ω_k in the range from 0.2 to 7.0 with 0.2 increment, and solving (22) yields

$$K(s) = \begin{bmatrix} 0.574 + 3.478\frac{1}{s} - 0.013s & 0.854 + 1.282\frac{1}{s} + 0.072s \\ -0.977 - 1.447\frac{1}{s} - 0.107s & 0.356 + 0.644\frac{1}{s} + 0.005s \end{bmatrix}.$$

The tuning responses are shown in Figure 6, where the set-point change of loop 1 and loop 2 occur at $t = 16.67$ and $t = 21.67$. The results show that there is almost complete decoupling and the performances of the control system are excellent.

Example 3

Consider a 3×3 plant presented by Vasnani[8]

$$G(s) = \begin{bmatrix} \frac{119e^{-5s}}{21.7s + 1} & \frac{40e^{-5s}}{337s + 1} & \frac{-2.1e^{-5s}}{10s + 1} \\ \frac{77e^{-5s}}{50s + 1} & \frac{76.7e^{-3s}}{28s + 1} & \frac{-5e^{-5s}}{10s + 1} \\ \frac{93e^{-5s}}{50s + 1} & \frac{-36.7e^{-5s}}{166s + 1} & \frac{-103.3e^{-4s}}{23s + 1} \end{bmatrix}.$$

The multivariable PID controller obtained is

$$K(s) = \begin{bmatrix} 0.0247 + 0.0012\frac{1}{s} + 0.0365s & -0.0016 - 0.0001\frac{1}{s} - 0.0071s & -0.0011 - 0.0001\frac{1}{s} - 0.0097s \\ -0.0100 - 0.0008\frac{1}{s} + 0.0066s & 0.1028 + 0.0037\frac{1}{s} + 0.1270s & -0.0058 + 0.0001\frac{1}{s} + 0.0074s \\ 0.0087 + 0.0008\frac{1}{s} + 0.0041s & -0.0039 - 0.0005\frac{1}{s} - 0.0014s & -0.0469 - 0.0020\frac{1}{s} - 0.0763s \end{bmatrix}$$

For this tuning sequences, the closed-loop damping coefficient ζ was set at 0.707, the closed-loop natural frequencies ω_{01}, ω_{02}, ω_{03} of loop 1, 2 and 3 were chosen as 0.2, 0.4, 0.3 respectively, and ω_k was changed in the range from 0.05 to 0.55 with 0.05 increment. The responses for a set-point change in loop 1, 2 and 3 are shown in Figure 7. Comparing with the design in Vasnani[8], the proposed method gives significant improvement.

Example 4

Consider the Vinante and Luyben plant in Luyben[10]:

$$G(s) = \begin{bmatrix} \frac{-2.2e^{-s}}{1 + 7s} & \frac{1.3e^{-0.3s}}{1 + 7s} \\ \frac{-2.8e^{-1.8s}}{1 + 9.5s} & \frac{4.3e^{-0.35s}}{1 + 9.2s} \end{bmatrix}.$$

Based on the identified frequency responses, the multivariable PID controllers are determined by solving (22) with the least squares method. Choosing $\zeta = 0.707$, $\omega_{01} = 0.85$, $\omega_{02} = 1.2$, ω_k in the range from 0.1 to 4.6 with 0.1 increment, solving (22) yields

$$K(s) = \begin{bmatrix} -1.557 - 0.473\frac{1}{s} - 0.360s & 0.663 + 0.206\frac{1}{s} + 0.259s \\ -0.267 - 0.318\frac{1}{s} + 0.161s & 1.760 + 0.340\frac{1}{s} - 0.157s \end{bmatrix}.$$

The Nyquist array and Gershgorin bands of the actual open-loop transfer matrix GK and those of desired open-loop transfer matrix Q are shown in Figure 8, and the least squares error matrix of above solution is

$$J(K) = \begin{bmatrix} 7.743 & -6.19 \\ -6.19 & 5.403 \end{bmatrix}$$

It follows that the criterion (24) is not satisfied for $\varepsilon = 0.05$. One more compensator is employed to the PID controller to enhance the performance. Solving (26) with least squares method for $a = 0.566$ yields

$$K'(s) = \frac{1}{s + 0.566} \begin{bmatrix} -2.043 - 0.231\frac{1}{s} - 1.422s - 0.405s^2 & 1.147 - 0.088\frac{1}{s} + 0.668s + 0.238s^2 \\ -1.064 - 0.149\frac{1}{s} + 0.862s - 0.193s^2 & 1.845 + 0.166\frac{1}{s} + 0.743s + 0.151s^2 \end{bmatrix}$$

The resultant least squares error decreases to

$$J(K') = \begin{bmatrix} 1.329 & -0.793 \\ -0.793 & 0.736 \end{bmatrix},$$

and the Nyquist array and Gershgorin bands of the new open-loop transfer matrix GK', which are much narrower

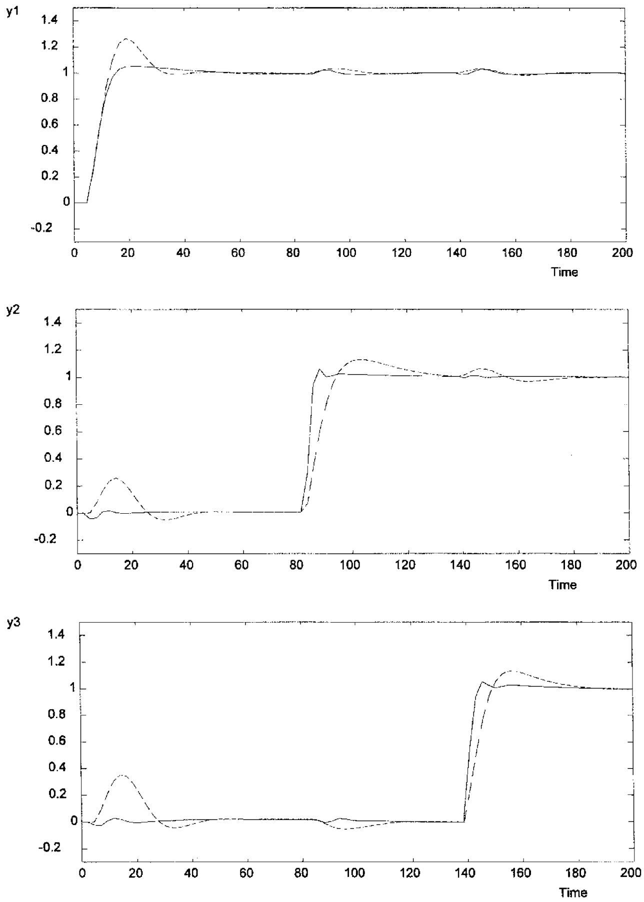

Figure 7. Responses of set-point change in loops for Example 3. —— proposed method; - - - - Vasnani's method.

than the previous design, are shown in Figure 9. The set-point change responses of loop 1 and loop 2 for both controllers are shown in Figure 10. For comparison, the results of BLT tuning method of Luyben[10] are also given in Figure 10, where the parameters of multiloop PI controllers are set as $K_c = (-1.07, 1.97)$ and $T_i = (7.10, 2.58)$. The results indicate that the proposed method gives significant improvement both in the decoupling and the performances of the control system.

LARGE DEAD TIME SYSTEMS

For processes with large dead time, the proposed method

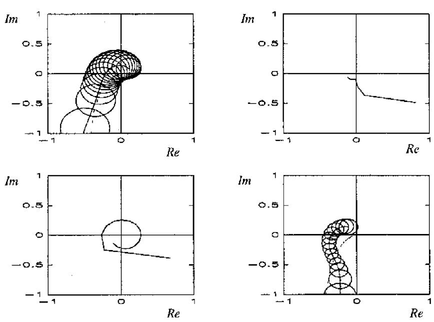

Figure 8. Open-loop Nyquist array with Gershgorin bands for Example 4. (PID controller without compensator). - - - - desired Q, —— designed GK.

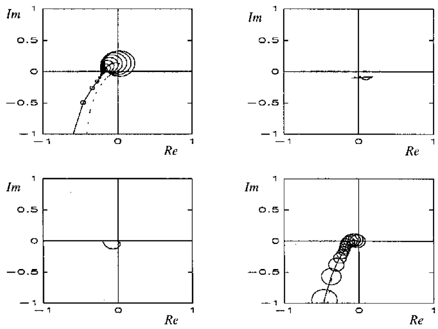

Figure 9. Open-loop Nyquist array with Gershgorin bands for Example 4. (PID controller with compensator). - - - - desired Q, —— designed GK'.

can still be used. However, the large delay will cause significant reduction of the process critical frequency, causing the designed system to be very sluggish. A well-known scheme for dead time compensation is the Smith Predictor[21]. In this section, how to synthesize the Smith Predictor for a multivariable control system from the frequency response is discussed.

For output prediction, a model of the process is required. It is known[22,23] that most industrial processes can be approximately described by a low order plus dead time model

$$G_m(s) = \frac{ds + 1}{as^2 + bs + c} e^{-Ls}. \tag{27}$$

Given the process frequency response $G(j\omega_i)$, $i = 1, 2, \ldots, M$, it follows from (27) that

$$G(j\omega_i) = \frac{j\omega_i d + 1}{(j\omega_i)^2 a + j\omega_i b + c} e^{-j\omega_i L}, \quad i = 1, 2, \ldots, M. \tag{28}$$

The determination of the parameters a, b, c and d in (28) seems to be a nonlinear problem[6]. However, it can be simplified as follows. Taking the magnitude of both sides of

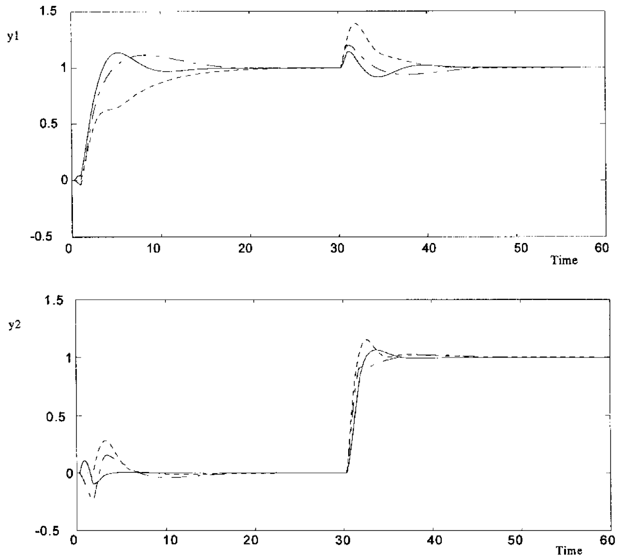

Figure 10. Responses of set-point change in loops for Example 4. - - - - PID controller; —— PID + one stage compensator; . . . . . . BLT tuning.

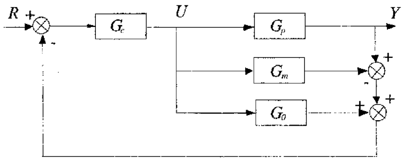

Figure 11. The Smith Predictor for multivariable system.

(28) yields.

$$\begin{bmatrix} \omega_i^4 & \omega_i^2 & 1 & \dfrac{-\omega_i^2}{|G(j\omega_i)|^2} \end{bmatrix} X = \frac{1}{|G(j\omega_i)|^2}, \quad i = 1, 2, \ldots, M, \tag{29}$$

where $X = [x_1 x_2 x_3 x_4]^T = [a^2 \; b^2 - 2ac \; c^2 \; d^2]^T$. Equation (29) is a system of linear equations in X and can be solved for X with the linear least squares method. For a stable system, the original model parameter a, b, c and d can be recovered from X as

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \sqrt{x_1} \\ \sqrt{x_2 + 2\sqrt{x_1 x_3}} \\ \sqrt{x_3} \\ \sqrt{x_4} \end{bmatrix}. \tag{30}$$

In addition, the argument relation in (28) gives,

$$\omega_i L = \tan^{-1}(d\omega_i) - \arg[G(j\omega_i)] - \tan^{-1}\left(\frac{b\omega_i}{c - a\omega_i^2}\right),$$
$$i = 1, 2, \ldots, M. \tag{31}$$

Obviously, L can thus be estimated again with the least squares method.

In the earlier section on process frequency response, the frequency response matrix of a multivariable process $G(j\omega) = \{g_{ij}(j\omega)\}$ is obtained. Applying the proposed method to all the elements of the frequency response matrix will yield the transfer function matrix of the process $G_m(s) = \{g_{ij}(s)\}$.

The scheme of the Smith Predictor for a multivariable system is shown in Figure 11, where $G_p(s)$ is the process, $G_m(s)$ is the model and the dead-time free part of the model $G_0(s)$ is determined as

$$G_0(s) = G_m(s)|_{L_{ij}=0}. \tag{32}$$

The primary controller $G_c(s)$ is designed for $G_0(s)$. Applying the controller design method in previous section yields

$$G_0(s) G_c(s) = G_0(S) \begin{bmatrix} I & \dfrac{I}{s} & sI \end{bmatrix} \begin{bmatrix} K_P \\ K_I \\ K_D \end{bmatrix} = Q(s), \tag{33}$$

where $Q(s)$ is the desired open-loop transfer matrix

$$Q(s) = diag\left\{\frac{\omega_{0i}^2}{s^2 + 2\zeta\omega_{0i}s}\right\}, \tag{34}$$

then the parameters (K_P, K_I, K_D) of the primary controller can be determined by solving (33) with least squares method for $s = j\omega_k$, $k = 1, 2, \ldots, n$. For this design, the damping rate ζ is usually valued as 0.707, the nature frequency ω_{0i} is chosen as 1.0–3.0 times of the least loop ultimate frequency $\omega_{u\,min}$, larger than the choice given in the design section, and the frequencies ω_k are usually chosen in the range from the low frequency to the largest loop ultimate frequency $\omega_{u\,max}$.

As a demonstration, consider a multivariable process

$$G_p(s) = \begin{bmatrix} \dfrac{2e^{-4s}}{s+1} & \dfrac{0.5e^{-4s}}{1.5s+1} \\ \dfrac{0.8e^{-5s}}{1.5s+1} & \dfrac{3e^{-5s}}{s+1} \end{bmatrix}.$$

The largest normalized dead-time of the process is 5. Choosing $\zeta = 0.707$, $\omega_{01} = 0.25$, $\omega_{02} = 0.25$, ω_k in the range from 0.1 to 0.7 with 0.1 increment, and applying the method in Section 4 yields

$$K(s) =$$
$$\begin{bmatrix} 0.145 + 0.090\dfrac{1}{s} + 0.312s & -0.022 - 0.015\dfrac{1}{s} - 0.063s \\ -0.031 - 0.024\dfrac{1}{s} - 0.072s & 0.113 + 0.058\dfrac{1}{s} + 0.263s \end{bmatrix}$$

The set-point change responses of loop 1 and loop 2, which are somewhat sluggish, are shown in Figure 12.

A Smith Predictor is employed to compensate for the long dead-time. It follows from the modelling procedure that the estimated transfer function model for this process is

$$G_m(s) = \begin{bmatrix} \dfrac{1.997e^{-4.02s}}{0.997s+1} & \dfrac{0.501e^{-4.01s}}{1.500s+1} \\ \dfrac{0.803e^{-4.99s}}{1.498s+1} & \dfrac{3.010e^{-5.02s}}{1.002s+1s} \end{bmatrix},$$

which is very accurate.

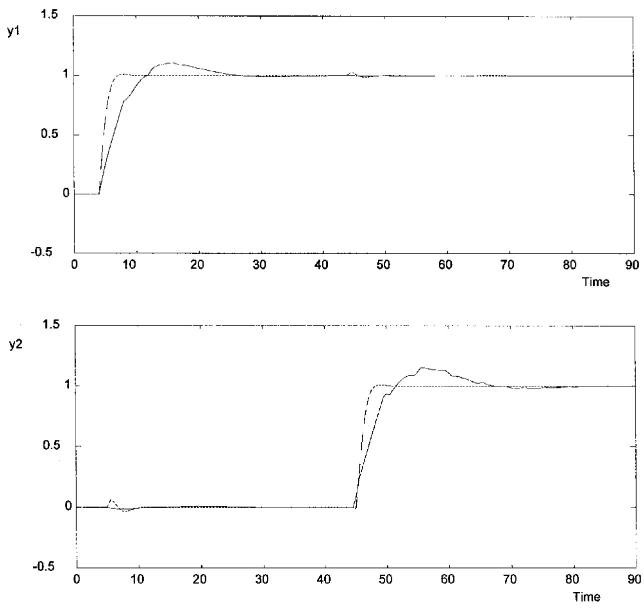

Figure 12. Responses of set-point change in loops for large dead time system. —— without Smith Predictor, - - - - with Smith Predictor.

Based on the dead-time-free part $G_c(s)$ of the model, the primary controller is tuned as

$$G_c(s) = \begin{bmatrix} 0.257+0.568\frac{1}{s}-0.104s & -0.003-0.094\frac{1}{s}-0.014s \\ -0.004-0.151\frac{1}{s}-0.021s & 0.171+0.379\frac{1}{s}-0.068s \end{bmatrix}$$

For this tuning, the damping coefficient ζ was set at 0.707, the natural frequencies ω_{01}, ω_{02} of loop 1 and 2 were both chosen as 1.5, and ω_k was changed in the range from 0.1 to 0.7 with 0.1 increment. The responses for a set-point change in loop 1 and 2 are also shown in Figure 12. Comparing with the previous design, the results are even better.

CONCLUSION

The tuning of cross-coupled multivariable process controllers has always been a difficult problem. In this paper, the new autotuning method of multivariable controllers from the frequency response of the process is presented and various typical processes have been employed to illustrate the effectiveness of the method.

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