ISA Transactions 99 (2020) 351–360

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Practice article

Fractional order automatic tuning of $PI^{\lambda}D$ controller for stable processes

Rishika Trivedi, Prabin Kumar Padhy *

Department of Electronics & Communication Engineering, PDPM, IIITDM, Jabalpur, Madhya Pradesh, India

ARTICLE INFO

Article history:
Received 14 January 2019
Received in revised form 9 September 2019
Accepted 9 September 2019
Available online 20 September 2019

Keywords:
Fractional order controller
Automatic tuning
Modified relay

ABSTRACT

This paper presents modified fractional order relay based automatic tuning of $PI^{\lambda}D$ controller for stable processes with load disturbance. The modified relay comprises of PI^{λ}controller introduced with the ideal relay in series. The proportional gain of PI^{λ} controller is assumed as unity and the integral time constant is chosen such that, the system attains minimum 0° phase margin. λ of PI^{λ} controller helps in reducing the harmonics in the output which ultimately leads to improvement in the accuracy of the proposed scheme. The limit cycle, thus, obtained helps in extracting the critical frequency and peak amplitude which helps in controller design. The designing of $PI^{\lambda}D$ controller is based on the measured parameters from modified relay. Subsequently, the derivative time constant is derived in the design of $PI^{\lambda}D$ controller using phase margin criteria and Nyquist curve is used to obtain the proportional gain. The integral time constant for $PI^{\lambda}D$ controller is same as that of PI^{λ} controller for auto tuning test. The auto tuning method and $PI^{\lambda}D$ controller design, thus proposed, is validated using typical examples through simulation.

1. Introduction

Though, fractional calculus was developed as a dedicated branch of mathematics [1,2], fractional order systems along with their applications has become a trending fields of research, nowadays [1–5]. The study of fractional calculus and fractional order controllers (FOC) in various fields of engineering have conquered immense significance due to the improved comprehension of the prospective of fractional calculus [6]. Various controller structures are proposed till date for FOC, for instance, TID controller [7], fractional order PID controller (FOPID) [8–10] along with its subcategories such as FOPI controller [11,12] and FOPD controller [13,14] along with their tuning rules [15,16].

The literature have also focused on designing a robust FOC [17–20]. Furthermore, the literature have demonstrated the practical applications of FOC as shown in [2,21–25]. FOC are preferred over integer order controllers as they are more robust and less sensitive to parametric variations. The fractional order (FO) of the integral and derivative respectively, provides additional degrees of freedom [10,26], which, leads to flexibility in system design. Hence, more precise control of dynamic systems is obtained. FOPID controller has memory because of fractional integro-differential operations. Therefore, it avoids instantaneous actions and memorizes the past states [27]. Nevertheless, FOPID controller are tedious in tuning and realization. The FOC portraits its main benefit by enhancing the outcome of the closed-loop dynamic system by increasing the robustness [1,28]. In [29,30], robustness is thoroughly analysed and the sensitivity to gain variations is demonstrated. It has resulted in studying various aspects like analysis, tuning, design and realization of such controllers.

The auto tuning technique suggested by Åstöm and Hägglund [31] involves critical gain (g_c) and critical frequency(f_c). They introduced a relay based automatic tuning method with prescribed gain and phase margin. Thereafter, Scali et al. [32] proposed a relay based identification process of an entirely unidentified process which needs various relay experiments to identify the time delay of the modified relay. Subsequently, an integrator was introduced along with the classical relay to improve the precision of the describing function [33]. This identification method used attenuation characteristics of the integrator to approximate a real process by a low order transfer function. The authors have used the pole-zero cancellation technique for designing PID controllers. However, the approximations used by the authors are unsuccessful in fetching the desirable results for many typical processes. Chen & Moore [34] proposed a method for designing a robust PID controller by using iso-damping property of the relay in feedback. However, the response of the closed loop obtained through this method is oscillatory.

* Correspondence to: Department of Electronics & Communication Engineering, PDPM, IIITDM Jabalpur, Dumna Airport Road, P.O.: Khamaria, Jabalpur–482005, Madhya Pradesh, India.

E-mail addresses: rishika.t@iiitdmj.ac.in (R. Trivedi), prabin16@iiitdmj.ac.in (P.K. Padhy).

https://doi.org/10.1016/j.isatra.2019.09.011
0019-0578/© 2019 ISA. Published by Elsevier Ltd. All rights reserved.

Later, Ho et al. [35] combined the idea of iterative feedback tuning with relay based auto tuning to achieve specified phase margin and bandwidth. Their method outperformed auto tuning method of standard relay which incorporates Zeigler–Nichols technique. Byeon et al. [36] introduced a FO integrator along with a relay to acquire the response in frequency domain of the process at a known phase shift. Jeng et al.introduced a modified relay feedback scheme based on evaluation of gain and phase margin (PM) for tuning the controller parameters [37]. Later, auto tuning is suggested for PI controller on the basis of the event. It is demonstrated for FOPDT processes which includes two relay feedback experiment [38]. Thereafter, Bajarangbali and Majhi [39] used relay with hysteresis to evaluate the undetermined parameters of the process. The method proposed helps in reducing the effect of measurement noise. Bazanella et al. [40] proposed a reaction curve experiment based on which PID controller is tuned traditionally. In their method, modified relay feedback is used, through which a transfer function is introduced in the loop possessing a fixed phase in an arbitrary large range of frequencies.

This paper presents an automatic tuning scheme which includes FO modified relay in which PI^{λ} controller is introduced in series with the ideal relay. The efficacy of the proposed modified relay is substantiated by the harmonic analysis on the output signal using Fast Fourier Transform (FFT). The harmonic contents in the output signal of the proposed modified relay are compared with the harmonic contents in the output signal of its integer order counterpart, relay with hysteresis, ideal relay with FO integrator and ideal relay. The main challenge in harmonic analysis is to choose an appropriate value of λ of PI^{λ} controller. Hence, λ is obtained through an analytical approach. The analytical approach is based on harmonic analysis for different λ. Then, the parameters of the controller are designed strictly adhering to the phase margin criteria. Thereafter, the relay and PI^{λ} controller connected in series are removed and $PI^{\lambda}D$ controller is connected whose parameters are designed according to the phase margin criteria.

The advantages of the proposed methodology are as follows: Primarily, the proposed method demonstrates that the harmonic content in the output signal of the proposed modified relay is reduced for an obtained optimal value of λ. It is then compared to the auto tuning method proposed in [41]. Therefore, a conclusion is drawn that the harmonic contents in the output signal are further reduced because of λ in PI^{λ} controller. Furthermore, the precision in the quantification of g_c and f_c are enhanced by the proposed method which helps in obtaining a more symmetrical and smoother limit cycle. Lastly, the proposed method works efficiently without the prior information of the process which is to be controlled. Also, the valid scale for λ is proposed for which $PI^{\lambda}D$ controller outperforms PID controller. Six examples and an experimental setup are used to validate the proposed methodology.

2. Analysis of proposed auto tuning scheme

The automatic tuning method proposed is based on FO modified relay as shown in Fig. 1. The proposed FO relay consists of unity proportional gain, PI^{λ} controller, $C_N(s)$, connected in series with the ideal relay, N. In the proposed method, $C_N(s)$ and $C(s)$ are defined as:

$$C_N(s) = \left(1 + \frac{1}{T_i s^{\lambda}}\right) = \left(\frac{T_i s^{\lambda} + 1}{T_i s^{\lambda}}\right) \tag{1}$$

$$C(s) = K_c\left(1 + \frac{1}{T_i s^{\lambda}} + \frac{T_d s}{1 + \beta T_d s}\right) = C_{FPID} \tag{2}$$

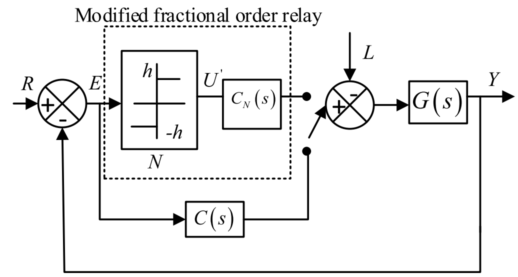

Fig. 1. Proposed fractional order relay based auto tuning scheme.

Table 1
Harmonic analysis for different values of λ.

λ	μ_3/μ_1	μ_5/μ_1	μ_7/μ_1
0.8	0.071696	0.036273	0.006317
0.9	0.058153	0.047901	0.006846
1	0.225886	0.087706	0.030014
1.1	0.019981	0.038695	0.015313
1.2	0.169543	0.115938	0.013224

where, K_c is the proportional gain, λ is the FO of the integral, T_i and T_d are the integral and derivative time constant respectively, β is the derivative filter constant which is neglected during controller design analysis.

PI^{λ} controller performs best with the proper and precise tuning of λ. Therefore, choosing the value of λ plays a crucial role in the proposed FO relay based controller auto tuning scheme. The ratio between the amplitude of the various harmonic components and the fundamental component are chosen as a performance criterion for accuracy measurement of the modified relay. Henceforth, simulation is carried out in MATLAB/Simulink using FOMCON [42] toolbox to obtain the value of λ through analytical approach. Harmonic analysis using Fast Fourier Transform (FFT) is taken in account for simulation in MATLAB/Simulink. Table 1 gives the insight of harmonic analysis, where μ is the amplitude of the harmonic component. It can be concluded from Fig. 2, that for $\lambda = 1.1$, the ratio between the amplitude of the various harmonic components and the fundamental harmonic component is minimum as compared to the other values of λ. Hence, an accurate closed loop performance can be predicted for modified relay at $\lambda = 1.1$, obtained through analytical approach and, thus, it can be assured that the accuracy test can be performed for the above mentioned value.

2.1. Accuracy test of the proposed methodology

The accurate results of the analysis of describing function are obtained once the limit cycle approaches sinusoidal behaviour which is not observed commonly for most of the processes. The approximation of describing function results in estimation error of g_c and f_c. The proposed scheme shows a decrease in the percentage harmonic content as compared to the modified relay proposed in [41] because of λ of the integral in PI^{λ} controller. Therefore, the proposed modified FO relay improves the accuracy of auto tuning.

Let the input to the ideal relay be the error signal described by $e(t) = \mu \sin(\omega t)$. A square wave is obtained as the relay output whose, Fourier series expansion can be represented as:

$$x'(t) = \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \sin((2\kappa - 1)\,\omega t), \tag{3}$$

where, $\mu'_{2\kappa-1} = \frac{4h}{\pi(2\kappa-1)}$ represents the peak amplitude of the harmonic contents of $x'(t)$ and h is the height of the relay. The

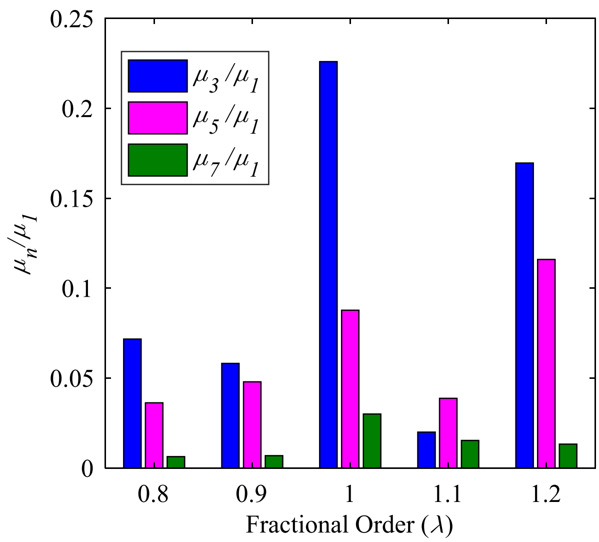

Fig. 2. Harmonic analysis for different values λ.

output of the modified relay is given by:

$$x(t) = \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \sin((2\kappa-1)\omega t) + \frac{1}{T_i} {}_aI_t^{\lambda} \left(\sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \sin((2\kappa-1)\omega t) \right), \tag{4}$$

where, ${}_aI_t^{\lambda}$ is the FO integral w.r.t time and λ is the fractional order.

$$x(t) = \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \sin((2\kappa-1)\omega t) + \frac{1}{T_i} {}_aI_t^{\lambda} \left(\sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \frac{e^{j(2\kappa-1)\omega t} - e^{-j(2\kappa-1)\omega t}}{j2} \right) \tag{5}$$

Eq. (5) can be transformed to,

$$x(t) = \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \sin((2\kappa-1)\omega t) + \frac{1}{j2T_i[(2\kappa-1)\omega]^{\lambda}} \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \left((j)^{-\lambda} e^{j(2\kappa-1)\omega t} - (-j)^{-\lambda} e^{-j(2\kappa-1)\omega t} \right) \tag{6}$$

Substituting $(j)^{-\lambda} = \cos\left(\frac{\lambda\pi}{2}\right) - j\sin\left(\frac{\lambda\pi}{2}\right)$, $(-j)^{-\lambda} = \cos\left(\frac{\lambda\pi}{2}\right) + j\sin\left(\frac{\lambda\pi}{2}\right)$, Eq. (6) becomes,

$$x(t) = \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \sin((2\kappa-1)\omega t) + \frac{1}{j2T_i[(2\kappa-1)\omega]^{\lambda}} \times \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \left(\left(\cos\left(\frac{\lambda\pi}{2}\right) - j\sin\left(\frac{\lambda\pi}{2}\right) \right) e^{j(2\kappa-1)\omega t} - \left(\cos\left(\frac{\lambda\pi}{2}\right) + j\sin\left(\frac{\lambda\pi}{2}\right) \right) e^{-j(2\kappa-1)\omega t} \right) \tag{7}$$

Eq. (7) can be rewritten as:

$$x(t) = \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \sin((2\kappa-1)\omega t) + \frac{1}{T_i[(2\kappa-1)\omega]^{\lambda}} \sum_{\kappa=1}^{\infty} \mu'_{2\kappa-1} \sin\left((2\kappa-1)\omega t - \frac{\lambda\pi}{2} \right) \tag{8}$$

The amplitude of the different frequency components in the output signal of the FO modified relay is given by:

$$\mu_{2\kappa-1} = \frac{4h}{\pi(2\kappa-1)^{\lambda+1}} \sqrt{(2\kappa-1)^{2\lambda} + \frac{1}{(T_i\omega^{\lambda})^2}} \tag{9}$$

Table 2(a) shows the harmonic analysis of ideal relay, relay with hysteresis [39] and integer order modified relay [41], whereas, Table 2(b) shows the harmonic analysis of ideal relay connected in series with the FO integrator (where $\lambda = 0.5$) [36] and the proposed FO modified relay. $\lambda = 1.1$ is chosen for harmonic analysis of proposed methodology which is obtained through analytical approach as discussed earlier. The proposed FO modified relay introduces an improvement in the harmonic content from 11% to 9%, 4% to 3.4% and 2% to 1.6% for 3rd, 5th and 7th harmonic respectively.

2.2. Controller design

The primary aim is to obtain an efficient controller through auto tuning test. It is done with the help of limit cycle obtained as the output of modified relay for a known phase ϕ'. As per the Nyquist stability criterion, the condition for limit cycle existence is,

$$NC_N G(j\omega_{cr}) = -1, \tag{10}$$

where, ω_{cr} is critical frequency of the output limit cycle of the FO relay, $C_N(s)$ is the controller as in Eq. (1) used for auto tuning process and the process transfer is given by G. From Eq. (1),

$$|C_N(s)| = \frac{\sin\left(\frac{\lambda\pi}{2}\right)}{T_i\omega_{cr}'^{\lambda}\sin\phi'} \tag{11}$$

and

$$\phi = \tan^{-1}\left(\frac{\sin\left(\frac{\lambda\pi}{2}\right)}{T_i\omega_{cr}'^{\lambda} + \cos\left(\frac{\lambda\pi}{2}\right)} \right) \tag{12}$$

Therefore, it can be concluded from Eqs. (10)–(12), that,

$$G(j\omega_{cr}) = \frac{T_i\omega_{cr}^{\lambda}\sin\phi}{N\sin\left(\frac{\lambda\pi}{2}\right)} e^{j(-180+\phi)} \tag{13}$$

It can be noted from Eq. (13), that,

$$T_i = \frac{\sin\left(\frac{\lambda\pi}{2}\right) - \cos\left(\frac{\lambda\pi}{2}\right)\tan(\phi')}{\omega_{cr}'^{\lambda}\tan(\phi')} \tag{14}$$

Also,

$$\sin\left(\frac{\lambda\pi}{2}\right) - \cos\left(\frac{\lambda\pi}{2}\right)\tan(\phi') > 0 \tag{15}$$

and λ exists in the range of (0.85, 2). So, to restrict λ well within the range,

$$\lambda' = \frac{2\phi' + 1.7\pi}{2\pi} \tag{16}$$

Therefore, a closed loop performance using modified relay is predicted at $\lambda = 1.1$ which is obtained through analytical approach from Fig. 2. Henceforth, the value of λ is updated to λ'

Table 2
Harmonic Analysis of Relays.

(a) Harmonic analysis for ideal relay, relay with hysteresis [39] and integer order modified relay [41]

Ideal relay			Relay with hysteresis [39]			Integer order modified relay [41]		
μ_3/μ_1	μ_5/μ_1	μ_7/μ_1	μ_3/μ_1	μ_5/μ_1	μ_7/μ_1	μ_3/μ_1	μ_5/μ_1	μ_7/μ_1
0.3333	0.2	0.1429	0.336	0.1864	0.0328	0.1111	0.0400	0.0204

(b) Harmonic analysis for ideal relay + fractional order integrator [36] and proposed fractional order modified relay.

Ideal relay + FO integrator [36]			Proposed FO modified relay		
μ_3/μ_1	μ_5/μ_1	μ_7/μ_1	μ_3/μ_1	μ_5/μ_1	μ_7/μ_1
0.1188	0.038	0.016	0.0995	0.0340	0.01679

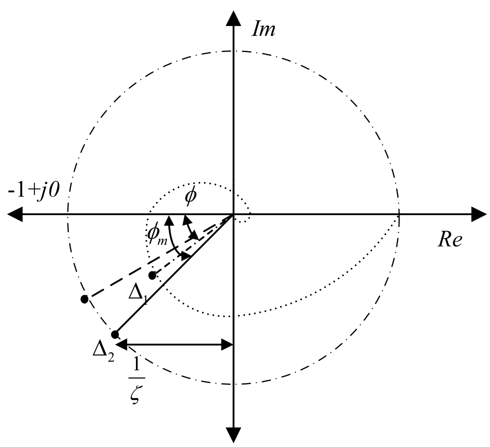

Fig. 3. Nyquist curve of uncompensated process (…), process with fractional order modified relay (—) and the process with the controller (-)

using Eq. (16). Therefore, $C_N(s)$ is known initially for auto tuning test. It can be observed from the Nyquist curve, in Fig. 3, that a point Δ_1 is obtained in third quadrant which has a phase lag of ϕ with the negative real axis. Also, a gain crossover point Δ_2 is observed in Fig. 3, which is responsible for loop phase margin $\phi_m > 0°$ for a user defined $\phi' \geq 0°$. The design of the controller can be categorized into two parts. Firstly, a minimum phase margin of 0° is to be maintained by achieving a phase lag of $-180r + \phi$. Secondly, the desirable closed loop response is obtained by an accurate choice of ζ, such that the phase margin (ϕ_m) requirements are satisfied, that is, $\phi_m > \cos^{-1}\left(\frac{1}{\zeta}\right)$ where $\zeta > 1$ and the

$$Re\left(CG\left(j\omega_{cr}\right)\right) = \frac{1}{\zeta}. \tag{17}$$

So, using Eqs. (1), (2), (13) and (17), it is obtained,

$$\left|C_{FPID}\left(j\omega_{cr}\right)\right| = \frac{k_c}{\cos\phi}\left(1 + \frac{1}{T_i\omega^{\lambda}}\cos\left(\frac{\lambda\pi}{2}\right)\right) \tag{18}$$

and

$$\angle C_{FPID}\left(j\omega_{cr}\right) = \phi \tag{19}$$

Solving Eqs. (18) and (19), derivative time constant and proportional gain is obtained,

$$\left.\begin{aligned} K_c &= \frac{4h}{\pi A\zeta}\frac{\sin\left(\frac{\lambda\pi}{2}\right)}{\sin\phi\ \left(T_i\omega^{\lambda} + \cos\left(\frac{\lambda\pi}{2}\right)\right)} \\ T_d &= \frac{\sin\left(\frac{\lambda\pi}{2}\right)}{T_i\omega^{\lambda+1}} \end{aligned}\right\} \tag{20}$$

for $\lambda' \in (0.85, 2)$. Since, $\phi' \geq 0°$, the lower range of λ' can be defined using Eq. (16) as 0.85. For, series form of $PI^{\lambda}D$ controller, the controller in Eq. (2) is as follows:

$$C_s(s) = K_{cs}\left(1 + \frac{1}{T_i s^{\lambda}}\right)\left(+\frac{T_{ds}s}{1 + \beta T_{ds}s}\right) \tag{21}$$

where, K_{cs} is the proportional gain and T_{ds} is the derivative time constant. Then, the parameters of the controller in Eq. (22) are designed with a similar approach as adopted for parallel form of $PI^{\lambda}D$ controller. The expression for T_i is same as Eq. (14), whereas K_{cs} and T_{ds} are obtained as follows:

$$\left.\begin{aligned} K_{cs} &= \frac{4h}{\pi A\zeta} \\ T_{ds} &= \frac{\tan(\phi)}{\omega_{cr}} \end{aligned}\right\} \tag{22}$$

for $\lambda' \in (0.85, 2)$.

For auto tuning and controller design, the steps are encapsulated as:

1. The initial step of auto tuning requires the value of T_i, which is chosen between 10–20 for the optimized value of λ obtained in Section 2.1 for stable processes. For a known value of $\phi' \geq 0°$, the value of T_i is updated using Eq. (14) and value of λ is updated to λ' using Eq. (16).
2. Now, amplitude A and ω_{cr} is noted to begin with the second step of auto tuning.
3. Finally, $PI^{\lambda}D$ controller is designed by obtaining its parameters for fine tuning using Eqs. (14), (16) and (20) for a chosen value of ζ.

3. Results & discussions

To validate the proposed methodology, this section considers six stable processes. The value of $T_i = 20$ is chosen for FO relay based auto tuning scheme. The limit cycle is obtained through which ω_{cr} is obtained. Then, the controller parameters are designed for $h = 0.5$. β is kept as 1% of T_d in $PI^{\lambda}D$ controller design. Fig. 4 shows the comparison of limit cycle obtained from ideal relay, relay with hysteresis [39], integer order modified relay [41] and proposed FO modified relay. The comparison of the output signal of ideal relay, relay with hysteresis [39], integer order modified relay [41] and proposed FO modified relay is shown in Fig. 5. Table 9 gives a detailed comparison of the simulation results based on rise time (t_r), settling time (t_s), overshoot (M_p), integral absolute error (IAE) and integral square error (ISE) of different tuning methods for various examples.

Example 1. A stable process of third order with recurrent roots is considered [34,41].

$$G_1(s) = \frac{1}{(s+1)^3}e^{-s}$$

For a known value of $\phi' = 28°$, the updated value of T_i is 2.2832. The limit cycle, thus, induced by modified relay, introduces a phase lag of $\phi = 36.30°$. The value of ζ is chosen as 1.954

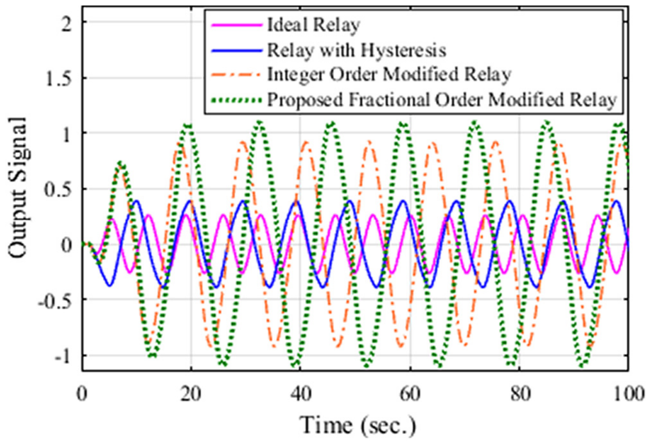

Fig. 4. Comparison of limit cycle obtained from ideal relay, relay with hysteresis [39], integer order modified relay [41] and proposed fractional order modified relay.

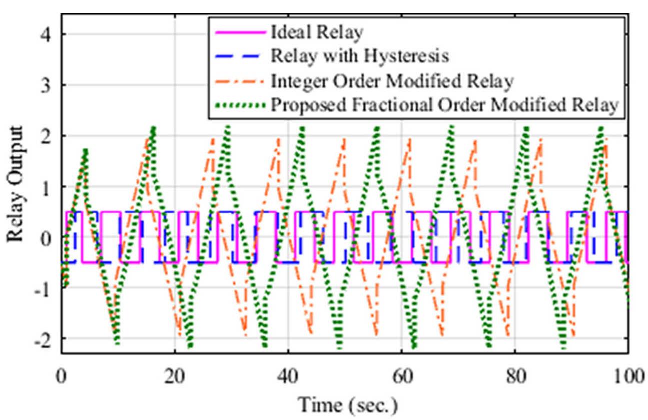

Fig. 5. Comparison of output signal of ideal relay, relay with hysteresis [39], integer order modified relay [41] and proposed fractional order modified relay.

Table 3
Controller parameters for Example 1.

Method	Controller parameters
Proposed	$0.9842\left(1+\frac{1}{2.2832s^{1.0056}}+\frac{0.9640s}{0.009s+1}\right)$
Padhy & Majhi [41]	$1.0749\left(1+\frac{1}{3.0513s}+\frac{0.7088s}{0.007s+1}\right)$
Chen & Moore [34]	$1.024\left(1+\frac{1}{1.241s}+1.539s\right)$

and $PI^{\lambda}D$ controller parameters are designed using Eqs. (14), (16) and (20). Table 3 depicts $PI^{\lambda}D$controller parameters estimated using proposed methodology as well as those suggested by Padhy and Majhi [41] and Chen and Moore [34].

Figs. 6–8 illustrates the closed loop step response for $G_1(s)$, with +30% and −30% variation in K, respectively. A step load disturbance of 0.5 is applied at 40 s. Table 9 shows that the proposed method outperforms [41] in terms of t_r, t_s and ISE, whereas when the proposed method is compared to [34], the proposed method gives much lesser t_s, M_P, IAE and ISE. It can be noted from Figs. 7 and 8 that for the variation in steady state gain the proposed method contributes a smoother response along with the improved disturbance handling as compared to the method proposed in [41] and [34].

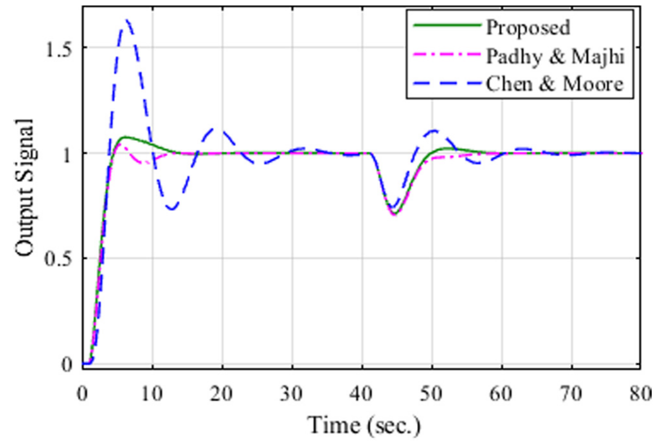

Fig. 6. Comparison of step response of the proposed controller, controller proposed in [41] and controller of [34] for Example 1.

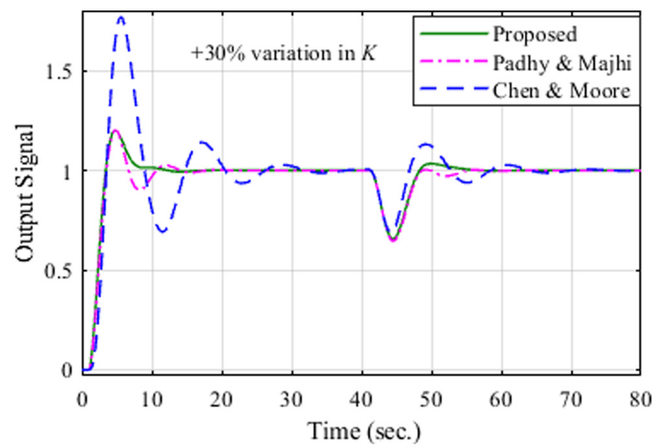

Fig. 7. Comparison of step response of the proposed controller, controller proposed in [41] and controller of [34] for Example 1 with +30% variation in K.

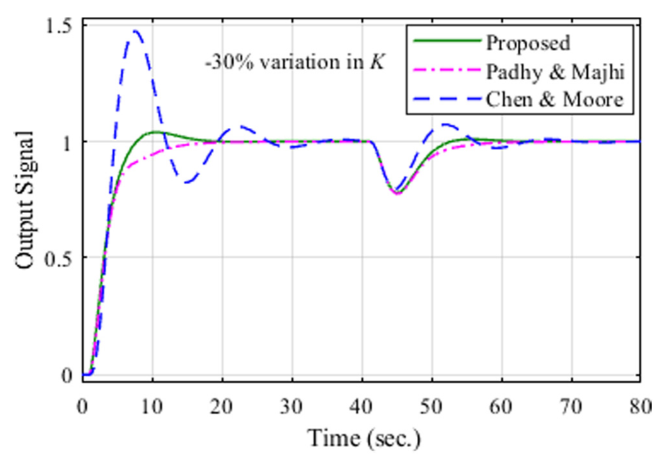

Fig. 8. Comparison of step response of the proposed controller, controller proposed in [41] and controller of [34] for Example 1 with −30% variation in K.

Example 2. A stable high order process is considered [34,37,41].

$$G_2(s)=\frac{1}{(s+1)^5}$$

Table 4
Controller parameters for Example 2.

Method	Controller parameters
Proposed	$1.0044\left(1+\dfrac{1}{2.9643s^{1.0056}}+\dfrac{1.1863s}{0.01s+1}\right)$
Padhy & Majhi [41]	$1.0749\left(1+\dfrac{1}{3.8470s}+\dfrac{0.8857s}{0.008s+1}\right)$
Chen & Moore [34]	$0.924\left(1+\dfrac{1}{1.961s}+1.969s\right)$
Jeng et al. [37]	$1.207\left(1+\dfrac{1}{3.364s}+0.913s\right)$

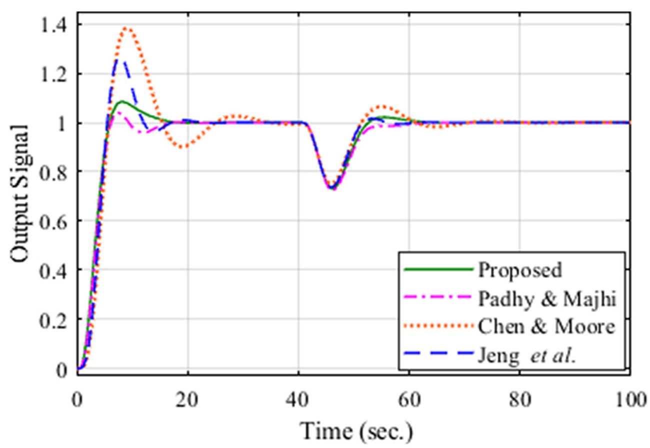

Fig. 9. Comparison of step response of the proposed controller, controller proposed in [34,37,41] for Example 2.

The updated value of T_i is 2.9643 for the user defined $\phi' = 28°$. A phase lag of $\phi = 35.83°$ is induced by the limit cycle of the modified relay. ζ is chosen as 2.1 and the Eqs. (14), (16) and (20) are used for designing parameters of $PI^{\lambda}D$ controller. Table 4 gives $PI^{\lambda}D$ controller parameters estimated using proposed methodology as well as those suggested by Padhy & Majhi [41], Chen and Moore [34] and Jeng et al. [37].

Figs. 9–11 illustrates the closed loop step response for $G_2(s)$, with +30% and −30% variation in K, respectively. A step load disturbance of 0.5 is applied at 40 s. Table 9 shows that the proposed method outperforms [41] in terms of t_r, t_s and ISE, whereas when the proposed method is compared to [34] and [37], the proposed method gives much lesser t_s, M_p, IAE and ISE. It can be noted from Figs. 10 and 11 that for the variation in steady state gain the method proposed in this paper gives a smoother response and improved disturbance handling as compared to the method proposed in [34,41] and [37].

Example 3. A SOPDT process is considered [37].

$$G_3(s) = \frac{1}{(10s+1)(2s+1)}e^{-s}$$

With the help of user defined $\phi' = 20°$, the updated value of T_i is obtained as 5.7218. The induced limit cycle by modified relay, introduces a phase lag of $\phi = 26.93°$. $PI^{\lambda}D$ controller parameters are designed using Eqs. (14), (16) and (20) with the value of ζ as 1.22. Table 5 depicts $PI^{\lambda}D$ controller parameters estimated using proposed methodology as well as those suggested by Jeng et al. [37]. Figs. 12–14 illustrates the closed loop step response for $G_3(s)$, with +30% and −30% variation in K, respectively. A step load disturbance at 40 s of magnitude 0.5 is applied. The proposed method produces better response than the method proposed in [37] in terms of M_p, t_r, t_s, IAE and ISE as shown in Table 9.

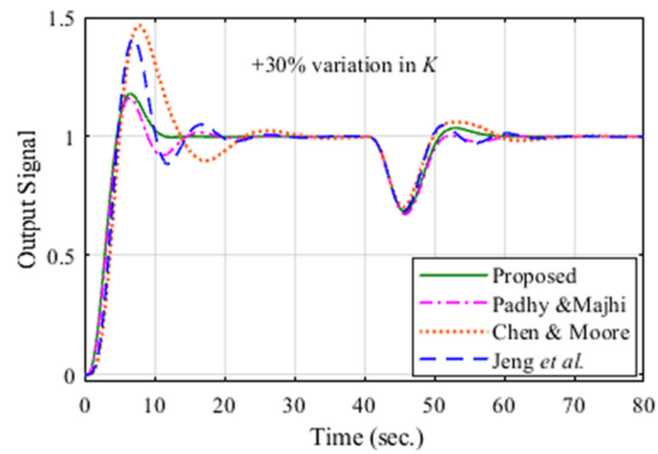

Fig. 10. Comparison of step response of the proposed controller, controller proposed in [34,37,41] for Example 2 with +30% variation in K.

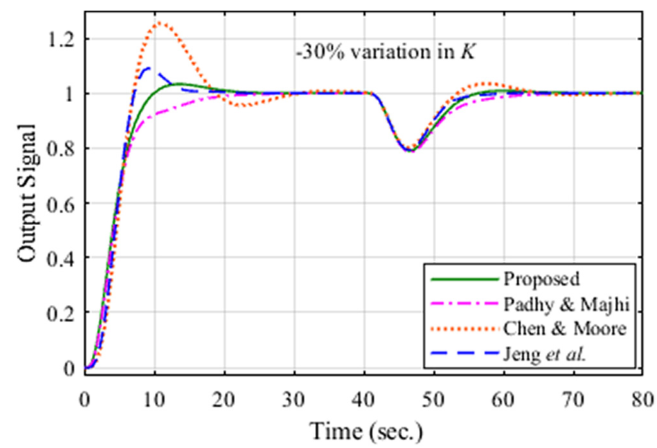

Fig. 11. Comparison of step response of the proposed controller, controller proposed in [34,37,41] for Example 2 with −30% variation in K.

Table 5
Controller parameters for Example 3.

Method	Controller parameters
Proposed	$6.4915\left(1+\dfrac{1}{5.7218s^{0.9611}}+\dfrac{1.1693s}{0.01s+1}\right)$
Jeng et al. [37]	$8.255\left(1+\dfrac{1}{5.634s}+1.409s\right)$

Example 4. A sixth order process is considered [35].

$$G_4(s) = \frac{1}{(s+1)^6}$$

The updated value of T_i is 3.5652 for the user defined $\phi' = 30°$. A phase lag of $\phi = 57.76°$ is induced by the limit cycle of the modified relay. ζ is chosen as 1.5 and the Eq. (14), (16) and (20) are used for designing parameters of $PI^{\lambda}D$ controller. Table 6 gives $PI^{\lambda}D$ parameters estimated using proposed methodology as well as those suggested by Ho et al. [35].

Fig. 15 exhibits the closed loop step response for $G_4(s)$. A step load disturbance is applied at 50 s of magnitude 0.5. The proposed method outperforms the method proposed in [35] in terms of M_p, t_s, IAE and ISE as shown in Table 9.

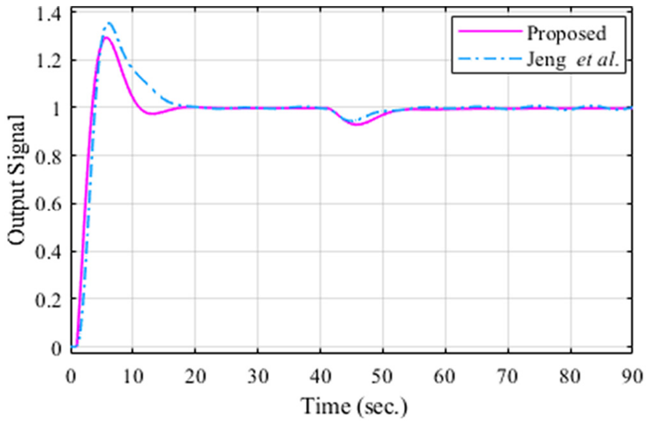

Fig. 12. Comparison of step response of the proposed controller, controller proposed in [37,41] for Example 3.

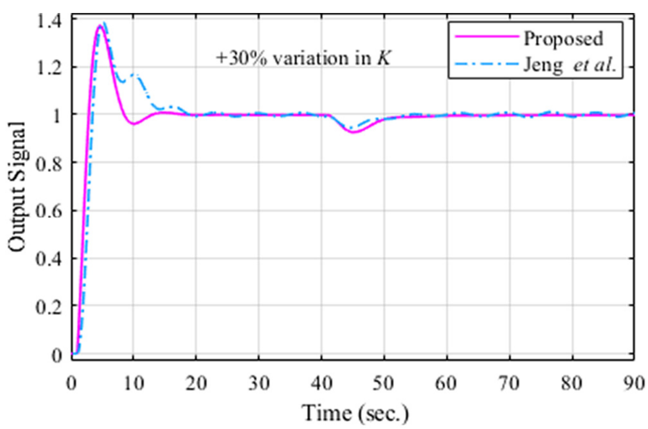

Fig. 13. Comparison of step response of the proposed controller, controller proposed in [37,41] for Example 3 with +30% variation in K.

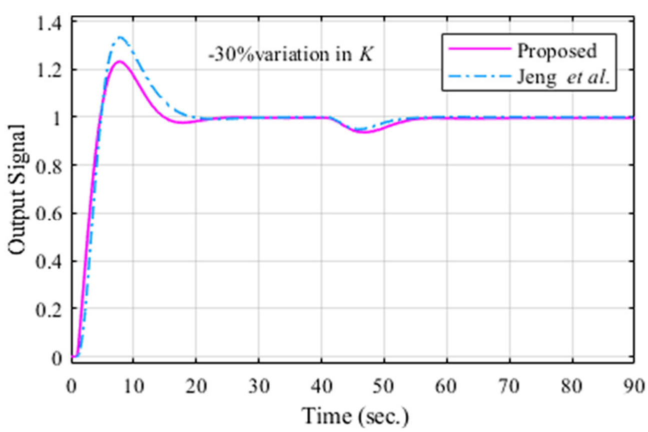

Fig. 14. Comparison of step response of the proposed controller, controller proposed in [37,41] for Example 3 with −30% variation in K.

Example 5. A linear representation of pitch angle of the aircraft is considered [40].

$$G_5(s) = \frac{1.15s + 0.18}{s^3 + 0.74s^2 + 0.92s}$$

The updated value of T_i is 0.3549 for the user defined $\phi' = 30°$. A phase lag of $\phi = 79.71°$ is induced by the limit cycle of the modified relay. ζ is chosen as 1.5 and the Eq. (14), (16) and (20)

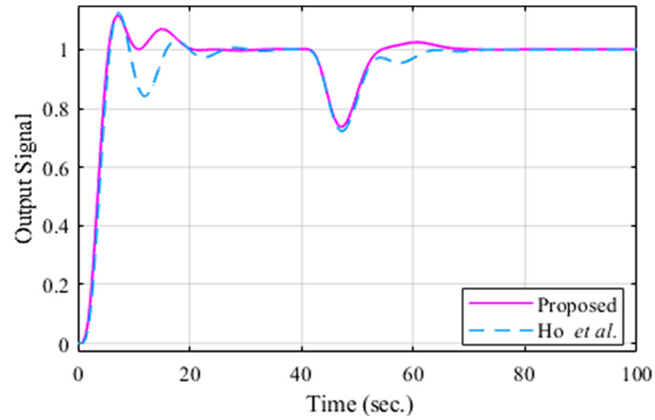

Fig. 15. Comparison of step response of the proposed controller and controller designed in [35] for Example 4.

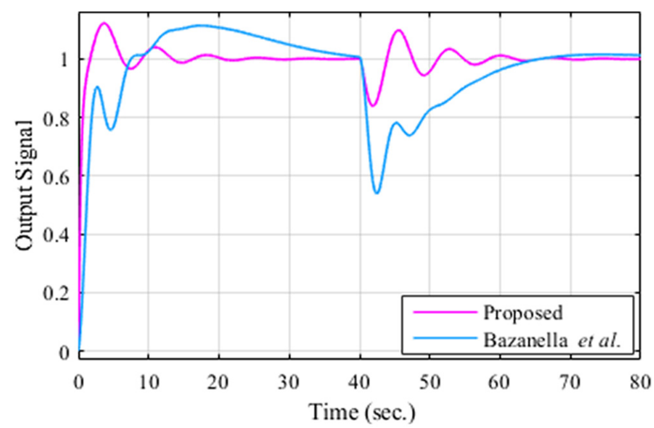

Fig. 16. Comparison of step response of the proposed controller and controller proposed in [40] for Example 5.

Table 6
Controller parameters for Example 4.

Method	Controller parameters
Proposed	$1.1781\left(1 + \frac{1}{3.5652s^{1.0167}} + \frac{1.9261s}{0.01s+1}\right)$
Ho et al. [40]	$1.3\left(1 + \frac{1}{5.3s} + \frac{1.3s}{0.13s+1}\right)$

Table 7
Controller parameters for Example 5.

Method	Controller parameters
Proposed	$0.7668\left(1 + \frac{1}{0.3549s^{1.0167}} + \frac{3.0954s}{0.03s+1}\right)$
Bazanella et al. [40]	$0.7738\left(1 + \frac{1}{5.2s} + \frac{0.155s}{0.000891s+1}\right)$

are used for designing parameters of $PI^{\lambda}D$ controller. Table 7 gives $PI^{\lambda}D$ parameters estimated using proposed methodology as well as those suggested by Bazanella et al. [40].

Fig. 16 illustrates the closed loop step response for $G_5(s)$. A step load disturbance of 0.5 is applied at 40 s. The proposed method produces better response than the method proposed in [40] in terms of M_p, t_r, t_s, ISE, IAE and disturbance handling as shown in Table 9.

Table 8
Controller parameters for Example 6.

Method	Controller parameters
Proposed	$0.3280\left(1+\frac{1}{0.3896s^{1.0333}}\right)$
SIMC tuning rules [38]	$1.0\left(1+\frac{1}{0.25s}\right)$

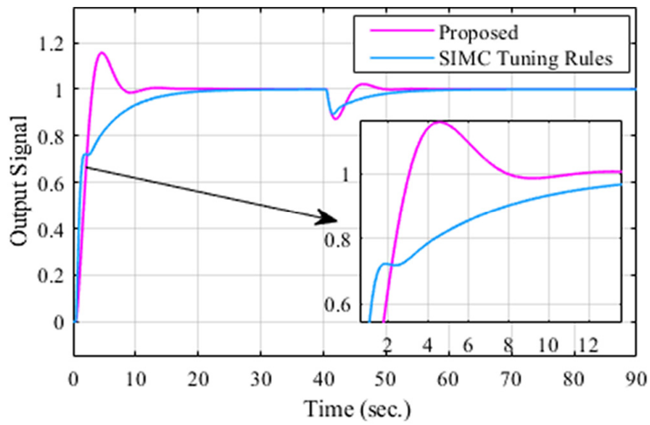

Fig. 17. Comparison of step response of the proposed controller and controller designed using SIMC tuning rules used in [38] for Example 6.

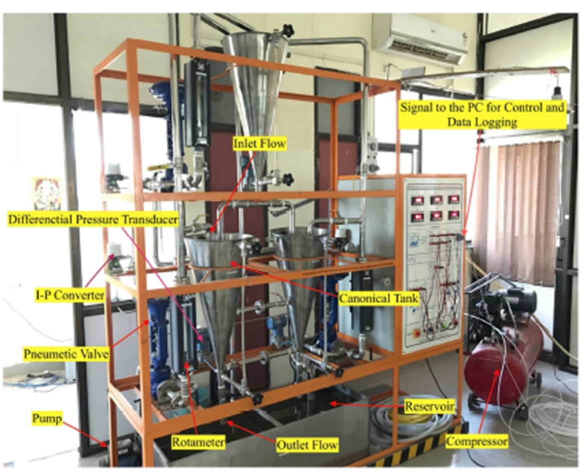

Fig. 18. Experimental setup of conical tank system.

Example 6. A stable FOPDT process is considered [43].

$$G_6(s) = \frac{1}{(s+1)}e^{-0.5s}$$

For a known value of $\phi' = 33°$, the updated value of T_i is 0.3896. The limit cycle, thus, induced by modified relay, introduces a phase lag of $\phi = 55.65°$. The value of ζ is chosen as 8 and PI^{λ} controller parameters are designed using Eqs. (14), (16) and (20). Table 8 depicts PI^{λ} parameters estimated using proposed methodology as well as those designed using SIMC tuning rules used in [38]. Fig. 17 illustrates the closed loop step response for $G_6(s)$. Table 9 shows that the response obtained using proposed method settles faster, giving less IAE and ISE. It also demonstrates better disturbance handling.

4. Experimental setup

A conical water tank system is considered as the experimental setup shown in Fig. 18. It consists of three conical tanks controlled individually. The water is supplied to the tanks (with 60 cm as the maximum liquid level of a tank) by the three different pumps with 0.5 hp individually. The pumps have 400 litre per hour as the utmost flow rate. The pneumatic valve and the manual valve control the inlet flow and the outlet flow respectively, which controls the level of water in the conical water tank system. The control signal for the pneumatic valve are obtained from PC via *I–P* converter. The current range of the input of *I–P* converter is 4 to 20 mA whereas 3 to 15 psi is the pressure range of the output. The differential pressure transducer is mounted at the extremity of the tank to measure the water level. The differential pressure transducer is limited to 0 to 4000 mmWc. The validation of the proposed work requires only one tank. The conical water tank system requires PC to CTS interfacing, which is achieved by using the software, namely, MATLAB/Simulink 2014a along with the data acquisition device, viz., VDPID-03. Following steps are considered for performing the experiment:

1. In the first step, process is identified using the input/output response of the open loop system for the process . After measuring the input/output response, the conical water tank system is identified to an equivalent SOPDT process model as shown below with the help of System Identification Toolbox in MATLAB.

$$G_e(s) = \frac{0.72}{(4.2068s+1)(4.9775s+1)}e^{-s}$$

2. In the second step, auto tuning of $PI^{\lambda}D$ is performed for the identified SOPDT model. The updated value of T_i is 3.4121 for the user defined $\phi' = 30°$ and a phase lag of $\phi = 52.55°$. The controller parameters designed using proposed modified FO relay and proposed by [41] are given in Table 10. The response of the conical water tank system is demonstrated in Fig. 19 and is compared with the output response of the controller designed by Padhy & Majhi [41]. Fig. 18 shows the actual image of conical water tank system.
3. The designed controller is then implemented to the conical water tank system in step 3. A saturation is introduced with $PI^{\lambda}D$ controller varying from 0 to 100 because of valve limitations.

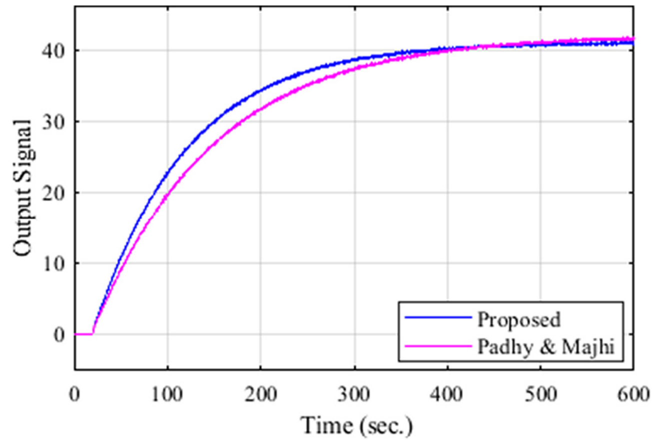

Fig. 19. Comparison of step response of the proposed controller and controller designed in [41] for conical water tank system.

5. Conclusion

This paper presents an auto tuning scheme on the basis of FO amended relay for $PI^{\lambda}D$ controller. The proposed scheme consists

Table 9
Comparison of performance indices for examples.

Process	Method	t_r (s)	t_s (s)	M_p (%)	IAE	ISE
$\frac{1}{(s+1)^3}e^{-s}$	Proposed	2.350	22.25	6.989	4.440	2.453
	Padhy & Majhi [41]	2.386	22.415	3.646	4.386	2.527
	Chen & Moore [34]	1.712	35.89	63.115	8.371	4.146
$\frac{1}{(s+1)^5}$	Proposed	3.477	28.489	8.152	5.550	2.954
	Padhy & Majhi [41]	3.566	29.061	3.646	5.581	3.065
	Chen & Moore [34]	2.962	35.585	38.194	6.731	3.931
	Jeng et al. [37]	2.881	51.318	25.949	6.311	3.58
$\frac{1}{(10s+1)(2s+1)}e^{-s}$	Proposed	1.880	25.787	29.221	4.151	2.094
	Jeng et al. [37]	1.915	39.1	36.301	5.153	2.829
$\frac{1}{(s+1)^6}$	Proposed	2.961	34.845	11.798	6.023	3.223
	Ho et al. [35]	2.998	51.633	13.068	6.918	3.557
$\frac{1.15s+0.18}{s^3+0.74s^2+0.92s}$	Proposed	0.8464	35.142	11.798	1.833	0.3017
	Bazanella et al. [37]	5.474	49.982	10.145	7.888	2.069
$\frac{1}{s+1}e^{-0.5s}$	Proposed	1.552	36.188	21.341	2.548	1.318
	SIMC tuning rules [38]	10.663	46.897	0.312	4.799	1.527

Table 10
Controller parameters for conical water tank system.

Method	Controller parameters
Proposed	$4.1550\left(1+\frac{1}{3.4121s^{1.0167}}+\frac{3.5877}{0.03s+1}\right)$
Padhy & Majhi [41]	$1.9215\left(1+\frac{1}{6.0269s}+\frac{1.5064s}{0.01s+1}\right)$

of two parts. Firstly, relay based auto tuning is done. Secondly, parameters of the controller are tuned to keep the phase margin greater than 0°. It can be concluded from Tables 2(a) and 2(b) that PI^λ controller connected in series with the ideal relay gives substantial decrease in the harmonic content of the output signal of the proposed modified relay as compared to its integer order counterpart, relay connected in series with FO integrator, relay with hysteresis and ideal relay. During auto tuning process, the proposed modified relay shows 2% and 24% reduction in the content of the most significant harmonic, i.e. third harmonic as compared to its integer order counterpart (from Fig. 2) and ideal relay, respectively. Then, tuning of the controller parameters is taken in account. The proposed methodology has been validated using six typical stable processes and an experimental setup. On comparison with the existing auto tuning schemes for PID controller, the proposed methodology for auto tuning of $PI^\lambda D$ controller outperforms considering M_p, t_r, t_s and disturbance handling. This work focuses mainly on designing of $PI^\lambda D$ controller, so, the future scope of the work considers designing of $PI^\lambda D^\mu$ controller which means it will incorporate designing of an additional parameter , that is, FO of the differentiator. It will be a challenge to design FO differentiator by maintaining the FO of the differentiator in the specified bounds.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Monje CA, Chen Y, Vinagre BM, Xue D, Feliu V. Fractional-order systems and controls: fundamentals and applications. London: Springer London; 2010.
[2] Luo Y, Chen YQ, Ahn HS, Pi YG. Fractional order robust control for cogging effect compensation in PMSM position servo systems: Stability analysis and experiments. Control Eng Pract 2010;18:1022–36.
[3] Chen Y. Fractional calculus applications in automatic control and robotics. In: 41st IEEE conf. decis. control tutor. work. 2002.
[4] Badri P, Sojoodi M. Robust fixed-order dynamic output feedback controller design for fractional-order systems. IET Control Theory Appl 2018;12:1236–43.
[5] Chen W, Dai H, Song Y, Zhang Z. Convex Lyapunov functions for stability analysis of fractional order systems. IET Control Theory Appl 2017;11:1070–4.
[6] Monje CA, Vinagre BM, Feliu V, Chen YQ. Tuning and auto-tuning of fractional order controllers for industry applications. Control Eng Pract 2008;16:798–812.
[7] Lurie BJ. Three-parameter tunable tilt-integral-derivative (TID) controller. US5371670, 1994.
[8] Podlubny I. Fractional-order systems and PIλDμ controllers. IEEE Trans Automat Contr 1999;44:208–14.
[9] Hamamci SE. An algorithm for stabilization of fractional-order time delay systems using fractional-order PID controllers. IEEE Trans Automat Contr 2007;52:1964–9.
[10] Tan N, Yeroglu C. Note on fractional-order proportional–integral–differential controller design. IET Control Theory Appl 2011;5:1978–89.
[11] Luo Y, Chen Y. Stabilizing and robust fractional order PI controller synthesis for first order plus time delay systems. Automatica 2012;48:2159–67, Pergamon.
[12] Tavazoei MS. From traditional to fractional PI control: A key for generalization. IEEE Ind Electron Mag 2012;6:41–51.
[13] Badri V, Tavazoei MS. On tuning fractional order [proportional–derivative] controllers for a class of fractional order systems. Automatica 2013;49:2297–301.
[14] Luo Y, Chen Y. Fractional order [proportional derivative] controller for a class of fractional order systems. Automatica 2009;45:2446–50.
[15] Padula F, Visioli A. Optimal tuning rules for proportional-integer-derivative and fractional-order proportional-integer-derivative controllers for integral and unstable processes. IET Control Theory Appl 2012;6:776–86.
[16] Verma B, Padhy PK. Optimal PID controller design with adjustable maximum sensitivity. IET Control Theory Appl 2018;12:1156–65.
[17] Shen J, Lam J. State feedback H$_\infty$ control of commensurate fractional-order systems. Int J Syst Sci 2014;45:363–72.
[18] Shen J, Lam J. H$_\infty$ model reduction for positive fractional order systems. Asian J Control 2014;16:441–50.
[19] Wang D-J, Gao X-L. H∞ design with fractional-order PDμ controllers. Automatica 2012;48:974–7.
[20] Wang J, Lu J, Li C. Observer-based robust stabilisation of a class of non-linear fractional-order uncertain systems: an linear matrix inequalitie approach. IET Control Theory Appl 2012;6:2757–64.
[21] Monje CA, Ramos F, Feliu V, Vinagre BM. Tip position control of a lightweight flexible manipulator using a fractional order controller. IET Control Theory Appl 2007;1:1451–60.
[22] Tavazoei MS, Haeri M, Jafari S, Bolouki S, Siami M. Some applications of fractional calculus in suppression of chaotic oscillations. IEEE Trans Ind Electron 2008;55:4094–101.
[23] Zhang BT, Pi Y. Robust fractional order proportion-plus-differential controller based on fuzzy inference for permanent magnet synchronous motor. IET Control Theory Appl 2012;6:829.
[24] Calderón AJ, Vinagre BM, Feliu V. Fractional order control strategies for power electronic buck converters. Signal Process 2006;86:2803–19.
[25] Luo Ying Chen, Yang Quan Ahn, Pi YG Hyo Sung. Fractional order periodic adaptive learning compensation for state-dependent periodic disturbance. IEEE Trans Control Syst Technol 2012;20:465–72.
[26] El-Khazali R. Fractional-order PIλDμ controller design. Comput Math with Appl 2013;66:639–46.

[27] Chopade AS, Khubalkar SW, Junghare AS, Aware MV, Das S. Design and implementation of digital fractional order PID controller using optimal pole-zero approximation method for magnetic levitation system. IEEE/CAA J Autom Sinica 2018;5:977–89.
[28] Kuttomparambil Abdulkhader H, Jacob J, Mathew AT. Fractional-order lead–lag compensator-based multi-band power system stabiliser design using a hybrid dynamic GA-PSO algorithm. IET Gener Transm Distrib 2018;12:3248–60.
[29] Liu L, Zhang S, Xue D, Chen YQ. General robustness analysis and robust fractional-order PD controller design for fractional-order plants. IET Control Theory Appl 2018;12:1730–6.
[30] Valério D, Sà da Costa J. Introduction to single-input, single-output fractional control. IET Control Theory Appl 2011;5:1033–57.
[31] Åström KJ, Hägglund T. Automatic tuning of simple regulators with specifications on phase and amplitude margins. Automatica 1984;20:645–51.
[32] Claudio Scali, Gabriele Marchetti A, Semino D. Relay with additional delay for identification and autotuning of completely unknown processes. Ind Eng Chem 1999;38:1987–97.
[33] Tang Wei, Shi Songjiao, Wang Mengxiao. Model identification and PID control of long time-delay processes (Cat. No.02EX527), In: Proc. 4th world congr. intell. control autom, vol. 2, IEEE; 2002, p. 974–8.
[34] Chen Y, Moore KL. Relay feedback tuning of robust PID controllers with iso-damping property. IEEE Trans Syst Man Cybern Part B 2005;35:23–31.
[35] Ho WK, Hong Y, Hansson A, Hjalmarsson H, Deng JW. Relay auto-tuning of PID controllers using iterative feedback tuning. Automatica 2003;39:149–57.
[36] Byeon J, Kim JS, Sung SW, Lee J. Fractional order integrator for the relay feedback identification of a process Nyquist point in the third quadrant. Korean J Chem Eng 2011;28:2227–9.
[37] Jyh-Cheng Jeng, Hsiao-Ping Huang A, Lin F-Y. Modified relay feedback approach for controller tuning based on assessment of gain and phase margins. Ind Eng Chem 2006;45:4043–51.
[38] Beschi M, Dormido S, Sanchez J, Visioli A. An automatic tuning procedure for an event-based PI controller. In: 52nd IEEE conf. decis. control. IEEE; 2013, p. 7437–42.
[39] Bajarangbali Majhi S. Systems science & control engineering: An open access identification of underdamped process dynamics. Syst Sci Control Eng An Open Access J 2014;2583:541–8.
[40] Bazanella AS, Pereira LFA, Parraga A. A new method for PID tuning including plants without ultimate frequency. IEEE Trans Control Syst Technol 2017;25:637–44.
[41] Kumar Padhy P, Majhi S. Improved automatic tuning of PID controller for stable processes. ISA Trans 2009;48:423–7.
[42] Tepljakov A, Petlenkov E, Belikov J. FOMCON: Fractional-order modeling and control toolbox for MATLAB. In: Proc 18th int conf mix des integr circuits syst - mix 2011. 2011. p. 684–9.
[43] Padhy PK, Majhi S. Relay based PI–PD design for stable and unstable FOPDT processes. Comput Chem Eng 2006;30:790–6.