# AUTOMATIC TUNING OF SIMPLE REGULATORS

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Abstract. This paper proposes methods for automatic tuning of simple regulators. The methods are robust and they require little apriori knowledge about the process. The paper presents the ideas behind the methods and analysis. Apart from being useful in themselves, the methods will also give a solution to the longstanding problem of safe initialization of more sophisticated adaptive controllers.

Keywords: Adaptive control, Limit cycles, PID control, Relay control.

## 1. INTRODUCTION

The majority of the regulators used in industry are of the PI(D) type. A large industrial plant may have hundreds of regulators. Many instrument engineers and plant personnel are used to select, install and operate such regulators. Many different methods have been proposed for tuning PI(D) regulators. The Ziegler-Nichols (1943) method is one of the more popular schemes. In spite of this it is common experience that many regulators are in practice poorly tuned. One reason is that simple robust methods for tuning the regulators have not been available. This paper addresses the problem of finding automatic tuning methods. The methods proposed are simple to implement using micro processors. They offer the possibilities to provide automatic tuning tools for a large class of common control problems.

The methods are based on a simple identification method which gives critical points on the Nyquist curve of the open loop transfer function. The key idea is a scheme which provides automatic excitation of the process which is nearly optimal for estimating the desired process characteristics.

The methods proposed are primarily intended to tune simple regulators of the PI(D) type. In such applications they will of course inherit the limitations of the PI(D) algorithms. They will not work well for problems where more complicated regulators are required. The technique may however also be applied to more complicated regulators. The experiences obtained so far from experimentation in laboratory and industry indicate that the simple versions of the algorithms work very well and that they are robust. It thus appears worthwhile to explore these algorithms further.

The proposed algorithms may be used in several different ways. They may be incorporated in single loop controllers to provide an option for automatic tuning. They may also be used to provide a solution to the long standing problem of safe initialization of more complicated adaptive or self-tuning schemes. When combined with a bandwidth self-tuner like the one discussed in Aström (1979) it is e.g. possible to obtain an adaptive regulator which may set a suitable closed loop bandwidth automatically.

The paper is organized as follows. The basic idea is presented in Section 2. The estimation is done by introducing relay feedback which brings the system into a stable limit cycle. The amplitude of the limit cycle can easily be controlled so that it is within acceptable limits. The period and the amplitude of the limit cycle allows determination of the desired process characteristics. Practical aspects on the implementation are also given. The properties of the closed loop system obtained with a linear system under relay feedback are discussed in the following sections. Simple expressions for the period of the limit cycle are derived in Section 3, and the results are interpreted in the frequency domain in Section 4. In Section 5, stability conditions for the limit cycle are given. The paper concludes with a discussion of possible uses and extensions of the proposed methods.

## 2. PRINCIPLES

A regulator with automatic tuning may be viewed as composed of four subsystems, an ordinary feedback regulator with adjustable parameters, a perturbation generator, a parameter estimator, and a block which performs design calculations. See Fig. 1. The perturbation generator provides testsignals which make it possible to estimate the relevant parameters of the process.

The system works in the following way. The process is excited from the perturbation generator. Relevant process dynamics is estimated from the response of the process to the excitation. The regulator parameters are calculated from the dynamics. The perturbation generator, the estimator and the design calculations are then disconnected and the system operates like an ordinary fixed gain regulator.

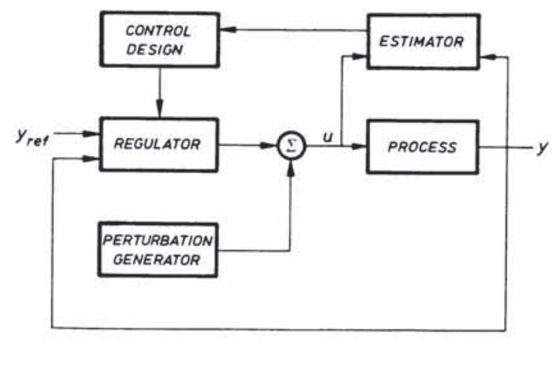


Fig. 1. Block diagram of a self-tuner.

Many schemes of this type have been proposed. See e.g. Aström (1983). All schemes do however require considerable apriori information. Typically it is necessary to know the order of the magnitude of the timeconstants. The novel schemes discussed in this paper build upon the ideas in Aström (1981, 1982), Hägglund (1981) and Aström and Hägglund (1983). The characteristic feature of this approach is that it gives a very simple system which does not require much prior information.

## The basic idea

The schemes are based upon design methods where the process dynamics is described by a few features of the Nyquist curve of the open loop transfer function. Typically knowledge of the critical point, i.e. the first point where the Nyquist curve intersects the negative real axis, is used. See Fig. 2. This point is characterized by the the critical gain, $k_c$ and the critical frequency $\omega_c$ or the critical period $t_c = 2\pi/\omega_c$. The Ziegler-Nichols method is a typical example of design methods of this type but there are many related methods. See Aström (1982) and Aström and Hägglund (1983). To use such design methods it is necessary to find an estimation method which determines the critical point. In principle this can be done using frequency analysis by sweeping over a frequency range until a point with a phaseshift of $\pi$ is obtained. Such an approach is however time consuming and not easy to implement.

Ziegler-Nichols originally proposed to determine the critical point by introducing proportional feedback and increasing the regulator gain until an oscillation is obtained. This procedure is not easy to do automatically. It is in particular difficult to arrange it in such a way that the amplitude of the oscillation is kept at a reasonable level.

Our techniques is based on a new method for automatic determination of the critical points. The method is based on the observation that a system with a phase lag of at least $\pi$ at high frequencies may oscillate with period $t_c$ under relay control. To determine the critical point in Fig. 2, the system is connected in a feedback loop with a relay as is shown in Fig. 3. The error e is then a periodic signal, and the parameters $k_c$ and $\omega_c$ can be determined approximatively from the first harmonic component of the oscillation.

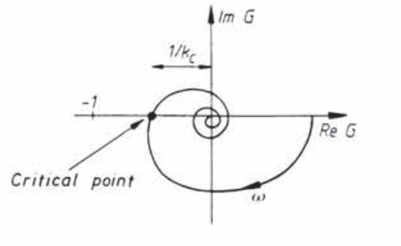


Fig. 2. Nyquist curve of the process.

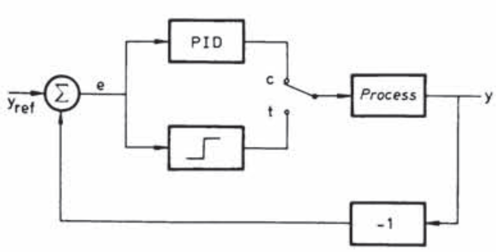


Fig. 3. Relay control of the process.

Let d be the relay amplitude and let a be the amplitude of the first harmonic of the error signal. A simple Fourier series expansion of the relay output then shows that the relay may be described by the equivalent gain

$$k_c = \frac{4d}{\pi a} \tag{1}$$

A more accurate analysis is given in the next section.

The period of the oscillation can easily be determined by measuring the times between zero-crossings. The amplitude may be determined by measuring the peak-to-peak values. These estimation methods are very easy to implement because they are based on counting and comparisons only. More elaborate estimation schemes may also be used to determine the amplitude and the period of the oscillation.

Notice that the technique will automatically generate an input signal to the process which has a significant frequency content at $\omega_c$. This ensures that the critical point can be determined accurately.

There are many variations of the scheme. Other points on the Nyquist curve can be estimated by introducing known dynamics and hysteresis in the relay. See Hägglund (1981).

## Tuning algorithms

A tuning algorithm based on the ideas described above will now be developed. The following example shows how the parameters of the PID regulator can be determined to obtain a desired phase margin of the system.

Example. Consider a process with the transfer function G(s). The loop transfer function with PID control is

$$G_0(s) = k\left(1 + sT_d + \frac{1}{sT_i}\right) G(s) \tag{2}$$

Assume that the Nyquist curve of G intersects the negative real axis when $\omega = \omega_c$. Requiring that the argument of the loop transfer function $G_0$ is $\phi_m - \pi$ at $\omega_c$ the following condition is obtained

$$\omega_c T_d - \frac{1}{\omega_c T_i} = \tan \phi_m$$

There are many $T_d$ and $T_i$ which satisfy this condition. One possibility is to choose $T_i$ and $T_d$ so that

$$T_i = \alpha\, T_d \tag{3}$$

where $\alpha$ is a design parameter. The derivation time $T_d$ is then given by

$$T_d = \frac{\tan \phi_m + \sqrt{\frac{4}{\alpha} + \tan^2 \phi_m}}{2\,\omega_c} \tag{4}$$

Simple calculations show that the loop transfer function has unit gain at $\omega_c$ if the regulator gain is chosen as

$$k = \frac{\cos \phi_m}{|G(i\omega_c)|} = k_c \cos \phi_m \tag{5}$$

where $k_c$ is the critical gain. The design rules are thus given by the equations (3), (4) and (5). □

Tuning algorithms of the above type have been successfully tested on laboratory processes as well as industrial processes. Examples are given in Åström and Hägglund (1983).

## 3. DETERMINATION OF LIMIT CYCLE PERIOD

The purpose of this and the following sections is to analyse the characteristics of a linear system under relay feedback. In particular we wish to determine the conditions for an oscillation to occur and the period of the oscillation.

We will start by investigating a linear system under relay control. In particular we will analyse the conditions for existence of a periodic solution. A crude answer was given by the describing function analysis in Section 2. More accurate results are given in this section. Consider a system described by

$$\begin{cases} \dfrac{dx}{dt} = Ax + Bu \\ y = Cx \end{cases} \tag{6}$$

Let the system be controlled by a relay with hysteresis i.e.

$$u(t) = \begin{cases} d_1 \text{ if } e>\varepsilon \text{ or } (e>-\varepsilon \text{ and } u(t-)=d_1) \\ -d_2 \text{ if } e<\varepsilon \text{ or } (e<\varepsilon \text{ and } u(t-)=d_2) \end{cases} \tag{7}$$

where $e = -y$.

Conditions for relay oscillations have been given by Hamel (1949) and Tsypkin (1958). Equivalent results will be given here with different derivations and different interpretations. The key result is given by the following theorem.

THEOREM 1. Consider the system (6) with the feedback law (7). Assume that the matrix $\Phi$-I is regular. A necessary condition for a limit cycle with period T is then

$$\begin{cases} C[I - \Phi]^{-1}[\Phi_2\Gamma_1 d_1 - \Gamma_2 d_2] = -\varepsilon \\ C[I - \Phi]^{-1}[-\Phi_1\Gamma_2 d_2 + \Gamma_1 d_1] = \varepsilon \end{cases} \tag{8}$$

where

$$\Phi = e^{AT} \quad \Phi_1 = e^{A\tau} \quad \Phi_2 = e^{A(T-\tau)}$$
$$\Gamma_1 = \int_0^{\tau} e^{As}ds\, B \quad \Gamma_2 = \int_0^{T-\tau} e^{As}ds\, B \tag{9}$$

Proof. Assume that a limit cycle exists, where the relay switches twice per period. The general form of the signals u and y are then as shown in Fig. 4. Integration of the state equations over one period gives

$$x(t_{2k+1}) = \Phi_1 x(t_{2k}) + \Gamma_1 u(t_{2k})$$
$$x(t_{2k+2}) = \Phi_2 x(t_{2k+1}) + \Gamma_2 u(t_{2k+1}) \tag{10}$$

where the matrices $\Phi_1$, $\Phi_2$, $\Gamma_1$ and $\Gamma_2$ are given by (9). Notice that the matrices $\Phi_1$ and $\Phi_2$ commute. The condition that the relay switches at times $t_k$ can be expressed as

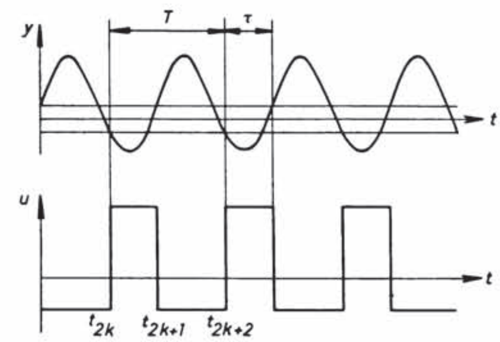


Fig. 4. Signals under limit cycle conditions.

$$y(t_{2k}) = Cx(t_{2k}) = -\varepsilon$$
$$y(t_{2k+1}) = Cx(t_{2k+1}) = \varepsilon$$

If a limit cycle exists then the state will be a periodic function. Introducing

$$x(t_{2k+2}) = x(t_{2k})$$
$$u(t_{2k}) = d_1$$
$$u(t_{2k+1}) = -d_2$$

into (9) then gives

$$x(t_{2k}) = a_1 = [I - \Phi]^{-1}[\Phi_2\Gamma_1 d_1 - \Gamma_2 d_2]$$
$$x(t_{2k+1}) = a_2 = [I - \Phi]^{-1}[-\Phi_1\Gamma_2 d_2 + \Gamma_1 d_1] \tag{11}$$

and (8) after straightforward calculations. □

## 4. FREQUENCY DOMAIN INTERPRETATIONS

It is intuitively appealing to reformulate the result in the frequency domain. The key observation is that under limit cycle conditions the behaviour of the systems (6), (7) can be described as a multivariable linear time invariant discrete time system. (The stroboscopic transformation.) The inputs are $u(t_{2k})$ and $u(t_{2k+1})$ and the outputs are $y(t_{2k+1})$ and $y(t_{2k+2})$. Introduce

$$z_k = \begin{pmatrix} x(t_{2k-1}) \\ x(t_{2k}) \end{pmatrix} \quad u_k = \begin{pmatrix} u(t_{2k}) \\ u(t_{2k+1}) \end{pmatrix} \quad y_k = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix} x_k$$

Equation (10) can then be written as

$$z_{k+1} = \begin{pmatrix} \Phi & 0 \\ 0 & \Phi \end{pmatrix} z_k + \begin{pmatrix} \Gamma_1 & \Phi_1\Gamma_2 \\ \Phi_2\Gamma_1 & \Gamma_2 \end{pmatrix} u_k$$
$$y_k = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix} z_k$$

This is a timeinvariant discrete time system. Let the pulse transfer function of the system be

$$H(z) = \begin{bmatrix} C[zI-\Phi]^{-1}\Gamma_1 & C[zI-\Phi]^{-1}\Phi_1\Gamma_2 \\ C[zI-\Phi]^{-1}\Phi_2\Gamma_1 & C[zI-\Phi]^{-1}\Gamma_2 \end{bmatrix} \tag{12}$$

Putting z = 1 in (12) it follows that the condition (8) can be written as

$$H(1)\cdot\begin{bmatrix} d_1 \\ -d_2 \end{bmatrix} = \begin{bmatrix} \varepsilon \\ -\varepsilon \end{bmatrix} \tag{13}$$

Symmetric oscillations

The case $d_1 = d_2 = d$ is of particular interest. It follows from (9) that

$$\Gamma_1 = \Gamma_2 = \Gamma$$

$$\Phi_1 = \Phi_2 = \Phi^{1/2}$$

in this case. Equation (8) then reduces to

$$C[I - \Phi]^{-1}[I - \Phi^{1/2}]\,\Gamma d = \varepsilon$$

or

$$C[I + \Phi^{1/2}]^{-1}\Gamma d = \varepsilon \tag{14}$$

This condition can also be written as

$$F(T) = C[I+e^{AT/2}]^{-1}\int_0^{T/2} e^{As}ds\,B = \varepsilon/d \tag{15}$$

Repeating the derivation it is also found that it is not necessary to assume that the matrix $\Phi$-I is regular.

The following result can now be established.

THEOREM 2. Let $H(\tau,z)$ be the pulse transfer function for zero-order-hold sampling of the system (6). A necessary condition for existence of periodic limit cycles under relay control (7) with $d_1 = d_2 = d$ is

$$H(T/2,\ -1) = -\frac{\varepsilon}{d} \tag{16}$$

where T is the period of oscillation.

Proof. The pulse transfer function is given by

$$H(\tau,z) = C(zI-e^{A\tau})\int_0^{\tau} e^{As}ds\ B$$

The theorem then follows from (15). □

Remark 1. This result has a strong intuitive appeal. The condition (16) can be written down directly by considering the discrete time system obtained by sampling at the times when the relay switches. The z-transforms of the input signals u and output signals y are given by

$$Z\{u\} = \frac{d}{z+1}, \qquad Z\{y\} = -\frac{\varepsilon}{z+1}$$

The propagation of the signal u is under stationary conditions described by the gain $H(\tau,-1)$. The condition (16) then simply tells that the signals match for $\tau = T/2$.

Remark 2. The condition (16) specializes to

$$H(T/2,-1) = 0 \tag{17}$$

when $\varepsilon = 0$, which also has a direct physical interpretation.

Special conditions for special systems

Consider in particular stable systems with non-negative impulse responses. When the matrix $\Phi$ has all its eigenvalues inside the unit disc, the equation (14) can be expanded in a converging series

$$C[I+\Phi^{1/2}]^{-1}\Gamma = C\Gamma - C\Phi^{1/2}\Gamma + C\Phi\Gamma - \ldots = \frac{\varepsilon}{d}$$

This can also be written as

$$\sum_{n=0}^{\infty} (-1)^n g(n+1) = \frac{\varepsilon}{d} \tag{18}$$

where $\{g(n)\}$ is the impulse response of the discrete system obtained when the system (6) and (7) is sampled with period T/2.

The condition (18) can be illustrated graphically. The pulse response can be read off directly from a plot of the step response S(t) of the system as is shown in Fig. 5. The left hand side of (18) can then be interpreted as the sum of the vectors shown in the figure. Bounds on T can easily be established from this construction. For systems with monotone step responses, the period of oscillation T must e.g. be smaller than the time $2\cdot t_o$, where $t_o$ is given by

$$S(t_0) = \frac{1}{2}\left[S(\infty) + \frac{\varepsilon}{d}\right] \tag{19}$$

The describing function approximation

Having obtained the exact formulas, it is possible to investigate the precision of the describing function approximation. It follows from sampled data theory that

$$H(\tau,z) = \frac{1}{\tau}\sum_{-\infty}^{\infty}\frac{1}{s+in\omega_s}\cdot$$

$$\cdot\left[1 - e^{-\tau(s+in\omega_s)}\right] G(s+in\omega_s)$$

where $\omega_s = 2\pi/\tau$. Put $sh = i\pi$ and $e^{s\tau} = z$. Examine the case $z = -1$ i.e. $s = i\pi/\tau$.

$$H(\tau,-1) = \frac{4}{\pi}\sum_{-\infty}^{\infty}\frac{2}{1+2n}\ \mathrm{Im}\ G\left[\frac{(1+2n)\pi}{h}\right]$$

$$= \sum_{0}^{\infty}\frac{4}{\pi[1+2n]}\ \mathrm{Im}\ G\left(\frac{\pi+2n\pi}{\tau}i\right) \tag{20}$$

Approximating $H(\tau,-1)$ by the first term of the series expansion the condition (16) becomes

$$\frac{4}{\pi}\ \mathrm{Im}\ G\left[i\ \frac{2\pi}{T}\right] = -\frac{\varepsilon}{d}$$

which corresponds to the describing function analysis. The period is thus given by the point where the imaginary part of the Nyquist curve equals $-\pi\varepsilon/4d$.

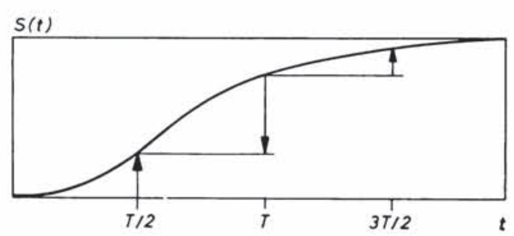


Fig. 5 A step response giving bounds on T.

**Relations with the Tsypkin locus**

The necessary conditions for periodic solutions under relay control were expressed by Tsypkin in terms of the function $\Lambda(s)$ defined by

$$\mathrm{Re}\ \Lambda(i\omega) = \frac{4}{\pi}\sum_{k=0}^{\infty} \mathrm{Re}\ G(i\omega(2k+1))$$

$$\mathrm{Im}\ \Lambda(i\omega) = \frac{4}{\pi}\sum_{n=0}^{\infty} \frac{1}{2n+1}\ \mathrm{Im}\ G(i\omega(2n+1))$$

See Tsypkin (1958). Frequencies of possible limit cycles are the points where the graph of $\{\Lambda(i\omega), 0 \le \omega < \infty\}$ intersects the describing function of the relay. It follows from (18) that

$$\mathrm{Im}\ \Lambda(i\omega) = H\left(\frac{\pi}{\omega}, -1\right)$$

Condition (16) is thus equivalent to Tsypkin's condition.

Notice that the condition based on $H(\tau,-1)$ can only be used when the nonlinearity is such that u is piecevise constant. Tsypkin's method applies to arbitrary nonlinearities.

## 5. STABILITY OF THE LIMIT CYCLE

Conditions for local stability of the limit cycle given by Theorem 1 will now be explored. The following result holds.

THEOREM 3. Consider the system (6) with the feedback (7). Assume that the matrix $\Phi$-I is regular and that $\tau$ and T are such that (8) is satisfied. Let the matrix

$$W = \left[I - \frac{w_2 C}{Cw_2}\right]\Phi_2\left[I - \frac{w_1 C}{Cw_1}\right]\Phi_1 \qquad (21)$$

where

$$w_1 = \Phi_1(Aa_1 + Bd_1)$$

$$w_2 = \Phi_2(Aa_2 - Bd_2)$$

$$a_1 = [I - \Phi]^{-1}[\Phi_2\Gamma_1 d_1 - \Gamma_2 d_2]$$

$$a_2 = [I - \Phi]^{-1}[-\Phi_1\Gamma_2 d_2 + \Gamma_1 d_1]$$

have all its eigenvalues inside the unit disc. The limit cycle is then locally stable.

Proof. It follows from the proof of Theorem 1 that

$$x(t_{2k}) = a_1 \text{ and } x(t_{2k+1}) = a_2$$

for the limit cycle. Compare with equation (11). Consider a solution to (6) and (7) where the initial condition is perturbed from $a_1$ to $a_1 + \delta a_1$. Hence

$$x(\tau+\delta\tau) = e^{A(\tau+\delta\tau)}(a_1+\delta a_1) + \int_0^{\tau+\delta\tau} e^{As}ds\ Bd_1$$

$$= \Phi_1 a_1 + \Gamma_1 d_1 + \Phi_1\delta a_1 + \Phi_1(Aa_1+Bd_1)\delta\tau + O(\delta^2) \qquad (22)$$

where $O(\delta^2)$ denotes terms of second and higher order in $\delta$. The value of $\delta\tau$ where the control signal switches is given by

$$y(\tau+\delta\tau) = Cx(\tau+\delta\tau) = \varepsilon$$

Hence

$$C\Phi_1\delta a_1 + C\Phi_1(Aa_1+Bd_1)\ \delta\tau + O(\delta^2) = 0$$

or

$$\delta\tau = \frac{-C\Phi_1\delta a_1}{C\Phi_1(Aa_1+Bd_1)} + O(\delta^2) = -\frac{C\Phi_1\delta a_1}{Cw_1} + O(\delta^2)$$

Equation (22) can then be written as

$$x(\tau+\delta\tau) = a_2 + \left[I - \frac{w_1 C}{Cw_1}\right]\Phi_1\delta a_1 + O(\delta^2)$$

Repeating the same analysis for the time interval where $u(t) = -d_2$ we find that the relay switches at $T+\delta T$ and that

$$x(T+\delta T) = x(t_{2k+2}) + W\cdot\delta a_1 + O(\delta^2)$$

For small perturbations in the initial conditions the changes in the state at the switching instants are thus governed by the difference equation

$$\delta a_{2k+2} = W\cdot\delta a_{2k}$$

The sequence $\{\delta a_{2k}\}$ then converges to zero exponentially to zero since W is a constant matrix with eigenvalues inside the unit disc. □

Remark 1. In the symmetric case, $d_1 = d_2$, the stability condition is that the matrix

$$W^{1/2} = \left[I - \frac{wC}{Cw}\right]\Phi^{1/2} \qquad (23)$$

where

$$w = \Phi^{1/2}\left[-A\ [I + \Phi^{1/2}]^{-1}\Gamma + B\right] =$$

$$= \left[I + \Phi^{1/2}\right]^{-1}B \qquad (24)$$

has all its eigenvalues inside the unit disk.

Remark 2. The condition that the matrix $W^{1/2}$ has all its eigenvalues inside the the unit disc can be given a system theoretic interpretation. First observe that $W^{1/2}$ can be interpreted as the dynamics matrix of the system

$$z(t+1) = \Phi^{1/2}z(t) + wu(t)$$

$$y(t) = C\Phi^{1/2}z(t) \qquad (25)$$

$$u(t) = -\frac{1}{Cw}y(t)$$

The condition that the matrix $W^{1/2}$ has all eigenvalues inside the unit disc is then equivalent to the condition that the closed loop system (25) is stable.

Notice that the pulse transfer function from u to y is given by

$$H_1(z) = C\Phi^{1/2}\left[zI - \Phi^{1/2}\right]^{-1}w \qquad (26)$$

**Examples**

We now have the tools for exploring when there will be a stable limit cycle. A few examples illustrate the results. For simplicity we consider only the symmetric case $d_1 = d_2 = d$.

Example 1. Consider a linear system with the transfer function

$$G(s) = \frac{b}{s+a} \qquad a, b > 0.$$

It follows that

$$H(\tau, -1) = -\frac{b(1-e^{-a\tau})}{a(1+e^{-a\tau})}$$

The period of the oscillation is given by

$$T = 2\tau = -\frac{2}{a}\ln\frac{bd-a\varepsilon}{bd+a\varepsilon} \approx \frac{4\varepsilon}{bd}$$

The limit cycle is stable because it follows from (21) that $W = 0$.

Notice that the period is proportional to $\varepsilon$ for small $\varepsilon$. Also notice that there will not be an oscillation for $\varepsilon = 0$ because $H(\tau,-1)$ is always different from zero. □

Example 2. Consider a double integrator which has the transfer function

$$G(s) = \frac{1}{s^2}$$

The pulse transfer function is

$$H(\tau, z) = \frac{\tau^2}{2}\,\frac{z+1}{(z-1)^2}$$

Hence $H(\tau,-1)$ is zero for all $\tau$. Without hysteresis we thus find that there may be periodic solutions for any value of $\tau$. The amplitude and the period depend on the initial conditions. The periodic solutions are not stable. □

Example 3. Consider the linear system in Example 1, combined with a time delay

$$G(s) = \frac{b}{s+a}\,e^{-st_o} \qquad a, b, t_o > 0.$$

The value of the pulse transfer function for $z = -1$ is

$$H(\tau, -1) = \frac{b}{a}\cdot\frac{e^{-a\tau}(2e^{at_o}-1) - 1}{1+e^{-a\tau}}$$

The period of the oscillation is given by

$$T = 2\tau = -\frac{2}{a}\ln\left|\frac{bd-a\varepsilon}{bd(2e^{at_o}-1)+a\varepsilon}\right|$$

It can be shown that the matrix $W^{1/2}$ has all its eigenvalues inside the unit disc. The limit cycle is thus stable according to Theorem 3. □

**When will there be oscillations?**

It follows from the analysis that there will be periodic oscillations for large classes of systems. Consider e.g. stable systems with monotone step responses. Equation (16) will always have a solution if $d\cdot G(0) > \varepsilon$, where $G(0)$ is the static gain of the process. It also follows that the period of the oscillation will increase with increasing $\varepsilon$. Since systems with monotone step responses can be approximated by a first order system with time delay for large $\tau$, it also follows from Example 3 that the limit cycle will be stable at least if $\varepsilon$ is (and thus also T is) sufficiently large. Compare also with Fig. 5. Notice however that there are systems like the double integrator in Example 2 where there will not be a stable limit cycle.

## 6. CONCLUSIONS

This paper has attempted to develop procedures for automatic tuning of simple regulators. These procedures are based on a combination of analysis and heuristics. The algorithms have been shown to be robust and simple. They can be used directly as tuning devices for simple regulators and as start-up procedures for other adaptive schemes. It is straight forward to extract a more general pattern from the results of the paper. Design procedures were first developed. The design methods were then analysed to determine the conditions when they will work and when they will not. Next we attempt to find criteria for those conditions. The conditions are then systematically explored to find heuristic rules to govern the operation of the complete system. The approach which can be applied to a wide variety of problems seems to offer interesting possibilities to combine analytical and heuristical approaches. Experiments with algorithms of this type have shown them to be useful and to have interesting properties.

## ACKNOWLEDGEMENTS

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