

Self-tuning Method for PI Regulators of PMSM Servo System Based on Frequency-response Characteristic

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Abstract--An on-line adaptive control based on parameter identification is proposed to solve the problem that motor parameters vary with the changing working environments, which results in the precision of control system to be affected. The motor resistance and inductance are identified by using recursive least squares method with forgetting factor. The parameters of PI regulator are obtained based on the requirement of the system for the frequency domain characteristics.

Index Terms--PMSM, PI controller, Frequency domain characteristics, self-tuning.

I. INTRODUCTION

With the development of the permanent magnet materials, motor manufacturing, control theory, power electronics, as well as the digital signal processors, permanent magnet synchronous motor (PMSM) AC servo system is widely used in all kinds of industry areas. Permanent magnet synchronous motor has a simple structure, small size, light weight, low loss, high efficiency, reliable operation, etc., and along with the development of rare-earth permanent magnet materials, permanent magnet synchronous motor will have a good prospect in the field of AC servo application [1].

In AC servo systems, precise motor parameter values are necessary in the design of the PI controller. However, the motor stator resistance and inductance parameters vary with the temperature rising and flux saturation, resulting in the precision of control system to be affected. Especially, the influence on PMSM parameter values from the temperature are the most significant and the most frequently [2]. To solve this problem, an on-line adaptive control based on parameter identification is proposed. Using recursive least squares method with

forgetting factor, the motor resistance and inductance are identified [3]. Based on the requirement of the system for the frequency domain characteristics, the parameters of PI regulator are obtained in real-time operating conditions. Simulation results and experiments illustrate that the system has good adaptive capacity when the motor parameters change.

II. MATHEMATICAL MODEL OF PMSM SPEED REGULATION SYSTEM

A. Model of PMSM

In the conditions of ignoring the core saturation of motor stator, regardless of eddy current and hysteresis losses, assuming there is no damper windings on the rotor and the permanent magnet is no damping effect as well as the induced electromotive force waveform in phase winding is sine wave [4], the voltage equation of PMSM in the synchronous rotating coordinate system is as follows:

$$\begin{cases} u_d = R_s i_d + L_d \frac{di_d}{dt} - \omega_s L_q i_q \\ u_q = R_s i_q + L_q \frac{di_q}{dt} + \omega_s L_d i_d + \omega_s \Psi_f \end{cases} \quad (1)$$

Torque equation:

$$T_{em} = \frac{3}{2} p \Psi_f i_q = K_t i_q \quad (2)$$

Equation of motion:

$$\frac{d\omega_m}{dt} = \frac{K_t}{J} i_q - \frac{B}{J} \omega_m - \frac{1}{J} T_L \quad (3)$$

where, u_d and u_q are voltages on d-axis and q-axis, i_d and i_q are currents on d-axis and q-axis, L_d and L_q are inductance on d-axis and q-axis, R_s is the stator phase resistance, ω_s is electric angular velocity of motor, ω_m is mechanical angular velocity of motor, ψ_f is permanent magnet flux linkage, T_{em} is electromagnetic torque, p is

number of pole-pairs, K_t is torque constant, J is moment of inertia, B is friction coefficient, T_L is load torque.

Using $i_d=0$ control, decoupled model of PMSM can be obtained via Laplace transform on formula (1) to formula (3), this model is as follows:

$$G_{c_pmsm}(s) = \frac{i_q(s)}{u_q(s)} = \frac{1}{sL_q + R_s} \quad (4)$$

$$G_{s_pmsm}(s) = \frac{\omega_m(s)}{T_{em}(s)} = \frac{1}{sJ + B} \quad (5)$$

B. Model of PMSM Double Closed Loop Speed Regulation System

The block diagram of PMSM double closed loop speed regulation system is as follows:

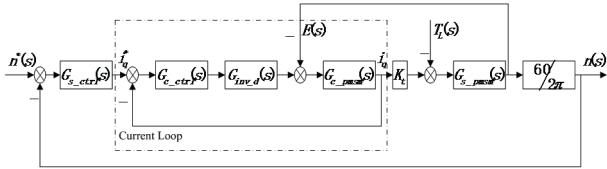


Fig. 1. Block Diagram of PMSM Speed Regulation System

where, current regulator and speed regulator are the classic PI regulator, the model is as follows:

$$G_{c_ctrl}(s) = K_{pc} + \frac{K_{ic}}{s} \quad (6)$$

$$G_{s_ctrl}(s) = K_{ps} + \frac{K_{is}}{s} \quad (7)$$

SVPWM control is suitable for the control of instantaneous current due to the advantages of large linear control range, small harmonic distortion and fast transient response [5], so the control system uses the SVPWM algorithm to control the inverter. The inverter based on SVPWM algorithm is equivalent to first order inertial link:

$$G_{inv} = \frac{1}{sT_s + 1} \quad (8)$$

where, T_s is control cycle of inverter. Considering sampling delay, dead time and computation delay, it is equivalent to a delay link [6], the delay link can be approximately equivalent to a first order inertial link:

$$G_d(s) = \frac{1}{sT_d + 1} \quad (9)$$

In summary, considering the delay and dead time, the model of SVPWM voltage source inverter model is as follows:

$$G_{inv_d}(s) = \frac{1}{(sT_s + 1)(sT_d + 1)} \quad (10)$$

III. PARAMETER TUNING OF PI REGULATOR BASED ON FREQUENCY DOMAIN CHARACTERISTICS

A. Parameter Tuning of PI Regulator in Current Loop

Inner ring in figure 1 is current loop, in which not only the feedback current $i_q(s)$, there is the motor back electromotive force $E(s)$, but in the double closed loop control system, the electromagnetic time constant is much smaller than the mechanical time constant, so it can be considered that the speed is substantially constant, in the analysis of the dynamic performance of the current loop, and the back electromotive force $E(s)$ which is proportional to speed remain unchanged and regarded as an external disturbance that can be eliminated by the current regulator through closed loop control, so it is often ignored in the analysis of current loop. Therefore open loop transfer function of current loop is following:

$$G_{co}(s) = G_{c_ctrl}(s)G_{inv_d}(s)G_{c_pmsm}(s) = \left(K_{pc} + \frac{K_{ic}}{s} \right) \left(\frac{1}{(sT_s + 1)(sT_d + 1)} \right) \left(\frac{1}{sL_q + R_s} \right) \quad (11)$$

Open loop cut-off frequency and phase margin of system directly affect the rapidity and stability of regulation process of the closed-loop system [7], parameter of PI regulator in current loop is designed to obtain the open loop cut-off frequency ω_{cc} and phase margin γ_{cc} that satisfy the requirement of current loop dynamic performance of current loop. Compared with open loop cut-off frequency ω_{cc} of current loop, $\frac{1}{T_s}$

$\frac{1}{T_d}$ are often very large, therefore, the $G_{inv_d}(s)$ link has a great influence on the high frequency characteristics of the current loop, and the effect of the low and middle frequency response characteristics of current loop is negligible. So we can only consider the $G_{c_pmsm}(s)$ link in the analysis of the low and middle frequency of current loop, the open loop transfer function of current loop is as follows:

$$G_{co}(s) = \left(K_{pc} + \frac{K_{ic}}{s} \right) \left(\frac{1}{sL_q + R_s} \right) \quad (12)$$

where, ω_{cc} and γ_{cc} should be satisfied.

$$\begin{cases} |G_{co}(j\omega_{cc})| = 1 \\ \gamma_{cc} = \pi + \angle G_{co}(j\omega_{cc}) \end{cases} \quad (13)$$

According to the formula (12) and formula (13), the PI parameters of the current regulator are calculated.

$$\begin{cases} K_{pc} = Q_c \sqrt{\frac{(\omega_{cc}L_q)^2 + R_s^2}{1+Q_c^2}} \\ K_{ic} = \omega_{cc} \sqrt{\frac{(\omega_{cc}L_q)^2 + R_s^2}{1+Q_c^2}} \end{cases} \quad (14)$$

where, $Q_c = \tan(\gamma_{cc} + \arctan \frac{\omega_{cc}L_q}{R_s} - \frac{\pi}{2})$.

B. Parameter Tuning of PI Regulator in Speed Loop

Speed regulator is the dominant regulator in speed regulation system, speed loop is shown in figure 1 for outer ring, is the key link to realize excellent speed regulation performance, so it is especially important to tune the parameters of speed loop PI regulator.

In the equation of motion, the friction coefficient B has a little influence on speed loop, and it can be neglected.

Since the adjusting process of current loop is much faster than the changing process of speed loop, this in the analysis of the speed loop, which is equivalent to a pure-order inertia link, and so on the analysis of the speed loop, the current loop is equivalent to a first-order inertia link:

$$G_c(s) = \frac{\omega_{cb}}{s + \omega_{cb}} \quad (15)$$

where, ω_{cb} is closed loop bandwidth of current loop.

Compared with open loop cut-off frequency ω_{sc} of speed loop, ω_{cb} is often very large, therefore, the current loop $G_c(s)$ link has a great influence on the high frequency characteristics of the speed loop, and the effect of the low and middle frequency response characteristics of speed loop is negligible [8]. So we can only consider the $G_{c_pmsm}(s)$ link in the analysis of the low and middle frequency of speed loop, the open loop transfer function of speed loop is as follows:

$$\begin{aligned} G_{so}(s) &= G_{c_ctrl}(s)G_{s_pmsm}(s) \frac{60}{2\pi} \\ &= \left(K_{ps} + \frac{K_{is}}{s} \right) \left(\frac{60K_t}{2\pi s J} \right) \\ &= K_s \frac{sK_{ps} + K_{is}}{s^2} \end{aligned} \quad (16)$$

where, $K_s = \frac{60K_t}{2\pi J}$.

Assume that the open loop cut-off frequency for ω_{sc} , phase margin for γ_{sc} , there are:

$$\begin{cases} |G_{so}(j\omega_{sc})| = 1 \\ \gamma_{sc} = \pi + \angle G_{so}(j\omega_{sc}) \end{cases} \quad (17)$$

According to the formula (16) and formula (17), the PI parameters of the speed regulator are calculated.

$$\begin{cases} K_{ps} = \frac{\omega_{sc} \tan \gamma_{sc}}{K_s \sqrt{1 + \tan^2 \gamma_{sc}}} \\ K_{is} = \frac{\omega_{sc}^2}{K_s \sqrt{1 + \tan^2 \gamma_{sc}}} \end{cases} \quad (18)$$

As it can be seen from the formula (14) and formula (18), the PI parameters of current regulator and speed regulator are not only related with the desired cutoff frequency and phase margin, but also with the stator winding resistance R_s , inductance of d-axis and q-axis L_q , L_d , moment of inertia J . But as the rise in motor temperature and the change of magnetic saturation degree, the resistance and inductance of PMSM will change, so it is necessary to identify the motor parameters online, in order to obtain accurate PI parameter.

IV. PARAMETER IDENTIFICATION OF PMSM

A. Recursive Least Squares Algorithm with Forgetting Factor

Batch least-square method is a kind of one-step finished algorithm for off-line identification, the more observations obtained in advance, the higher the accuracy of estimates, but the more memory occupied, if the number of observations adds, we need to recalculation leading to the increase in the amount of calculation. Recursive least squares algorithm is the algorithm which correction parameter estimates when it get a new observed data, when the identified system is in operation, such as time goes on, new observed data is introduced

again and again, we can obtain satisfactory identification result. Meanwhile, in order to prevent saturation of recursive data and algorithm from losing correction ability, forgetting factor is introduced to diminishing the role of the old data.

There is the least square format for the system model [9].

$$y(k) = \varphi^T(k)\theta \quad (19)$$

where, $y(k)$ is output sequence of system, $\varphi^T(k)$ is input sequence of system, θ is sequence of parameter to be identified.

The expression of recursive least squares algorithm with forgetting factor is as follows:

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + M(k)[y(k) - \varphi^T(k)\hat{\theta}(k-1)] \\ M(k) = \frac{N(k-1)\varphi(k)}{\rho + \varphi^T(k)N(k-1)\varphi(k)} \\ N(k) = \frac{1}{\rho}[I - M(k)\varphi^T(k)]N(k-1) \end{cases} \quad (20)$$

where, $\hat{\theta}$ is estimate of parameter to be identified, M and N are intermediate variable matrix of recursive process, I is unit matrix, ρ is forgetting factor, generally $0.95 < \rho < 0.999$.

B. On-line Identification Parameter of PMSM

In on-line identification parameter of PMSM based on recursive least squares algorithm, the key is how to convert PMSM model for linear equation in which parameter to be identified is unknown vector.

Therefore, the formula (1) is discretized, then the least square parameter estimation equation with forgetting factor is obtained.

$$\begin{bmatrix} u_q(k) - \omega_s(k)\Psi_f \\ u_d(k) \end{bmatrix} = \begin{bmatrix} i_q(k) & \omega_s i_d(k) & \frac{i_q(k) - i_q(k-1)}{h} \\ i_d(k) & \frac{i_d(k) - i_d(k-1)}{h} & -\omega_s i_q(k) \end{bmatrix} \begin{bmatrix} R_s \\ L_d \\ L_q \end{bmatrix} \quad (21)$$

where, h is calculation step of computer,

$$\begin{cases} y(k) = \begin{bmatrix} u_q(k) - \omega_s(k)\Psi_f \\ u_d(k) \end{bmatrix} \\ \varphi(k) = \begin{bmatrix} i_q(k) & i_d(k) \\ \omega_s i_d(k) & \frac{i_d(k) - i_d(k-1)}{h} \\ \frac{i_q(k) - i_q(k-1)}{h} & -\omega_s i_q(k) \end{bmatrix} \\ \hat{\theta}(k) = \begin{bmatrix} R_s & L_d & L_q \end{bmatrix}^T \end{cases} \quad (22)$$

Formula (21) and formula (22) are the least square mathematical model for the identification of the stator resistance R_s and inductance of q-axis and d-axis L_q, L_d .

The PMSM parameter that is obtained by identify and the setting value of frequency domain index put into the calculation formula of PI parameter, namely formula (14) and formula (18), tuning value of PI parameter can be obtained. When the PMSM parameters changes with the change of working conditions, we can correct identification values and then adjust the PI parameters by updating real-time data, we can overcome the mismatch caused by PMSM parameters change, and make the system work in the best operating point.

V. SIMULATION AND EXPERIMENTAL RESULT

We will build a simulation platform by Matlab / Simulink as shown in figure 2, to verify the validity of the control algorithm. PMSM parameters are rated speed is 3200rpm, rated current is 66.7A, number of pole-pairs is 4, armature resistance is 3.56mΩ, inductance of q-axis is 19.5uH, inductance of d-axis is 17.9uH, permanent magnet flux linkage is 0.03Wb, torque constant is 0.06N·m/A, moment of inertia is 0.23Kg·cm².

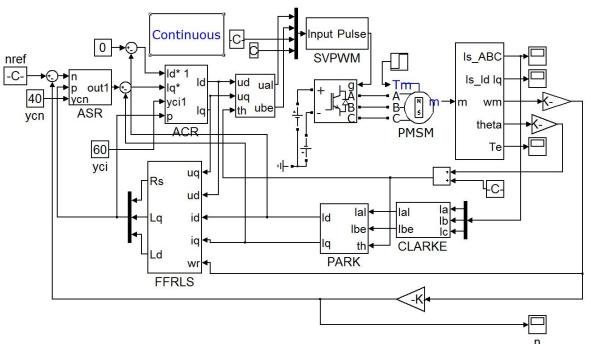


Fig. 2. Block Diagram of Simulation Model

Figure 3 is identification result based on the recursive least squares algorithm, three parameters are identified in this part, R_s , L_q , L_d . It can be seen clearly by figure 3 that identification results of resistance and inductance are basically consistent with the actual value.

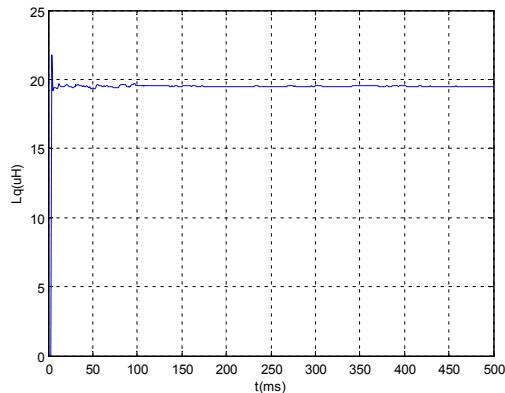
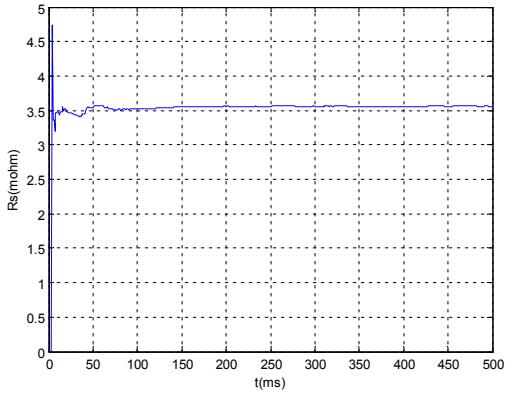


Fig. 3. Simulation Waveform of Resistance and Inductance

Cut-off frequency and phase margin set to 2513rad/s , and 50° in current loop of system, Cut-off frequency and phase margin set to 100rad/s , and 40° in speed loop. PI parameters of current regulator and speed regulator are

obtained according to formula (14) and formula (18), Bode diagrams of current loop and speed loop are shown in figure 4 and figure 5 by simulation. The figure 4 and figure 5 show that tuning PI parameters can satisfy the requirement of the system frequency domain.

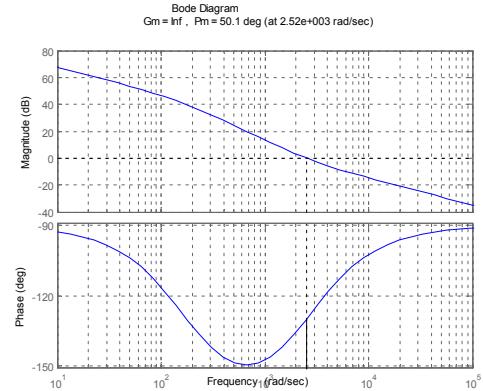


Fig. 4. Bode Diagrams of Current Loop

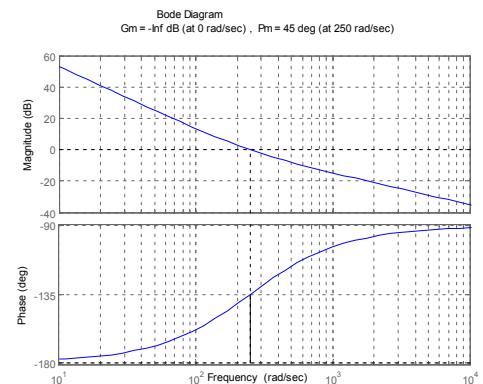
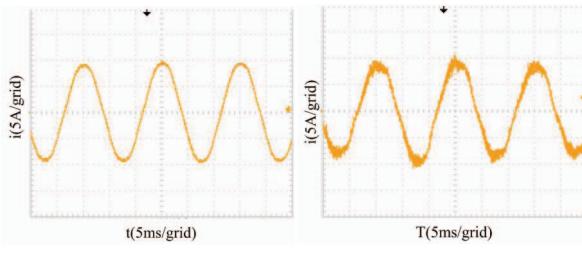


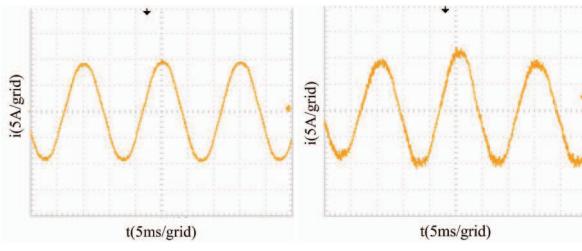
Fig. 5. Bode Diagrams of Speed Loop

Adaptive control algorithm is verified on the hardware platform with DSP TMS320F2812 as the core. SPWM frequency is 10kHz , voltage and current sampling period is $100\mu\text{s}$. This article has carried on the two kinds of experiments: ① According to the PMSM parameters and frequency domain performance index, the PI regulator parameters are calculated and the motor speed is adjusted to 1000rpm , then phase current waveform is measured just on the power and running after 2h , shown in figure 6. ② According to the parameter identification results and tuning PI regulator parameters by setting cut-off frequency and phase margin, the motor speed is adjusted to 1000rpm by self-tuning method for PI regulators, then phase current waveform is measured just on the power and running after 2h , shown in figure 7.



(a)Traditional PI Regulator just on Power (b)After Running 2h

Fig. 6. Experiment 1 Waveform



(a)Self-tuning PI Regulator just on Power (b)After Running 2h

Fig. 7. Experiment 2 Waveform

Comparing figure 6 (a) and figure 7 (a) can be seen, the current waveform obtained by the two methods are almost the same when system is just on power, and illustrate values of resistance and inductance are accurate by the online identification. Comparing figure 6 (b) and figure 7 (b) can be seen, the traditional PI regulator cannot meet the requirements of the system due to values of resistance and inductance changes, but adaptive PI regulator can still make the system stable operation after system running 2h.

VI. CONCLUSIONS

This paper puts forward self-tuning method for PI regulators of PMSM servo system based on frequency-response characteristic and gets mathematical expression of PI regulator parameters. For motor parameters changing as the operating environment changes, this paper puts forward a kind of online identification method based on recursive least square method, that can obtain accurate motor parameters, realized the real-time updating PI regulators parameters of speed regulate system. Simulation and experimental results show that his method can effectively solve the problem of system performance degradation due to motor

parameter changes by adjusting speed regulate system PI parameters in real-time.

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