

# Observer-Based Inertial Identification for Auto-Tuning Servo Motor Drives

Sheng-Ming Yang

Dept. of Mechanical and Electro-mechanical Engineering  
Tamkang University  
Tamsui, Taipei County  
Taiwan

Yu-Jye Deng

Behavior Tech Computer Co.  
2F, 51 Tung-Hsing Rd, Taipei  
Taiwan

**Abstract** - In servo motor drive applications variation of inertia degrades drive's performance. Motor response is affected not only by external disturbance input but also by mechanical parameters such as inertia and friction. These factors must all be considered in order for accurate inertia identification and drive tuning. In this paper, an observer-based auto-tuning scheme for servo motor drives is presented. This scheme is consisted of a state estimator to estimate motor disturbance and two adaptive controllers to separately adjust drive inertia and friction to their correct value. The servo control loop is tuned automatically with the inertia found. The experimental results show that this auto-tuning scheme can achieve good performance and that the scheme is able to estimate the mechanical parameters accurately.

**Keywords** —servo motor drive; auto-tune; disturbance estimator; inertia identification

## I. INTRODUCTION

In servo motor drive applications, variation of load inertia degrades drive's performance. High performance drives generally requires identification of moment of inertia of the whole system including the motor and the load. Once the drive inertia is identified the motion control loop gains can be tuned automatically in order to maintain a consistent dynamic response.

Motor response is affected not only by external disturbance input but also by mechanical parameters such as inertia and friction. These factors must all be considered in order for accurate inertia identification and drive tuning. In comparison to disturbance input, friction is more predictable and sometime can be treated with simple measurement and compensation. Therefore, the schemes which use disturbance torque estimator are commonly employed for drive inertia identification.

There were many studies on the identification of drive inertia for auto-tuning of servo or speed-controlled motor drives. A model-free method was presented which used motor speed response to estimate inertia [1]. Because this scheme does not consider external disturbance, the results are dependent on the operating condition when the disturbance exists. Model reference adaptive control was also applied to find drive inertia [2][3]. However, these methods either excluded the effect from the external disturbance or did not include friction in the model.

Many disturbance observer based auto-tuning schemes have been proposed in the past [4-9]. These methods generally used a disturbance estimator to estimate the external disturbance

input. Because the estimated disturbance is also affected by mechanical parameters, inertia can then be identified after the estimator. The disturbance estimator has various forms, for example, inverse mechanical system model [4][5], full or reduced state estimator [6][7], extended Kalman filter [8][9] ect. Most of these methods need complex software for implementations and are cumbersome for practical industrial applications.

In this paper, an alternative observer-based auto-tuning scheme for servo motor drives is presented. A state estimator is used to estimate the mechanical parameter-influenced external disturbance. The average of the actual disturbance is calculated and then used to correct the output of the state estimator so that the influence from the actual disturbance is removed. Then, two adaptive controllers following the state estimator are used to separately adjust the drive inertia and friction to their correct value. The servo control loop is tuned automatically with the inertia found.

## II. SYSTEM MODEL

Figure 1 shows a typical model for servo motor drives. The bandwidth of the motor current control loop is generally much higher than the bandwidth of the position and velocity loop, therefore current controller can be approximate with unity gain and excluded from the model. The equations of the system can be expressed as

$$\frac{d\theta_r}{dt} = \omega_r \quad (1)$$

$$J \frac{d\omega_r}{dt} + b \cdot \omega_r = k_t \cdot i - T_L \quad (2)$$

where  $\omega_r$  is the speed,  $J$  is the equivalent motor and load inertia,  $b$  is the viscous damping coefficient,  $k_t$  is the motor torque constant,  $i$  is the current, and  $T_L$  is the external disturbance torque. Because  $k_t$  does not change with motor operating condition in general, it is treated as a constant in the following analysis. It can be seen from Fig. 1 that there are two inputs to the motor drive, i.e. the generated torque  $T_m$  and the external disturbance torque  $T_L$ . Both of these inputs affect motor response. Motor torque can be measured via current feedback with good accuracy, but the external disturbance can not be measured directly. However, as mentioned in the previous section, it can be estimated from motor current and velocity or position feedback with various estimation schemes.

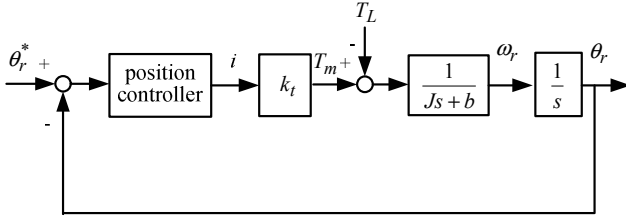


Figure 1. Servo motor drive model

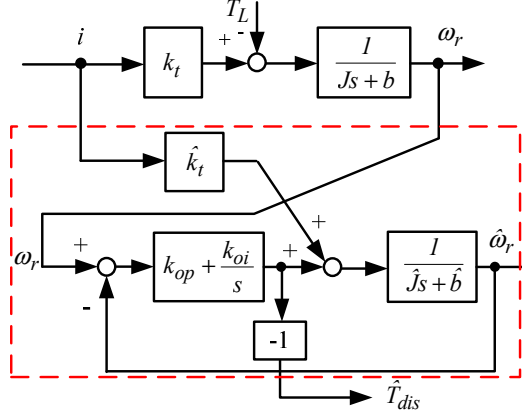


Figure 2. Disturbance estimator

Because variation of mechanical parameters  $J$  and  $b$  also affects the response,  $J$  can be identified indirectly from the estimated disturbance provided that the influences from the actual external disturbance and friction are removed. In the following section, a disturbance torque estimator which estimates the mechanical parameter-influenced external disturbance torque is presented.

### III. DISTURBANCE TORQUE ESTIMATOR

From Eqs.(1)-(2) a full state estimator for  $\theta_r$  and  $\omega_r$  can be constructed as follows,

$$\begin{bmatrix} \dot{\hat{\theta}}_r \\ \dot{\hat{\omega}}_r \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\hat{b}}{\hat{J}} \end{bmatrix} \begin{bmatrix} \hat{\theta}_r \\ \hat{\omega}_r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\hat{k}_t}{\hat{J}} \end{bmatrix} i + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} (\theta_r - \hat{\theta}_r) \quad (3)$$

where the symbols with '^' on top are the estimated values, and  $k_1, k_2$  are the gains of the estimator. Note that the motor position is measured in the estimator. By using the relation between motor speed and position, i.e.  $\omega_r = s\theta_r$ , the estimator can be rearranged into the block diagram shown in Fig.2. The controller of the estimator becomes a PI since the velocity is used as the measured input instead of the position. The proportional and the integral gain of the modified estimator are denoted as  $k_{op}$  and  $k_{oi}$  for simplicity, and  $k_{op} = \hat{J}k_1$ ,  $k_{oi} = \hat{b}k_1 + \hat{J}k_2$ .

Comparing Fig. 2 and Eq. (2) it can be seen that the signal just after the PI controller is the equivalence of  $-T_L$ . Note that since the estimate is a function of  $T_L$ ,  $\hat{J}$  and  $\hat{b}$ ,  $\hat{T}_{dis}$  is used instead of  $T_L$  so as to distinguish it from the actual external

disturbance. Let the errors of the mechanical parameters be  $\Delta J = J - \hat{J}$  and  $\Delta b = b - \hat{b}$ , respectively, and  $\hat{k}_t = k_t$  as explained in the previous section. Then,  $\hat{T}_{dis}$  can be expressed as a function of  $T_L$ ,  $\Delta J$ , and  $\Delta b$  as follows,

$$\hat{T}_{dis} = \frac{k_{op} \cdot s + k_{oi}}{\hat{J}s^2 + (\hat{b} + k_{op})s + k_{oi}} (\Delta J \cdot s\omega_r + \Delta b \cdot \omega_r + T_L) \quad (4)$$

Eq.(4) denotes the condition upon which influences from  $T_L$ ,  $\Delta J$ , and  $\Delta b$  on  $\hat{T}_{dis}$  exists.  $\hat{T}_{dis}$  is affected by  $\Delta J$  only when the motor is acceleration/or deceleration. On the other hand it is affected by  $\Delta b$  as long as the motor is not at rest. If both  $T_L$  and  $\Delta b$  can be found and their influences removed, then  $\Delta J$  can be identified when as the motor is in acceleration and deceleration region.

### IV. ACTUAL EXTERNAL DISTURBANCE AND FRICTION

It can be seen from Eq.(4) that both  $T_L$  and  $\Delta b$  can be estimated when the motor is running at constant speed. But, estimation of rapidly changing  $T_L$  and separating it from the friction is very difficult unless the motor is at standstill. Therefore, in the following, it is assumed that the actual disturbance torque vary much slowly than the servo motor's cycle time, then, its average over a servo cycle can be calculated.

#### A. External Disturbance

From Eq.(2), the relationship between the average motor current and  $T_L$  when the motor is running at constant speed in a servo cycle can be written as

$$T_L = k_t \cdot \bar{i}_1 - b \cdot \omega_{r1} \quad (5)$$

where  $\bar{i}_1$  is the average current and  $\omega_{r1}$  is the motor speed. If the average current at another speed  $\omega_{r2}$  is also measured, and its average current is  $\bar{i}_2$ ,

$$T_L = k_t \cdot \bar{i}_2 - b \cdot \omega_{r2} \quad (6)$$

Then, combine Eqs. (5) and (6) and eliminate the terms consisting  $b$ ,  $T_L$  can be written as

$$\hat{T}_L = \frac{k_t (\bar{i}_1 - (\omega_{r1}/\omega_{r2}) \cdot \bar{i}_2)}{1 - (\omega_{r1}/\omega_{r2})} \quad (7)$$

This result shows that the external disturbance torque, when varying slowly, can be calculated with the average current measured at two different motor speeds. The estimation is performed over each servo cycle. Although only slow varying  $T_L$  can be estimated accurately, the scheme is sufficient for most servo motor drive applications.

#### B. Friction

After  $T_L$  is removed,  $\hat{T}_{dis}$  becomes a function of  $\Delta J$  and  $\Delta b$  only, and is denoted as  $\hat{T}'_{dis}$  for convenience. Again,  $\Delta b$  can be estimated when the motor is running at constant speed. A simple MIT rule is used to design an adaptive controller for  $\hat{b}$  [10]. Also, it is reasonable to set the convergence rate of

$\hat{b}$  to much slower than the dynamic response of the  $\hat{T}_{dis}$  estimator. Then, from Eq. (4)  $\hat{T}_{dis}'$  can be simplified to

$$\hat{T}_{dis}' \approx \Delta b \cdot \omega_r = (b - \hat{b})\omega_r \quad (8)$$

Because  $\hat{T}_{dis}'$  represents the model error, the sensitivity of  $\hat{b}$  relative to the model error is

$$\frac{\partial \hat{T}_{dis}'}{\partial \hat{b}} = -\omega_r \quad (9)$$

Hence, the adaptation rule for  $\hat{b}$  can be expressed as

$$\frac{d\hat{b}}{dt} = -k_b \cdot \hat{T}_{dis}' \cdot \left( \frac{\partial \hat{T}_{dis}'}{\partial \hat{b}} \right) = k_b \cdot \hat{T}_{dis}' \cdot \omega_r \quad (10)$$

where  $k_b$  is the adaptation rate. Note that the above equation can also be expressed in the  $s$ -domain for convenience as follows

$$\hat{b} = \frac{k_b}{s} \cdot \hat{T}_{dis}' \cdot \omega_r + b_o \quad (11)$$

where  $b_o$  is the initial  $\hat{b}$ .

The upper part of Fig. 3 shows the block diagram of the adaptive controller for  $\hat{b}$ . Note that the speed command  $\omega_r^*$  is used in the adaptive controller instead of  $\omega_r$ . In practice, there is no difference of either use  $\omega_r^*$  or  $\omega_r$  in the controller. However, since motor speed information is generally obtained by differentiation of the position feedback, use of  $\omega_r^*$  can prevent quantization error caused by the inadequate resolution of the position sensor.

## V. INERTIA IDENTIFICATION AND AUTO-TUNING

Also from Eq.(4), motor and load inertia can be estimated when the motor is in acceleration and deceleration regions. The sensitivity of  $\hat{J}$  relative to the model error is

$$\frac{\partial \hat{T}_{dis}'}{\partial \hat{J}} = -\alpha_r \quad (12)$$

where  $\alpha_r$  is the motor acceleration rate. The adaptive controller expressed in the  $s$ -domain is

$$\hat{J} = \frac{k_j}{s} \cdot \hat{T}_{dis}' \cdot \alpha_r + J_o \quad (13)$$

where  $J_o$  is the initial  $\hat{J}$ ,  $k_j$  is the adaptation rate. The lower diagram in Fig. 3 shows the block diagram of the adaptive controller for  $\hat{J}$ . Similarly, the acceleration command  $\alpha_r^*$  is used in the adaptive controller instead of  $\alpha_r$ . Also as show in Fig. 3,  $\hat{J}$  and  $\hat{b}$  adaptive controller were drawn in parallel. However,  $\hat{J}$  control is executed when the motor is in acceleration and deceleration regions, and  $\hat{b}$  control is executed when the motor is running at constant speed. The estimated parameter holds at the last updated value when its

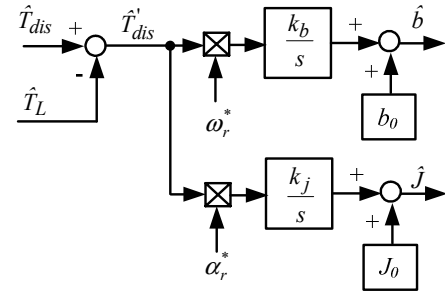


Figure 3. Adaptive controller for  $\hat{b}$  and  $\hat{J}$

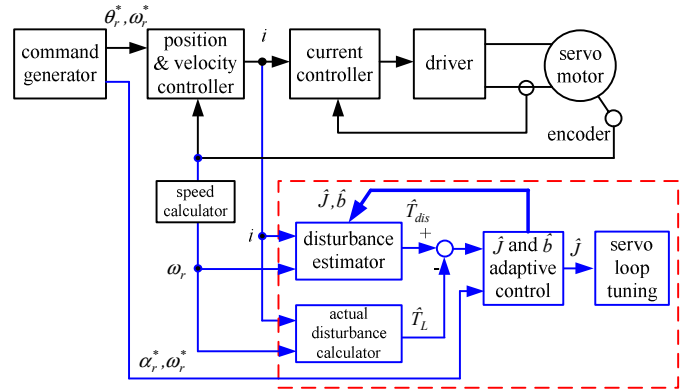


Figure 4. Observer-based inertia estimator and auto-tuning servo motor drive

corresponding control is not executing. Also, it can be shown that the estimated  $\hat{J}$  and  $\hat{b}$  are globally stable.

The block diagram shown in Figure 4 summarizes the proposed auto-tuning scheme. Motor current and speed information are used to estimate the mechanical parameter-influenced external disturbance  $\hat{T}_{dis}$ . This state estimator requires information on  $\hat{J}$  and  $\hat{b}$ . The actual disturbance torque is calculated with Eq. (7), and then subtracted from  $\hat{T}_{dis}$ . After the influence of  $T_L$  is removed,  $\hat{T}_{dis}'$  becomes a function of  $\hat{J}$  and  $\hat{b}$  only. Two adaptive controllers are then used to find  $\hat{J}$  and  $\hat{b}$ . The identified  $\hat{J}$  and  $\hat{b}$  are used for the disturbance estimator. Also,  $\hat{J}$  is used to calculate new servo loop gains. Note that the gains are calculated every servo cycle, and updated when the motor is stopped to prevent sudden change in motor dynamic response.

## VI. EXPERIMENTAL RESULTS

The proposed control schemes were implemented with a TMS320F2812 based DSP controller for experimental verifications. A 400 Watt PMSM motor was used in the experiments. A load motor was coupled to the test motor so that load torque can be applied. Sampling frequency of the servo control loop was about 5 KHz. Parameters of the motor and load can be found in Appendix A.

Figure 5-7 shows the verifications of the influence of  $\hat{J}$ ,  $\hat{b}$ , and  $\hat{T}_L$  on  $\hat{T}_{dis}$ , respectively. The adaptive controllers were not running in these experiments. Figure 5 shows the motor velocity and  $\hat{T}_{dis}$  for various  $\hat{J}$  when  $\hat{b} = b$ ,  $T_L = 0$ , and  $\omega_r = 1000\text{rpm}$ . The total inertia was about 4 times the motor's original inertia, i.e.  $J \approx 0.001\text{kg}\cdot\text{m}^2$ .  $\hat{J}$  was purposely set to  $J$ ,  $2J$  and  $0.5J$ , respectively. It can be seen that  $\hat{T}_{dis}$  had noticeable error when the motor was in the acceleration and deceleration regions. The errors were approximately equal to  $\Delta J/\alpha_r$ . Also from the bottom two curves,  $\hat{T}_{dis}$  changed its polarity as  $\hat{J}$  varied from  $2J$  to  $0.5J$ .

Figure 6 shows the motor velocity and  $\hat{T}_{dis}$  when  $\hat{b}$  was purposely set to zero for  $\hat{J} = J$ ,  $T_L = 0$ , and  $\omega_r = 1000\text{rpm}$ . It can be seen that  $\hat{T}_{dis}$  had error when the motor was rotating, and the error was approximately proportional to motor speed as expected. Figure 7 shows the motor velocity and  $\hat{T}_{dis}$  when a  $-0.5\text{Nm}$  load was applied to the motor for  $\hat{J} = J$ ,  $\hat{b} = b$ , and  $\omega_r = 1000\text{rpm}$ . It can be seen that  $\hat{T}_{dis}$  was approximately constant regardless of the motor speed, and its value was about the same as the external load applied. Note that the results given in Fig. 5-7 are all consistent with Eq. (4).

Figure 8 shows the motor velocity  $\omega_r$ , inverse of  $\hat{J}$ ,  $\hat{b}$ , and  $\hat{T}_L$  when the motor was running back and forth between 1000 and  $-1000\text{ rpm}$ . A load inertia of about 10 times the motor inertia was mounted on the motor for testing. The initial  $J_o$  was set to the motor's original inertia,  $b_o$  was set to 0, and no disturbance torque was applied. Note that  $1/\hat{J}$  is shown in the figure because it is more convenience for the control program to calculate  $1/\hat{J}$  instead of  $\hat{J}$ . It can be seen from Fig. 8 that in the beginning, motor speed response had significant overshoot due to the incorrect  $\hat{J}$  and  $\hat{b}$  used for position loop tuning. But, after several cycles,  $\hat{J}$  and  $\hat{b}$  approached their actual value and the overshoots in  $\omega_r$  diminished. Also notice  $\hat{T}_L$  stayed around zero all the time.

Figure 9 also shows  $\omega_r$ ,  $1/\hat{J}$ ,  $\hat{b}$ , and  $\hat{T}_L$  when the motor was running back and forth between 1000 and  $-1000\text{ rpm}$ . In the beginning, both  $\hat{J}$  and  $\hat{b}$  were tuned to their correct value already, and no disturbance torque was applied. At the instance shown in the figure a disturbance torque of about  $-0.4\text{Nm}$  was applied. It can be seen that  $\hat{T}_L$  returned to its correct value after just one servo cycle. Both  $\hat{J}$  and  $\hat{b}$  were disturbed at the instant load torque was applied, however, they also returned to their correct value after  $\hat{T}_L$  was correctly estimated and removed from  $\hat{T}_{dis}$ .

## VII. CONCLUSIONS

An observer-based auto-tuning scheme for servo motor

drives is presented in this paper. A state estimator is used to estimate the mechanical parameter-influenced external disturbance. It was found that this estimate is a function of the actual external disturbance, drive inertia, and friction. Under the assumption that the actual disturbance is slow varying, its average over a servo cycle can be calculated and then removed from the estimated disturbance. Then, two adaptive controllers are used to separately adjust the drive inertia and friction to their corresponding values. Finally, the servo control loop gains are tuned automatically with the inertia found. The experimental results have verified the relationships between the estimated disturbance and the actual disturbance, motor inertia, and friction. The results also show that the auto-tuning scheme can achieve good performance and that the scheme is able to estimate the mechanical parameters accurately. The scheme is practical for implementation since it can be realized with simple software for easy installation. Because of the assumption on the external disturbance, larger error may exist if the scheme is applied for applications with rapidly changing disturbance.

## APPENDIX A

The motor used in the experiments was a three-phase PMSM motor, its parameters are:

Output power	400 Watt
Motor inertia	0.000245 $\text{kg}\cdot\text{m}^2$
Load inertia	0~10 times motor inertia
Viscous friction coefficient	$1.5 \times 10^{-4} \text{ Nm/rad/s}$

## REFERENCES

- [1] B. Zhang, Y. Li and Y. Zuo, "A DSP-based fully digital PMSM servo drive using on-line self-tuning PI controller", Proceeding of the 3<sup>rd</sup> Power Electronics and Motion Control Conference, vol. 2, Aug. 2000, pp. 1012-1017.
- [2] Y. Guo, L. Huang, Y. Qiu, and M. Maramatsu, "Inertia identification and auto-tuning of induction motor using MRAS", Proceeding of the 3<sup>rd</sup> Power Electronics and Motion Control Conference, Aug. 2000, pp. 1006-1011.
- [3] K. Fujita and K. Sado, "Instantaneous speed detection with parameter identification for ac servo system", IEEE Transactions on Industry Applications, vol. 28, no. 4, Jul. 1992, pp. 864-872.
- [4] I. Awaya, Y. Kato, I. Miyake, and M. Ito, "New motion control with inertia identification function using disturbance observer", International Conference on Power Electronics and Motion Control, Nov. 1992, pp. 77-81.
- [5] S. Komada, M. Ishida, K. Ohnishi, and T. Hori, "Motion control of linear synchronous motors based on disturbance observer", IEEE IAS Annual Meeting, Nov. 1990, pp. 27-30.
- [6] K.B. Lee, J.H. Song, I. Choy, and J.Y. Yoo, "An inertia identification using ROELO for low speed control of electric machine", IEEE APEC, Feb. 2003, pp. 1052-1055.
- [7] R.D. Lorenz, "Observers and state filters in drives and power electronics", IEEE IAS OPTIM 2002, May 2002.
- [8] S.J. Hong, H.W. Kim, and S.K. Sul, "A novel inertia identification method for speed control of electric machine", IEEE IECON, Aug. 1996, pp. 1234-1239.
- [9] M. De Campos, E.G. Caratti and H.A. Grundling, "Design of a position servo with induction motor using self-tuning regulator and Kalman filter", IEEE IAS Annual Meeting, Oct. 2000, pp. 1613-1618.
- [10] K.J. Astrom and B. Wittenmark, *Adaptive Control*, Addison-Wesley, 1989.

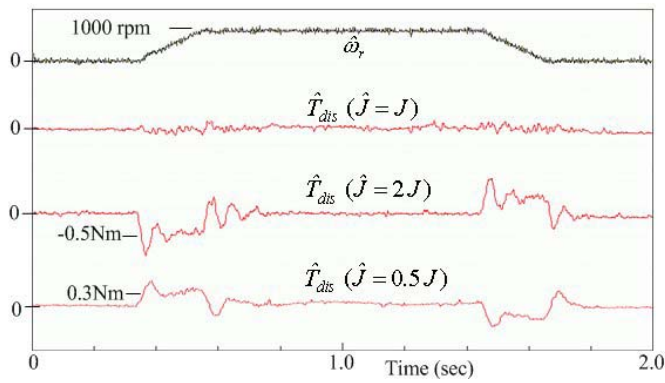


Figure 5. Motor velocity and  $\hat{T}_{dis}$  for various  $\hat{J}$  when  $\hat{b} = b$ ,  $T_L = 0$ , and  $\omega_r = 1000$  rpm (adaptive controllers were not running)

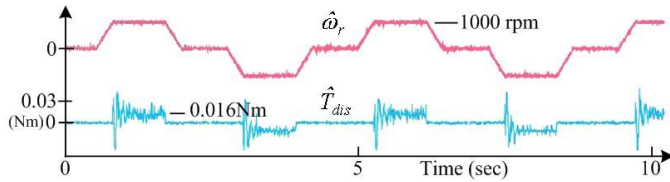


Figure 6. Motor velocity and  $\hat{T}_{dis}$  when  $\hat{J} = J$ ,  $\hat{b} = 0$ ,  $T_L = 0$ , and  $\omega_r = 1000$  rpm (adaptive controllers were not running)

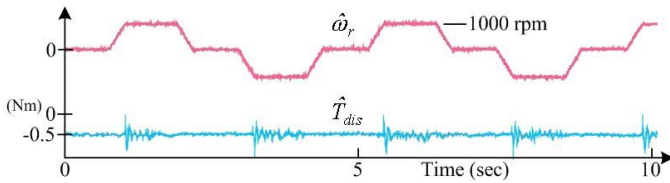


Figure 7. Motor velocity and  $\hat{T}_{dis}$  when  $\hat{J} = J$ ,  $\hat{b} = b$ ,  $T_L = -0.5$  Nm, and  $\omega_r = 1000$  rpm (adaptive controllers were not running)

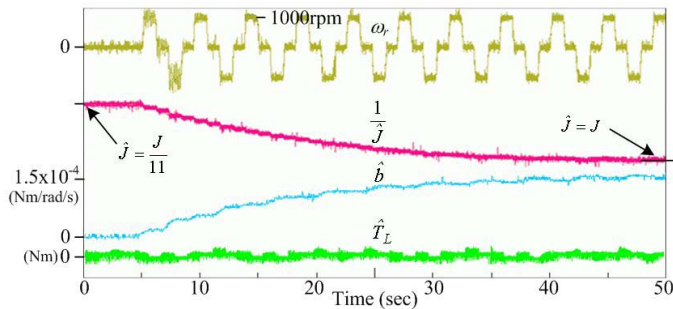


Figure 8. Motor velocity,  $1/\hat{J}$ ,  $\hat{b}$ , and  $\hat{T}_L$  when the motor was running between 1000 and -1000 rpm,  $J_o = J/11$ ,  $b_o = 0$ , and  $T_L = 0$

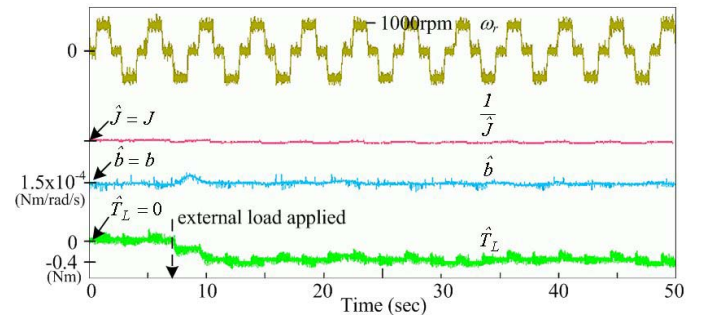


Figure 9. Motor velocity,  $1/\hat{J}$ ,  $\hat{b}$ , and  $\hat{T}_L$  when the motor was running between 1000 and -1000 rpm and an external load of  $T_L = -0.4$  Nm was applied suddenly