Investigation into the Effects of Spatial Correlation on MIMO Channel Estimation and Capacity

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Abstract—**This paper reports on investigations into the effects of spatial correlation on channel estimation and capacity of a Multiple Input Multiple Output (MIMO) system. Minimum Mean Square Error (MMSE) method is applied for channel estimation under a correlated channel scenario. Simulation results for a 4x4 MIMO system with the Jakes model describing a channel between a base station and a mobile station demonstrate a trade-off between the effects of spatial correlation, channel estimation and capacity. For low values of Signal to Noise Ratio (SNR), spatial correlation decreases channel estimation errors and improves capacity. For high values of SNR, spatial correlation has a less pronounced effect on improving channel estimation and thus the MIMO capacity is adversely affected by the decreased channel matrix rank.**

Keywords-Spatial Correlation ; MIMO; Channel Estimation; Channel Capacity; Trade-off

I. INTRODUCTION

It has been shown in recent works that a wireless communication system can achieve an increased capacity by using multiple element antennas at the transmitter and receiver accompanied by a suitable signal processing scheme. Such an unconventional wireless system, named a multiple input multiple output (MIMO) system, offers an increased capacity without the need of increasing an operational bandwidth.

In order to realize the potential of MIMO, two conditions have to be fulfilled. One requires the presence of a rich scattering environment, and the other one concerns the accurate channel state information (CSI) to be available at the receiver [1]. In the first condition, the rich scattering environment is needed to support the formation of statistically independent virtual channels over which the parallel data transmission can take place. In this case, the lack of (spatial) correlation between the virtual channels leads to the increased capacity. In turn, the availability of accurate CSI is needed to decode the received signal and to practically reach the MIMO capacity.

It has to be noted that to obtain CSI, a number of channel estimation methods can be applied. The methods based on the use of a pilot signal, also known as a training sequence, are the most popular. In [2] and [3], the superiority of minimum mean square error (MMSE) method over other training-based channel estimation methods has been shown. In [4], [5], it has been proved that while applying the MMSE method the presence of spatial correlation can help to achieve a better quality CSI.

From the above deliberations, a tradeoff is apparent. The presence of spatial correlation is detrimental to obtaining independent virtual channels which are behind the increased capacity. However, at the same time it is of importance to achieving better quality CSI which is behind the practical realization of an increased data throughput. Note that many works (for example, [6] [7]) concerning MIMO capacity neglect this issue and consider the presence of spatial correlation as jeopardizing the channel capacity. This conclusion is valid under an idealized scenario that the perfect knowledge of channel state information (CSI) is available to the receiver. In this case, the reduced capacity is explained by the reduced channel matrix rank [8].

In this paper, we focus our considerations on the spatial correlation and its effect on MIMO channel estimation and capacity. In the undertaken investigations, the Jakes fading model [9] [10] is applied to characterize a spatial correlated narrow-band MIMO channel. The MMSE channel estimation algorithm is applied to estimate CSI. A closed-form expression for MIMO channel capacity that includes the CSI estimation error under the condition of a correlated channel is derived.

The rest of the paper is organized as follow. In section II, a MIMO system model and closed-form expression for channel capacity that includes the channel estimation error is introduced. The correlated channel model is described in section III. The MMSE channel estimation is described in section IV. Simulation results are presented in section V and Section VI concludes the paper.

II. SYSTEM MODEL & CHANNEL CAPACITY

A. *MIMO system model*

In this work, a flat block-fading narrow-band MIMO system with M_t transmit and M_r receive antennas is considered. The relationship between the received signals and the transmitted signals is given by (1):

$$Y = HS + V \qquad (1)$$

where Y is the M_r x N complex matrix representing the received signals; S is the M_t x N complex matrix representing transmitted signals; H is the M_r x M_t complex channel matrix and V is the M_r x N complex zero-mean white noise matrix.

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B. Channel capacity

Here, we assume that the channel matrix is not accurately known because of estimation errors. In this case, the operation of the MIMO system described by (1) has to be modified to:

$$\mathrm{Y} = \hat{\mathrm{H}}\mathrm{S} + \mathrm{eS} + \mathrm{V} \tag{2}$$

where $\hat{\mathrm{H}} = \mathrm{H} + \mathrm{e}$ and e is the channel estimation error at the receiver. The mean square error of estimation is expressed as:

$$\sigma_{\mathrm{e}}^2 = E\{\left\|\mathrm{H} - \hat{\mathrm{H}}\right\|_{\mathrm{F}}^2\} \tag{3}$$

where $\| \ \|_F^2$ stands for the Frobenius norm and *E{.}* is a statistical expectation. σ_e^2 is the parameter indicating the channel estimation quality.

For the case that the channel state information is perfectly known at the receiver, the MIMO capacity in a flat fading channel is given as [11] [12] [13]:

$$C = E(\log_2\{\det[I_{M_r} + \frac{P}{M_t\sigma_n^2}(HH^{\perp})]\}) \tag{4}$$

where $\perp$ stands for the pseudo inverse operation, σ_n^2 is the power of noise and P is the total transmitted power. According to [14], the lower bound of channel capacity of MIMO system over a flat fading channel with channel estimation error can be written as:

$$\begin{aligned} C &= E(\log_2\{\det[I_{M_r} + \frac{P\hat{H}\hat{H}^{\perp}}{M_t(\sigma_n^2 + P\sigma_e^2)}]\}) \\ &= E(\log_2\{\det[I_{M_r} + \frac{P\hat{H}\hat{H}^{\perp}}{M_t\sigma_n^2}\frac{1}{1+P\frac{\sigma_e^2}{\sigma_n^2}}]\}) \end{aligned} \tag{5}$$

in which, SNR is given as $\frac{P}{\sigma_n^2}$.

Equation (5) indicates that for a fixed value of SNR, the capacity is a function of estimated channel matrix $\hat{H}$ and the channel estimation error σ_e^2. Therefore, the spatial correlation and the quality of channel estimation influence the MIMO capacity.

III. CORRELATION CHANNEL MODEL

Here, the Jakes fading model [10], [11] is applied to describe a spatial correlated MIMO channel. An uplink case between a base station (BS) and a mobile station (MS) is assumed. The BS antennas are assumed at a large height above the ground where the influence of scatterers close to receiver is negligible. In turn, MS is assumed to be surrounded by many scatterers distributed within a "circle of influence" [9]. For this case, the signal correlation coefficients at the BS and MS, ρ^{BS} and ρ^{MS}, can be obtained from [10], [15] and are given as:

$$\begin{aligned} \rho^{MS}(\delta) &= J_0[2\pi\delta/\lambda\ ] \\ \rho^{BS}(\delta) &= J_0[\frac{2\pi}{\lambda}\delta\,\gamma_{\max}\ \cos(\theta)]\exp(-j\frac{2\pi}{\lambda}\delta\sin(\theta)) \end{aligned} \tag{6}$$

where, δ is the antenna spacing distance; λ is the wavelength of the carrier; γ_{max} is the maximum angular spread (AS); θ is the AoA of LOS and J_0 is the Bessel function of 0-th order.

Using $\rho^{BS}(\delta)$ and $\rho^{MS}(\delta)$, the correlation matrices R_{BS} and R_{MS} for BS and MS links can be generated and the channel matrix can be obtained as given by [16]:

$$H = R_{BS}^{1/2} H_g R_{MS}^{1/2} \tag{7}$$

where H_g is a matrix with i.i.d. Gaussian entries.

The channel matrix H in (7) can be used in (4) to calculate the MIMO capacity for the case of correlated channel.

IV. MMSE CHANNEL ESTIMATION METHOD

For the training based channel estimation, the relationship between the received signals and the training sequences is given as (6):

$$\mathrm{Y} = \mathrm{HP} + \mathrm{V} \tag{8}$$

where P is the M_t x N complex training matrix, which includes training sequences (pilot signals).The goal is to estimate the complex matrix H from the knowledge of Y and P.

Assuming the training matrix is known, the channel matrix H can be estimated using the minimum mean-square error (MMSE) method, as described in [2] [3]

$$\hat{H} = \frac{\rho}{M_t} YP^H (R_H^{-1} + \frac{\rho}{M_t} PP^H)^{-1} \tag{9}$$

with MSE estimation error given by:

$$J_{MMSE} = E\{\left\|H - \hat{H}\right\|_F^2\} = tr\{(R_H^{-1} + \frac{1}{\sigma_n^2 M_r} PP^H)^{-1}\} \tag{10}$$

where ρ is the signal to noise ratio (SNR) and tr {.} denotes the trace of matrix, and $R_H = E\{H^H H\}$ is the channel correlation matrix. A method to obtain R_H practically is given in [2] [3].

Using the eigenvalue decomposition, R_H can be expressed as (11):

$$R_H = Q\Lambda Q^H \tag{11}$$

In (11) Q is the unitary eigenvector matrix and Λ is the diagonal matrix with nonnegative eigenvalues.

By substituting (11) into (10), one can get the error as given by:

$$J_{MMSE} = tr\{(\Lambda^{-1} + \frac{\rho}{M_t} Q^H PP^H Q)^{-1}\} \tag{12}$$

To minimize the estimation error (12), $Q^H PP^H Q$ needs to be diagonal [2], [3], [15]. To satisfy this condition, the training sequence developed in [17] [18] can be used. Under this condition, the MSE can be expressed as:

$$J_{MMSE} = \sum_{i=0}^{M_t-1} \sum_{j=0}^{M_r-1} \frac{1}{\frac{\rho}{M_t}\beta_i + (\lambda_i(R_t)\lambda_j(R_r))^{-1}} \qquad (13)$$

where R_t and R_r are spatial correlation matrices at the transmitter and receiver, respectively; β_i is the power of training sequence. λ_k stands for the k-th largest eigenvalue. Following the channel model introduced in section III, R_t and R_r have to be replaced by R_{BS} and R_{MS}, respectively. Here, J_{MMSE} is considered to be a function of the variable σ_e^2, as shown in equation (5).

V. Simulation Results

In this section, we discuss the simulation results for MIMO capacity under the condition of channel estimation error. A 4x4 MIMO system with antenna spacing of half-wavelength both in the transmitting and receiving arrays is assumed. The minimum mean square error (MMSE) channel estimation method is applied to estimate the Jakes model describing channel between the BS and MS, as shown in the previous sections. The channel capacity is determined using equation (5), in which the estimated channel matrix $\hat{H}$ is given by (9). The actual channel matrix H can be generated using (7). For simulation purposes, R_H is obtained using the actual channel matrix H. The mean square channel estimation error σ_e^2 is given as J_{MMSE} and is obtained from equation (13). 10000 realizations are used to obtain the value of capacity in (5).

Numerical analysis are presented in the form of 3-D graphs to show the relationship between channel estimation mean square error (MSE), signal to noise ratio (SNR), spatial correlation and channel capacity. Here, the level of spatial correlation is indicated by the value of angle spread. As we know, angle spread value is the parameter that affects correlation. Larger angle spreads result in smaller correlation, while smaller angle spreads lead to more severe correlation. Here we fix some parameters such as the antenna spacing normalized against the wavelength. As a result, the varied values of angle spread reflect different levels of spatial correlation.

Figure 1 shows the relationship between channel estimation MSE, angle spread and SNR. As observed in Figure 1, the mean square error decreases quickly when SNR increases. On the other hand, when the angle spread value becomes larger, the performance of channel estimation gets worse. This means that a small spatial correlation undermines channel estimation quality. This is apparent at low SNR. Similar findings have been reported in [4] [5].

In Figure 2, the relationship between the MIMO channel capacity, SNR and the angle spread (characterizing the spatial correlation level) is investigated. Note that Figure 2a and 2b represent the same graph viewed at two different angles.

In Figure 2a, the channel capacity shows two trends along the angle spread (spatial correlation). At a relatively low SNR mostly corresponding to yellow and orange color zone, as angle spread decreases (spatial correlation becomes more severe) channel capacity increases. This disobeys the conventional knowledge that spatial correlation reduces the channel capacity. On the other hand, at high SNR corresponding to blue and purple color zones, channel capacity decreases when angle spread becomes smaller (the level of spatial correlation becomes higher). This obeys the conventional knowledge of relationship between spatial correlation and channel capacity. In this case, there should be a joint cross point for the two different trends. This joint point stands for the trade-off of spatial correlation effects on MIMO channel capacity and channel estimation accuracy.

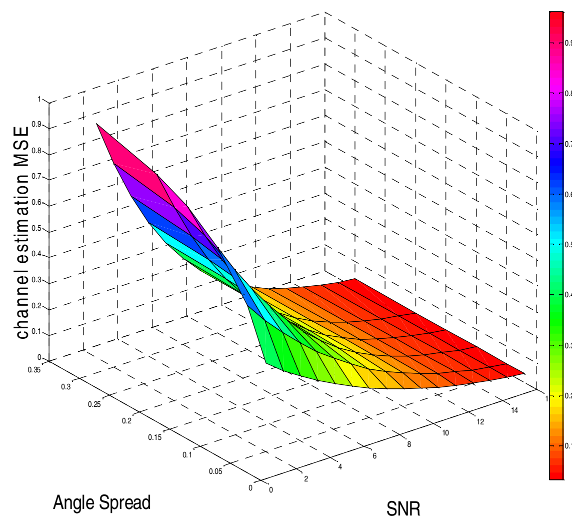

Figure 1. MSE vs angle spread vs SNR

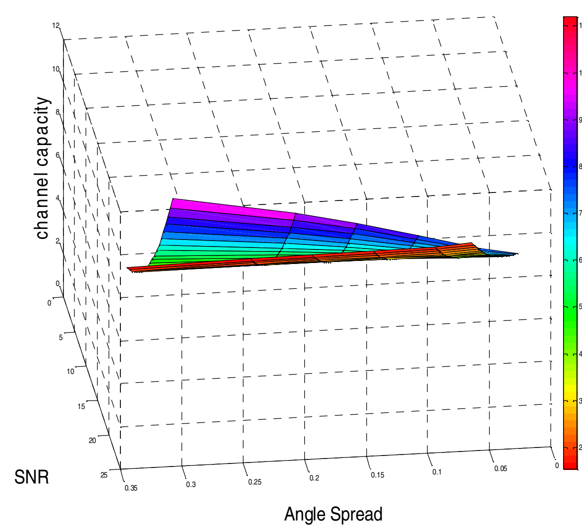

Figure 2a. channel capacity vs angle spread vs SNR

Figure 2b represents a different view of results shown in Figure 2a to better identify the joint point. The view angle is from one side of the cube, which makes a 2-D like graph. As one can see, the orange and yellow zones correspond to the capacity rising trend while blue and purple zones correspond to the capacity declining trend. The cross joint point is at about SNR equal to 8dB. This point reveals the trade off that at a low SNR, spatial correlation helps improve the channel capacity because its positive influence on channel estimation accuracy. This effect is more pronounced than the adverse effect concerning the channel matrix rank reduction. In turn, at high SNR this adverse effect becomes dominant. Consequently, the channel capacity decreases as spatial correlation is further increased.

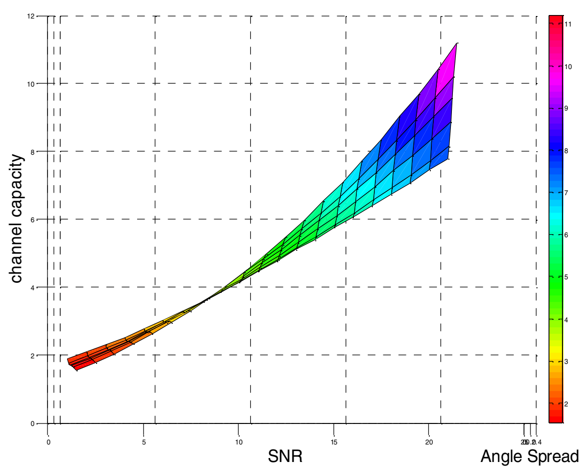

Figure 2b. channel capacity vs angle spread vs SNR

VI. Conclusion

In this paper, the effect of spatial correlation on the accuracy of MIMO channel estimation and the resulting MIMO channel capacity has been investigated. The undertaken analysis has revealed that at low SNR the conventional view that that the presence of spatial correlation decreases the MIMO channel capacity is invalid. In this case, the spatial correlation helps to increase capacity. This is because at low SNR, the presence of spatial correlation improves the channel estimation accuracy. This is more significant than the reduction of the channel matrix rank on MIMO capacity. For high values of SNR, the impact of channel matrix rank reduction due to the increase in spatial correlation on channel capacity becomes more pronounced. In this case, the channel estimation helped by spatial correlation becomes less important. The general view of the wireless communication community is that the benefits of MIMO should be mainly considered under high SNR scenarios. Under such assumption, the realization of high capacity of MIMO requires low spatial correlation, as confirmed by the presented results.

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