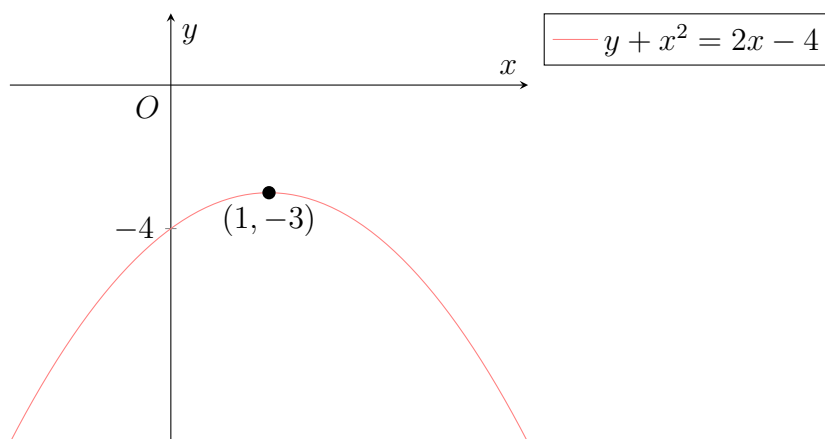
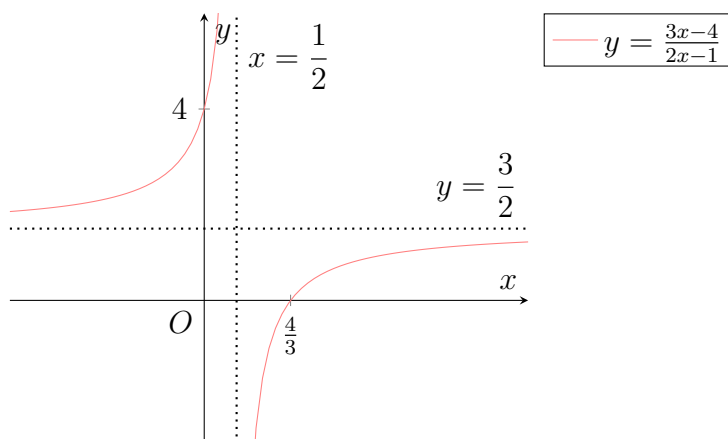


Problem 1.

Sketch clearly labelled diagrams of each of the following curves, giving exact values of axial intercepts, stationary points and equations of asymptotes, if any.

(a) $y + x^2 = 2x - 4$

(b) $y = \frac{3x - 4}{2x - 1}$

Solution**Part (a)****Part (b)**

Problem 2.

On separate diagrams, sketch the graphs of

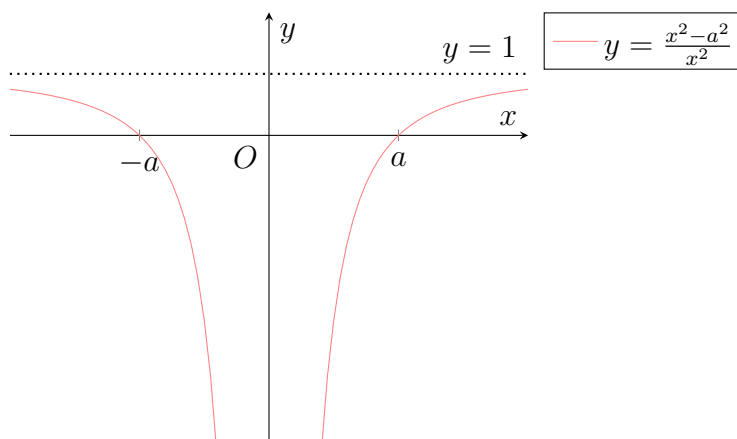
(a) $y = \frac{x^2 - a^2}{x^2}, a > 0$

(b) $y = \frac{x - 1}{2x(x + 3)}$

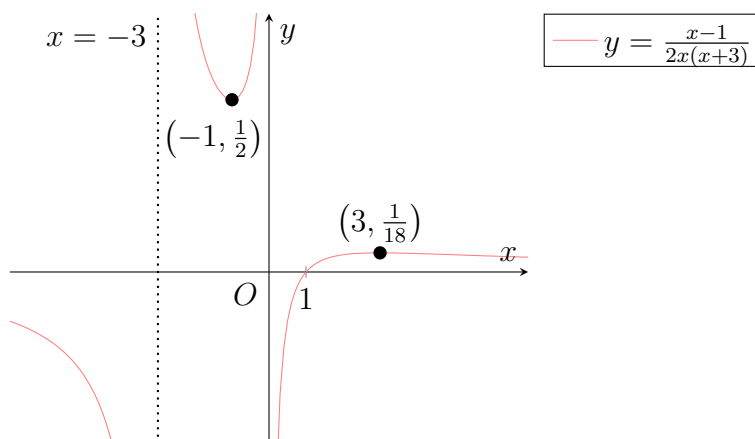
Indicate clearly the coordinates of axial intercepts, stationary points and equations of asymptotes, if any.

Solution

Part (a)



Part (b)



Problem 3.

The curve C has equation $y = \frac{ax^2 + bx - 2}{x + 4}$, where a and b are constants. It is given that $y = 2x - 5$ is an asymptote of C .

- Find the values of a and b .
- Sketch C .
- Using an algebraic method, find the set of values that y cannot take.
- By drawing a sketch of another suitable curve in the same diagram as your sketch of C in part (b), deduce the number of distinct real roots of the equation $x^3 + 6x^2 + 3x - 2 = 0$.

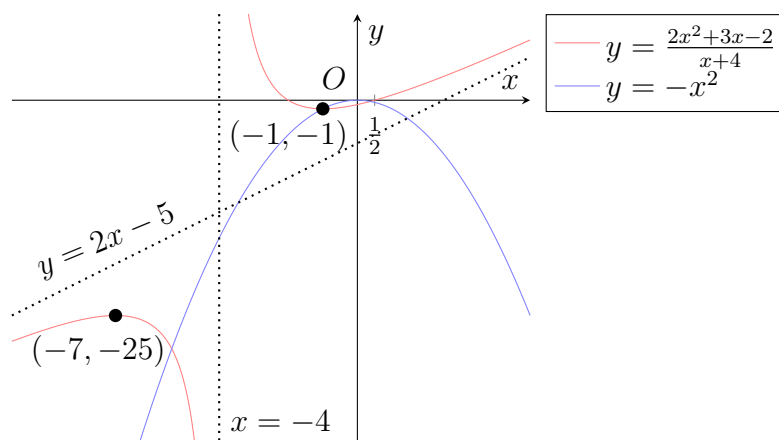
Solution**Part (a)**

Since $y = 2x - 5$ is an asymptote of C , $\frac{ax^2 + bx - 2}{x + 4}$ can be expressed in the form $2x - 5 + \frac{k}{x + 4}$, where k is a constant.

$$\begin{aligned}\frac{ax^2 + bx - 2}{x + 4} &= 2x - 5 + \frac{k}{x + 4} \\ \Rightarrow ax^2 + bx - 2 &= (2x - 5)(x + 4) + k \\ \Rightarrow ax^2 + bx - 2 &= 2x^2 + 3x - 20 + k\end{aligned}$$

Comparing coefficients of x^2 , x and constant terms, we have $a = 2$, $b = 3$ and $k = 18$.

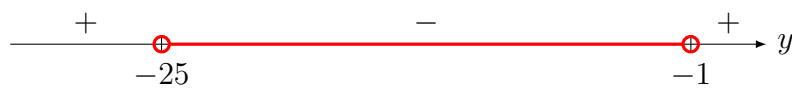
$$a = 2, b = 3$$

Part (b)

Part (c)

$$\begin{aligned}
y &= \frac{2x^2 + 3x - 2}{x + 4} \\
\Rightarrow (x + 4)y &= 2x^2 + 3x - 2 \\
\Rightarrow 2x^2 + (3 - y)x - (2 + 4y) &= 0
\end{aligned}$$

For values that y cannot take on, there exist no solutions to $2x^2 + (3 - y)x - (2 + 4y) = 0$. Hence, $\Delta < 0$.

$$\begin{aligned}
&\Delta < 0 \\
\Rightarrow (3 - y)^2 - 4(2)(-(2 + 4y)) &< 0 \\
\Rightarrow y^2 + 26y + 25 &< 0 \\
\Rightarrow (y + 25)(y + 1) &< 0
\end{aligned}$$


We thus see that y cannot take on a value between -25 and -1.

$$\boxed{\{y \in \mathbb{R}: -25 < y < -1\}}$$

Part (d)

$$\begin{aligned}
&x^3 + 6x^2 + 3x - 2 = 0 \\
\Rightarrow \frac{x^3 + 6x^2 + 3x - 2}{x + 4} &= 0 \\
\Rightarrow \frac{x^3 + 4x^2}{x + 4} + \frac{2x^2 + 3x - 2}{x + 4} &= 0 \\
\Rightarrow x^2 + \frac{2x^2 + 3x - 2}{x + 4} &= 0 \\
\Rightarrow C &= -x^2
\end{aligned}$$

Plotting $y = -x^2$ on the same digram, we see that there are 3 intersections between $y = x^2$ and C . Hence, there are 3 distinct real roots to $x^3 + 6x^2 + 3x - 2 = 0$.