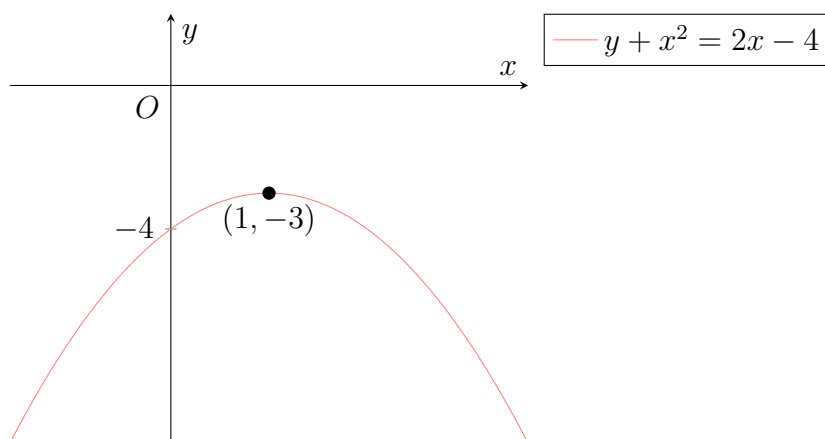
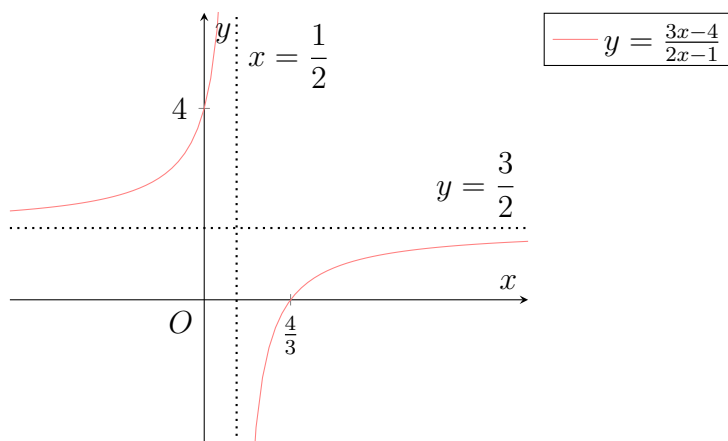


**Problem 1.**

Sketch clearly labelled diagrams of each of the following curves, giving exact values of axial intercepts, stationary points and equations of asymptotes, if any.

(a)  $y + x^2 = 2x - 4$

(b)  $y = \frac{3x - 4}{2x - 1}$

**Solution****Part (a)****Part (b)**

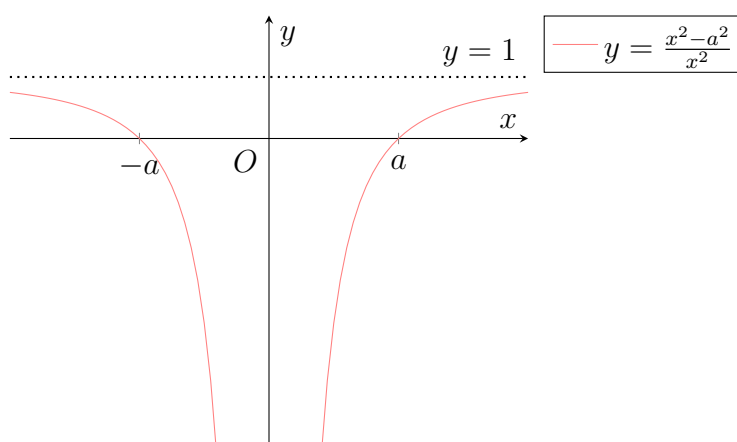
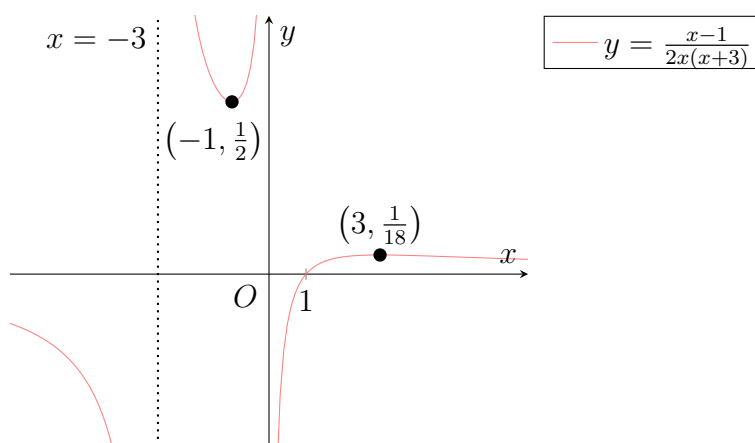
**Problem 2.**

On separate diagrams, sketch the graphs of

(a)  $y = \frac{x^2 - a^2}{x^2}, a > 0$

(b)  $y = \frac{x - 1}{2x(x + 3)}$

Indicate clearly the coordinates of axial intercepts, stationary points and equations of asymptotes, if any.

**Solution****Part (a)****Part (b)**

**Problem 3.**

The curve  $C$  has equation  $y = \frac{ax^2 + bx - 2}{x + 4}$ , where  $a$  and  $b$  are constants. It is given that  $y = 2x - 5$  is an asymptote of  $C$ .

- Find the values of  $a$  and  $b$ .
- Sketch  $C$ .
- Using an algebraic method, find the set of values that  $y$  cannot take.
- By drawing a sketch of another suitable curve in the same diagram as your sketch of  $C$  in part (b), deduce the number of distinct real roots of the equation  $x^3 + 6x^2 + 3x - 2 = 0$ .

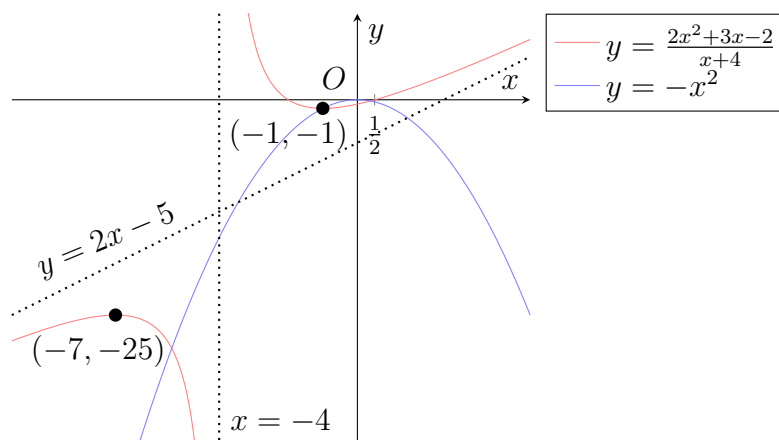
**Solution****Part (a)**

Since  $y = 2x - 5$  is an asymptote of  $C$ ,  $\frac{ax^2 + bx - 2}{x + 4}$  can be expressed in the form  $2x - 5 + \frac{k}{x + 4}$ , where  $k$  is a constant.

$$\begin{aligned}\frac{ax^2 + bx - 2}{x + 4} &= 2x - 5 + \frac{k}{x + 4} \\ \implies ax^2 + bx - 2 &= (2x - 5)(x + 4) + k \\ \implies ax^2 + bx - 2 &= 2x^2 + 3x - 20 + k\end{aligned}$$

Comparing coefficients of  $x^2$ ,  $x$  and constant terms, we have  $a = 2$ ,  $b = 3$  and  $k = 18$ .

$a = 2, b = 3$

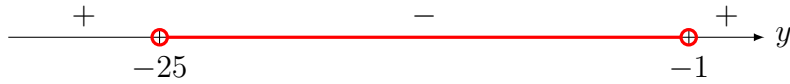
**Part (b)**

**Part (c)**

$$\begin{aligned}
y &= \frac{2x^2 + 3x - 2}{x + 4} \\
\Rightarrow (x + 4)y &= 2x^2 + 3x - 2 \\
\Rightarrow 2x^2 + (3 - y)x - (2 + 4y) &= 0
\end{aligned}$$

For values that  $y$  cannot take on, there exist no solutions to  $2x^2 + (3 - y)x - (2 + 4y) = 0$ . Hence,  $\Delta < 0$ .

$$\begin{aligned}
&\Delta < 0 \\
\Rightarrow (3 - y)^2 - 4(2)(-(2 + 4y)) &< 0 \\
\Rightarrow y^2 + 26y + 25 &< 0 \\
\Rightarrow (y + 25)(y + 1) &< 0
\end{aligned}$$



We thus see that  $y$  cannot take on a value between -25 and -1.

$$\boxed{\{y \in \mathbb{R}: -25 < y < -1\}}$$

**Part (d)**

$$\begin{aligned}
&x^3 + 6x^2 + 3x - 2 = 0 \\
\Rightarrow \frac{x^3 + 6x^2 + 3x - 2}{x + 4} &= 0 \\
\Rightarrow \frac{x^3 + 4x^2}{x + 4} + \frac{2x^2 + 3x - 2}{x + 4} &= 0 \\
\Rightarrow x^2 + \frac{2x^2 + 3x - 2}{x + 4} &= 0 \\
\Rightarrow C &= -x^2
\end{aligned}$$

Plotting  $y = -x^2$  on the same digram, we see that there are 3 intersections between  $y = x^2$  and  $C$ . Hence, there are 3 distinct real roots to  $x^3 + 6x^2 + 3x - 2 = 0$ .