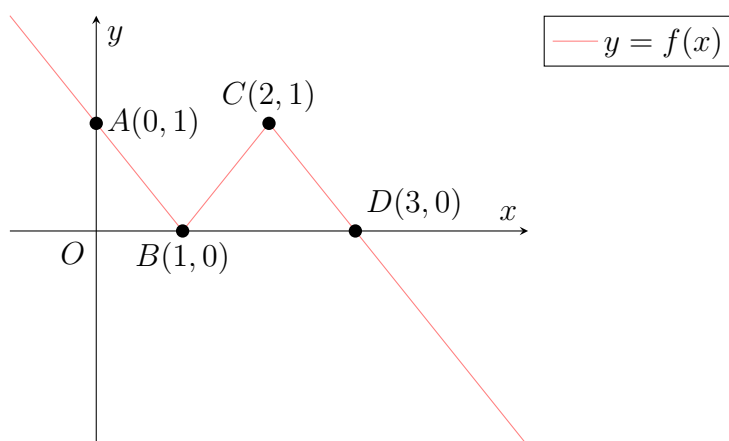
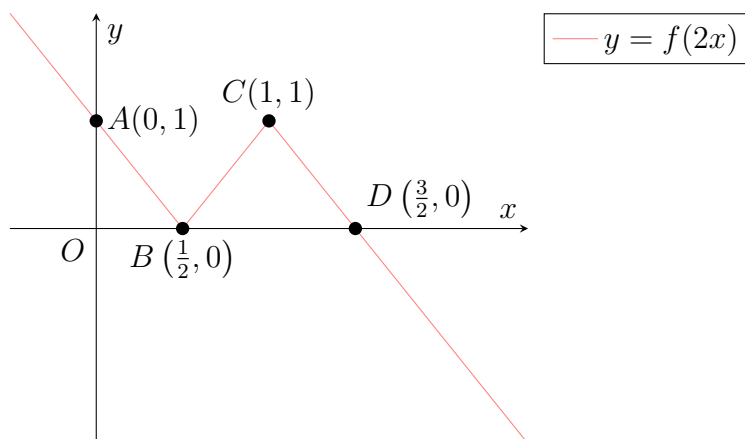


Problem 1.

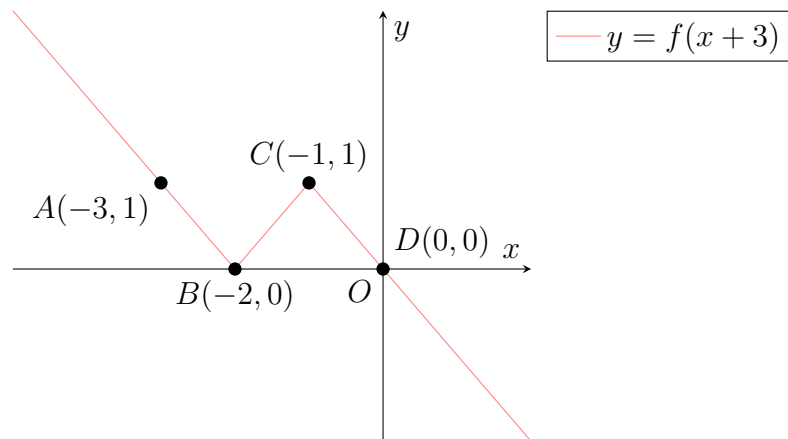
The graph of $y = f(x)$ is shown here. The points A , B , C and D have coordinates $(0, 1)$, $(1, 0)$, $(2, 1)$ and $(3, 0)$ respectively. Sketch, separately, the graphs of

- (a) $y = f(2x)$
- (b) $y = f(x + 3)$
- (c) $y = 1 - f(x)$
- (d) $y = 3f\left(\frac{x}{2} - 1\right)$

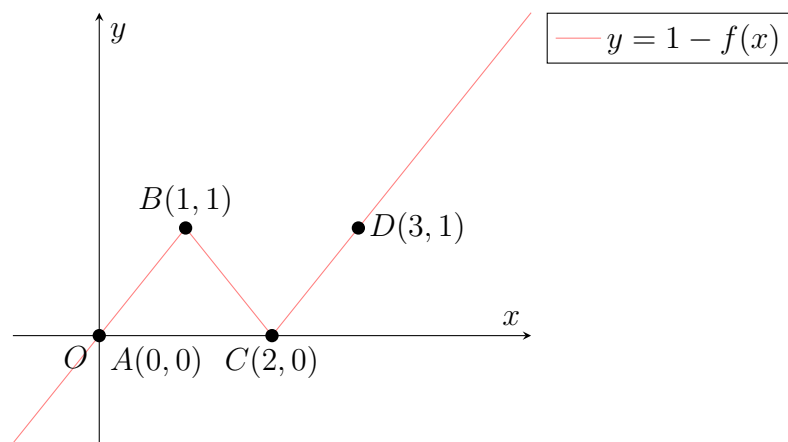
stating, in each case, the coordinates of the points corresponding to A , B , C and D .

Solution**Part (a)**

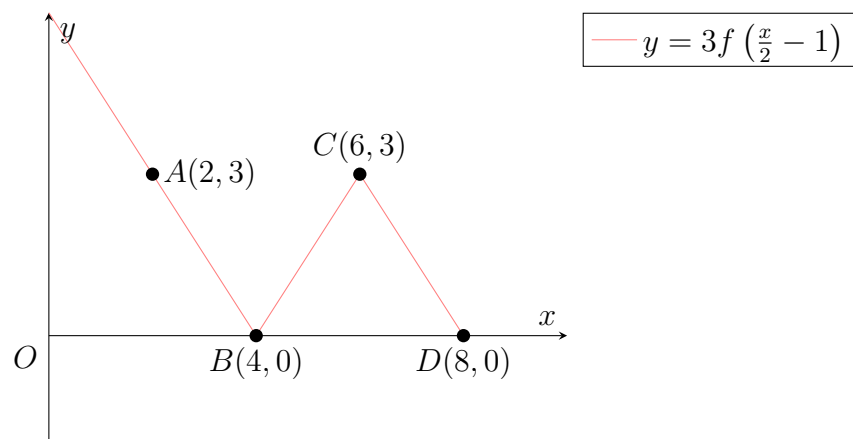
Part (b)



Part (c)



Part (d)

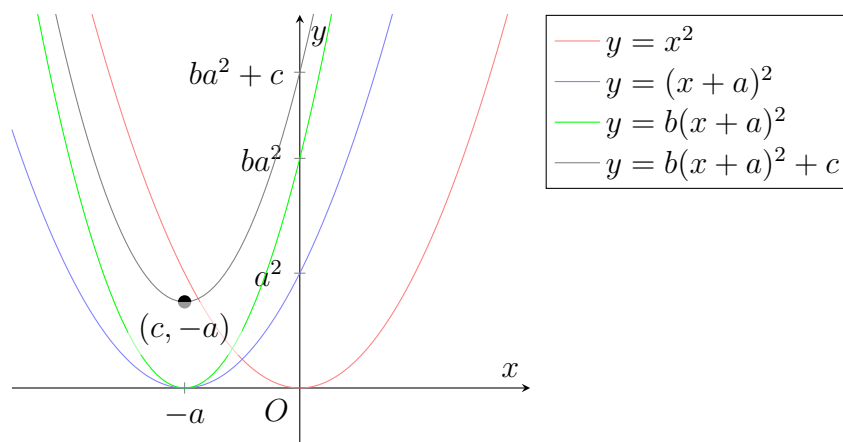


Problem 2.

Sketch, on a single clear diagram, the graphs of

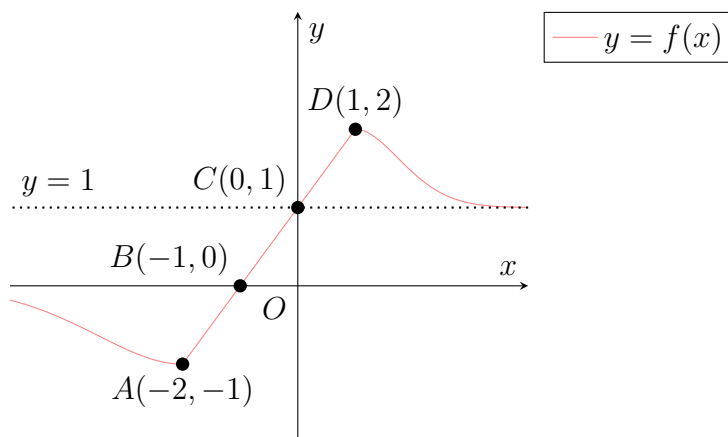
- (a) $y = x^2$
- (b) $y = (x + a)^2$
- (c) $y = b(x + a)^2$
- (d) $y = b(x + a)^2 + c$

(Assume constants $a > 0$, $c > 0$ and $b > 1$)

Solution

Problem 3.

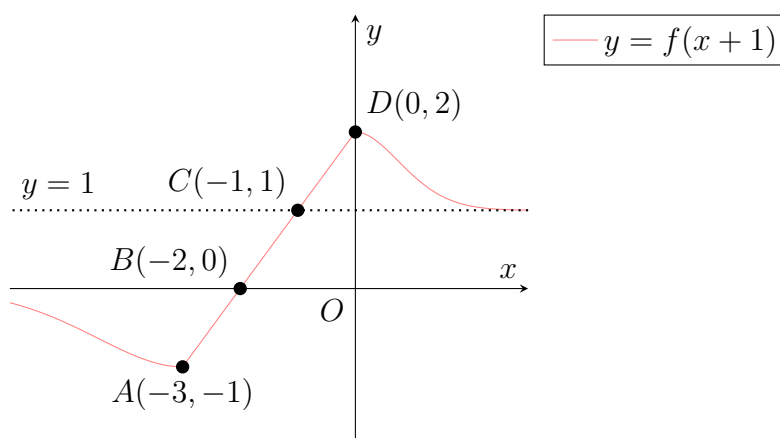
The graph below has equation $y = f(x)$. It has asymptotes $y = 1$ and $y = 0$, a maximum point at $D(1, 2)$, a minimum point at $A(-2, -1)$, cuts the x -axis at $B(-1, 0)$ and cuts the y -axis at $C(0, 1)$.

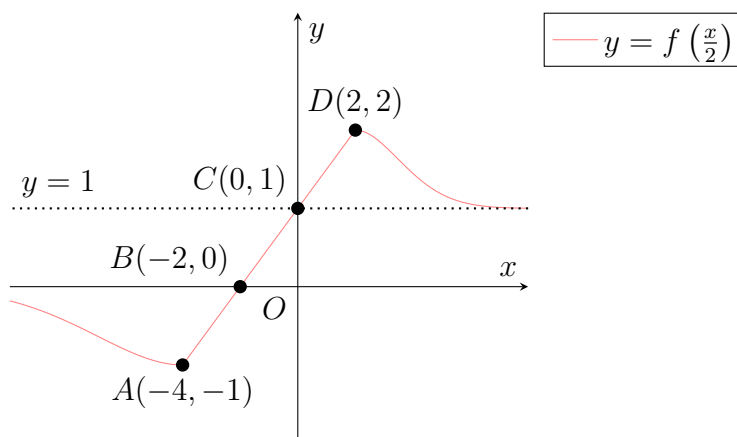
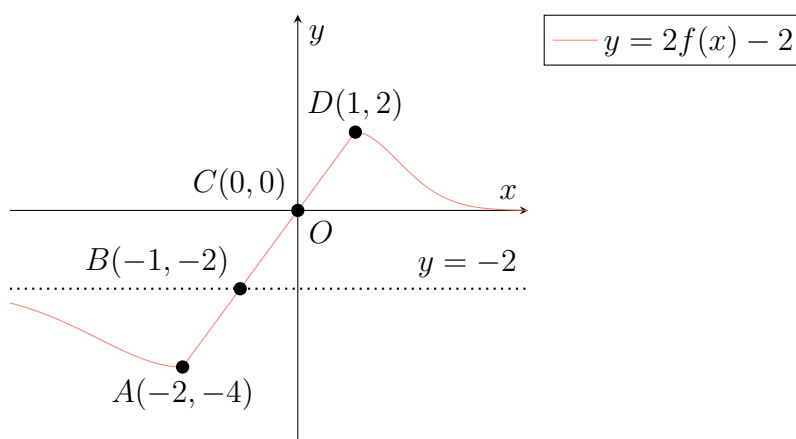


Sketch on separate diagrams the graphs of the following curves, labelling each curve clearly, indicating the horizontal asymptotes and showing the coordinates of the points corresponding to points A , B , C and D .

- (a) $y = f(x + 1)$
- (b) $y = f\left(\frac{x}{2}\right)$
- (c) $y = 2f(x) - 2$

Find the number of solutions to the equation $f(x) = af(x)$ where $a \geq 2$.

Solution**Part (a)**

Part (b)**Part (c)**

All points with a y -coordinate of 0 are invariant under the transformation $f(x) \mapsto af(x)$. Since there is only one such point ($B(-1, 0)$), there is only 1 solution to the equation $f(x) = af(x)$, where $a \geq 2$.

There is 1 solution.

Problem 4.

The curve with equation $y = x^2$ is transformed by a translation of 2 units in the positive x -direction, followed by a stretch with scale factor $\frac{1}{2}$ parallel to the y -axis, followed by a translation of 6 units in the negative y -direction. Find the equation of the new curve in the form $y = f(x)$ and the exact coordinates of the points where this curve crosses the x - and y -axes.

Solution

$$\begin{array}{c} y = x^3 \\ \downarrow x \mapsto x - 2 \\ y = (x - 2)^3 \\ \downarrow y \mapsto 2y \\ 2y = (x - 2)^3 \\ \downarrow y \mapsto y + 6 \\ 2(y + 6) = (x - 2)^3 \\ \boxed{y = \frac{1}{2}(x - 2)^3 - 6} \end{array}$$

When $x = 0$, $y = \frac{1}{2}(-2)^3 - 6 = -10$. When $y = 0$, $x = 2 + \sqrt[3]{12}$.

The curve crosses the x -axis at $(2 + \sqrt[3]{12}, 0)$ and the y -axis at $(0, -10)$.

Problem 5.

Find the values of the constants A and B such that $\frac{x^2 - 4x}{(x - 2)^2} = A + \frac{B}{(x - 2)^2}$ for all values of x except $x = 2$.

Hence state a sequence of transformations by which the graph of $y = \frac{x^2 - 4x}{(x - 2)^2}$ may be obtained from the graph of $y = \frac{1}{x^2}$.

Solution

$$\begin{aligned}\frac{x^2 - 4x}{(x - 2)^2} &= \frac{(x - 2)^2 - 4}{(x - 2)^2} \\ &= 1 + \frac{-4}{(x - 2)^2}\end{aligned}$$

$$\boxed{A = 1, B = -4}$$

$$\begin{aligned}y &= \frac{1}{x^2} \\ \downarrow x &\mapsto x - 2 \\ y &= \frac{1}{(x - 2)^2} \\ \downarrow y &\mapsto \frac{1}{4}y \\ y &= \frac{4}{(x - 2)^2} \\ \downarrow y &\mapsto -y \\ y &= \frac{-4}{(x - 2)^2} \\ \downarrow y &\mapsto y - 1 \\ y &= 1 + \frac{-4}{(x - 2)^2}\end{aligned}$$

1. Translate the curve 2 units in the positive x -direction.
2. Stretch the curve with a scale factor of 4 parallel to the y -axis.
3. Reflect the curve about the x -axis.
4. Translate the curve 1 unit in the positive y -direction.

Problem 6.

The transformations A , B , C and D are given as follows:

- A : A reflection about the y -axis.
- B : A translation of 2 units in the positive x -direction.
- C : A scaling parallel to the y -axis by a factor of 3.
- D : A translation of 1 unit in the positive y -direction.

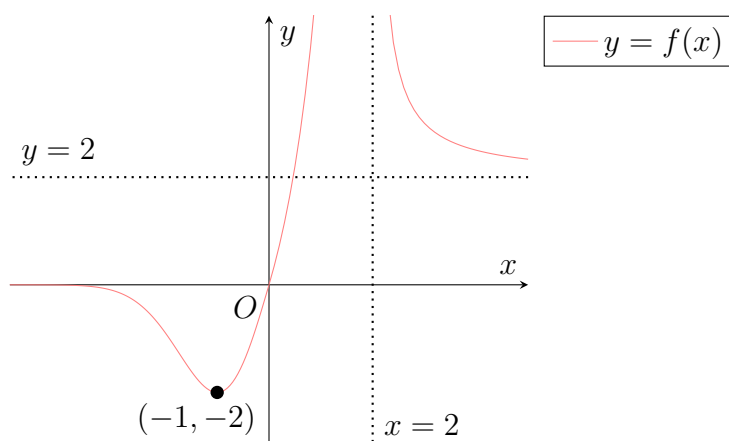
A curve undergoes the transformations A , B , C and D in succession, and the equation of the resulting curve is $y = 3\sqrt{2-x} + 1$. Determine the equation of the curve before the transformations were effected.

Solution

$$\begin{aligned} A: x &\mapsto -x &\implies A^{-1}: x &\mapsto -x \\ B: x &\mapsto x - 2 &\implies B^{-1}: x &\mapsto x + 2 \\ C: y &\mapsto \frac{1}{3}y &\implies C^{-1}: y &\mapsto 3y \\ D: y &\mapsto y - 1 &\implies D^{-1}: y &\mapsto y + 1 \end{aligned}$$

$$\begin{aligned} y &= 3\sqrt{2-x} + 1 \\ &\downarrow D^{-1} \\ y + 1 &= 3\sqrt{2-x} + 1 \\ &\downarrow C^{-1} \\ 3y + 1 &= 3\sqrt{2-x} + 1 \\ &\downarrow B^{-1} \\ 3y + 1 &= 3\sqrt{2-(x+2)} + 1 \\ &\downarrow A^{-1} \\ 3y + 1 &= 3\sqrt{2-(-x+2)} + 1 \end{aligned}$$

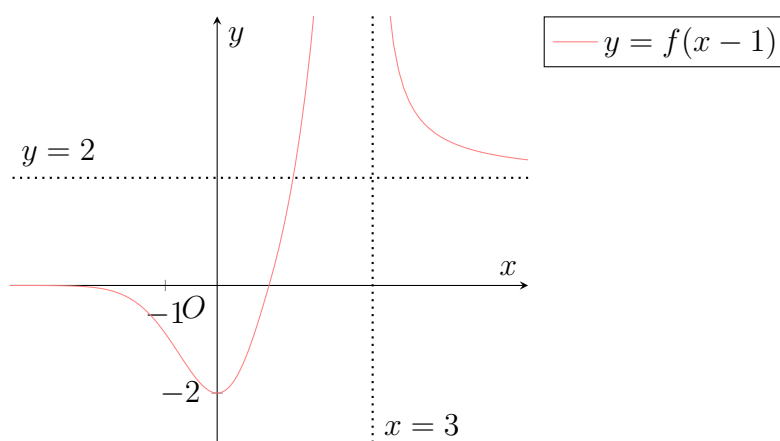
The original curve has equation $y = \sqrt{x}$.

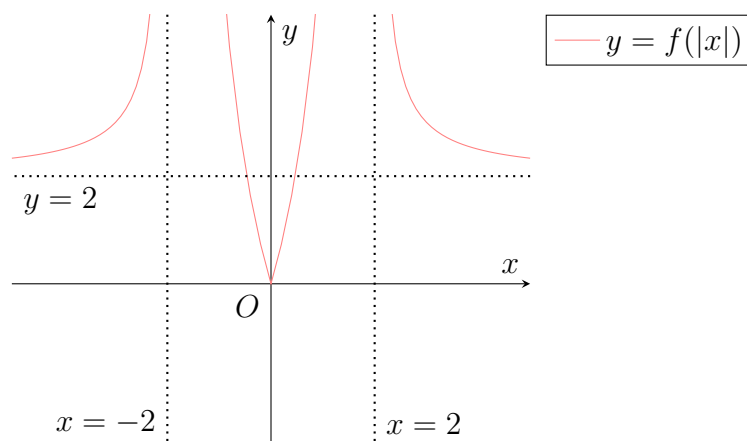
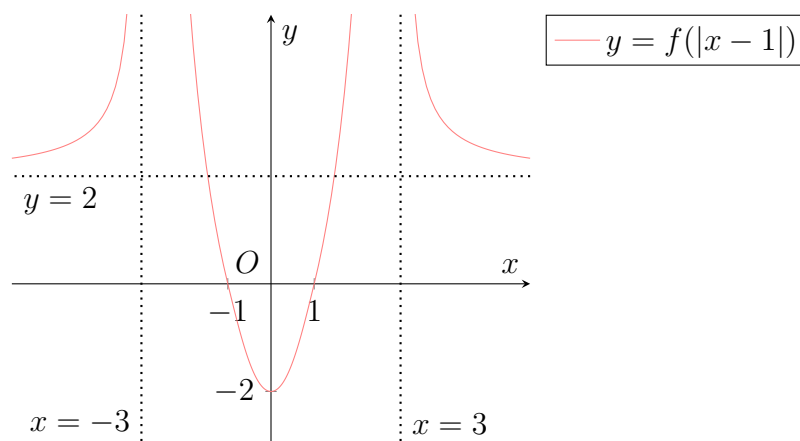
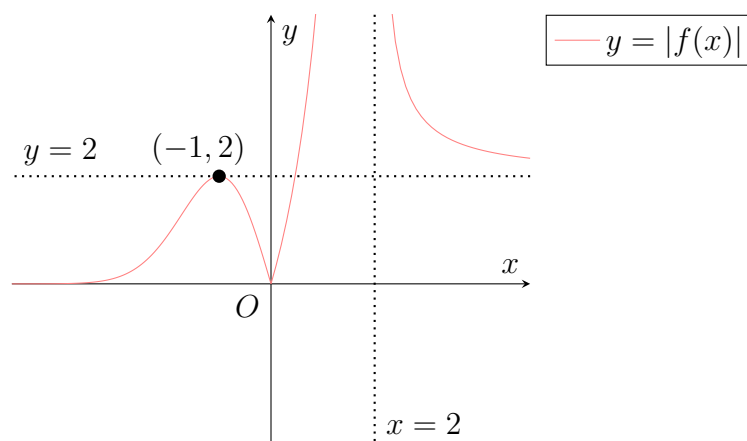
Problem 7.

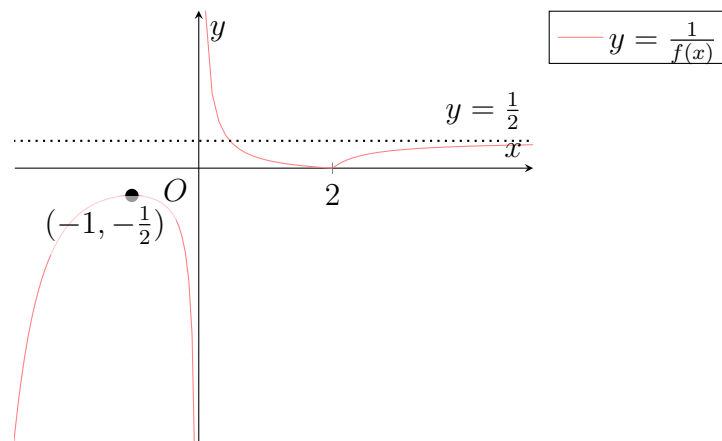
The diagram shows the graph of $y = f(x)$. The curve passes through the origin and has minimum point $(-1, -2)$. The asymptotes are $x = 2$, $y = 0$ and $y = 2$.

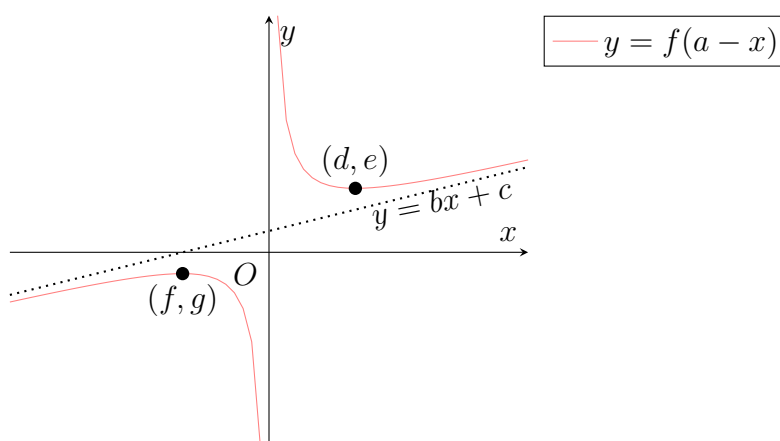
Sketch, on separate diagrams, the graphs of

- (a) $y = f(x - 1)$
- (b) $y = f(|x|)$
- (c) $y = f(|x - 1|)$
- (d) $y = |f(x)|$
- (e) $y = \frac{1}{f(x)}$

Solution**Part (a)**

Part (b)**Part (c)****Part (d)**

Part (e)

Problem 8.

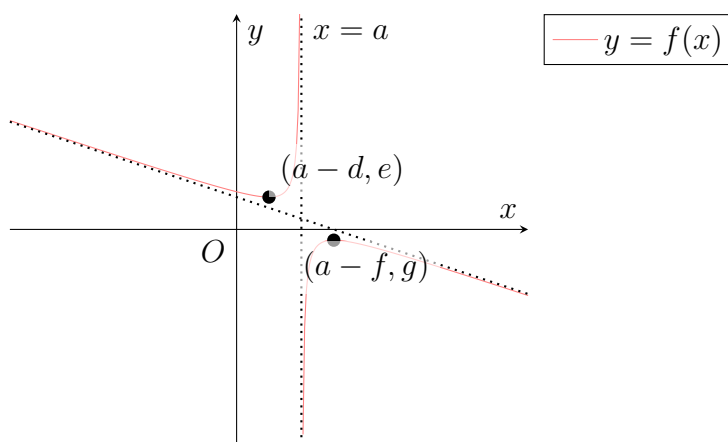
The graph of $y = f(a - x)$ is shown in the figure, where $a > 0$. The curve has asymptotes $x = 0$, $y = bx + c$, a minimum point at (d, e) and a maximum point at (f, g) .

Given $a > d$, sketch separately, the graphs of

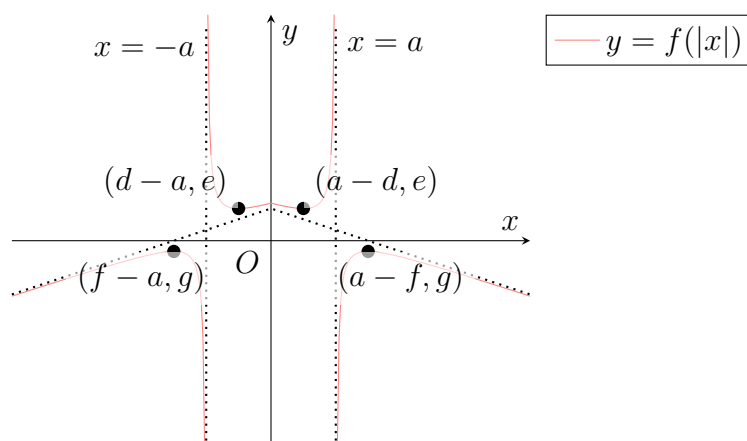
- (a) $y = f(x)$
- (b) $y = f(|x|)$
- (c) $y = \frac{1}{f(x)}$

Solution

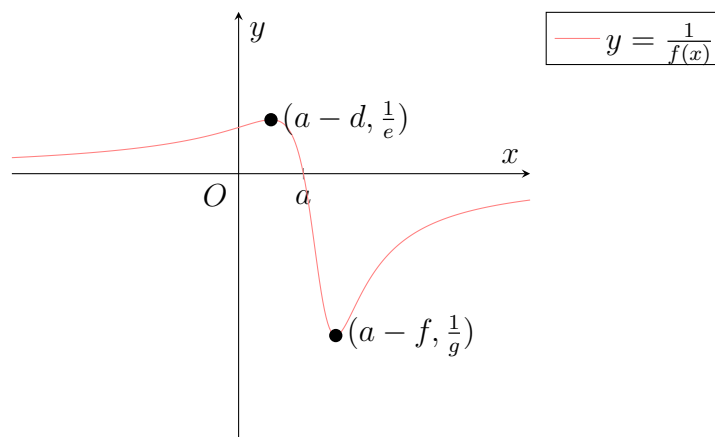
Part (a)



Equation of asymptote: $y = b(a - x) + c$

Part (b)

Equation of asymptotes: $y = b(a+x) + c$, $y = b(a-x) + c$

Part (c)

Problem 9.

A curve C_1 is defined by the parametric equations

$$x = t(t + 2), \quad y = 2(t + 1)$$

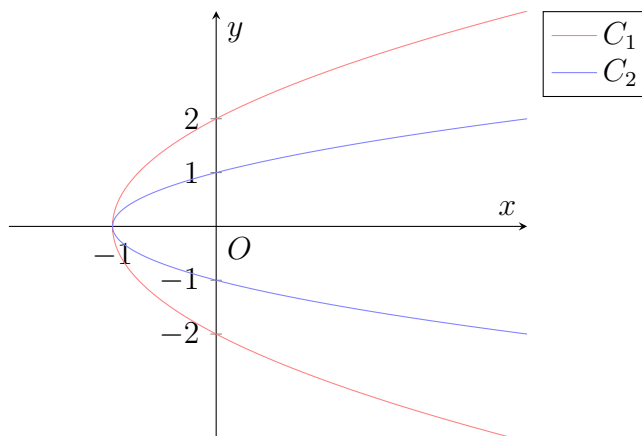
- (a) Find the axial intercepts of the curve.
- (b) Sketch C_1 .
- (c) A curve C_2 is defined by the parametric equations $x = t(t + 2)$, $y = t + 1$. Describe a geometrical transformation which maps C_1 to C_2 . Hence sketch the curve C_2 in the same diagram as C_1 .
- (d) Show that the Cartesian equation of the curve C_1 is given by $y^2 = 4(x + 1)$.

Solution**Part (a)**

Consider $x = 0$. Then $t(t + 2) = 0$, whence $t = 0$ or $t = -2$. When $t = 0$, $y = 2$. When $t = -2$, $y = -2$. Hence, the curve intercepts the y -axis at $(0, 2)$ and $(0, -2)$.

Consider $y = 0$. Then $t = -1$, whence $x = -1$. Hence, the curve intercepts the x -axis at $(-1, 0)$.

The axial intercepts of the curve are $(0, 2)$, $(0, -2)$ and $(-1, 0)$.

Part (b)**Part (c)**

Scale by a factor of $\frac{1}{2}$ parallel to the y -axis.

Part (d)

$$\begin{aligned}y^2 &= (2(t+1))^2 \\&= 4(t^2 + 2t + 1) \\&= 4(t(t+1) + 1) \\&= 4(x+1)\end{aligned}$$