

**Problem 1.**

Without using a calculator, sketch the following graphs of conics.

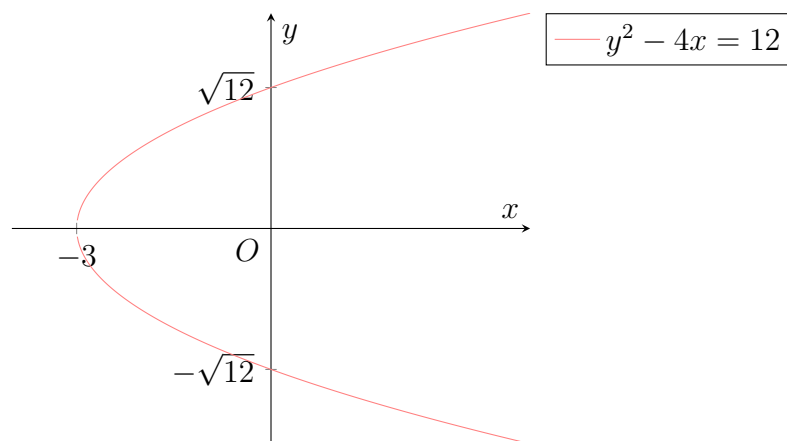
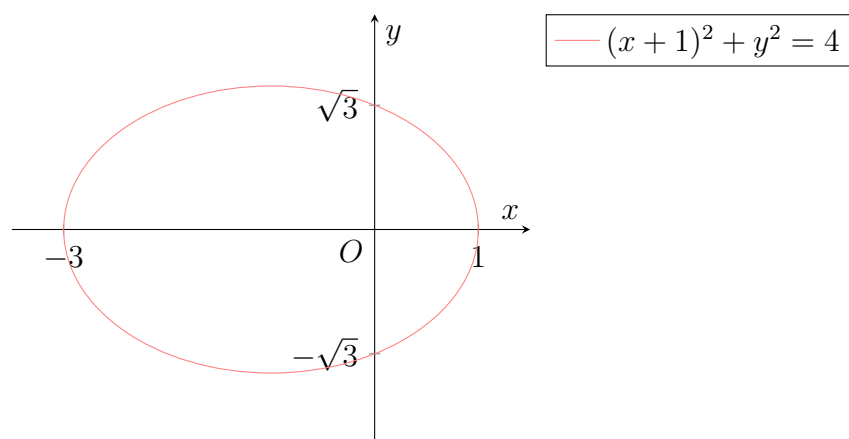
(a)  $y^2 - 4x = 12$

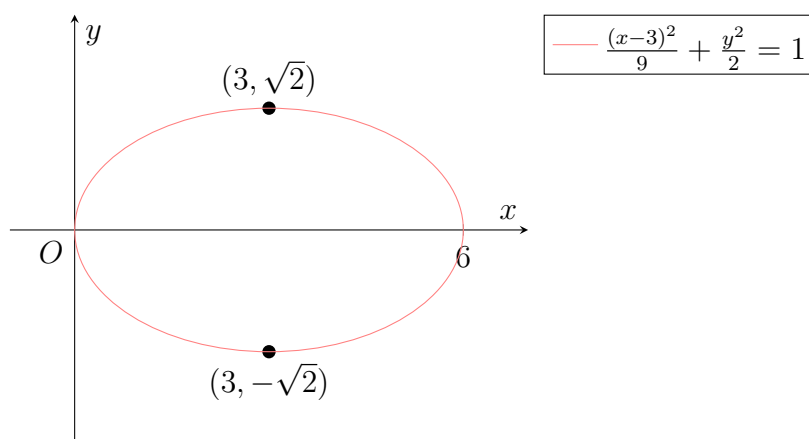
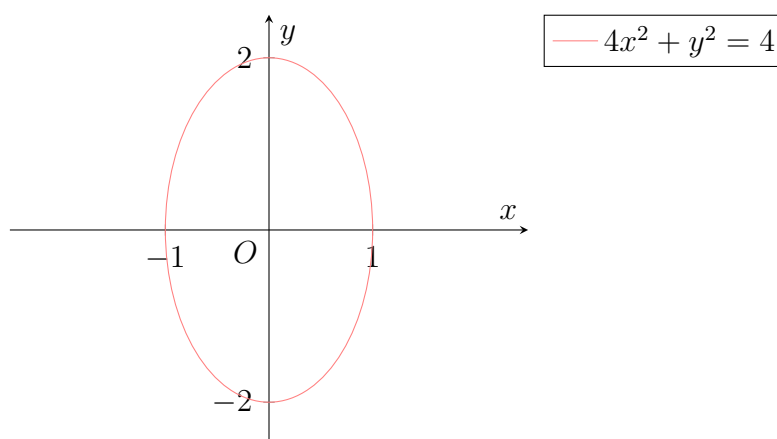
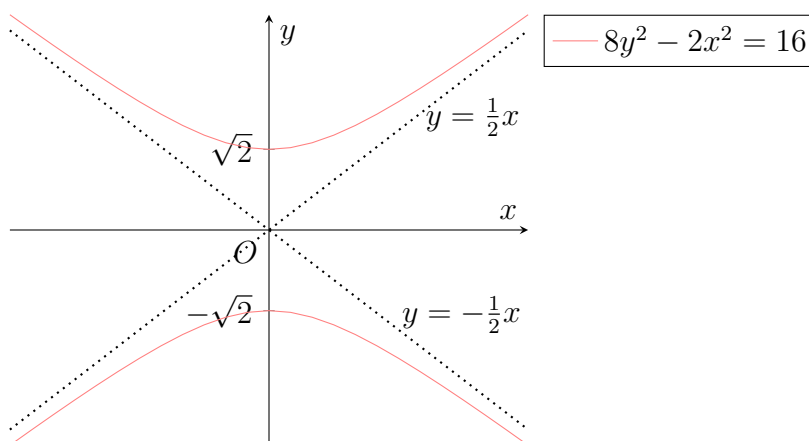
(b)  $(x + 1)^2 + y^2 = 4$

(c)  $\frac{(x - 3)^2}{9} + \frac{y^2}{2} = 1$

(d)  $4x^2 + y^2 = 4$

(e)  $8y^2 - 2x^2 = 16$

**Solution****Part (a)****Part (b)**

**Part (c)****Part (d)****Part (e)**

## Problem 2.

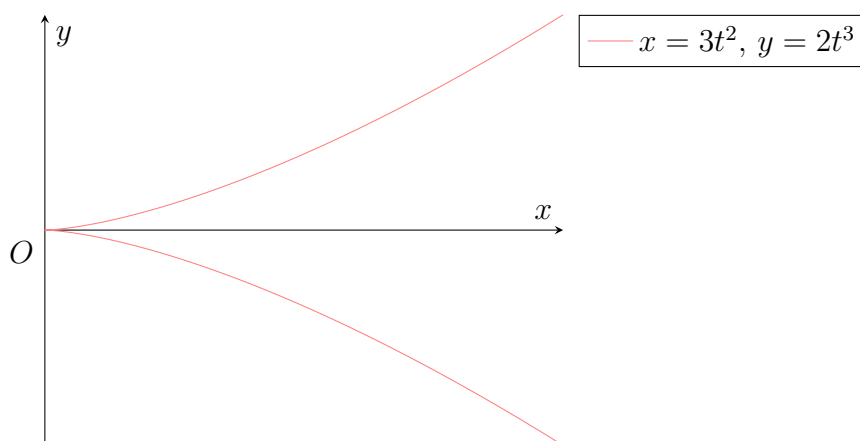
Sketch the curves defined by the following parametric equations, indicating the coordinates of any intersection with the axes.

(a)  $x = 3t^2$ ,  $y = 2t^3$

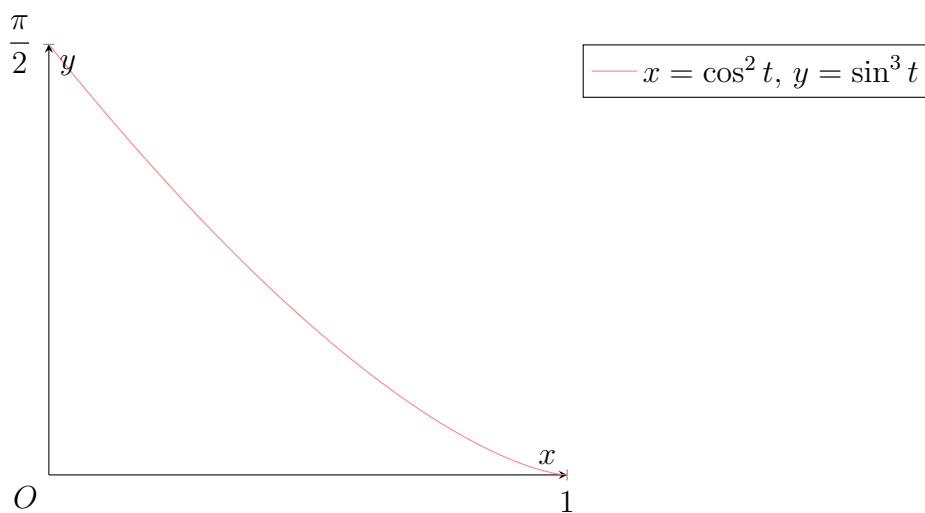
(b)  $x = \cos^2 t$ ,  $y = \sin^3 t$ ,  $0 \leq t \leq \frac{\pi}{2}$

### Solution

#### Part (a)



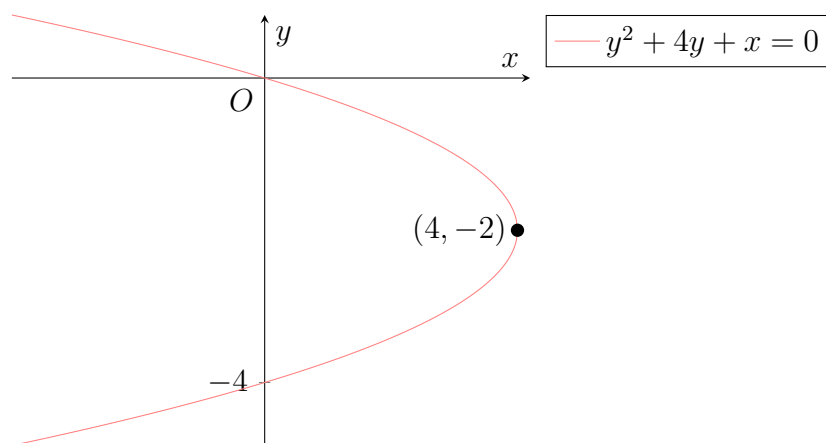
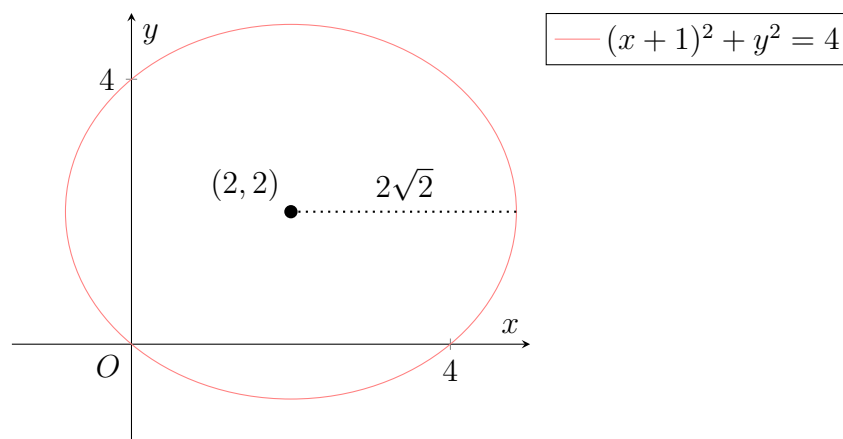
#### Part (b)

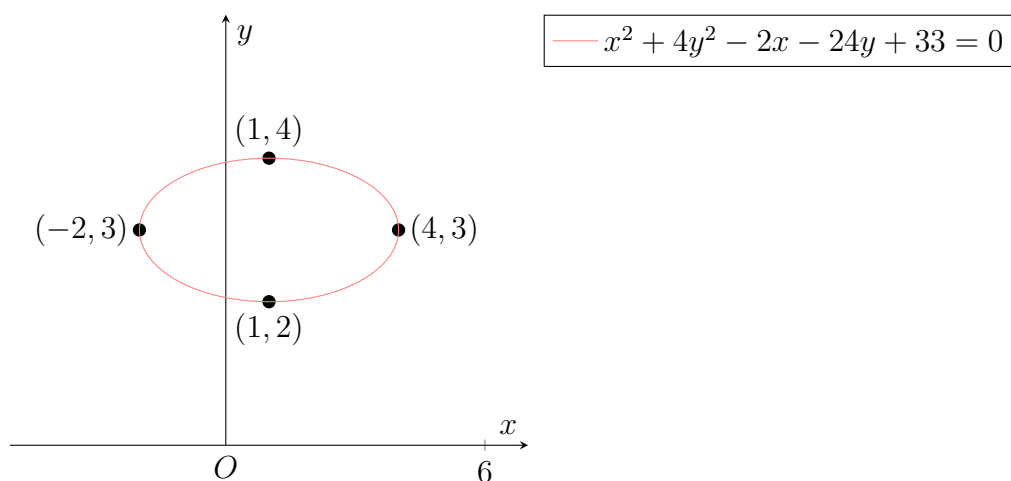
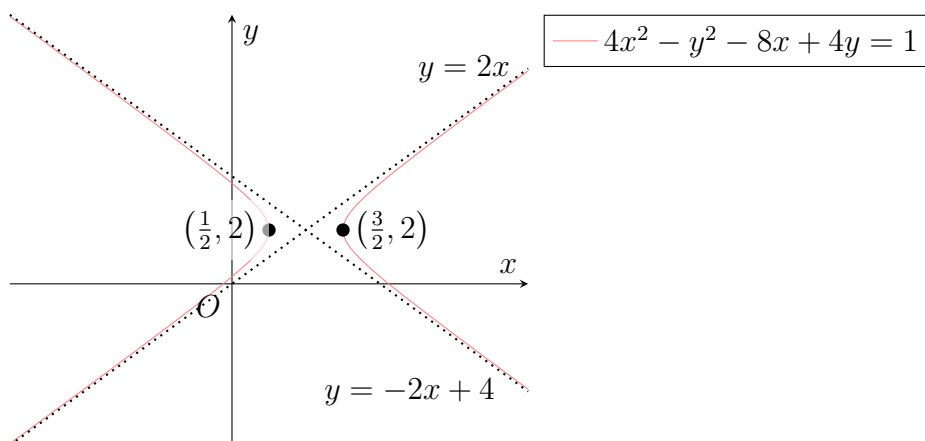
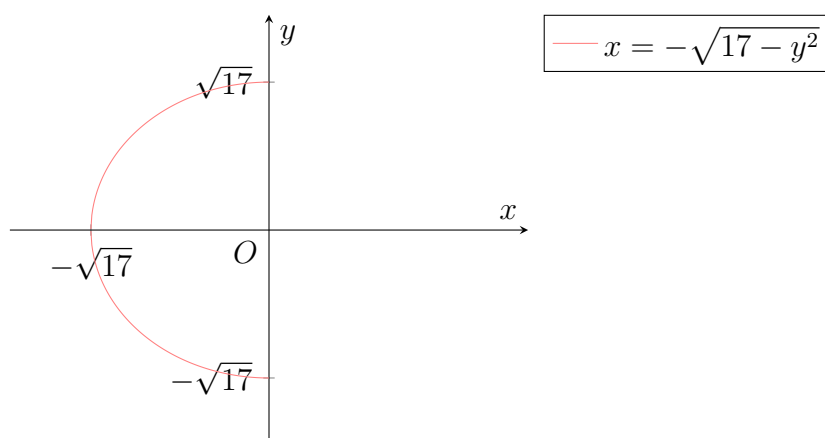


**Problem 3.**

Without using a calculator, sketch the following graphs of conics.

- (a)  $y^2 + 4y + x = 0$
- (b)  $x^2 + y^2 - 4x - 4y = 0$
- (c)  $x^2 + 4y^2 - 2x - 24y + 33 = 0$
- (d)  $4x^2 - y^2 - 8x + 4y = 1$
- (e)  $x = -\sqrt{17 - y^2}$

**Solution****Part (a)****Part (b)**

**Part (c)****Part (d)****Part (e)**

**Problem 4.**

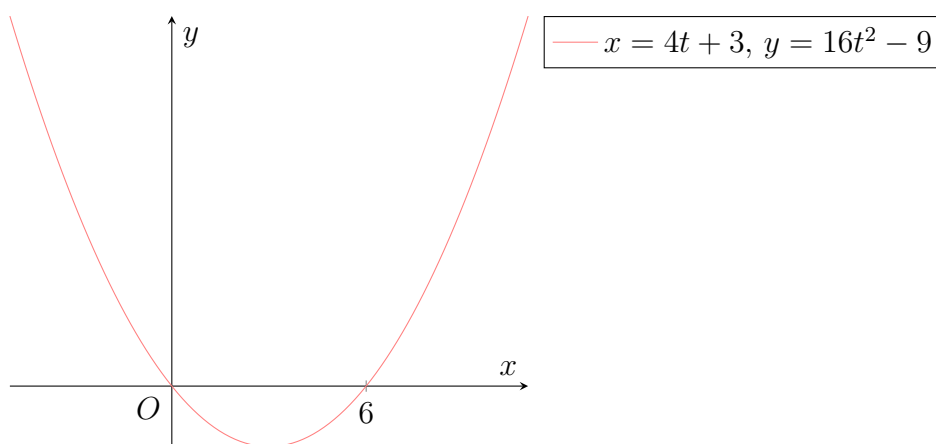
Sketch the curves defined by the following parametric equations. Find also their respective Cartesian equations.

(a)  $x = 4t + 3, y = 16t^2 - 9, t \in \mathbb{R}$

(b)  $x = t^2, y = 2 \ln t, t \geq 1$

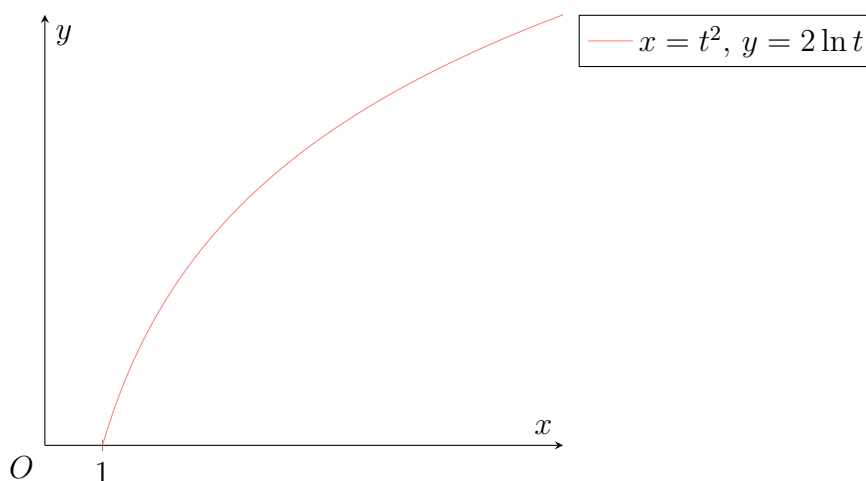
(c)  $x = 1 + 2 \cos \theta, y = 2 \sin \theta - 1, 0 \leq \theta \leq \frac{\pi}{2}$

(d)  $x = t^2, y = \frac{2}{t}, t \neq 0$

**Solution****Part (a)**

Since  $x = 4t + 3$ , we have  $t = \frac{1}{4}(x - 3)$ . Thus,  $y = 16 \left( \frac{1}{4}(x - 3) \right)^2 - 9 = (x - 3)^2 - 9$ .

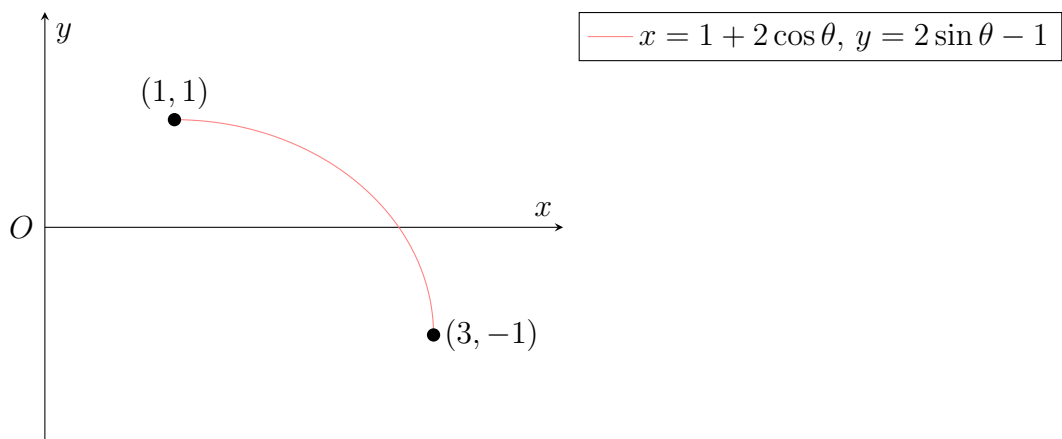
$$y = (x - 3)^2 - 9$$

**Part (b)**

Since  $x = t^2$  and  $t \geq 1 > 0$ , we have  $t = \sqrt{x}$ . Thus,  $y = 2 \ln t = 2 \ln \sqrt{x} = \ln x$ .

$$y = \ln x$$

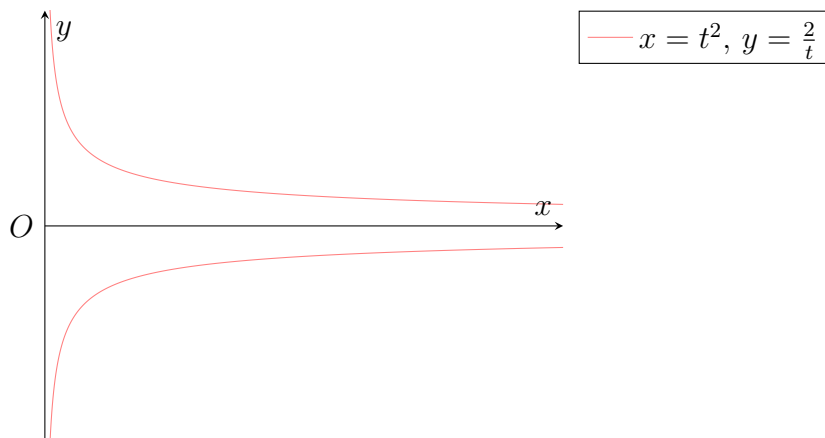
**Part (c)**



We have  $2 \cos \theta = x - 1$  and  $2 \sin \theta = y + 1$ . Squaring both equations and adding them, we obtain  $4 \cos^2 \theta + 4 \sin^2 \theta = (x - 1)^2 + (y + 1)^2$ , which simplifies to  $(x - 1)^2 + (y + 1)^2 = 4$ .

$$(x - 1)^2 + (y + 1)^2 = 4$$

**Part (d)**



Since  $x = t^2$ , we have  $t = \pm \sqrt{x}$ . Hence,  $y = \pm \frac{2}{\sqrt{x}}$ .

$$y = \pm \frac{2}{\sqrt{x}}$$

**Problem 5.**

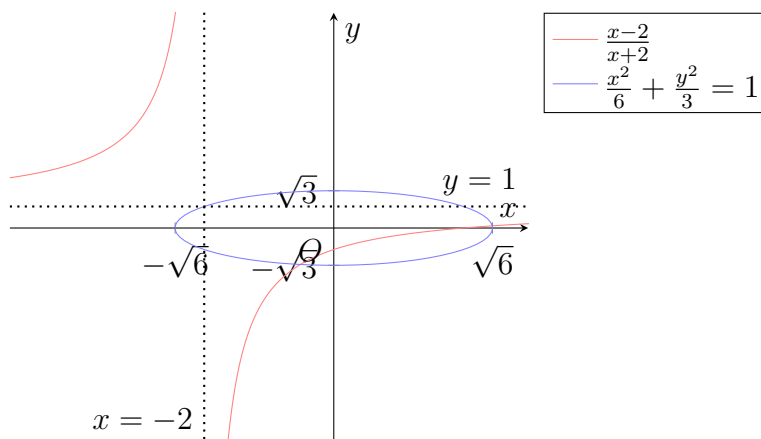
The curve  $C_1$  has equation  $y = \frac{x-2}{x+2}$ . The curve  $C_2$  has equation  $\frac{x^2}{6} + \frac{y^2}{3} = 1$ .

- Sketch  $C_1$  and  $C_2$  on the same diagram, stating the exact coordinates of any points of intersections with the axes and the equations of any asymptotes.
- Show algebraically that the  $x$ -coordinates of the points of intersection of  $C_1$  and  $C_2$  satisfy the equation  $2(x-2)^2 = (x+2)^2(6-x^2)$ .
- Use your calculator to find these  $x$ -coordinates.

Another curve is defined parametrically by

$$x = \sqrt{6} \cos \theta, \quad y = \sqrt{3} \sin \theta, \quad -\pi \leq \theta \leq \pi$$

- Find the Cartesian equation of this curve and hence determine the number of roots to the equation  $\sqrt{3} \sin \theta = \frac{\sqrt{6} \cos \theta - 2}{\sqrt{6} \cos \theta + 2}$  for  $-\pi \leq \theta \leq \pi$ .

**Solution****Part (a)****Part (b)**

From  $C_1$ , we have  $y(x+2) = x-2$ . Hence,

$$y^2(x+2)^2 = (x-2)^2$$

From  $C_2$ , we have  $x^2 + 2y^2 = 6$ . Hence,

$$y^2 = \frac{6-x^2}{2}$$

Putting both equations together, we have

$$\begin{aligned} (x-2)^2 &= \frac{(6-x^2)(x+2)^2}{2} \\ \implies 2(x-2)^2 &= (6-x^2)(x+2)^2 \end{aligned}$$



**Part (c)**

$$x = -0.515 \vee x = 2.45$$

**Part (d)**

Since  $x = \sqrt{6} \cos \theta$  and  $y = \sqrt{3} \sin \theta$ , we have  $x^2 = 6 \cos^2 \theta$  and  $2y^2 = 6 \sin^2 \theta$ . Adding both equations together, we have

$$\begin{aligned} x^2 + 2y^2 &= 6 \cos^2 \theta + 6 \sin^2 \theta \\ &= 6 \\ \implies \frac{x^2}{6} + \frac{y^2}{3} &= 1 \end{aligned}$$

$$\frac{x^2}{6} + \frac{y^2}{3} = 1$$

This is the equation that gives  $C_1$ . We further observe that the equation  $\sqrt{3} \sin \theta = \frac{\sqrt{6} \cos \theta - 2}{\sqrt{6} \cos \theta + 2}$  simplifies to  $y = \frac{x - 2}{x + 2}$ . This is the equation that gives  $C_2$ . Since there are two intersections between  $C_1$  and  $C_2$ , there are thus two roots to the equation  $\sqrt{3} \sin \theta = \frac{\sqrt{6} \cos \theta - 2}{\sqrt{6} \cos \theta + 2}$ .

$$\text{There are 2 roots to } \sqrt{3} \sin \theta = \frac{\sqrt{6} \cos \theta - 2}{\sqrt{6} \cos \theta + 2}.$$