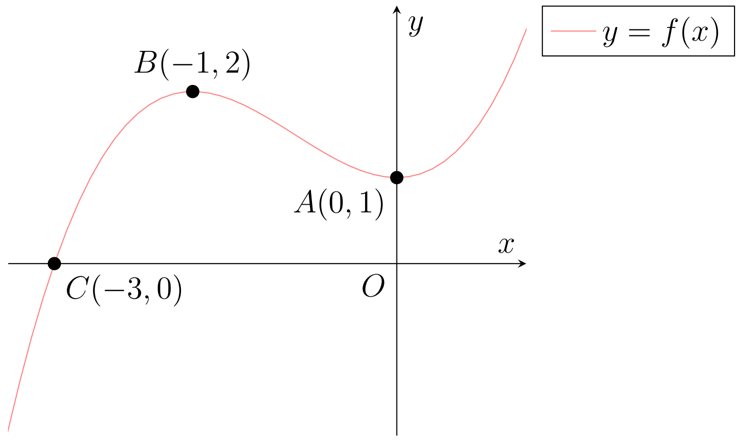
Problem 1.



The diagram shows the graph of $y = f(x)$. The points A, B and C have coordinates $(0, 1)$, $(-1, 2)$ and $(-3, 0)$ respectively. Sketch, separately, the graphs of

(a) $y = f(-x)$

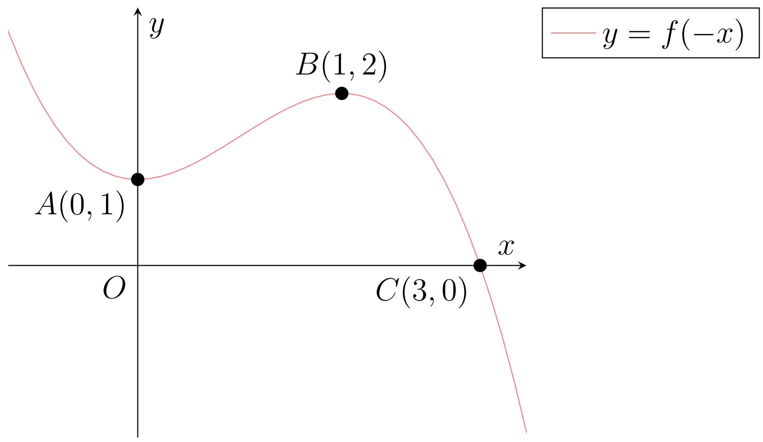
(b) $y = 3 - 2f(x)$

(c) $y = 3f(\frac{x}{2} + 1)$

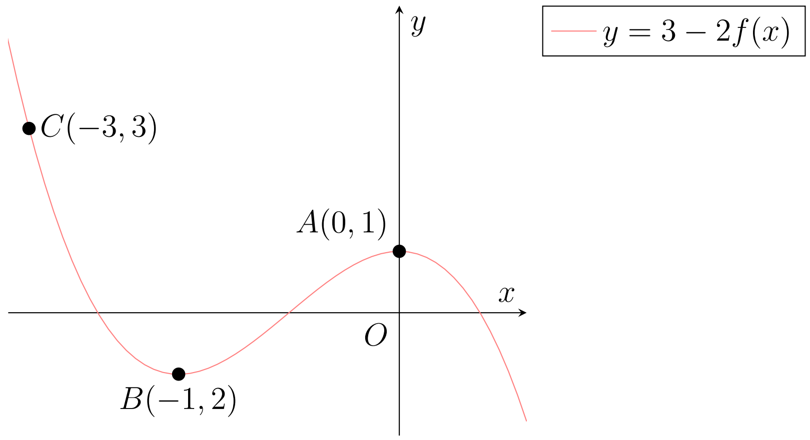
showing in each case the coordinates of the points corresponding to A, B and C.

Solution

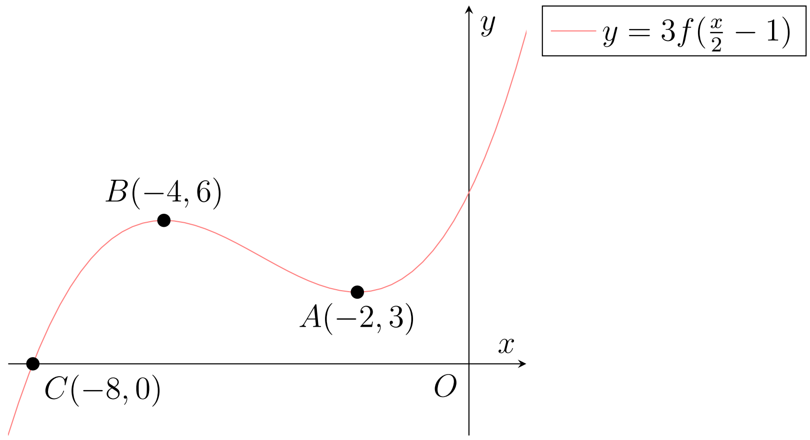
Part (a)



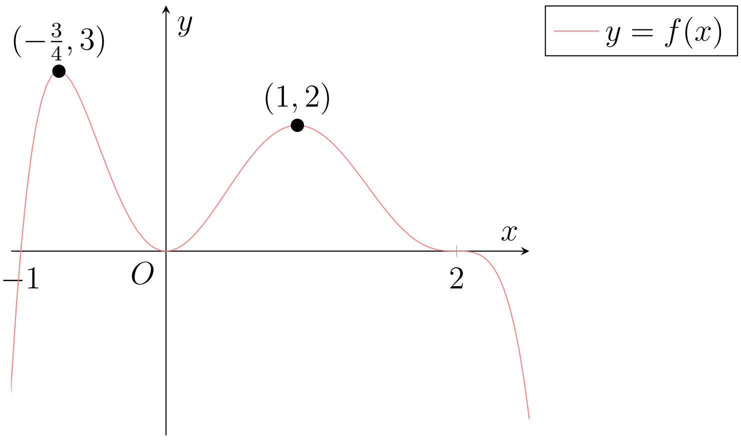
Part (b)



Part (c)



Problem 2.



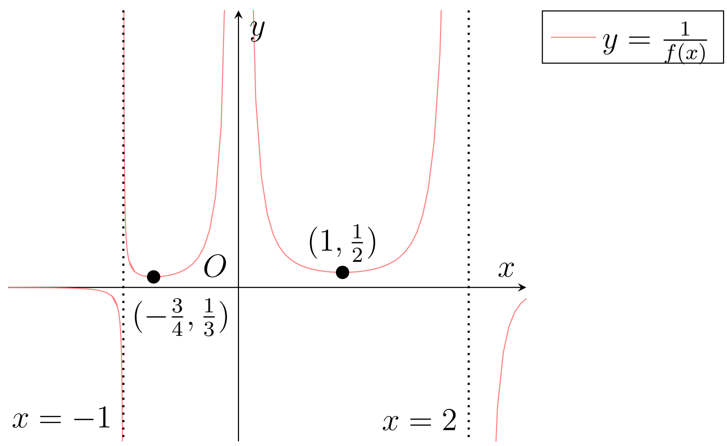
The curve shown is the graph of $y = f(x)$. The x-axis is a tangent at the origin and at $(2, 0)$. The curve has two maximum points at $(-\dfrac{3}{4}, 3)$ and $(1, 2)$. On two separate diagrams, sketch the graphs of the following equations. Show clearly the shapes of the graphs where they meet the x-axis and any asymptotes.

(a) $y = \dfrac{1}{f(x)}$, $x \neq -1, 0, 2$

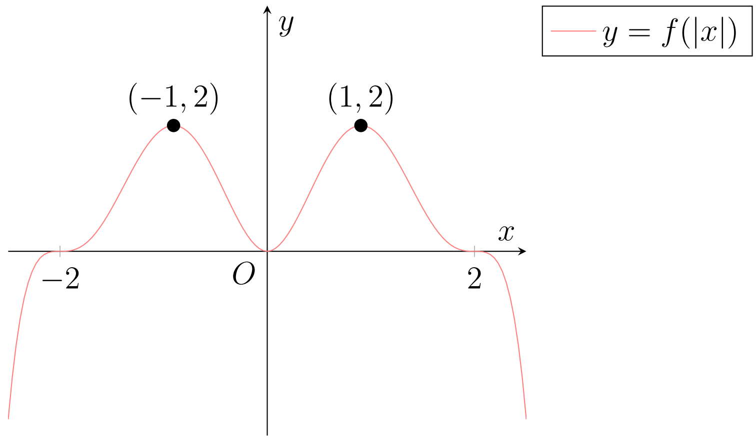
(b) $y = f(|x|)$

Solution

Part (a)



Part (b)



Problem 3.

A graph with equation $y = f(x)$ undergoes transformation A followed by transformation B where A and B are described as follows:

- A: a translation of 1 unit in the positive direction of the x-axis
- B: a scaling parallel to the x-axis by a factor $\dfrac{1}{2}$

The resulting equation is $y = 4x^2 - 4x + 1$. Find the equation $y = f(x)$.

Solution

$$\begin{aligned} A\colon x \mapsto x - 1 &\implies A^{-1}\colon x \mapsto x + 1 \\ B\colon x \mapsto 2x &\implies B^{-1}\colon x \mapsto \frac{1}{2}x \end{aligned}$$

$$\begin{gathered} y = 4x^2 - 4x + 1 \\ \big\downarrow B^{-1} \\ y = 4\left(\frac{1}{2}x\right)^2 - 4\left(\frac{1}{2}x\right) + 1 \\ \big\downarrow A^{-1} \\ y = 4\left(\frac{1}{2}(x+1)\right)^2 - 4\left(\frac{1}{2}(x+1)\right) + 1 \end{gathered}$$

$$\begin{aligned} & y = 4\left(\frac{1}{2}(x+1)\right)^2 - 4\left(\frac{1}{2}(x+1)\right) + 1 \\ \implies & y = (x+1)^2 - 2(x+1) + 1 \\ \implies & y = x^2 + 2x + 1 - 2x - 2 + 1 \\ \implies & y = x^2 \end{aligned}$$

$$\boxed{y = x^2}$$