Problem 1.

A traveller just returned from Germany, France and Spain. The amount (in dollars) that he spent each day on housing, food and incidental expenses in each country are shown in the table below.

Country	**Housing**	**Food**	**Incidental Expenses**
Germany	28	30	14
France	23	25	8
Spain	19	22	12

The traveller's records of the trip indicate a total of \$191 spent for housing, \$430 for food and \$180 for incidental expenses. Calculate the number of days the traveller spent in each country.

He did his account again and the amount spent on food is \$337. Is this record correct? Why?

Solution

Let g, f and s represent the number of days the traveller spent in Germany, France and Spain respectively.

$$\begin{cases} 23f + 28g + 19s = 391 \\ 25f + 30g + 22s = 430 \\ 8f + 14g + 12s = 180 \end{cases}$$

This gives the unique solution $g = 4$, $f = 8$ and $s = 5$.

$$\boxed{\text{The traveller spent 4 days in Germany, 8 days in France and 5 days in Spain.}}$$

$$\begin{cases} 23f + 28g + 19s = 391 \\ 25f + 30g + 22s = 337 \\ 8f + 14g + 12s = 180 \end{cases}$$

This gives the unique solution $g = 66$, $f = -27$ and $s = -44$

$$\boxed{\text{The record is incorrect as } f \text{ and } s \text{ must be positive.}}$$

Problem 2.

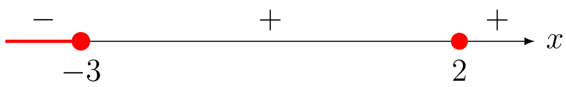
(a) Solve algebraically $x^2 - 9 \geq (x+3)\left(x^2 - 3x + 1\right)$.

(b) Solve algebraically $\dfrac{7-2x}{3-x^2} \leq 1$.

Solution

Part (a)

$$
\begin{aligned}
& & x^2 - 9 &\geq (x+3)\left(x^2 - 3x + 1\right) \\
&\implies & (x+3)(x-3) &\geq (x+3)\left(x^2 - 3x + 1\right) \\
&\implies & (x+3)\left(x^2 - 4x + 4\right) &\leq 0 \\
&\implies & (x+3)(x-2)^2 &\leq 0
\end{aligned}
$$



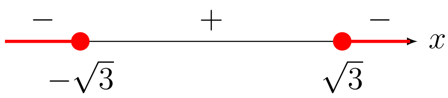
$$\boxed{x \leq -3 \lor x = 2}$$

Part (b)

$$
\begin{aligned}
& & \frac{7-2x}{3-x^2} &\leq 1, \qquad x \neq \pm\sqrt{3} \\
&\implies & \frac{7-2x}{3-x^2} - \frac{3-x^2}{3-x^2} &\leq 0 \\
&\implies & \frac{x^2 - 2x + 4}{3-x^2} &\leq 0
\end{aligned}
$$

Observe that $x^2 - 2x + 4 = (x-1)^2 + 3 > 0$. Hence,

$$
\begin{aligned}
\frac{1}{3-x^2} &\leq 0 \\
3 - x^2 &\leq 0
\end{aligned}
$$



$$\boxed{x < -\sqrt{3} \lor x > \sqrt{3}}$$

Problem 3.

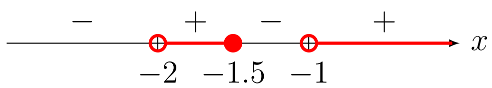
(a) Without using a calculator, solve the inequality $\dfrac{3x+4}{x^2+3x+2} \geq \dfrac{1}{x+2}$.

(b) Hence, deduce the set of values of x that satisfies $\dfrac{3\,|x|+4}{x^2+3\,|x|+2} \geq \dfrac{1}{|x|+2}$.

Solution

Part (a)

$$
\begin{aligned}
&& \frac{3x+4}{x^2+3x+2} &\geq \frac{1}{x+2}, && x \neq -2 \\
\implies && \frac{3x+4}{(x+2)(x+1)} &\geq \frac{1}{x+2}, && x \neq -1 \\
\implies && (3x+4)(x+2)(x+1) &\geq (x+2)(x+1)^2 \\
\implies && (x+2)(x+1)(2x+3) &\geq 0
\end{aligned}
$$



$$\boxed{-2 < x \leq -\frac{3}{2} \vee x > -1}$$

Part (b)

Observe that $|x|^2 = x^2$. Hence, with the map $x \mapsto |x|$, we obtain

$$-2 < |x| \leq -\frac{3}{2} \vee |x| > -1$$

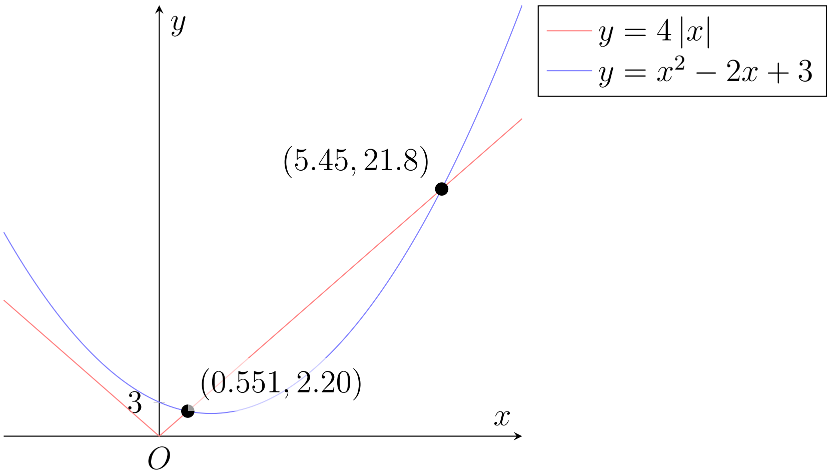
Since $|x| > 0$, we have that $|x| > -1$ is satisfied for all real x.

$$\boxed{\text{The solution set is } \mathbb{R}}$$

Problem 4.

On the same diagram, sketch the graphs of $y = 4|x|$ and $y = x^2 - 2x + 3$. Hence or otherwise, solve the inequality $4|x| \geq x^2 - 2x + 3$.

Solution



$$\boxed{0.551 \leq x \leq 5.45}$$