

Problem 1.

Without using a calculator, sketch the following graphs of conics.

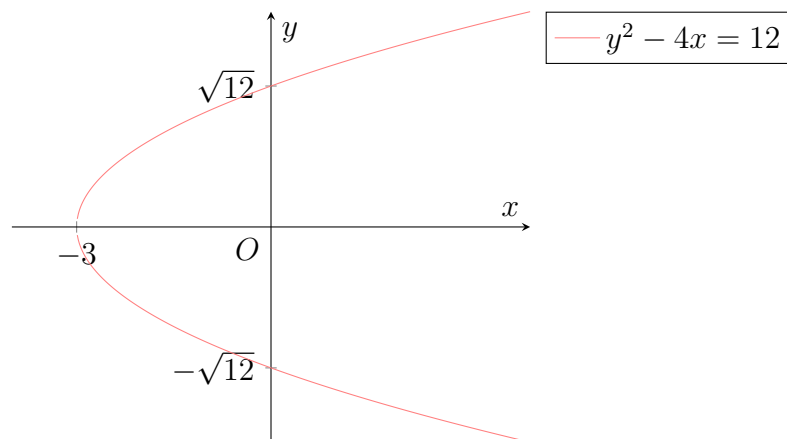
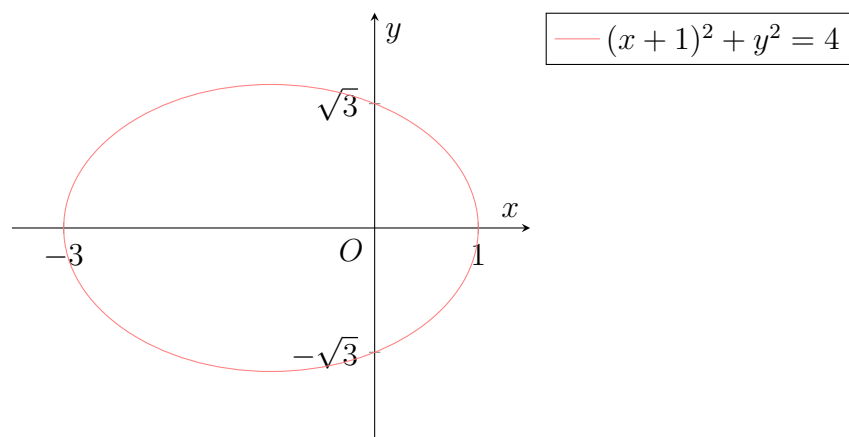
(a) $y^2 - 4x = 12$

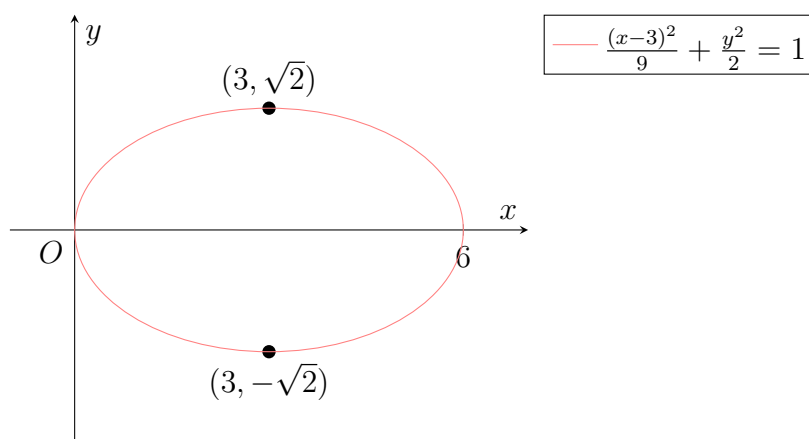
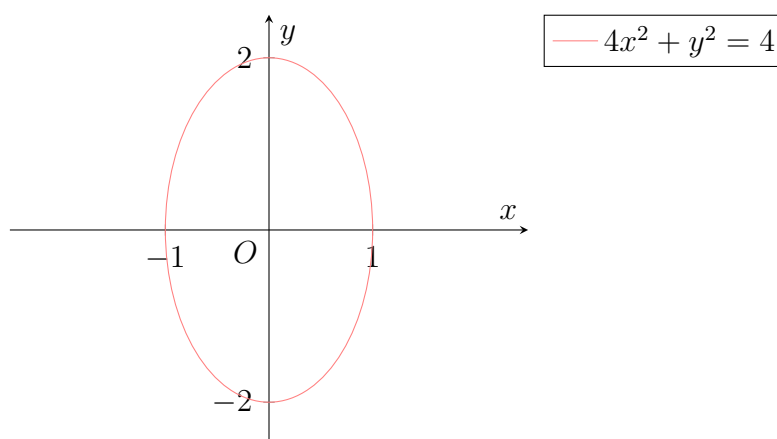
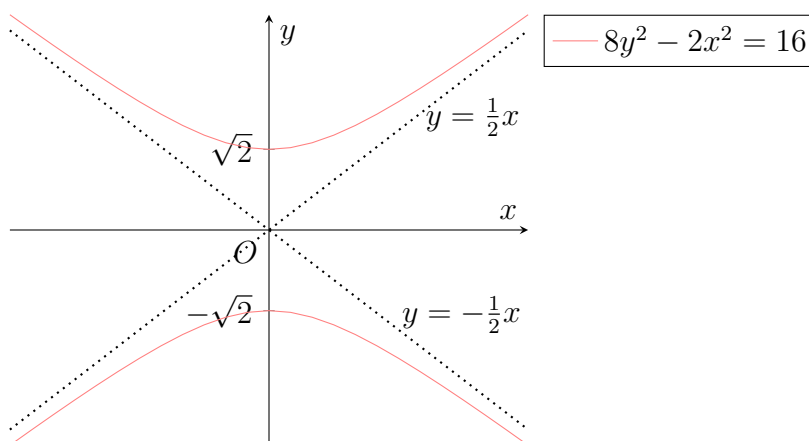
(b) $(x + 1)^2 + y^2 = 4$

(c) $\frac{(x - 3)^2}{9} + \frac{y^2}{2} = 1$

(d) $4x^2 + y^2 = 4$

(e) $8y^2 - 2x^2 = 16$

Solution**Part (a)****Part (b)**

Part (c)**Part (d)****Part (e)**

Problem 2.

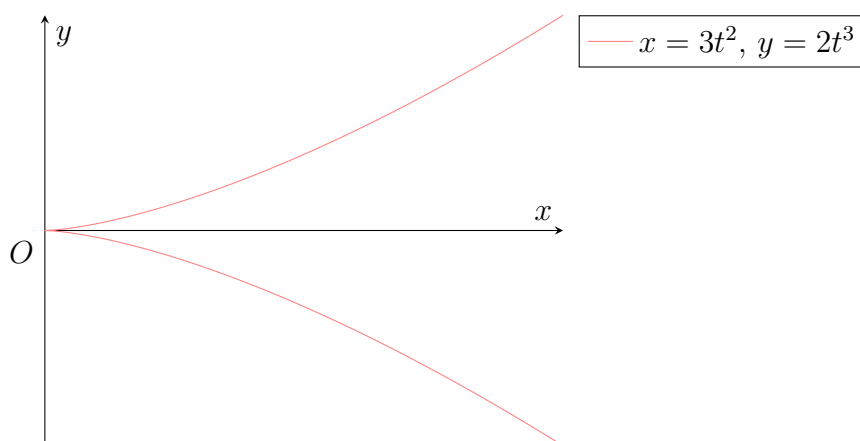
Sketch the curves defined by the following parametric equations, indicating the coordinates of any intersection with the axes.

(a) $x = 3t^2, y = 2t^3$

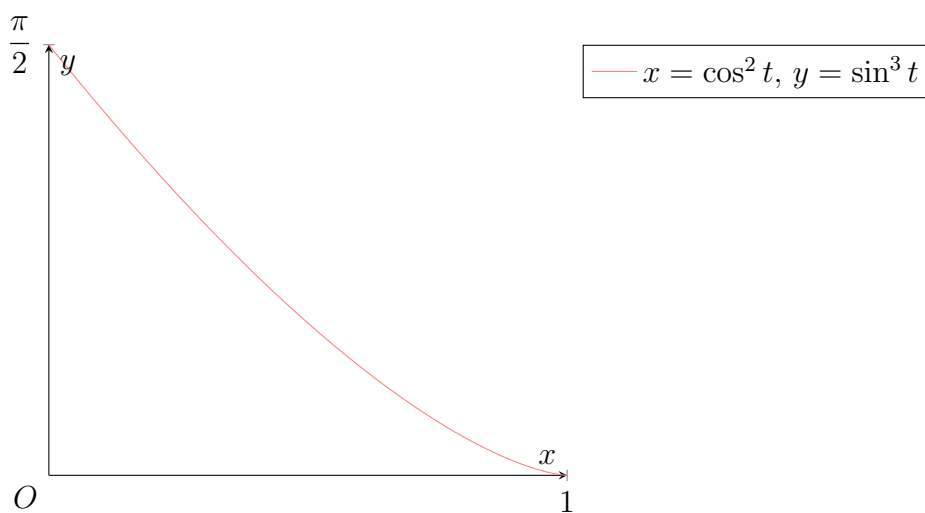
(b) $x = \cos^2 t, y = \sin^3 t, 0 \leq t \leq \frac{\pi}{2}$

Solution

Part (a)



Part (b)



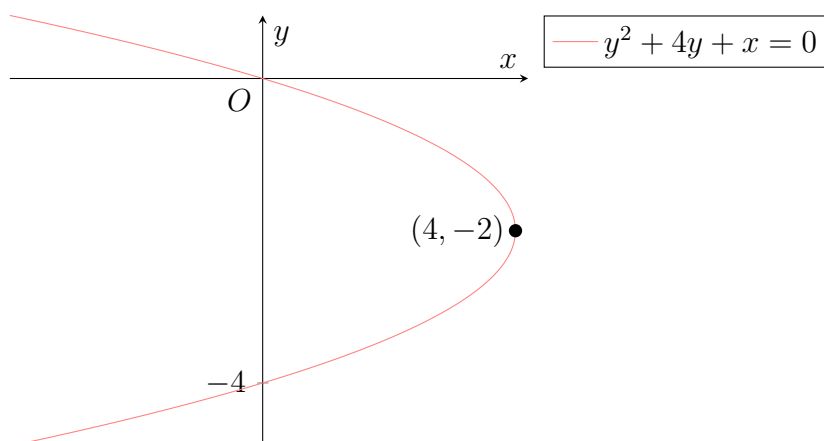
Problem 3.

Without using a calculator, sketch the following graphs of conics.

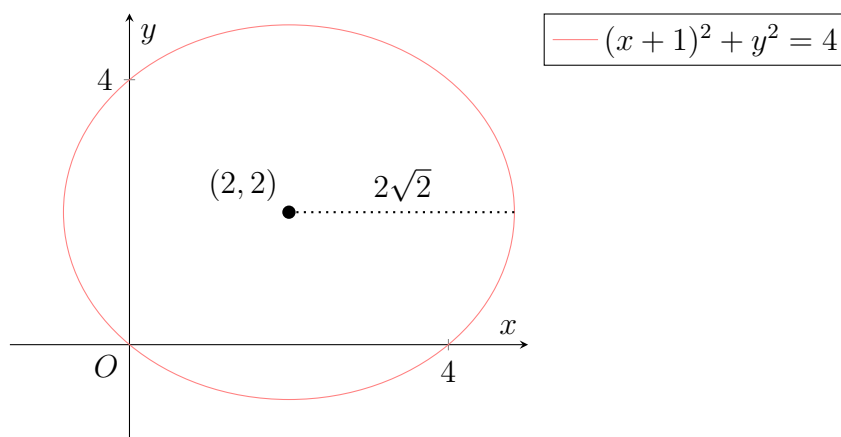
- (a) $y^2 + 4y + x = 0$
- (b) $x^2 + y^2 - 4x - 4y = 0$
- (c) $x^2 + 4y^2 - 2x - 24y + 33 = 0$
- (d) $4x^2 - y^2 - 8x + 4y = 1$
- (e) $x = -\sqrt{17 - y^2}$

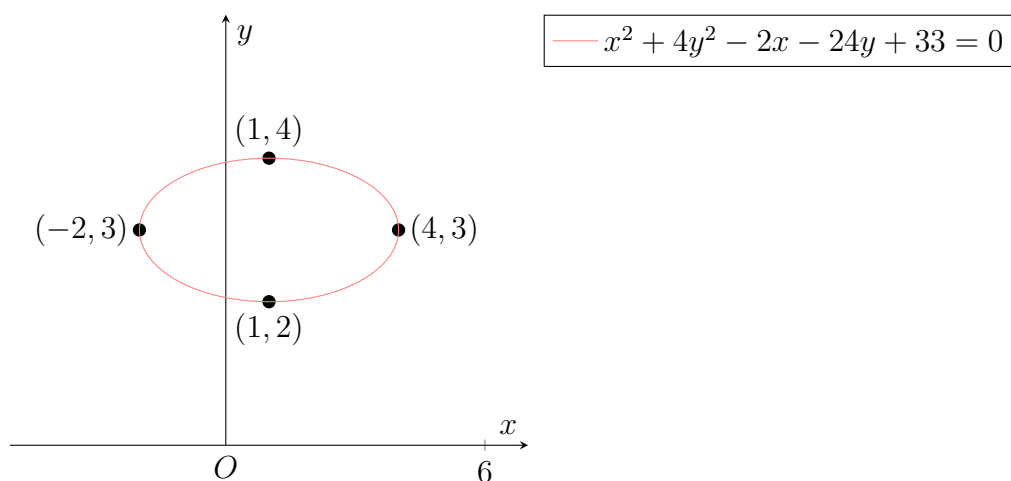
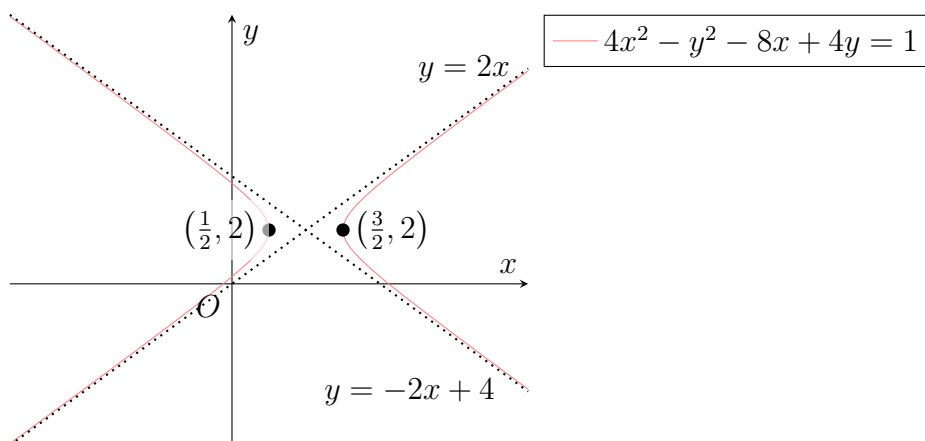
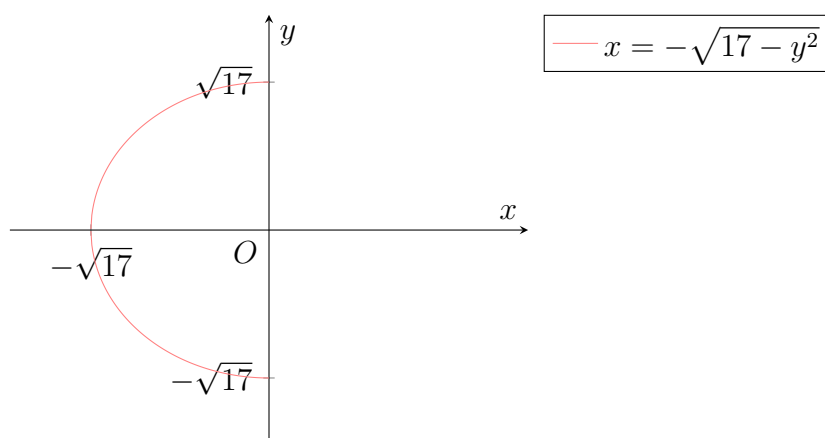
Solution

Part (a)



Part (b)



Part (c)**Part (d)****Part (e)**

Problem 4.

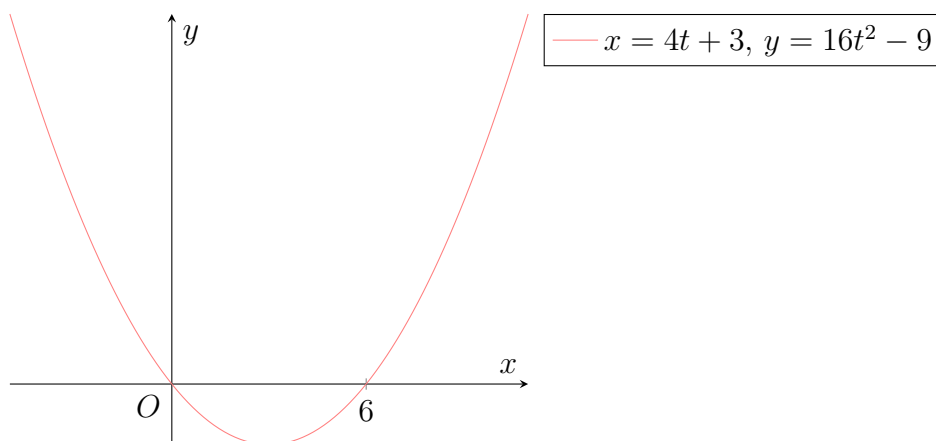
Sketch the curves defined by the following parametric equations. Find also their respective Cartesian equations.

(a) $x = 4t + 3, y = 16t^2 - 9, t \in \mathbb{R}$

(b) $x = t^2, y = 2 \ln t, t \geq 1$

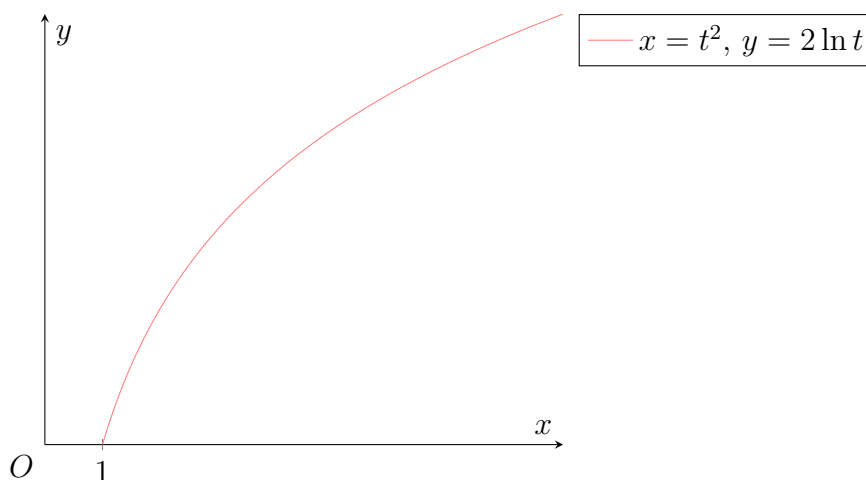
(c) $x = 1 + 2 \cos \theta, y = 2 \sin \theta - 1, 0 \leq \theta \leq \frac{\pi}{2}$

(d) $x = t^2, y = \frac{2}{t}, t \neq 0$

Solution**Part (a)**

Since $x = 4t + 3$, we have $t = \frac{1}{4}(x - 3)$. Thus, $y = 16 \left(\frac{1}{4}(x - 3) \right)^2 - 9 = (x - 3)^2 - 9$.

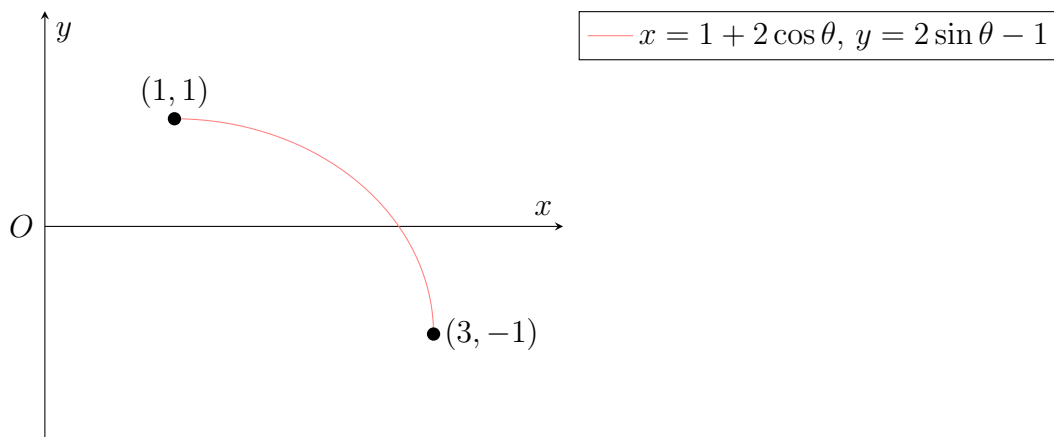
$$y = (x - 3)^2 - 9$$

Part (b)

Since $x = t^2$ and $t \geq 1 > 0$, we have $t = \sqrt{x}$. Thus, $y = 2 \ln(t) = 2 \ln(\sqrt{x}) = \ln(x)$.

$$y = \ln x$$

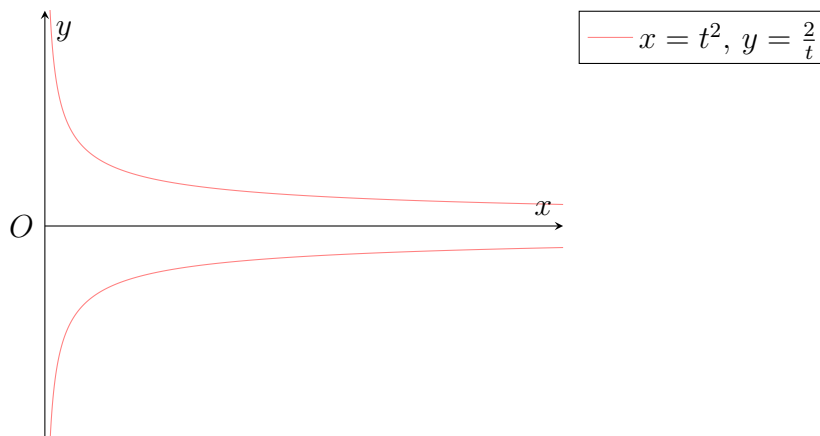
Part (c)



We have $2 \cos \theta = x - 1$ and $2 \sin \theta = y + 1$. Squaring both equations and adding them, we obtain $4 \cos^2 \theta + 4 \sin^2 \theta = (x - 1)^2 + (y + 1)^2$, which simplifies to $(x - 1)^2 + (y + 1)^2 = 4$.

$$(x - 1)^2 + (y + 1)^2 = 4$$

Part (d)



Since $x = t^2$, we have $t = \pm \sqrt{x}$. Hence, $y = \pm \frac{2}{\sqrt{x}}$.

$$y = \pm \frac{2}{\sqrt{x}}$$

Problem 5.

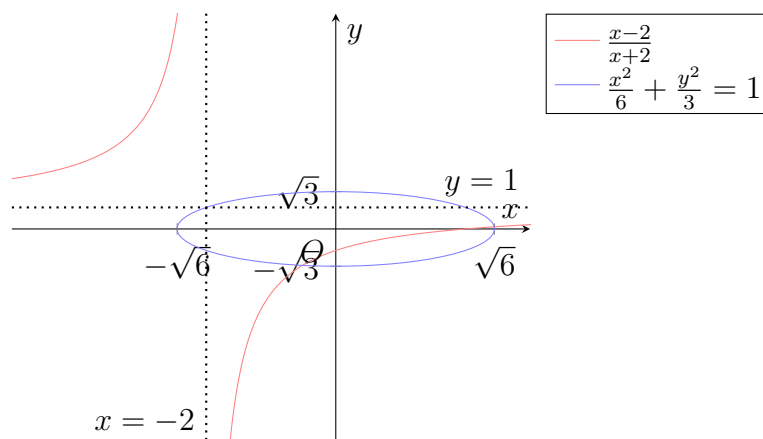
The curve C_1 has equation $y = \frac{x-2}{x+2}$. The curve C_2 has equation $\frac{x^2}{6} + \frac{y^2}{3} = 1$.

- Sketch C_1 and C_2 on the same diagram, stating the exact coordinates of any points of intersections with the axes and the equations of any asymptotes.
- Show algebraically that the x -coordinates of the points of intersection of C_1 and C_2 satisfy the equation $2(x-2)^2 = (x+2)^2(6-x^2)$.
- Use your calculator to find these x -coordinates.

Another curve is defined parametrically by

$$x = \sqrt{6} \cos \theta, \quad y = \sqrt{3} \sin \theta, \quad -\pi \leq \theta \leq \pi$$

- Find the Cartesian equation of this curve and hence determine the number of roots to the equation $\sqrt{3} \sin \theta = \frac{\sqrt{6} \cos \theta - 2}{\sqrt{6} \cos \theta + 2}$ for $-\pi \leq \theta \leq \pi$.

Solution**Part (a)****Part (b)**

From C_1 , we have $y(x+2) = x-2$. Hence,

$$y^2(x+2)^2 = (x-2)^2$$

From C_2 , we have $x^2 + 2y^2 = 6$. Hence,

$$y^2 = \frac{6-x^2}{2}$$

Putting both equations together, we have

$$\begin{aligned} (x-2)^2 &= \frac{(6-x^2)(x+2)^2}{2} \\ \implies 2(x-2)^2 &= (6-x^2)(x+2)^2 \end{aligned}$$

Part (c)

$$x = -0.515 \vee x = 2.45$$

Part (d)

Since $x = \sqrt{6} \cos \theta$ and $y = \sqrt{3} \sin \theta$, we have $x^2 = 6 \cos^2 \theta$ and $2y^2 = 6 \sin^2 \theta$. Adding both equations together, we have

$$\begin{aligned} x^2 + 2y^2 &= 6 \cos^2 \theta + 6 \sin^2 \theta \\ &= 6 \\ \implies \frac{x^2}{6} + \frac{y^2}{3} &= 1 \end{aligned}$$

$$\frac{x^2}{6} + \frac{y^2}{3} = 1$$

This is the equation that gives C_1 . We further observe that the equation $\sqrt{3} \sin \theta = \frac{\sqrt{6} \cos \theta - 2}{\sqrt{6} \cos \theta + 2}$ simplifies to $y = \frac{x - 2}{x + 2}$. This is the equation that gives C_2 . Since there are two intersections between C_1 and C_2 , there are thus two roots to the equation $\sqrt{3} \sin \theta = \frac{\sqrt{6} \cos \theta - 2}{\sqrt{6} \cos \theta + 2}$.

$$\text{There are 2 roots to } \sqrt{3} \sin \theta = \frac{\sqrt{6} \cos \theta - 2}{\sqrt{6} \cos \theta + 2}.$$