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习题4

2.解 $E(X_1)=\sum x_i p(x_i)=50$

$E(X_2)=50$

$D(X_1)=E(X-E(X))^2=1$

$D(X_2)=2 \quad > D(X_1)$

∴方法甲的测定精度更高

5.解 $f(x)=\begin{cases}\frac{1}{\pi\sqrt{1-x^2}} & -1<x<1\\ 0 & 其它\end{cases}$

$E(X)=\int_{-1}^{1} x\cdot\frac{1}{\pi\sqrt{1-x^2}}dx=-\frac{1}{2\pi}\int_{-1}^{1}(1-x^2)^{-\frac{1}{2}}d(1-x^2)=0$

~~D(X)~~, $E(X^2)=\int_{-1}^{1}x^2\cdot\frac{1}{\pi\sqrt{1-x^2}}dx$

$=2\int_0^1 x^2\frac{1}{\pi\sqrt{1-x^2}}dx=\frac{1}{2}$

$D(X)=E(X^2)-E^2(X)=\frac{1}{2}-0=\frac{1}{2}$

6.解 $E(X)=\sum_{x=0}^{k} x\cdot\frac{1}{1+a}\left(\frac{a}{1+a}\right)^{x}=$ 令 $q=\frac{a}{1+a}$

令 $\sum_{x=0}^{k}xq^x=S=0+1q^1+2q^2+\cdots+xq^x$

$qS=\quad q^1+1q^2+\cdots+(x-1)q^x+xq^{x+1}$

$-(q-1)S=q^2+q^3+\cdots+q^x-xq^{x+1}$

$(1-q)S=\frac{q^2(q^{x-1}-1)}{q-1}=\frac{q^2-q^{x+1}}{1-q} \qquad S=\frac{q^2-q^{x+1}}{(1-q)^2}-\frac{xq^{x+1}}{1-q}$

$\therefore E(X)=\frac{1}{1+a}\cdot\left(q^2-(1+a)^2\cdot\left(\frac{a}{1+a}\right)^{x+1}\right)=\cancel{\frac{q^2a^2}{1+a}-(1+a)\cdot\left(\frac{a}{1+a}\right)^{x+1}}$

$=a$

$D(X)=E(X^2)-E^2(X)=a+a^2$

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29 解 $E(X)=7300$ $D(X)=700^2$

记 $\varepsilon=2100$

$$P(|X-E(X)|<\varepsilon) \geq 1-\frac{D(X)}{\varepsilon^2} = 1-\frac{(700)^2}{(2100)^2} = \frac{8}{9}$$

32 解 当 $18\cdot(10000-x) = (2500-18)\cdot x$ 时亏本

即 $x \geq 72$ 时保险公司亏本

设出意外的人数为 X, $X_i = \begin{cases} 1, & 0.006 \\ 0, & 0.994 \end{cases}$ $E(X_i)=0.006$

$E(X) = E(X_1)+E(X_2)+\cdots+E(X_{10000}) = 10000 \times 0.006 = 60$

X 近似服从 $N(60, 59.64)$

$$P(X \geq 72) = P\left(\frac{X-60}{\sqrt{59.64}} \geq \frac{12}{\sqrt{59.64}}\right) = 1-\phi(1.55) = 0.061$$

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24. 解 $f_X(x)=\int_0^2 \frac{1}{8}x+\frac{1}{8}y\ dy=\frac{1}{4}x+\frac{1}{4}$ $\quad f_Y(y)=\frac{1}{4}y+\frac{1}{4}$

$E(X)=\int_0^2 \frac{1}{4}x+\frac{1}{4}\ dx=1 \quad E(Y)=\int_0^2 \frac{1}{4}y+\frac{1}{4}\ dy=1$

$E(XY)=\int_0^2 dx\int_0^2 \frac{1}{8}(x+y)\cdot xy\ dy=1$

$cov(X,Y)=E(XY)-E(X)E(Y)=0$

$\rho_{XY}=0$

(1) $E(Z)=E(\frac{1}{3}X+\frac{1}{2}Y)=\frac{1}{3}E(X)+\frac{1}{2}E(Y)=\frac{1}{3}\cdot 1+\frac{1}{2}\cdot 0=\frac{1}{3}$

26 解 $D(Z)=D(\frac{1}{3}X+\frac{1}{2}Y)=D(\frac{1}{3}X)+D(\frac{1}{2}Y)+2\cdot\frac{1}{6}cov(X,Y)$

$=\frac{1}{9}D(X)+\frac{1}{4}D(Y)+\frac{1}{3}cov(X,Y)$

$cov(X,Y)=\rho\sqrt{D(X)}\sqrt{D(Y)}=-\frac{1}{2}\times 3\times 4=-6$

$=3$

(2) $Z\sim N(\frac{1}{3},5) \quad X\sim N(1,9)$

~~E(XZ)=~~ $\rho_{XZ}=\frac{E(XZ)-E(X)E(Z)}{\sqrt{5}\cdot\sqrt{9}}=\frac{E(XZ)-\frac{1}{3}}{3\sqrt{5}}$

$E(XZ)=E(X\cdot(\frac{1}{3}X+\frac{1}{2}Y))=E(\frac{1}{3}X^2+\frac{1}{2}XY)=\frac{1}{3}E(X^2)+\frac{1}{2}E(XY)=$

$=\frac{1}{3}(DX+E^2X)+\frac{1}{2}E(XY)=\frac{10}{3}+\frac{1}{2}\times(cov(X,Y)+E(X)E(Y))$

$=\frac{1}{3}$

$\therefore \rho_{XZ}=0$

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14 $f_X(x) = \int_0^1 x+y \, dy = x+\frac{1}{2}$

$f_Y(y) = \int_0^1 x+y \, dx = y+\frac{1}{2}$

$E(X) = \int_0^1 (x+\frac{1}{2})x \, dx = \frac{7}{12}$ $\qquad E(Y) = \int_0^1 (y+\frac{1}{2})y \, dy = \frac{7}{12}$

$E(XY) = \int_0^1 dx \int_0^1 xy(x+y) \, dy = \frac{1}{3}$

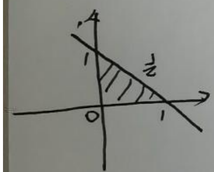
17. 解 $E(X+Y)^2 = E(X^2+Y^2+2XY)$

$= E(X^2)+E(Y^2)+2E(XY)$

$= E(X^2)+E(Y^2)+2[E(X)E(Y) + \mathrm{cov}(X,Y)]$

$= 2+2+2[0+0.5] = 5$

23. 解 $f(x,y) = \begin{cases} 2, & 0\le x\le 1 \quad 0\le y\le 1-x \\ 0 & 其它 \end{cases}$



$E(XY) = \int_0^1 dx \int_0^{1-x} 2xy \, dy = \frac{1}{12}$

$f_X(x) = \int_0^1 dx \int_0^{1-x} 2 \, dy = 2-2x$ $\qquad E(X^2) = \int_0^{1-x} (2-2x)x^2 \, dx = \frac{1}{6}$

$E(Y^2) = \frac{1}{6}$

$E(X) = \int_0^1 (2-2x)x \, dx = \frac{1}{3}$

$f_Y(y) = \int_0^{1-y} 2 \, dx = 2-2y$ $\qquad E(Y) = \int_0^1 (2-2y)y \, dy = \frac{1}{3}$

$\mathrm{Cov}(X,Y) = E(XY)-E(X)E(Y) = -\frac{1}{36}$

$D(X) = EX^2 - E^2X = \frac{1}{18}$ $\qquad D(Y) = \frac{1}{18}$

$\rho_{XY} = \dfrac{-\frac{1}{36}}{\sqrt{\frac{1}{18}}\sqrt{\frac{1}{18}}} = -\frac{1}{2}$