山东大学

习题3.1

1. $\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix} = 3\times(\overset{-13}{-3-10}) - 0 + 4\times(10-0) = 1$

$\quad\quad -39 \quad\quad +40$

$\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix} = -0 + 3\times|-3| - 5\times(6-8) = 1$

$\quad\quad -9 \quad +(-10)$

3T $\begin{vmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{vmatrix} = 2\times(-1-6) - (-2)\times(-3-2) + 3\times(9-1) = 0$

$\quad\quad -14 \quad -10 \quad +24$

$\begin{vmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{vmatrix} = -(-2)(\overset{-5}{-3-2}) + 1\times(-2-3) - 3\times(\underset{-5}{4-9}) = 0$

$\quad\quad -10 \quad -5 \quad +15$

10T. $\begin{vmatrix} 1 & -2 & 3 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -4 & 3 & 5 \\ 2 & 0 & 3 & 5 \end{vmatrix} = -3\times\begin{vmatrix} 1 & -2 & 2 \\ 2 & -4 & 5 \\ 2 & 0 & 5 \end{vmatrix}$

$= -3\times[2\times(-10+8) + 5\times(-4+4)]$

$= -3\times 2\times(-2) = 12$

12T 解

$$\begin{vmatrix} \cancel{3} & \cancel{0} & \cancel{0} & \cancel{0} \\ 7 & -2 & 0 & 0 \\ 2 & 0 & 3 & 0 \\ 3 & -8 & 4 & -3 \end{vmatrix} = 3\times\begin{vmatrix} -2 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} = 3\times(-2)\times 3\times(-3) \overset{27\times2\ 54}{=} 54$$

14T. $$\begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ \cancel{2} & \cancel{0} & \cancel{0} & \cancel{0} & 0 \\ 4 & 2 & 3 & 2 & 0 \end{vmatrix} = -2\times\begin{vmatrix} 3 & 2 & 4 & 0 \\ 0 & -4 & 1 & 0 \\ \cancel{-5} & \cancel{6} & \cancel{7} & \cancel{1} \\ 2 & 3 & 2 & 0 \end{vmatrix} = -2\times(-1)\times\begin{vmatrix} 3 & 2 & 4 \\ 0 & -4 & 1 \\ 2 & 3 & 2 \end{vmatrix}$$

$$= 2\times\left(3\times\overset{-11}{(-8-3)} + 2\times\overset{-18\times2\ 36}{(-16-2)}\right) = 2\times\overset{-69}{(-33-36)} = -138$$

15T. $= 1\times3\times(-2) + 0\times2\times0 + 4\times2\times5 - 0 - 5\times2\times1 - 0$

$= -6 + 40 - 10 = 24$

19T 行交换，变为负值；

20T. 2行×k 加到1行，不变

21T. 2行×k，值×k

22T 1行×k加到2行，不变

习题3.2

5T 解

$$\begin{vmatrix} 1 & 5 & -4 \\ -1 & -4 & 5 \\ -2 & -8 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 5 & -4 \\ 0 & 1 & 1 \\ 0 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 5 & -4 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{vmatrix} = 1\times1\times3=-3$$

11T 解

$$\begin{vmatrix} 3 & 4 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 3 \\ 6 & 8 & -4 & -1 \end{vmatrix} = \begin{vmatrix} 3 & 4 & -3 & 1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 3 \\ 0 & 0 & 2 & 1 \end{vmatrix} = -4\times\begin{vmatrix} 3 & 1 & -3 \\ -6 & -4 & 3 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= -4\times\begin{vmatrix} 3 & 1 & -3 \\ 0 & -2 & -3 \\ 0 & 2 & 1 \end{vmatrix} = -4\times\begin{vmatrix} 3 & 1 & -3 \\ 0 & -2 & -3 \\ 0 & 0 & -2 \end{vmatrix} = -4\times3\times-2\times-2=-48$$

21T
$$\begin{vmatrix} 2 & 6 & 0 \\ -1 & 3 & 2 \\ 3 & 9 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 0 \\ 0 & 0 & -2 \\ 3 & 9 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{vmatrix} = 0$$ ∴不可逆

24T
$$\begin{vmatrix} 4 & -7 & -3 \\ 6 & 0 & -5 \\ 2 & 7 & -2 \\ \cancel{-4} & \cancel{-14} & \cancel{4} \end{vmatrix} = \begin{vmatrix} 4 & -7 & -3 \\ 6 & 0 & -5 \\ 0 & \frac{21}{2} & -\frac{1}{2} \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 4 & -7 & -3 \\ 6 & 0 & -5 \\ 0 & 21 & -1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 4 & -7 & -3 \\ 0 & \frac{21}{2} & -\frac{1}{2} \\ 0 & 21 & -1 \end{vmatrix} = 0$$

∴不可 ~~逆~~ 线性相关

25T
$$\begin{vmatrix} 7 & -8 & 7 \\ -4 & 5 & 0 \\ -6 & 7 & -5 \end{vmatrix} = \begin{vmatrix} 7 & -8 & 7 \\ 0 & \frac{3}{7} & 4 \\ -6 & 7 & -5 \end{vmatrix} = \begin{vmatrix} 7 & -8 & 7 \\ 0 & \frac{3}{7} & 4 \\ 0 & \frac{1}{7} & 1 \end{vmatrix} = \begin{vmatrix} 7 & -8 & 7 \\ 0 & \frac{3}{7} & 4 \\ 0 & 0 & -\frac{1}{3} \end{vmatrix} \neq 0$$

∴线性无关

26T 解

$$-\tfrac{5}{3} \quad -5 \quad -\tfrac{10}{3} \quad \tfrac{10}{3}$$

$$6 \quad 4 \quad -4$$

$$\begin{vmatrix} 3 & 2 & -2 & 0 \\ 5 & -6 & -1 & 0 \\ -6 & 0 & 3 & 0 \\ \cancel{4} & \cancel{7} & \cancel{0} & \cancel{-2} \end{vmatrix} = -2\times\begin{vmatrix} 3 & 2 & -2 \\ 5 & -6 & -1 \\ -6 & 0 & 3 \end{vmatrix} = -2\begin{vmatrix} 3 & 2 & -2 \\ 5 & -6 & -1 \\ 0 & 4 & -1 \end{vmatrix} = -2\begin{vmatrix} 3 & 2 & -2 \\ 0 & -\frac{28}{3} & \frac{7}{3} \\ 0 & 4 & -1 \end{vmatrix}$$

$\neq 0$ ∴ 线性无关

29T 解

$$-1 \quad 0 \quad -1$$

$$\det B^4 = (\det B)^4 = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}^4 = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 0 \end{vmatrix}^4 = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{vmatrix}^4 = (-2)^4 = 16$$

31T 解

$$\det I = \det AA^{-1} = \det A \det A^{-1} = 1$$

$$\therefore \det A^{-1} = \frac{1}{\det A}$$

40T 解

a) $= \cancel{4}\,3$

b) $(\det B)^5 = -1$

c) $2\det A = -6$

d) $\det A^T \det B \det A = \det A \det A \det B = 9\times 4 = 36$

e) $\frac{1}{\det B}\det A \det B = -3$

习题 3.3

1. 解

$$\begin{bmatrix} 5 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\det A = 20-14=6$

$$x_1 = \frac{\det A_1(\vec{b})}{\det A} = \frac{\begin{vmatrix} 3 & 7 \\ 1 & 4 \end{vmatrix}}{6} = \frac{12-7}{6} = \frac{5}{6}$$

$$x_2 = \frac{\det A_2(\vec{b})}{\det A} = \frac{\begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix}}{6} = \frac{5-6}{6} = -\frac{1}{6}$$

5T 解

$$A=\begin{bmatrix} 1 & 1 & 0 \\ -3 & 0 & 2 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

$$\det A = -\begin{vmatrix} 0 & 1 & -2 \\ -3 & 0 & 2 \\ 1 & 1 & 0 \end{vmatrix} -2+(-1)(6)+0 = -2-6=-8$$

$$x_1 = \frac{\begin{vmatrix} 3 & 1 & 0 \\ 0 & 0 & 2 \\ 2 & 1 & -2 \end{vmatrix}}{-8} = \frac{-2(3-2)}{-8} = \frac{1}{4}$$

$$x_2 = \frac{\begin{vmatrix} 1 & 3 & 0 \\ -3 & 0 & 2 \\ 0 & 2 & -2 \end{vmatrix}}{-8} = \frac{\overset{-4\ -18}{-4-3(6-0)}}{-8} = \frac{-22}{-8} = \frac{11}{4}$$

$$x_3 = \frac{\begin{vmatrix} 1 & 1 & 3 \\ -3 & 0 & 0 \\ 0 & 1 & 2 \end{vmatrix}}{-8} = \frac{-(-3)\times(2-3)}{-8} = +\frac{3}{8}$$

11T 解

$C_{11}=0 \quad C_{12}=2\times5=-10$

$C_{13}=-1\times(5)=-5$

$C_{21}=-5\times(2+1)=-15$

$C_{22}=0 \quad C_{23}=0$

$C_{31}=-1\times0=0 \quad C_{32}=-1\times(5)=-5$

$C_{33}=1\times(-10)=-10$

$$\begin{bmatrix} 0 & -15 & 0 \\ -10 & 0 & -5 \\ -5 & 0 & -10 \end{bmatrix}$$

$\det A = -5\times(2+1)=-15$

$$\therefore A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 1/3 & 0 & 2/3 \end{bmatrix}$$

19T 解

$$\begin{vmatrix} 5 & 6 \\ 2 & 4 \end{vmatrix} = 20-12=8$$

21T 解

$$\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = 3$$

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补充练习

7T 解 $\det\begin{bmatrix} 1 & x & y \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{bmatrix} = x_1y_2 - x_2y_1 + (-1)(xy_2 - x_2y) + xy_1 - x_1y$

$= x_1y_2 - x_2y_1 - xy_2 + x_2y + xy_1 - x_1y$

$= (y_1 - y_2)x + (x_2 - x_1)y + x_1y_2 - x_2y_1 = 0$

经检验，过 (x_1,y_1) (x_2,y_2) 两点，得证.

14T 解

a) $\det\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} = \det AI = \det A$

b) $\det\begin{bmatrix} I & 0 \\ C & D \end{bmatrix} = \det ID = \det D$

c) $\det\begin{bmatrix} A & 0 \\ C & D \end{bmatrix} = \det AD = \det A \det D$ 之

~~d) det~~ $\det\begin{bmatrix} A & B \\ 0 & D \end{bmatrix} = \det AD = \det A \det D$

16T 解

a) $A = \begin{bmatrix} a & b & b & \cdots & b \\ a-b & b-a & 0 & \cdots & 0 \\ 0 & a-b & b-a & \cdots & 0 \\ & & \cdots & a-b & b-a \\ 0 & \cdots & & & \end{bmatrix}$

行倍数 $\times(-1)$ 加到另一行 不会改变值

b) $A = \begin{bmatrix} a & a+b & a+2b & a+3b & \cdots & a+(n-1)b \\ a-b & 0 & 0 & \vdots & & \vdots \\ 0 & a-b & 0 & \vdots & & \vdots \\ \vdots & 0 & \vdots & & a-b & \\ 0 & 0 & 0 & & & \end{bmatrix}$

拥有于倒置后的行变换

不变值

~~detA(b)~~ $\begin{bmatrix} a-b & b-a & 0 & 0 & \cdots & 0 \\ 0 & a-b & b-a & 0 & \cdots & 0 \\ \vdots\,0 & 0 & a-b & b-a & \cdots & 0 \\ b & b & b & b & \cancel{a-b}\ b & \cancel{b-a}\ a \end{bmatrix}$

$\begin{bmatrix} a-b & 0 & b-a & b-a \\ 0 & a-b & 0 & 0 \\ \vdots & \vdots & a-b & 0 \\ b & 2b & 3b & \cancel{a-b} = a+(n-1)b \end{bmatrix}$

阶梯形：

$\det A = (a-b)^{n-1}(a+(n-1)b)$