第二章

5. 解 $P(X=n)=(\frac{1}{4})^{n-1}\times\frac{3}{4}$

$P(X为偶数)=\frac{1}{4}\times\frac{3}{4}+(\frac{1}{4})^3\times\frac{3}{4}+\cdots+(\frac{1}{4})^{2n-1}\times\frac{3}{4}$

$=(\frac{1}{4}+(\frac{1}{4})^3+\cdots+(\frac{1}{4})^{2n-1})\times\frac{3}{4}$

$=\frac{3}{15}(1-(\frac{1}{4})^{2n})$

10. 解 设配有 n 为维修工

$P(\text{坏}\overset{X}{\geq}n\text{台})=\sum C_{100}^{n}(0.01)^n(0.09)^{100-n}\leq 0.01$

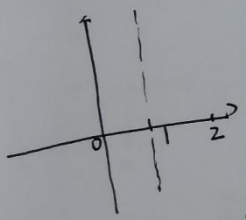
根据泊松定理，$\lambda=np=1$

$\sum_{k=n+1}^{\infty}\frac{1^k}{k!}e^{-1}\leq 0.01$. 解得 $k>4$ $n+1>4$ $n>3$

∴至少 4 为工人

15. 解 ∵ $f(1+x)=f(1-x)$

∴ $f(x)$ 关于 $x=1$ 对称



$\int_0^2 f(x)dx=0.6$

∴ $\int_0^1 f(x)dx=0.3$

又 $\int_{-\infty}^{1} f(x)dx=0.5$

∴ $P(X<0)=\int_{-\infty}^{0} f(x)dx=\int_{-\infty}^{1} f(x)dx-\int_0^1 f(x)dx$

$=0.2$

20. 解 $f(x) = \lambda e^{-\lambda x}$, $x>0$

$F(x) = 1-e^{-\lambda x}$, $x>0$

$F(x) = 1-e^{-0.1x}$

(1) $P(X>10) = 1-P(X\le 10) = 1-(1-e^{-0.1\times 10}) = e^{-1} = \frac{1}{e}$

(2) $P(10<X\le 20) = P(X\le 20) - P(X\le 10)$

$= (1-e^{-0.1\times 20}) - (1-e^{-0.1\times 10})$

$= \frac{1}{e} - \frac{1}{e^2}$

25. 解 $P(|X|>19.6) \ge 3\times\frac{1}{100}$

每一次, $P(-19.6<X<19.6) = F(19.6) - F(-19.6)$

$= \Phi(\frac{19.6}{10}) - \Phi(-\frac{19.6}{10}) \approx 0.94256$

~~$P(X\ge 3) = 1-[P(0)+P(1)+P(2)] = 1-(\overline{p})^3 - C_{100}^{\dagger}$~~

$P(X\ge 3) \approx \sum_{k=0}^{\infty} e^{-5.1744} \times \frac{(5.1744)^k}{k!} \approx 0.875348$

30. 解

$F(x) = \begin{cases} 0, & x<0 \\ \frac{1}{6}x, & 0\le x<6 \\ 1, & x\ge 6 \end{cases}$

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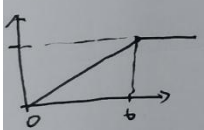

$Y=|X-3|$, $x<0$ 时, $y<0$

$0\le x<6$ 时, $0\le y<3$

$x\ge 6$ 时, $y\ge 3$

$\therefore F(Y) = \begin{cases} \frac{1}{3}y, & 0\le y<3 \\ 1, & y\ge 3 \end{cases}$

$f(Y) = \begin{cases} \frac{1}{3}, & 0\le y<3 \\ 0, & \text{其余} \end{cases}$

~~$x\ge 3$ 时~~ $F(Y) = P(Y\le y)$

~~$f(Y) =$~~

$= P(|X-3|\le y)$

$= P(3-y\le X\le 3+y)$

$= P(X\le 3+y) - P(X\le 3-y)$

~~$= P(Y\le X+3) - P(X\le -X+3)$~~

$0\le|X-3|<3$ 或 $0\le|X-3|<3$ 或 $|X-3|\ge 3$

$= F_Y(3+x) - F_Y(x-3)$

$= F_X(3+y) + F_X(3-y)$

$=$