习题3

1 $P(1<X\le 2, 3<Y\le 5)$

$= F(2,5) - F(2,3) - F(1,5) + F(1,3)$

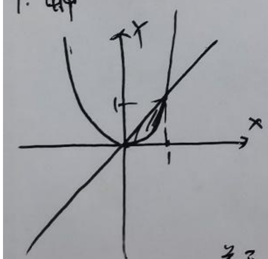
$= 2^{-4}+2^{-7}-2^{-5}-2^{-6}$

$= \frac{3}{128}$

2. 解

X \ Y	1	2	3	4	Y
1	$\frac{1}{4}$	0	0	0	$\frac{1}{4}$
2	$\frac{1}{8}$	$\frac{1}{8}$	0	0	$\frac{1}{4}$
3	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	0	$\frac{1}{4}$
4	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
X	$\frac{25}{48}$	$\frac{13}{48}$	$\frac{7}{48}$	$\frac{1}{16}$	1

9. 解



面积 $S=\int_0^1\int_{x^2}^{x} 1\,dy\,dx$

$=\int_0^1 (x-x^2)\,dx = \frac{1}{6}$

$\therefore f(x,y) = \begin{cases} 6, & (x,y)\in D \\ 0, & 其它 \end{cases}$

关于X的边缘概率密度，$f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy$

$=\int_{x^2}^{x} 6\,dy = 6(x-x^2)$

关于Y：$f_Y(y)=\int_{y}^{\sqrt{y}} 6\,dx = 6(\sqrt{y}-y)$，

$\therefore f_X(x) = \begin{cases} 6(x-x^2), & 0<x\le 1 \\ 0, & 其它 \\ \cancel{6(\sqrt{y}-y),\ 0\le} \end{cases}$

$f_Y(y) = \begin{cases} 6(\sqrt{y}-y), & 0\le y\le 1 \\ 0, & 其它 \end{cases}$

24. 解

X: 1 2 Y. 1 2 独立

$\frac{2}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{3}$

$F_U(U) = P(\max\{X,Y\} \le U) = P(X \le U, Y \le U) = F_X(U)\,F_Y(U)$

$\begin{cases} X=1 \\ Y=1 \end{cases}$ $U=V=1$, $P=\frac{4}{9}$ $\begin{cases} X=1 \\ Y=2 \end{cases}$ / $\begin{matrix} X=2 \\ Y=1 \end{matrix}$ $U=2$ $V=1$, $P=\frac{2}{9}+\frac{2}{9}=\frac{4}{9}$

$\begin{cases} X=2 \\ Y=2 \end{cases}$ $U=V=2$ $P=\frac{1}{9}$

∴

V \ U	1	2
1	$\frac{4}{9}$	$\frac{4}{9}$
2	0	$\frac{1}{9}$

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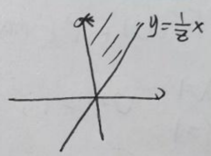
20. 解 $f(x)=\begin{cases}\lambda_1 e^{-\lambda_1 x}, & x>0\\ 0, & x\le 0\end{cases}$ $f(y)=\begin{cases}\lambda_2 e^{-\lambda_2 y}, & y>0\\ 0, & y\le 0\end{cases}$



$F(z)=P(\frac{X}{Y}\le z)$, $z>0$

$=\iint f(x,y)\,dx\,dy=\int_0^{+\infty}dx\int_{\frac{x}{z}}^{+\infty}f(x,y)\,dy$

$f(x,y)=f(x)f(y)=\begin{cases}\lambda_1\lambda_2 e^{-\lambda_1(x+y)}, & x>0,\ y>0\\ 0, & 其它\end{cases}$

$\therefore F(z)=\int_0^{+\infty}dx\int_{\frac{x}{z}}^{+\infty}\lambda_1\lambda_2 e^{-\lambda_1 x-\lambda_2 y}\,dy=\cancel{z}+\frac{\lambda_1 z}{\lambda_1 z+\lambda_2}$

$f(z)=-\frac{\lambda_1^2 z+\lambda_1\lambda_2-\lambda_1^2 z}{(\lambda_1 z+\lambda_2)^2}=+\frac{\lambda_1\lambda_2}{(\lambda_1 z)+\lambda_2)^2}$, $z>0$

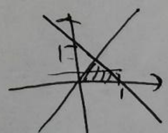
0, 其它

22. 解

(1) $f_X(x)=\begin{cases}1, & 0<x<1\\ 0, & 其它\end{cases}$ $f_{Y|X}(y|x)=\begin{cases}\frac{1}{x}, & 0<y<x\\ 0, & 其它\end{cases}$

$f(x,y)=f_X(x)\cdot f_{Y|X}(y|x)=\begin{cases}\frac{1}{x}, & \cancel{0<x<1,}\ 0<y<x<1\\ 0, & 其它\end{cases}$

(2) $f_Y(y)=\frac{f(x,y)}{f_{X|Y}(x|y)}=\int_y^1\frac{1}{x}\,dx=\begin{cases}-\ln y, & 0<y<1\\ 0, & 其它\end{cases}$

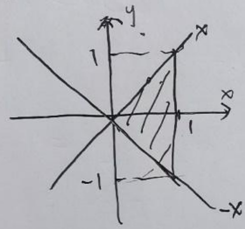


(3) $P(X+Y>1)=\int_0^{\frac{1}{2}}dy\int_y^{1-y}\frac{1}{x}\,dx$

$=\cancel{\ln 2-\frac{3}{8}}\ 1-\ln 2$

$0<y<1$时 / $-1<y<0$时

10.



$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\frac{1}{\int_{-y}^{1}dy \text{ 或 } \int_{y}^{1}dy}=\frac{1}{1-|y|},\quad |y|<x<1$$

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\frac{1}{\int_{-x}^{x}dy}=\frac{1}{2x},\quad \begin{cases}-x<y<x\\ 0<x<1\end{cases}$$

13. 解

(1) $\int_0^1 dx\int_0^1 Axy^2dy=1$

则 $\frac{1}{6}A=1,\quad A=6$

(2) $f_X(x)=\int_0^1 6xy^2dy=2x$

$f_Y(y)=\int_0^1 6xy^2dx=3y^2$

$f(x,y)=f_X(x)\cdot f_Y(y)$ ∴ 相互独立

16. 解 Z_1的可能取值为 2, 3, 4, 5

∴ Z_1	2 (1+1)	3 (1+2, 2+1)	4 (1+3, 2+2)	5 (2+3)
	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$

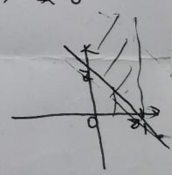
Z_2的可能取值为 1, 2, 3

1	2	3
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

19. 解

∵ X Y 相互独立

$$\therefore f(x,y)=f_X(x)f_Y(y)=\begin{cases}e^{-y}, & 0<x\le 1,\ y>0\\ 0, & 其它\end{cases}$$

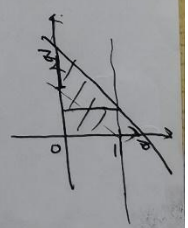


$F(z)=P(X+Y\le z)$

当 $z\le 0$ 时 $F(z)=0$

当 $0<z\le 1$ 时

$$F(z)=\int_0^z dx\int_0^{-x+z}e^{-y}dy=\int_0^z(1-e^{x-z})dx=z-1+e^{-z}$$



$f_Z(z)=1-e^{-z}$

当 $z>1$ 时

$$\therefore f(z)=\begin{cases}0, & z\le 0\\ 1-e^{-z}, & z>0,\ z<1\\ (e-1)e^{-z}, & z\ge 1\end{cases}$$

补: $z>1$ 时 $F(z)=\int_0^1dx\int_0^{-x+z}e^{-y}dy$

$=1-e^{1-z}+e^{-z}$