山东大学

14.解 (1) 已知 σ: 求 μ:

$$U=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\in(-u_{0.025},u_{0.025})\sim N(0,1)$$

$$\mu\in\left[\bar{x}-\frac{\sigma u_{0.025}}{\sqrt{n}},\ \bar{x}+\frac{\sigma u_{0.025}}{\sqrt{n}}\right]=[14.79,\ 14.99]$$

(2) $$U=\frac{\bar{x}-\mu}{S/\sqrt{n}}\sim t(n-1),$$

$$\mu\in\left[\bar{x}-\frac{S}{\sqrt{n}}u_{0.025},\ \bar{x}+\frac{\sigma}{\sqrt{n}}u_{0.025}\right]=[14.74,\ 15.04]$$

17. 解 $$U=\frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}\sim F(m-1,n-1)$$

$$\frac{\sigma_1^2}{\sigma_2^2}=\left[\frac{S_1^2}{S_2^2}\frac{1}{F_{\alpha/2}(m-1,n-1)},\ \frac{S_1^2}{S_2^2}\frac{1}{F_{1-\frac{\alpha}{2}}(m-1,n-1)}\right],\ \alpha=0.05,\ =[0.12],\ 5.55]$$

18. 解 H_0: 无显著差异. σ 不变

$$U=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\sim N(0,1)$$, ~~置信区间为~~ $u_{\alpha}=u_{0.025}=1.906$

$\bar{x}=50.45$. $$U=\frac{50.45-50}{0.8/\sqrt{n}}=1.8656<1.906.$$

∴无显著差异

19. 解 $$\bar{x}=\frac{969+645+\dots+683}{10}$$ $\sigma=56.25$ S:

$$U=\frac{\sum_{i=1}^{10}(x_i-\bar{x})^2}{\sigma^2}\sim\chi^2(n-1)$$, 区间为 $$\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\chi^2_{0.025}(9)},\ \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\chi^2_{0.975}(9)}\right]$$

落入否定域 不能说明

24. 解

$T = \frac{\bar{X}-\bar{Y}}{S_w\left(\sqrt{\frac{1}{m}+\frac{1}{n}}\right)}$，拒绝域 $|T| > t_{\alpha/2}(m+n-2) = t_{0.025}(18) = 2.1009$

$S_w = \sqrt{\frac{9S_1^2+9S_2^2}{18}}$ $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$，$\bar{Y}=\frac{1}{n}\sum_{i=1}^{n}Y_i$

解出 T 落出拒绝域

疗效无显著差异

7. 解 ∵ 无偏

∴ $E(C\sum_{i=1}^{n}X_i^2) = \theta^2$

$C \cdot E(\sum_{i=1}^{n}X_i^2) = C E(X_1^2 + X_2^2 + \cdots + X_n^2) = C[(D(X_1)+E^2(X_1)) + \cdots + (D(X_n)+E^2(X_n))]$

$= C[nD(X) + nE^2(X_1)] = C \cdot n \cdot \frac{15}{6}\theta^2 = \theta^2$

$\left(E(X_i^2) = \int_{\theta}^{2\theta} \frac{2x}{3\theta^2} \cdot x^2 dx = \frac{15}{6}\theta^2\right)$

∴ $C = \frac{2}{5n}$

9 解 (1) $E(\hat{\mu}) = E(\sum_{i=1}^{n} C_i X_i) = \sum_{i=1}^{n} E(C_i X_i) = E(C_1X_1) + E(C_2X_2) + \cdots + E(C_nX_n)$

$= C_1\mu + C_2\mu + \cdots + C_n\mu = (C_1 + \cdots + C_n)\mu = \mu$

∴ 是

(2) $D(\bar{X}) = \frac{1}{n} D(X_1 + \cdots + X_n) = \frac{\overbrace{6+\cdots+6}^{n}}{n} = 6$

若不为方而为其它，$\frac{n_1+\cdots+n_n}{n}$ 一定 >1，$\frac{n_1+\cdots+n_n}{n}6 > 6$

13. 解 $\bar{X} = \frac{1}{n}\sum X_i = 0.54375$ 　样本 $D(X) = E(X^2) - E^2(X) = \frac{25.05}{16} - (0.54375)^2 = 1.27$

μ: 　$U = \frac{\bar{X}-\mu}{S6/\sqrt{n}} = \frac{0.54375-\mu}{1.27/4} = \frac{0.54375-\mu}{0.3175} \in (-U_{0.025}, U_{0.025})$ 　1.906

$1-0.95 = 0.05$

∴ μ 为 $[-0.1082, 1.242]$

6^2 　$U = \frac{\sum(X_i-\bar{X})^2}{6^2} \sim \chi^2(15)$ 　为 $\left[\frac{\sum_{i=1}^{15}(X_i-\bar{X})^2}{\chi^2_{0.025}(15)}, \frac{\sum_{i=1}^{15}(X_i-\bar{X})^2}{\chi^2_{0.975}(15)}\right]$

$= [0.766, 3.554]$

2、解 (1) $E(X)=\int_a^b \frac{1}{b-a}\cdot x\,dx=\frac{a+b}{2}=\frac{1}{n}\sum_{i=1}^{n}X_i$

$E(X^2)=\int_a^b \frac{1}{b-a}x^2\,dx=\frac{1}{3}(a^2+ab+b^2)=\frac{1}{n}\sum_{i=1}^{n}X_i^2$ $\quad D(X)=\frac{1}{3}(a^2+ab+b^2)-\frac{1}{4}(a^2+2ab+b^2)=\frac{(a-b)^2}{12}$

$\therefore\ \hat{a}=\bar{X}-\sqrt{\frac{3}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2}$ $\quad \hat{b}=\bar{X}+\sqrt{\frac{3}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2}$

(2) $L(x_1,\cdots,x_n;a,b)=f(x_1;a,b)f(x_2;a,b)\cdots f(x_n;a,b)=\frac{1}{(b-a)^n}$

$\frac{\partial \ln L}{\partial a}=\frac{n}{b-a}>0$ $\quad \frac{\partial \ln L}{\partial b}=-\frac{n}{b-a}<0$, $b\geq a$

$\therefore\ a=\min\{x_1\cdots x_n\}$ $\quad b=\max\{x_1\cdots x_n\}$

5、解 (1) $f(x)=\begin{cases}\frac{\beta}{x^{\beta+1}}, & x>1\\ 0, & x\leq 1\end{cases}$

$E(X)=\int_1^{+\infty}\left(\frac{\beta}{x^{\beta+1}}\right)\cdot x\,dx=\frac{\beta}{\beta-1}=\frac{1}{n}\sum_{i=1}^{n}X_i$

$\therefore\ \beta=\frac{\bar{X}}{\bar{X}-1}$

(2) $L(\beta)=\frac{\beta}{x_1^{\beta+1}}\cdot\frac{\beta}{x_2^{\beta+1}}\cdots\frac{\beta}{x_n^{\beta+1}}$ $\quad \ln\beta^n-(\beta+1)\ln(x_1x_2\cdots x_n)$

~~$\ln L'(\beta)=-\ln(x_1x_2\cdots x_n)<0$ ∴ 当β最小时L最大~~

$(\ln L(\beta))'=\frac{n}{\beta}-\sum_{i=1}^{n}\ln X_i=0$ $\quad \beta=\frac{n}{\sum_{i=1}^{n}\ln X_i}$