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12. 解. $\bar{X} \sim (\mu, \frac{\sigma^2}{2n})$

$x_i + x_{n+i} - 2\bar{x} \sim (0, \frac{2(n-1)}{n}\sigma^2)$

$$\sum_{i=1}^{n} \frac{(x_i + x_{n+i} - 2\bar{x})^2}{\frac{2n-2}{n}\sigma^2} \sim \chi^2(n)$$

$\therefore Y / \frac{2n-2}{n}\sigma^2 \sim \chi^2(n)$, $E = n$

$\cancel{\therefore E(Y) = \frac{n}{(2n+2)\sigma^2}}$ $E(Y) = 2(n-1)\sigma^2$

13. 解 (1) $\bar{x} = \frac{1}{n}\sum x_i$

$\bar{y} = \frac{1}{n}\sum(x_i - a) = \frac{1}{n}\sum x_i - \frac{1}{n}\sum a = \frac{1}{n}\sum x_i - a$

证毕

$S_x^2 = S_y^2 = \frac{1}{n-1}\sum(\bar{x} - x_i)^2$

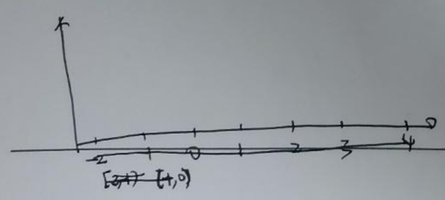
(2) $\bar{x} = \frac{1}{n}\sum x_i$

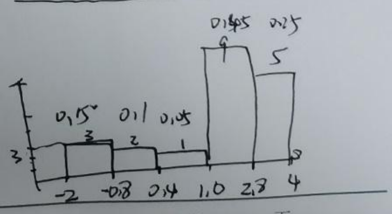
$\bar{y} = \frac{1}{n}\sum(\frac{1}{b}x_i - \frac{a}{b}) = \frac{1}{b}\frac{1}{n}\cdot\sum x_i - \frac{a}{b} = \frac{\frac{1}{n}\sum x_i - a}{b} = \frac{\bar{x} - a}{b}$

$S_x^2 = \frac{1}{n-1}\sum(b\bar{y} - by_i)^2 = b^2 \cdot \frac{1}{n-1}\sum(\bar{y} - y_i)^2 = b^2 S_y^2$

证毕

14 解 (1) -2, -2, -1, -0.5, 0, 0.5, 1, 1, 1, 1.5, 1.5, 2, 2, 2, 2.5, 3, 3, 3.4, 4





(2)

$$F_n(x)=\begin{cases}0, & x<-2\\ 1.325, & -2\le x<4\\ 1, & x\ge 4\end{cases}$$

(3) 不太一致. 当为均匀分布时一致(即每个值取到的概率相等)

n足够大时

15. 解 $\bar{X}=\frac{1}{n}\sum X_i \sim (\mu_1, \frac{1}{m}\sigma_x^2)$

$\bar{Y} \sim (\mu_2, \frac{1}{n}\sigma^2)$

$\bar{X}-\mu_1 \sim (0, \frac{1}{m}\sigma^2)$ $\qquad$ $\bar{Y}-\mu_2 \sim (0, \frac{1}{n}\sigma^2)$

$\alpha(\bar{X}-\mu_1)+\beta(\bar{Y}-\mu_2) \sim (0, \frac{\overset{(\frac{\alpha^2}{m}+\frac{\beta^2}{n})}{\cancel{\alpha^2+\beta^2}}}{n}\sigma^2)$

$S_x^2=\frac{1}{n-1}\sum_{i=1}^{m}(X_i-\bar{X})^2$ $\qquad$ $X_i-\bar{X} \sim (0, \frac{n+1}{n}\sigma^2)$

$$\frac{\sum (X_i-\bar{X})^2}{\frac{n+1}{n}\sigma^2} \sim \chi^2(m-1)$$

同理, $\dfrac{\sum (Y_i-\bar{Y})^2}{\frac{n+1}{n}\sigma^2} \sim \chi^2(n-1)$

原式 $\sim t(m+n-2)$

(2) $\frac{1}{\sqrt{8}}(X_1-X_2)^2 \sim N(0,1)$ $\frac{1}{\sqrt{8}}(X_3+X_4)^2 \sim N(0,1)$

$\therefore Z=\frac{(X_1-X_2)^2}{(X_3+X_4)^2}=\frac{\frac{1}{\sqrt{8}}(X_1-X_2^2)/1}{\frac{1}{\sqrt{8}}(X_3+X_4)^2/1} \sim F(1,1)$

10. 解

$\bar{X}=\frac{1}{n}(X_1+\cdots+X_n) \sim N(\mu, \frac{1}{n}6^2)$

$X_{n+1} \sim N(\mu, 6^2)$

$X_{n+1}-\bar{X} \sim N(0, (\frac{1}{n}+1)6^2)$

$\frac{X_{n+1}-\bar{X}}{\sqrt{\frac{n+1}{n}}\,6} \sim N(0,1)$

$S=\sqrt{\frac{1}{n-1}\sum(X_i-\bar{X})^2}$

$\because (X_i-\bar{X}) \sim N(0, \frac{n+1}{n}6^2)$

$\therefore \sum\frac{(X_i-\bar{X})^2}{\frac{n+1}{n}6^2} \sim \chi^2(n)$

$Y=\frac{X_{n+1}-\bar{X}}{\sqrt{\frac{1}{n-1}\sum(X_i-\bar{X})^2}}\cdot\sqrt{\frac{n}{n+1}}$

$=\frac{(X_{n+1}-\bar{X})/\sqrt{n+1/n}}{\sqrt{\sum(X_i-\bar{X})^2}\,\sqrt{\frac{1}{n-1}}}/6$

$=\frac{(X_{n+1}-\bar{X})/6\sqrt{n+1/n}}{(\sqrt{\sum(X_i-\bar{X})^2}/\sqrt{\frac{n+1}{n}}\,6)\cdot\sqrt{\frac{1}{n-1}}}$

$=\frac{\frac{(X_{n+1}-\bar{X})}{\sqrt{n+1/n}\;6}\cdot 6}{6\cdot\sqrt{\frac{n+1}{n}}\cdot\sqrt{\frac{\sum(X_i-\bar{X})^2}{\frac{n+1}{n}6^2}}\cdot\sqrt{\frac{1}{n-1}}} \sim t(n-1)$

11. 解

$Y_1 \sim N(\mu, \frac{6^2}{6})$ $Y_2 \sim N(\mu, \frac{1}{3}6^2)$

$Y_1-Y_2 \sim N(0, \frac{1}{2}6^2)$

$\frac{Y_1-Y_2}{6/\sqrt{2}} \sim N(0,1)$

$X_i-Y_2 \sim (0, \frac{4}{3}6^2)$

$\frac{X_i-Y_2}{26/\sqrt{3}} \sim N(0,1)$

$\sum_{i=7}^{9}\frac{(X_i-Y_2)^2}{46^2/3} \sim \chi^2(2)$

$Z=\frac{Y_1-Y_2}{S/\sqrt{2}}$

$=\frac{\frac{(Y_1-Y_2)}{6/\sqrt{2}}\cdot\sqrt{2}\cdot\frac{6}{\sqrt{2}}}{\sqrt{\frac{4}{3}\sum_{i=7}(X_i-Y_2)^2}/\sqrt{2}}$

$=\frac{\frac{Y_1-Y_2}{6/\sqrt{2}}\cdot\cancel{\sqrt{2}}\cdot 6}{\frac{26}{\sqrt{3}}\times\sqrt{\frac{\sum_{i=7}^{9}(X_i-Y_2)^2}{46^2/3}/2}}$

$=\frac{\frac{Y_1-Y_2}{6/\sqrt{2}}\cdot\frac{\sqrt{3}}{2}}{\sqrt{\frac{\sum_{i=7}^{9}(X_i-Y_2)^2}{46^2/3}/2}} \sim t(2)$

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习题5

3. (1) $P(|\bar{X}-10|>2) = P(|\frac{\bar{X}-10}{2}|>1) = 1-P(|\frac{\bar{X}-10}{2}|\le 1) = 1-(\Phi(\frac{\sqrt{5}}{4}) - \Phi(\frac{-\sqrt{5}}{4}))$

$\bar{X} \sim N(10, \frac{4}{5})$

$= \cancel{0,1587}\ 0,026$

(2) 原式 $= P(\max > 12) = 1-P(\max\{\} \le 12)$

$= 1-P(X_1\le 12, X_2 \le 12, \cdots, X_5 \le 12) = 1-F(X_1)F(X_2)\cdots F(X_5)$

$= 1-F^5(12) = 1-\Phi^5(1) = 0,5785$

(3) 原式 $= P(X_1>8, X_2>8, \cdots, X_5>8) = (1-P(X_1\le 8))\cdots(1-P(X_5\le 8))$

$= (1-F(8))^5 = (1-\Phi(-1))^5 = (1-(1-\Phi(1)))^5 = \Phi^5(1) = 0,421$

4. 解 $P(\sum X_i^2 < a) = P(\sum(\frac{X_i}{0.2})^2 < \frac{a}{0.04}) = P(\chi^2(8) < \frac{a}{0.04}) = 0,95$

$\therefore a = 15,507 \times 0,04 = 0,62$

7. 解 $\chi^2_{0,9}(16) = 9,312$ $\quad \chi^2_{0,95}(16) = 7,962$ $\quad \chi^2_{0,10}(16) = 23,542$ $\quad \chi^2_{0,05}(16) = 26,296$

$t_{0,95}(20) = -t_{0,05}(20) = -1,7247$ $\quad t_{0,05}(20) = 1,7247$

$F_{0,95}(4,6) = \frac{1}{F_{0,05}(6,4)} = 6,16$ $\quad F_{0,05}(4,6) = 4,53$ $\quad u_{0,025} = 0,490$ $\quad u_{0,05} = 0,4801$

9. 解 (1) $X_1 - X_2 \sim N(0,8)$ $\quad X_3 + X_4 \sim N(0,8)$

$\frac{1}{\sqrt{8}}(X_1-X_2) \sim N(0,1)$ $\quad \frac{1}{\sqrt{8}}(X_3+X_4) \sim N(0,1)$

$\therefore \frac{1}{8}((X_1-X_2)^2 + (X_3+X_4)^2) \sim \chi^2(2)$

$C = \frac{1}{8}$ $\quad n=2$