Prodotto righe per colonne

$$a^t b = \sum_{i=1}^{n} a_i b_i$$

riga colonna

$$a^t (b' + b'') = a^t b' + a^t b''$$

$$\sum a^t (b_i' + b_i'') = \sum a^t b_i' + \sum a^t b_i''$$ → Per proprietà delle sommatorie

$$(a'^t + a''^t) b = a'^t b + a''^t b$$

$$(\alpha a^T) b = \alpha (a^t b)$$

Scrittura sintetica di combinazioni lineari

$$\begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_1 + 3x_2 \\ 4x_1 + 5x_2 \\ 6x_1 + 7x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} x_1 + \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix} x_2$$

$$m \underbrace{(a, b, \dots)}_{n} \overbrace{\begin{pmatrix} \alpha \\ \beta \\ \vdots \end{pmatrix}}^{} m = a\alpha + b\beta + \dots$$

$$n$$

$$[a_i, b_i, \dots] \begin{bmatrix} \alpha \\ \beta \\ \vdots \end{bmatrix} = a_i \alpha + b_i \beta + \dots = a_i \alpha + b_i \beta + \dots$$

$$\begin{array}{ccc} & c_1 & c_2 \\ r_1 & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$}} \\ r_2 & & \end{array} \qquad (c_1, \dots, c_n) = \underset{m \times n}{A} = \begin{pmatrix} r_1 \\ \vdots \\ r_n \end{pmatrix}$$

$c_1, \dots, c_n$ sono lin indipendenti $\quad A \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \underline{0} \rightarrow Ax = \underline{0}$

$$c_1 x_1 + \dots + c_n x_n = \underline{0} \rightarrow c_1, \dots c_n = 0$$

lin. ind

↓

$$Ax = \underline{0}$$

$b = c_1, \dots, c_n$ base di R^n $\quad \forall b \in R^n$

$Ax = b$ unica se

se $c_1, \dots c_n$ è base di R^n

$\downarrow \quad \uparrow$

$Ax = \underline{0}$ unica sol $\rightarrow$ lin ind

$r_1, \dots, r_n$ L.I. $\quad \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{pmatrix} = y_1(2,3) + y_2(4,5) + y_3(6,7)$

$(y_1, y_2, y_3) \cdot \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{pmatrix}$

$y^T A = \underline{0}^T$ ha un'unica soluzione

$\updownarrow$

$r_1, \dots, r_n$ base di R^n $\quad \forall b^t \in R^n \quad y^T A = b^T$ unica sol

Se $r_1, \dots, r_n$ LD $\rightarrow \exists\, \bar{y}^T \neq 0 \mid \bar{y}^T A = 0$

$\rightarrow Ax = \bar{y}$ non ha soluzione

$\bar{y}^T A x = \bar{y}^T \bar{y} \neq 0$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \qquad c_1, c_2 \text{ LI} \rightarrow \exists A^{-1}$$

se c_1, c_2 LD $\rightarrow \nexists A^{-1}$

1) le colonne / righe di $A_{m \times m}$ sono lin ind

$\updownarrow$

2) $\exists A^{-1}$ $\quad$ A non singolare

$Ax = \underline{0}$ unica sol

$x = A^{-1}\underline{0} = \underline{0}$

$y^T A = \underline{0}^T \qquad y^T = \underline{0}^T A^{-1} = \underline{0}^T$

$$\begin{pmatrix} \overset{A}{r_1} \\ \hline r_2 \end{pmatrix} \left(c_1 \middle| c_2 \right) = \begin{pmatrix} r_1c_1 & r_1c_2 \\ r_2c_1 & r_2c_2 \end{pmatrix} =$$

$$= (Ac_1, Ac_2) = A(c_1, c_2, \dots)$$

$$Ax = I_2 \quad A(b,c) = (e_1, e_2) \quad \begin{cases} Ab = e_1 \\ Ac = e_2 \end{cases}$$

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

V sp. vett $\longrightarrow$ le operazioni elementari tra righe mantengono la lineare dipendenza

$$R_2 - 2R_1 \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right) \quad \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right)$$

$$R_1 - R_2 \quad \left(\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{array}\right)$$