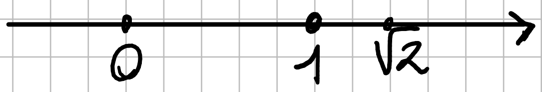
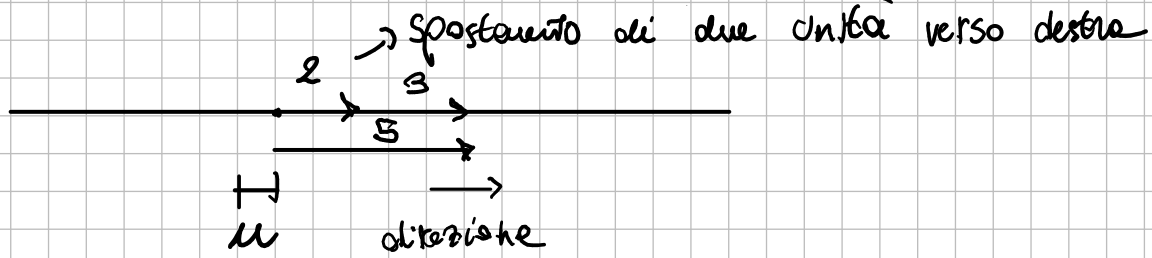
$\mathbb{R}$

$\mathbb{Q}$

$\sqrt{2} \in \mathbb{R}$, ma non a $\mathbb{Q}$





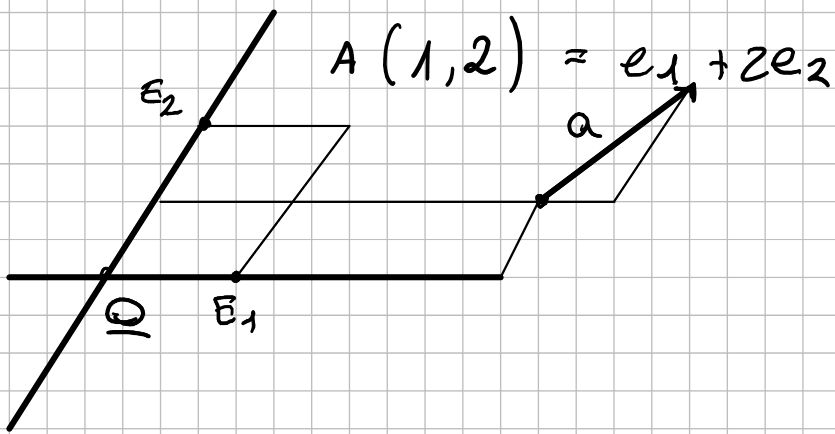
$\mathbb{R}^2$: insieme delle coppie ordinate di numeri reali

$(1,2) \neq (2,1)$ ordinato

$\underline{a} = (a_1, a_2)$

$a = b \iff \{a_i = b_i \quad i = 1,2$

$\underline{0} = (0,0) \quad e_1 = (1,0) \quad e_2 = (0,1)$



Somma $\quad (1,2) + (3,1) = (1+3, 2+1) = (4,3) \in \mathbb{R}^2$

$(\underline{a} + b)_i = a_i + b_i \quad i = 1,2$

Proprietà

- ammette un elemento nullo: $\underline{0}$
- ammette un elemento neutro: $(1,1)$

- Commutativa: $a+b = b+a \quad \forall a,b \in \mathbb{R}$
- Associativa: $a+(b+c) = (a+b)+c$
- ogni elemento ammette un inverso $\mid a+(-a) = \underline{0}$

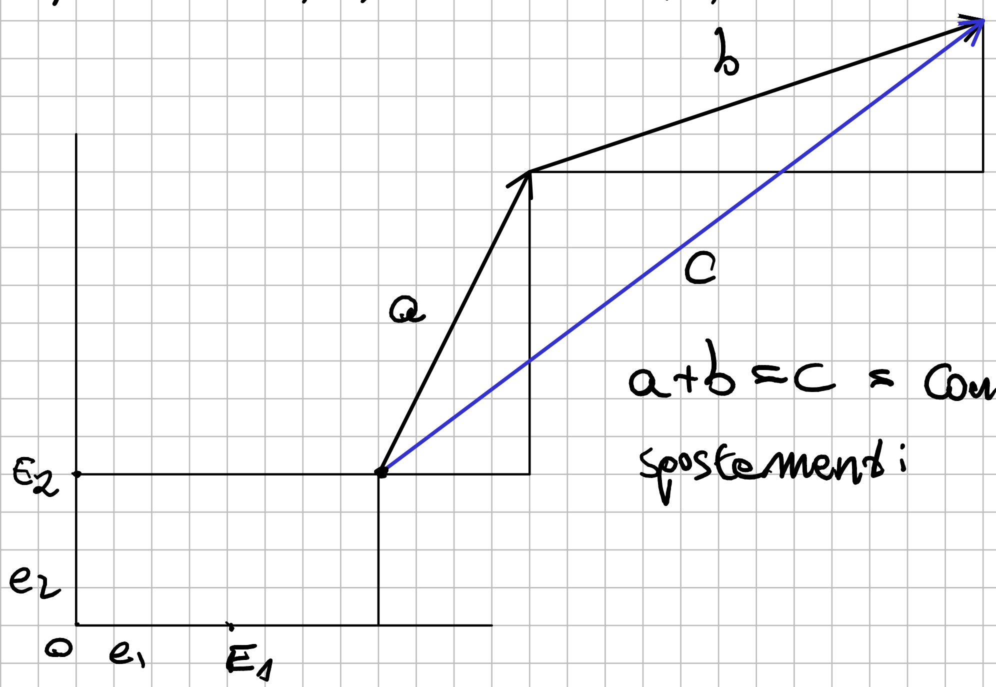
$$\left(-a + \left(-(-a) \right) = 0 \right)$$

$$(a_1, a_2) + (2,3) = (5,1)$$

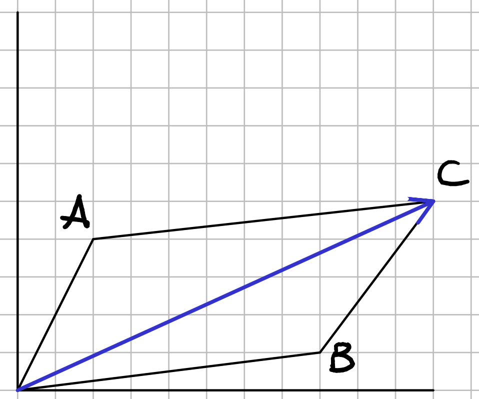
$$\begin{cases} a_1 + 2 = 5 \\ a_2 + 3 = 1 \end{cases} \qquad \begin{cases} a_1 = 3 \\ a_2 = 2 \end{cases}$$

oppure

$$(a_1, a_2) = (5,1) - (2,3) = (3,2)$$



$a+b=c$ = Concatenazione degli spostamenti

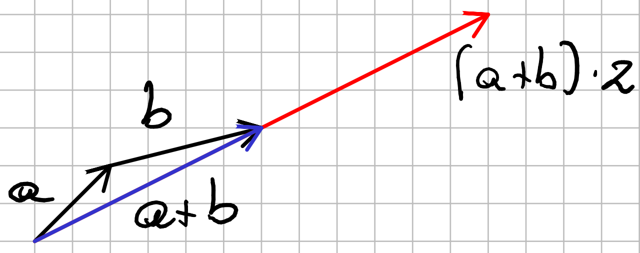


Prodotto Scalare

$2(3,4) = (6,8)$

$r \in \mathbb{R},\ a \in \mathbb{R}^2 \rightarrow ra \in \mathbb{R}^2 \mid (ra)_i = ra_i \quad i=1,2$

$r(a+b) = ra + rb$



ra ha stessa direzione di a, lunghezza del spostamento $= |r| \cdot \|a\|$

Il verso è uguale ad a se $r > 0$, opposto se $r < 0$

- $(r+s)a = ra + sa$
- $r(a+b) = ra + rb$
- $(rs)a = r(sa)$
- $1a = a$