CSC/MA/OR 565 - Spring 2018
Homework 3

1. Show that every simple graph G with 6 vertices has either a clique of size 3 or an independent set of size 3.

2. Prove or disprove: In a simple graph, every closed even trail of length more than 3 contains an even cycle.

3. Prove the following by strong induction on the length of the trail: The edge set of every closed trail can be partitioned into zero or more pairwise edge-disjoint cycles.

4. Suppose that T is a maximal trail in a simple graph G and that T has at least one edge and is not closed. Prove that the endpoints of T have odd degree.

5. If G is a graph with vertices $v_1, v_2, \ldots, v_n$ and A^k denotes the kth power of the adjacency matrix of G under matrix multiplication then

(***) $A^k[i, j]$ is the number of v_i, v_j-walks of length k in G.

Show how to use (***) to solve the following without multipltying matrices and prove your answer correct: Let A be the adjacency matrix of K_n. If $i = j$, then $A^3[i, j] =$ ______. Otherwise $A^3[i, j] =$ ______.

6. Draw a simple, connected graph with the following degree sequence, or prove that no such graph is possible:

a. (3, 3, 3, 2, 2, 2)

b. (7, 6, 5, 4, 3, 2, 1)

c. (3, 3, 2, 2, 1, 1)

d. (7, 6, 5, 4, 3, 3, 2)

e. (6, 6, 5, 4, 3, 3, 1)

7. How many *different* simple graphs are there with 5 edges and with vertex set $\{v_1, v_2, \ldots v_5\}$? (We are counting labeled graphs, not isomorphism classes)

8. Prove by contradiction: A graph with every vertex degree even has no cut-edge.