# Homework 3

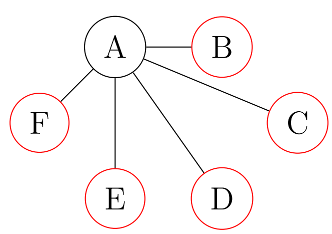
Dustin Lambright - dalambri
Aseem Raina - araina
Bihan Zhang - bzhang28
CSC 565 - Graph Theory

February 2, 2018

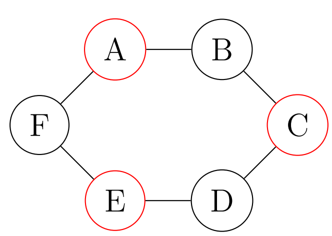
**Question 1.** Show that every simple graph $G$ with 6 vertices has either a clique of size 3 or an independent set of size 3.

By the definition of inependent sets and cliques, any graph G with a clique of size n will also have a clique of n-1, n-2, ... n-(n-1). The same goes for an independent set. Therefore, using contradiction, if a graph does not have a clique of size 3 and does not have an independent set of size 3, the statement is false.

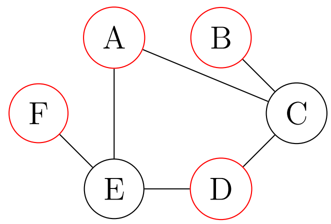
The graph $K_{1,5}$ has a maximum clique size of 2, but an independent set of 5. (displayed in red)




In an effort to reduce the size of the maximum independent set, edges are replaced, creating a cyclic graph:




This graph has a a maximum independent set of 3, and a maximum clique of 2. The only way to reduce the size of an independent set, i.e., $\{A, E, C\}$ is to connect any of those vertices with an edge. The connection of any of these vertices results in a clique of size 3. In an effort to maintain our clique constraint, if we remove the edges AF and AB to connect the independent set $\{A, E, C\}$, we end up with an independent set $\{A, F, B, D\}$.




The graph $C_6$ is the closest we can get to a graph without a clique of size 3 and without an independent set of size 3, but it has an independent set of size three, proving that a graph with a vertex count of 6 has to have an independent set of size 3 or a clique of size 3.

**Question 2.** Prove or disprove: In a simple graph, every closed even trail of length more than 3 contains an even cycle.

**Question 3.** Prove the following by strong induction on the length of the trail: The edge set of every closed trail can be partitioned into zero or more pairwise edge-disjoint cycles.

**Question 4.** Suppose that T is a maximal trail in a simple graph G and that T has at least one edge and is not closed. Prove that the endpoints of T have odd degree.

**Question 5.** If G is a graph with vertices v1, v2, . . . , vn and Ak denotes the kth power of the adjacency matrix of G under matrix multiplication then

(***) $A^k[i, j]$ is the number of vi, vj -walks of length k in G.

Show how to use (***) to solve the following without multipltying matrices and prove your answer correct: Let A be the adjacency matrix of Kn. If i = j, then A3 [i, j] =____ . Otherwise A3 [i, j] = ____.

**Question 6.** Draw a simple, connected graph with the following degree sequence, or prove that no such graph is possible:
a. (3, 3, 3, 2, 2, 2)
b. (7, 6, 5, 4, 3, 2, 1)
c. (3, 3, 2, 2, 1, 1)
d. (7, 6, 5, 4, 3, 3, 2)
e. (6, 6, 5, 4, 3, 3, 1)

**Question 7.** How many different simple graphs are there with 5 edges and with vertex set v1, v2, . . . v5? (We are counting labeled graphs, not isomorphism classes)

**Question 8.** Prove by contradiction: A graph with every vertex degree even has no cut-edge.