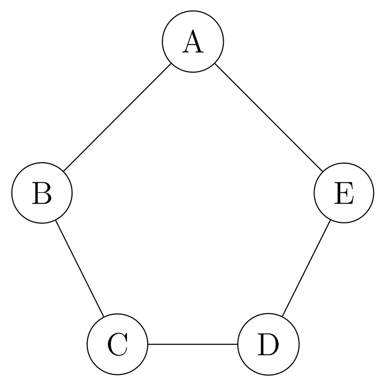
Homework 2

Team F
CSC 565 - Graph Theory

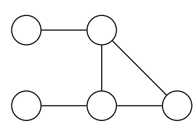
January 29, 2018

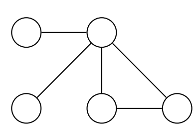
Question 1. Find a graph G with 5 vertices which has neither a clique of size 3 nor an independent set of size 3.

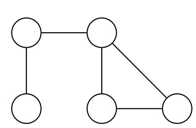


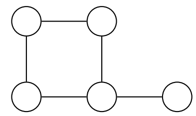
C5 has a maximal clique of 2 and a maximal independent set of 2.

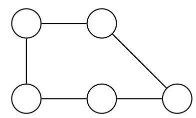
Question 2. Find all metamorphism classes of simple graphs with 5 vertices and 5 edges.

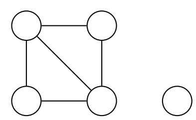




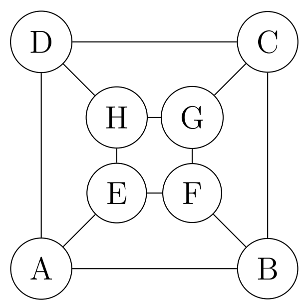


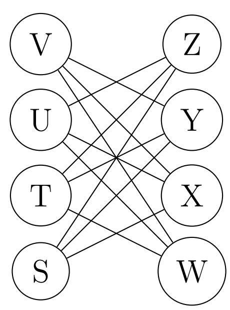




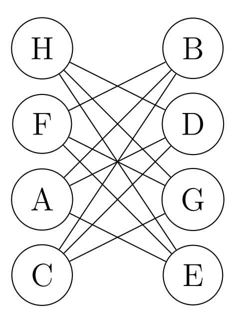


Question 3. Determine which pairs of graphs below are bipartite.





The first and second graph are isomorphic. The first can be relabeled to look like the second:



The third graph is not isomorphic to the first two graphs, because it is not a bipartite graph like the first two graphs (its non bipartite nature is easy discernible- it contains an odd cycle of 5, and by Theorem 1.2.18, a graph is bipartite iff it contains no odd cycles)

Question 4. Let A be the adjacency matrix of K_n. If i = j , then $A^2[i, j]$_____. Otherwise, $A^2[i, j] =$_____.

- If i = j , then $A^2[i, j]$ = (n-1)
- Otherwise, $A^2[i, j]$ = (n-2)

Question 5. Prove or disprove: if G is a disconnected graph, then $\overline{G}$, the complement of G, is connected.

i. Let u, v be on different components in G. Since there is no uv edge in G, there must be a uv edge in $\overline{G}$. Thus u v are connected in $\overline{G}$

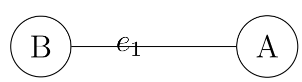
ii. Let u, v be on the same component in G. Let w be a vertex on a separate component in G. There must be a uw edge in $\overline{G}$. There must also be a vw edge in $\overline{G}$.

Because there is a uw path, and a vw path, by Lemma 1.2.5 there must also be a uv path. Thus u and v are connected in $\overline{G}$

Because there exists a path between any two vertices in $\overline{G}$ it is a connected graph.

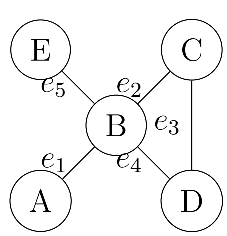
Question 6. Determine whether K_4 contains the following (give an example or a proof of non-existence);

a) A walk that is not a trail.



The walk B, e_1, A, e_1, B is a walk that is not a trail; it repeats the $e1$ edge.

b) A trail that is not closed and is not a path.



The walk A, e_1, B, e_2, C, e_3, D, e_4, B, e_5, E is a trail that is not closed and is not a path, as the vertex B is repeated.

c) A closed trail that is not a cycle.

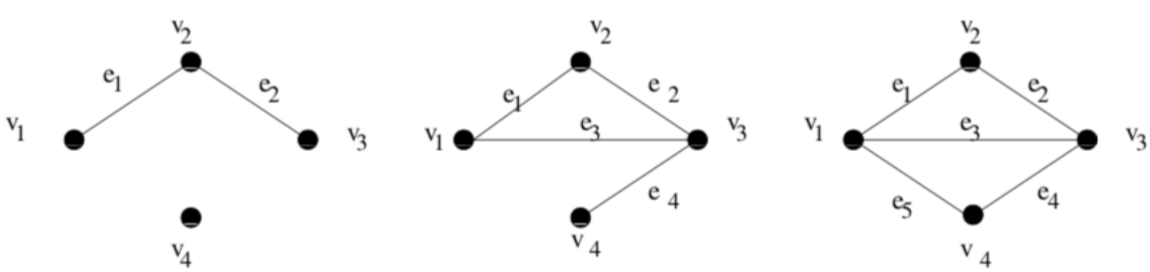
A

The closed trail of A alone is a closed trail that is not a cycle. The length of the walk is 0, so it is not a cycle. $v_0 = v_n$ so it is closed. It also does not repeat any edges, therefore it is a closed trail that is not a cycle.

Question 7. A graph is called chordal if it has no induced subgraph isomorphic to C_n for any $n \leq 4$. Which of the following 10-vertex graphs are chordal?

- K_{10} : Yes
- $K_{5,5}$: No - $K_{5,5}$ contains C_4 as an induced subgraph
- $K_{1,9}$: Yes - $K_{1,9}$ is trivially chordal, since it does not contain any cycles
- C_{10} : No - C_{10} contains itself as an induced subgraph
- P_{10} : Yes - P_{10} is trivially chordal, since it does not contain any cycles
- Petersen Graph: No - The Petersen graph has C_5 as an induced subgraph

Question 8. a. For each graph G below, find the incidence matrix $M(G)$. (See definition 1.1.17 and Example 1.1.19.)



Far Left

	e1	e2
v1	1	0
v2	1	1
v3	0	1
v4	0	0

Middle

	e_1	e_2	e_3	e_4
v_1	1	0	1	0
v_2	1	1	0	0
v_3	0	1	1	1
v_4	0	0	0	1

Far Right

	e_1	e_2	e_3	e_4	e_5
v_1	1	0	1	0	1
v_2	1	1	0	0	0
v_3	0	1	1	1	0
v_4	0	0	0	1	1

b. Recall that the transpose of a $p \times q$ matrix M is the $q \times p$ matrix M^T with $M^T[i, j] = M[j, i]$. For each graph G above in (a), find the matrix product $M(G) \cdot (M(G))^T$.

Far Left

1	1	0	0
1	2	1	0
0	1	1	0
0	0	0	0

Middle

2	1	1	0
1	2	1	0
1	1	3	1
0	0	1	1

Far Right

3	1	1	1
1	2	1	0
1	1	3	1
1	0	1	2