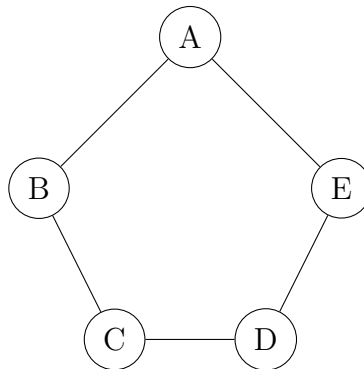


Homework 2

Team F
CSC 565 - Graph Theory

January 27, 2018

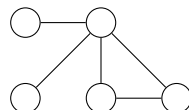
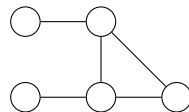
Question 1. Find a graph G with 5 vertices which has neither a clique of size 3 nor an independent set of size 3.

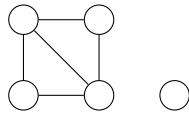
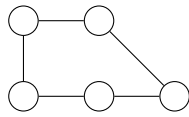
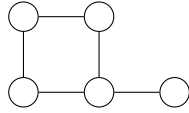
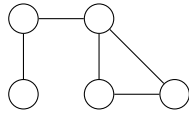


Question 4. Let A be the adjacency matrix of K_n . If $i = j$, then $A^2[i, j]$ ----- . Otherwise, $A^2[i, j] =$ -----.

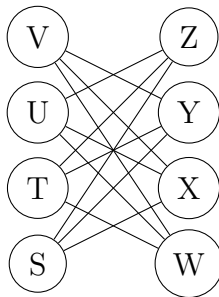
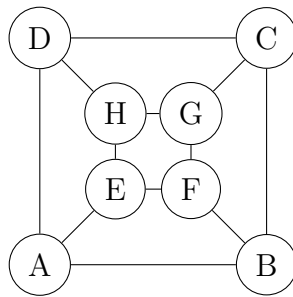
- If $i = j$, then $A^2[i, j] = (n-1)$
- Otherwise, $A^2[i, j] = (n-2)$

Question 2. Find all metamorphism classes of simple graphs with 5 vertices and 5 edges.

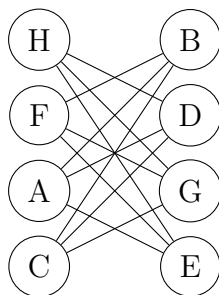




Question 3. Determine which pairs of graphs below are bipartite.



The first and second graph are isomorphic. The first can be relabeled to look like the second:



The third graph is not isomorphic, because it is not a bipartite graph like the first two graphs.

Question 5. Prove or disprove: if G is a disconnected graph, then \overline{G} , the complement of G , is connected.

i. Let u, v be on different components in G . Since there is no uv edge in G , there must be a uv edge in \overline{G} . Thus u, v are connected in \overline{G} .

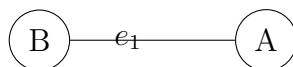
ii. Let u, v be on the same component in G . Let w be a vertex on a separate component in G . There must be a uw edge in \overline{G} . There must also be a vw edge in \overline{G} .

Because there is a uw path, and a vw path, by Lemma 1.2.5 there must also be a uv path. Thus u and v are connected in \overline{G} .

Because there exists a path between any two vertices in \overline{G} it is a connected graph.

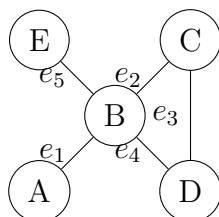
Question 6. Determine whether K_4 contains the following (give an example or a proof of non-existence);

a) A walk that is not a trail.



The walk B, e_1 , A, e_1 , B is a walk that is not a trail.

b) A trail that is not closed and is not a path.



The walk A, e_1 , B, e_2 , C, e_3 , D, e_4 , B, e_5 , E is a trail that is not closed and is not a path, as the vertex B is repeated.

c) A closed trail that is not a cycle.

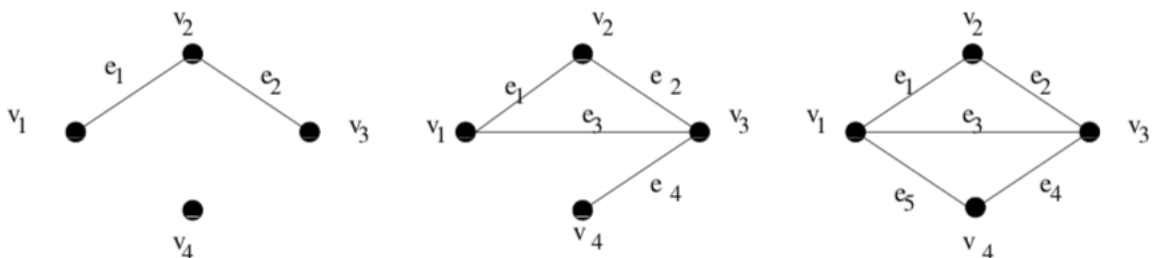


The closed trail of A alone is a closed trail that is not a cycle.

Question 7. A graph is called chordal if it has no induced subgraph isomorphic to C_n for any $n \leq 4$. Which of the following 10-vertex graphs are chordal?

- K_{10} : No
- $K_{5,5}$: No
- $K_{1,9}$: Yes
- C_{10} : No
- P_{10} : Yes
- Peterson Graph: No

Question 8. a. For each graph G below, find the incidence matrix $M(G)$. (See definition 1.1.17 and Example 1.1.19.)



Far Left

	e ₁	e ₂
v ₁	1	0
v ₂	1	1
v ₃	0	1
v ₄	0	0

Middle

	e ₁	e ₂	e ₃	e ₄
v ₁	1	0	1	0
v ₂	1	1	0	0
v ₃	0	1	1	1
v ₄	0	0	0	1

Far Right

	e ₁	e ₂	e ₃	e ₄	e ₅
v ₁	1	0	1	0	1
v ₂	1	1	0	0	0
v ₃	0	1	1	1	0
v ₄	0	0	0	1	1

b. Recall that the transpose of a $p \times q$ matrix M is the $q \times p$ matrix M^T with $M^T[i, j] = M[j, i]$. For each graph G above in (a), find the matrix product $M(G) \cdot (M(G))^T$.

Far Left

1	1	0	0
1	2	1	0
0	1	1	0
0	0	0	0

Middle

2	1	1	0
1	2	1	0
1	1	3	1
0	0	1	1

Far Right

3	1	1	1
1	2	1	0
1	1	3	1
1	0	1	2