

Homework 3

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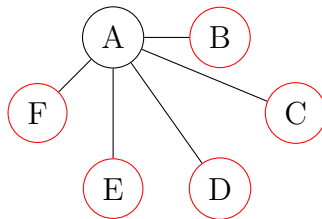
CSC 565 - Graph Theory

February 2, 2018

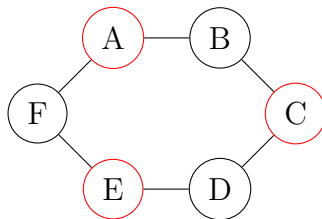
Question 1. Show that every simple graph G with 6 vertices has either a clique of size 3 or an independent set of size 3.

By the definition of independent sets and cliques, any graph G with a clique of size n will also have a clique of $n-1$, $n-2$, ... $n-(n-1)$. The same goes for an independent set. Therefore, using contradiction, if a graph does not have a clique of size 3 and does not have an independent set of size 3, the statement is false.

The graph $K_{1,5}$ has a maximum clique size of 2, but an independent set of 5. (displayed in red)

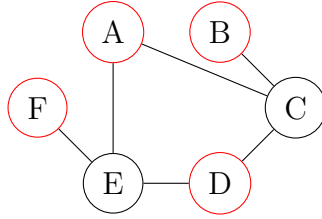


In an effort to reduce the size of the maximum independent set, edges are replaced, creating a cyclic graph:



This graph has a maximum independent set of 3, and a maximum clique of 2. The only way to reduce the size of an independent set, i.e., $\{A, E, C\}$ is to connect any of those

vertices with an edge. The connection of any of these vertices results in a clique of size 3. In an effort to maintain our clique constraint, if we remove the edges AF and AB to connect the independent set $\{A, E, C\}$, we end up with an independent set $\{A, F, B, D\}$.



The graph C_6 is the closest we can get to a graph without a clique of size 3 and without an independent set of size 3, but it has an independent set of size three, proving that a graph with a vertex count of 6 has to have an independent set of size 3 or a clique of size 3.