Saving and Liquidity Constraints

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Key aim of the paper

- This paper is concerned with the theory of optimal intertemporal consumption behaviour who are restricted in their ability to borrow to finance consumption
- General rule: Partial equilibrium. Start from a simple stochastic process for labor income. Derive appropriate policy rule/consumption function given borrowing constraint. Study the time-series behaviour of consumption, savings and assets.

Roadmap

1. PIH, very briefly
2. Some empirical evidence
3. Main results
4. Basic Model
5. Some stochastic labor income process, solution, simulations
6. Repeat 5 with some other stochastic process

The benchmark: Permanent Income Hypothesis

- Assume infinite horizon, stochastic income, quadratic utility, $\delta = r$ and no borrowing constraints
- We get: Consumption is a random-walk (Hall, 1978), depends only on annuity value of income. Consumption will only change when new/unanticipated information about permanent income is revealed
- Also, certainty equivalence: variance and higher moments of the income process do not matter for the determination of consumption

Empirical evidence, 1

- Campbell and Deaton (1989): Assume aggregate income process: $\Delta y_t = \lambda \Delta y_{t-1} + \epsilon_t$
- Apply it to the PIH model
- For plausible values of r and λ they found that permanent income (current consumption) can be twice as variable as current income. Data showed consumption is less variable than (current) income $\rightarrow$ Deaton's paradox (excessive smoothness of aggregate consumption, relative to PIH/LC)

Empirical evidence, 2

- Consumption is a random walk, cannot be predicted from past information
- Flavin (1981) and many subsequent authors: changes in consumption are positively relate to predictable changes in income
- Furthermore, Carroll and Summers (1989) show that consumption appears to track household income quite closely over life cycle
- Moreover, most versions of the life-cycle models predict a dissociation of consumption from income, and the existence of substantial asset at least at some points in the life-cycle.
    - Validity challenged. Most US household's hold very few assets, estimates vary but median household wealth, excluding pension

Towards borrowing constraints (and impatience)

- In summary, existing empirical evidence at the time showed consumption responds to predictable change in income $\rightarrow$ Euler equation violated
- Many households have little financial wealth, unlikely to be able to finance consumption by borrowing
- Large share of consumers do not have many financial assets despite strong savings incentives $\rightarrow$ households may be less patient than when $\delta = r$

Main results, 1

- Impatience + BC + prudence $\rightarrow$ behaviour of saving and accumulation quite sensitive to stochastic process generating income
- Stationary, i.i.d income: asset are buffer stock, consumers save and dissaves to smooth consumption (smoother than income), more prudent consumer + more uncertain income = more precautionary balance
- Positive serial correlation: desirability and feasibility of using assets as buffer diminishes.
- Random walk: consume all your income

Main results, 2

- Stationary income growth, growth mimics aggregate data: paradoxical result, savings is counter-cyclical
- Agent's income process independent, with negative serial correlation in growth rates: No savings in the aggregate
- Combination of aggregate and idiosyncratic income process: generates savings in the aggregate, can potentially reproduce stylized facts in the actual data

Basic model

Standard intertemporal utility maximization. Consumer receives y_t, stochastic. Given r.

Assumptions:

1. BC: $A_t \geq 0$
2. Prudence: $u''' \geq 0$
3. Impatience: $\delta \geq r$

Case 1: i.i.d. income shock

Suppose y_t is i.i.d with distribution $F(.)$. No persistence, no growth. Relaxed later.

$$u = E_t(\sum_{\tau=t}^{\infty}(1+\delta)^{t-\tau}v(c_\tau)) \tag{1}$$

$$\text{s.t.} \quad A_{t+1} = (1+r)(A_t + y_t - c_t) \tag{2}$$

The Euler Equation

Define, x_t as "cash on hands" as $x_t = A_t + y_t$

Hence,

$$\lambda(c_t) = max[\lambda(x_t), \beta E_t \lambda(c_{t+1})] \tag{3}$$

where $\quad \lambda(c_t) = \upsilon'(c_t)$

Solution methods

1. Solve simultaneously the two difference equations in Eq.(2) and Eq.(3)

2. Alternatively, work through the following value function

$$V(x) = \max_{0 \leq s \leq x} \left\{ v(x-s) + (1+\delta)^{-1} \int V[(1+r)s + y] dF(y) \right\} \tag{4}$$

Solution, 1: Buffer-stock behaviour

$$c = f(x) = x, \quad x \leq x^*$$

$$c = f(x) \leq x, \quad x \geq x^*$$

1. If total income below the critical level, everything is spent, and the household goes into the next period with no assets. If the total is greater that x^*, something will be held, positive level of assets carried forward
2. Assets are not desired for their own sake, but to buffer fluctuations in income. High income, saving. Low income, dissaving
3. Distribution of consumption not symmetric

Solution, 2

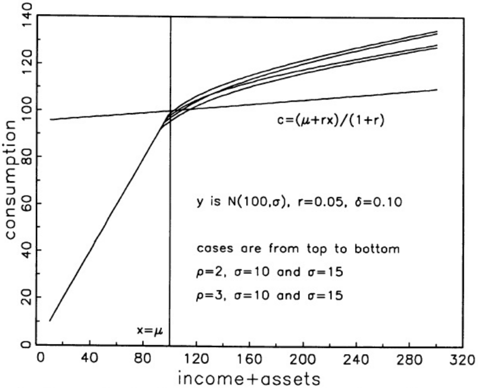

FIGURE 1.—Consumption functions for alternative utility functions and income dispersions.

Simulation

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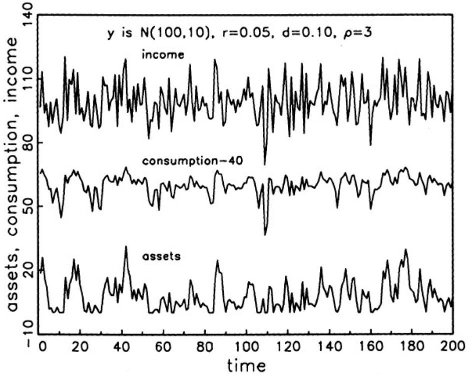

FIGURE 2.—Simulations of income, consumption, and assets, with white noise income.

1. 200 period simulation, consumption notably smoother
2. However, depends on sd of income and prudence though

Case 2: Stationary serially correlated income

Is i.i.d. a reasonable assumption? Maybe for farmers in LDC's whose income depends on weather. Even for them, many behavioural and technical responses are likely to generate correlated income processes. For advanced countries? maybe, not

Towards serially correlated income

Value function

Assume, income follows AR(1).

$$(y_t - \mu) = \phi(y_{t-1} - \mu) + \epsilon_t$$

Essentially, the same problem. But, with two state variables. Cash in hand today and income today used to predict expected marginal utility of consumption tomorrow.

$$V(x, y^{cur}) = \max_{0 \le s \le x} \left\{ \upsilon(x-s) + (1+\delta)^{-1} \int V[(1+r)s+y] dF(y|y^{cur}) \right\} \tag{5}$$

Solution: same buffer-stock behaviour

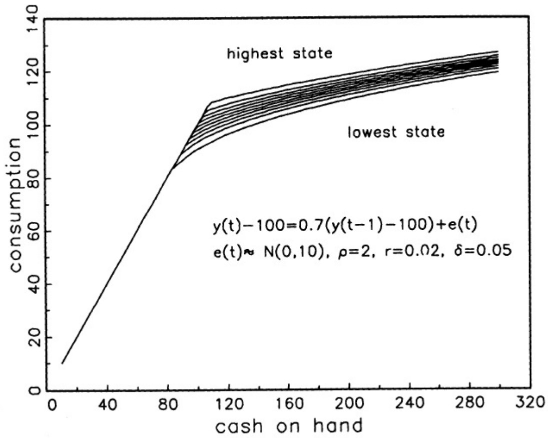

FIGURE 3.—Consumption and cash on hand for AR(1) income process.

Simulation, 1

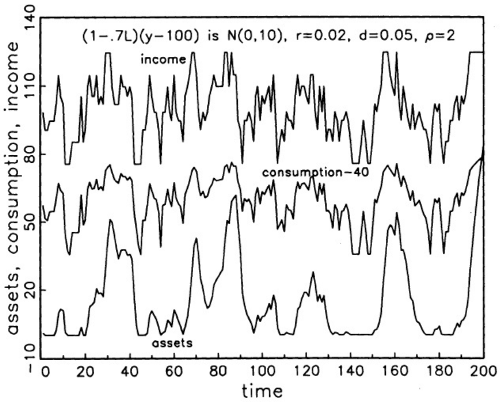

FIGURE 4.—Simulations of income, consumption, and assets with positively autocorrelated income.

Simulation, 2

1. Consumption again smoother than income, but less so. Sd(c) is 10.4, sd(y) is 14
2. Savings pro-cyclical, large assets are occasionally accumulated.
3. Quite long periods of no or close to zero assets, more likely the more positively autocorrelated is income
4. Asymmetric behaviour of consumption still prominent

Simlation, 3

TABLE I

STANDARD DEVIATIONS OF CONSUMPTION AND INCOME FOR AR(1) INCOME, PARAMETER ϕ

AR coeff ϕ	−0.4	0.0	0.3	0.5	0.7	0.9
1. sd(y)	10.9	10.0	10.5	11.5	14.0	22.9
2. est sd(y)	10.8	10.2	10.0	11.4	13.3	27.5
3. est sd(c)	4.6	5.1	6.7	7.6	10.4	25.9
ratio 3/2	0.43	0.50	0.67	0.67	0.78	0.94

Continue

1. For the i.i.d. case, optimal smoothing can remove half of the standard deviation of income, but as autocorrelation increases consumption becomes as noisy as income
2. Impatient consumers. Precautionary demand powerful motive for assets, but smoothing consumption over long autocorrelated swings require more consumption sacrifice.

Case 3: Random walks,1

However, most consumers in developed and developing economies can reasonably expect income to grow over time. If such expectations are held, analysis will be very different

- With income growth implications are different. For agents with $\delta > r$, individuals more likely to be liquidity constraint
- Assume isoelastic preferences with relative risk aversion parameter ρ
- Assume no uncertainty, no initial assets. Consumption should grow at rate $\rho^{-1}(r - g)$
- If income grows at $g > \rho^{-1}(r - g) \rightarrow$ borrowing will be required, more likely to be income constrained

Case 3: Random walks, 2

1. Logarithm of the income process is stationary in first-differences
2. Such models capable of modelling actual aggregate household income in the U.S.

Model

- Same problem as before but log od income follows a random walk. Assume isoelastic preferences

$$z_{t+1} = \frac{y_{t+1}}{y_t}$$

$$\lambda(\theta_t) = max[\lambda(_t), \beta E_t z_{t+1}^{\rho} \lambda(\theta_{t+1})]$$

where $\theta_t = \frac{c_t}{y_t}$ and $w_t = \frac{x_t}{y_t}$

$$w_{t+1} = 1 + (1 + r)(w_t + \theta_t) z_{t+1}^{-1}$$

Value function and intuition

$$V(w) = \max_{0 \leq \theta_t \leq x} \left\{ v(\theta_t) + (1+\delta)^{-1} \int V[1+(1+r)(w+\theta)z^{-1}]dF(z) \right\} \tag{6}$$

- With random walk we get the result that consumption is equal to income
- Presence of binding constraints makes it undesirable to undertake smoothing
- Suppose consumer has no asset, income growth well above average
- May seem like a good situation to save, but consumer is already liquidity constrained

Continue

- Additional income merely provides an opportunity to get closer to the ideal consumption path which would have been the case when no BC $\rightarrow$ good draws in income are spent, assets remain at zero
- But, saving is desirable when income is expected to be lower?
- However, RW implies that while a bad draw does indeed imply that income thereafter can expected to be permanently lower, expected growth rate of income is unchanged
- There is never any rational expectation that income will be lower

Case 4: Autocorrelated growth rates

- Post war US quarterly data, the growth of aggregate hh income better approximated by AR(1)
- For both GDP and income, growth shocks are persistent, positive shocks followed by positive shocks
- Incorporating such an income process is capable of generating some savings, even in the presence of BC, but don't behave in the same way as aggregate data

Model

- Take the same setup as in Case 3
- Two state Markov process with noise, keeps calculations simple
- State 1 = "boom", State 2 = "slump"

$$\text{when state 1} \quad \Delta \ln y_t = g_1 + \epsilon_t$$

$$\text{when state 2} \quad \Delta \ln y_t = g_2 + \epsilon_t$$

where $g_2 < 0 < g_1$ and ϵ_t is a gaussian white noise.

Transition probabilities are $\psi_1 = pr(s_t = 1 | s_{t-1} = 1)$ and $\psi_2 = pr(s_t = 2 | s_{t-1} = 2)$. Economy shows positive growth on average $(1 - \psi_2) g_1 + (1 - \psi_1) g_2 > 0$

Solution

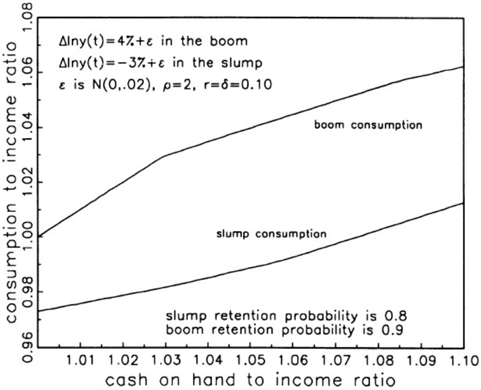

FIGURE 5.—Consumption to cash on hand ratios for Hamilton's model.

Simulation

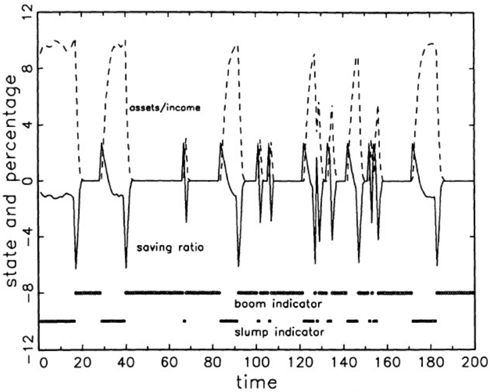

FIGURE 6.—Cyclical saving behavior in Hamilton's model.

Case 5: Individual and aggregate behaviour

- Failure of the representative model does not imply BC do not behave as described here
- One possibility: constrained consumers, responsible for large share of consumption but only small share of savings
- Cont: aggregate savings accounted by unconstrained consumers
- Second possibility: unlikely that micro level income process mirror time-series behaviour of aggregate income
- Hence, start from micro data and observed income process there

Cont.

- At the micro level, income less persistent than in the aggregate
- Year-to-year changes show significant negative autocorrelation, either because of measurement error or substantial transitory income in each year
- Start with: income process as having MA representation in first difference of logs MaCurdy(1982)

$$\Delta \ln y_t - \mu = \epsilon_t + \epsilon_{t-1}$$

Cont, 1

- MaCurdy estimates σ, the sd of ϵ, the innovation to the log income process to be 0.235
- This would imply sd of $\Delta \ln y_t$ to be 0.25. In this situation agents are unlikely to borrow.
- Deaton assumes σ to be between 0.10 and 0.15

Main conclusions

1. Consumption functions have the general shape as before
2. A high innovation now implies low income growth next period, so lower consumption/higher saving than before
3. But if each income process were independent, no variation in aggregate income growth rates as savings and dissavings would cancel out in aggregate

Cont, 2

Assume now that consumer receives aggregate shock with idiosyncratic component

$$\Delta \ln y_t - g = z_{1t} + z_{2t} + z_{3t}$$

where $z_{1t} = \epsilon_{1t} + \beta\epsilon_{1t-1}$, $z_{2t} = \epsilon_{2t}$ and $z_{3t} = \epsilon_{3t} + \epsilon_{3t-1}$

z_{1t} is common to all, others are idiosyncratic. Individual has no way of separating the three components, observes only their sum, which is IMA(1,1). Effectively implying, consumers do not observe aggregate shocks, even with a lag.

Cont, 3

- Individual growth is negatively correlated, aggregate growth positively correlated
- Simulations and subsequent aggregation shows that in such a situation savings ratios are procyclical
- Assets, however, are always positive
- Model provides a means of reconciling orthogonality condition failures in the micro and macro data

Conclusion, 1

1. i.i.d income: consumption very smooth, buffer stock behaviour strong
2. stationary but serially correlated: consumption more noisy, buffer stock behaviour less likely
3. random walk: limiting case, consumption = income
4. labor shock mimics aggregate growth: counter-cyclical savings
5. aggregate and individual combined: potential to generate savings in the aggregate, replicate data

Conclusion, 2

1. Story incomplete in many aspects (few financial assets, but high housing wealth and pension rights, need to integrate them into the analysis)
2. No claim that all agents are liquidity constrained
3. Relatively patient as well as impatient ones, the former ones likely to accumulate capital as in the standard LC model
4. Deaton believes that such people are in minority, although they hold disproportionate share of agg. savings and wealth accumulation
5. aggregate and individual combined: potential to generate savings in the aggregate, replicate data