CryptoVerif: Mechanising Game-Based Proofs

Part II

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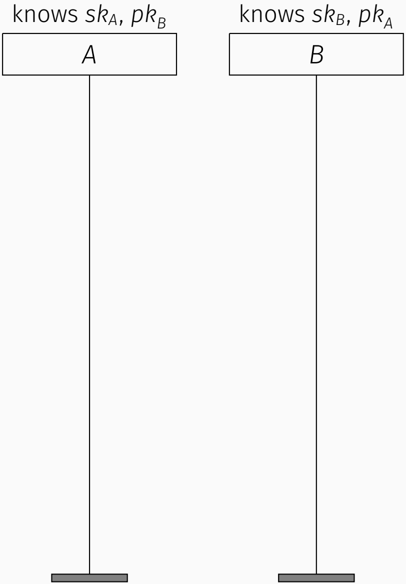
What to Expect from Part II

A more complex example, a protocol with multiple messages:
Signed Diffie-Hellman Authenticated Key Exchange

What's new?

- model a random oracle
- use a Computational Diffie-Hellman (CDH) assumption
- prove key secrecy using `query secret`
- prove authentication properties using correspondence between events
- model a Public-Key Infrastructure using a list

Signed Diffie-Hellman: Authenticated Key Exchange



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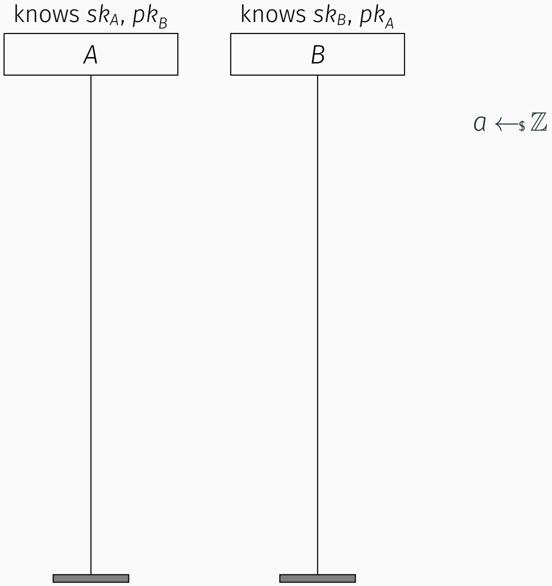

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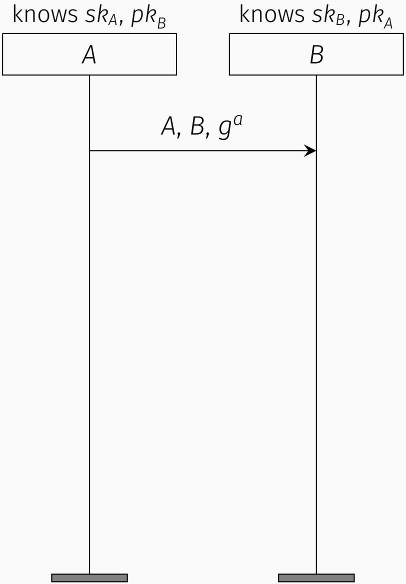

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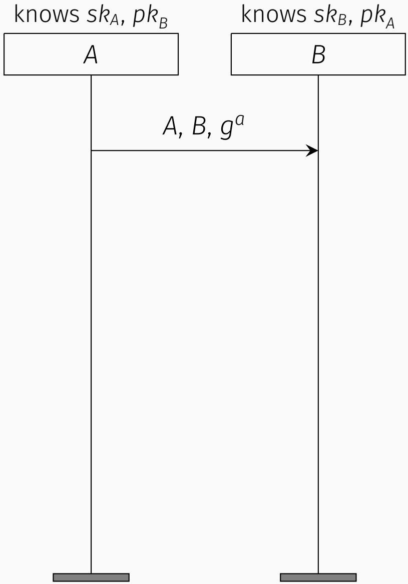

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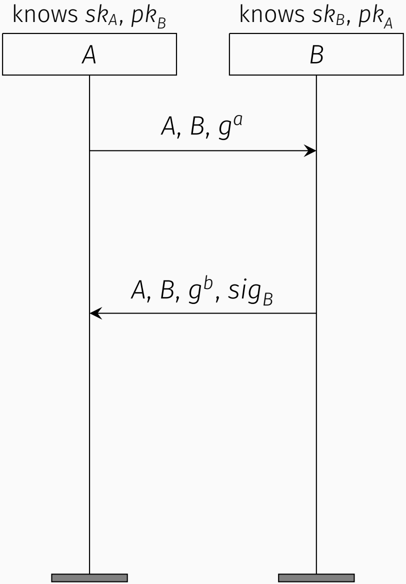

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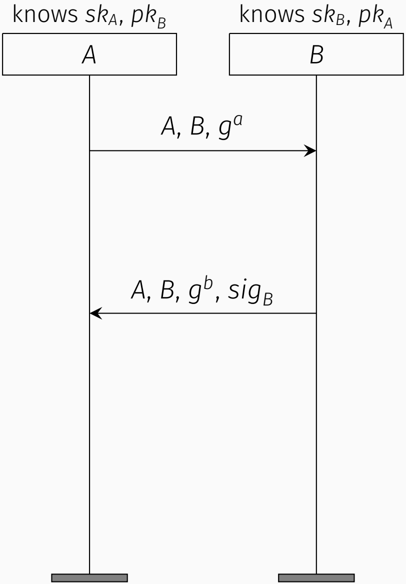

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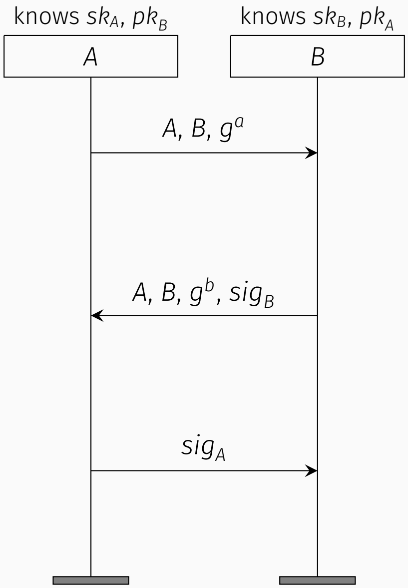

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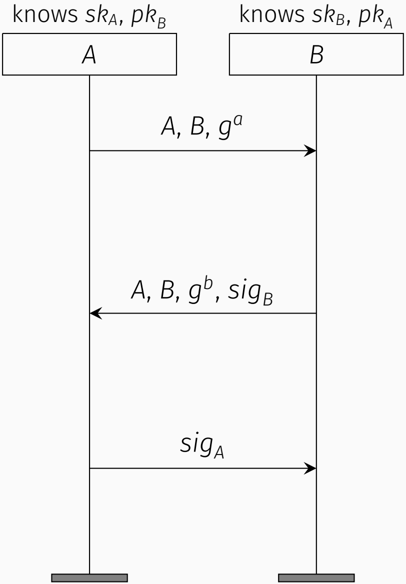

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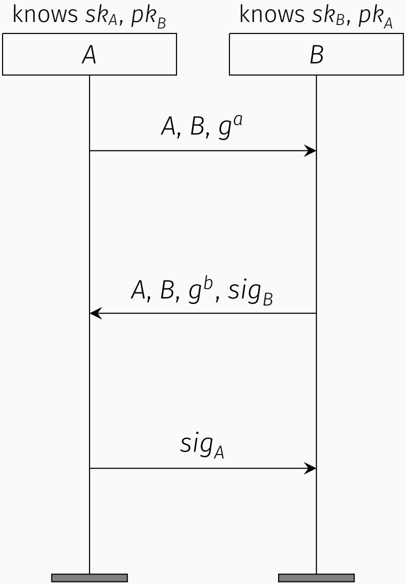

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Signed Diffie-Hellman: Security Properties

- The shared secrets k_A and k_B are secret (indistinguishable from random bitstrings of equal length)
  `query secret kA.` `query secret kB.`
- If A is convinced to have concluded a session with B using ephemerals g^a, g^b, then B actually started such a session
  ```
  query x:G, y:G; inj-event(endA(A, B, x, y)) ==>
  inj-event(beginB(A, B, x, y)).
  ```
- If B is convinced to have concluded a session with A using ephemerals g^a, g^b, then A is likewise convinced
  ```
  query x:G, y:G; inj-event(endB(A, B, x, y)) ==>
  inj-event(endA(A, B, x, y))
  ```

Cryptographic Assumptions

We use the following cryptographic assumptions to prove these security properties:

- hash is a random oracle
- (sign, verify) is a UF-CMA-secure probabilistic signature
- the CDH assumption holds in the group G

Types and Probabilities for the Signature

Types define names for subsets of the bitstrings. The annotations restrict them on a high level.

```
type keyseed [large,fixed].
type pkey [bounded].
type skey [bounded].
type message [bounded].
type signature [bounded].
```

We define names for probabilities. They will appear in the final probability bound.

```
proba Psign.     (* breaking the UF-CMA property *)
proba Psigncoll. (* probability of collision between
                    independently generated keys *)
```

Using the Macro: UF-CMA-secure Signature

```
expand UF_CMA_proba_signature(
  (* types, to be defined outside the macro *)
  keyseed,
  pkey,
  skey,
  message,
  signature,
  (* names for functions defined by the macro *)
  skgen,
  pkgen,
  sign,
  verify,
  (* probabilities, to be defined outside the macro *)
  Psign,
  Psigncoll
).
```

Functions Defined by the Signature Macro

In this example, we use a *probabilistic* signature. The macro makes this transparent for us, by defining the seed type and a `sign` wrapper function.

```
fun skgen(keyseed):skey.
fun pkgen(keyseed):pkey.

fun verify(message, pkey, signature): bool.
fun sign_r(message, skey, sign_seed): signature.

letfun sign(m: message, sk: skey) =
  r <-R sign_seed; sign_r(m, sk, r).
```

The macro in CryptoVerif's default library defines the equation for correctness (not shown here).

Diffie-Hellman Part I

```
type Z [large,bounded].
type G [large,bounded].

proba PCollKey1.
proba PCollKey2.
```

CryptoVerif's default library comes with several macros for groups. We'll use a basic group in which some collision probabilities are negligible.

```
expand DH_proba_collision(
  G,         (* type of group elements *)
  Z,         (* type of exponents *)
  g,         (* group generator *)
  exp,       (* exponentiation function *)
  exp',      (* exp. func. after transformation *)
  mult,      (* func. for exponent multiplication *)
  PCollKey1,(* g^(fresh x) collides with indep. Y *)
  PCollKey2 (* g^(fr. x * fr. y) coll. w/ indep. Y *)
).
```

Diffie-Hellman Part II

The macro defines the exponentiation function, a group generator, and equations for exponent multiplication. An extract:

```
fun exp(G, Z): G.
const g: G.

fun mult(Z, Z): Z.
equation builtin commut(mult).

equation forall a:G, x:Z, y:Z;
  exp(exp(a, x), y) = exp(a, mult(x, y)).
```

Diffie-Hellman Part III

Assumptions like CDH, DDH, GDH, ... must be instantiated with a separate macro. We use CDH, indicating the previously defined group:

```
proba pCDH. (* probability of breaking CDH in G *)
expand CDH(G, Z, g, exp, exp', mult, pCDH).
```

This macro implements a multi-key version of (simplified presentation):

$$\mathrm{Succ}_G^{\mathrm{CDH}}(t) = \max_{\mathcal{A}} \Pr_{x,y \leftarrow_\$ Z}[g^{xy} \leftarrow \mathcal{A}(g^x, g^y)] \text{ is negligible.}$$

Random Oracle Part I – Definition

A random oracle is an idealized random function that returns

- an independent uniformly random value on new input,
- the same value than before on previously seen input.

To model this, *all* calls, also adversarial ones, must be observed by the game.

```
type hashfunction [fixed].

expand ROM_hash(
  hashfunction, (* type for hash function choice *)
  G,            (* type of input *)
  key,          (* type of output *)
  h,            (* name of hash function *)
  hashoracle,   (* process defining the hash oracle *)
  qH            (* parameter: number of calls *)
).
```

Random Oracle Part II – Macro Internals

The macro defines the hash function. The first parameter models the choice of the specific hash function: The adversary could call `hash`, but does not know the value the protocol uses for the 1st parameter.

```
fun hash(hashfunction, G): key.
```

The macro defines the oracle we must expose such that the adversary can use the RO:

```
param qH.

let hashoracle(hf: hashfunction) :=
  foreach ih <= qH do
  Ohash(x: G) :=
    return(hash(hf, x)).
```

It allows `qH` calls, a parameter that will appear in the final probability formula.

Random Oracle Part III – Usage

In the initial game, we sample a random hash function

```
hf <-R hashfunction;
```

and use it in each call of hash:

```
kA <- hash(hf, gab);
```

We must include the process defined by the macro, such that the adversary can access the random oracle for its own calls:

```
run hashoracle(hf)
```

Random Oracle Part IV – Applying the Assumption

When applying the RO assumption, CryptoVerif replaces each call of the hash function

```
foreach i <= N do (* ... *) hash(hf, x) (* ... *)
```

by an array lookup, comparing with *all* other inputs:

```
find j <= N suchthat defined(x[j], k[j]) && x = x[j]
then k[j]
else k <-R key; k
```

There will be one `find` branch per hash call.

In particular, the `hash` call in the `hashoracle` process will be replaced by a table lookup, comparing with all hash inputs used in the entire game.

Setting up the Game

In the game setup, we create signature keypairs for the two honest parties. We can define functions (`letfun`) that CryptoVerif will inline.

```
letfun keygen() =
  rk <-R keyseed;
  sk <- skgen(rk);
  pk <- pkgen(rk);
  (sk, pk).
```

The initial game starts after the `process` keyword.

```
process
  Ostart() :=
    hf <-R hashfunction;
    let (skA: skey, pkA: pkey) = keygen() in
    let (skB: skey, pkB: pkey) = keygen() in
    return(pkA, pkB);
```

The Complete Main Process

```
param NA, NB, NK. (* number of calls *)

process
  Ostart() :=
    hf <-R hashfunction;
    let (skA: skey, pkA: pkey) = keygen() in
    let (skB: skey, pkB: pkey) = keygen() in
    return(pkA, pkB);
    (
     (foreach iA <= NA do run processA(hf, skA))
    |
     (foreach iB <= NB do run processB(hf, skB))
    |
     (foreach iK <= NK do run pki(pkA, pkB))
    |
      run hashoracle(hf) (* # of calls def. inside *)
    )
```

Public Key Infrastructure

We define a type for hosts, a list for (host, public key) tuples, and two honest hosts.

```
type host [bounded].
table keys(host, pkey).
const A, B: host. (* The two honest peers *)
```

We allow the adversary to register additional entries:

```
let pki(pkA: pkey, pkB: pkey) =

  Opki(hostZ: host, pkZ: pkey) :=
    if        hostZ = B then  insert keys(B, pkB)
    else if   hostZ = A then  insert keys(A, pkA)
    else                      insert keys(hostZ, pkZ).
```

We will use `get keys(=hostX, pkX)` to retrieve X's key.

Sequential Oracles in Processes

We expose one oracle for each protocol message.

`OA1, OA3, OAfin`, and `OB2, OBfin` can only be called in this order. A “session” identifier is implicit (the replication index).

```
let processA(...) =
  OA1(...) :=
    ...
    return(...);

  OA3(...) :=
    ...
    return(...);

  OAfin(...) :=
    ...
    return(...).
```

```
let processB(...) =
  OB2(...) :=
    ...
    return(...);

  OBfin(...) :=
    ...
    return(...)
```

1st and 2nd Message

Creating the 1st message. The adversary chooses A's peer.

```
let processA(hf:hashfunction, skA:skey) =
  OA1(hostX: host) :=
    a <-R Z;   ga <- exp(g,a);
    return(A, hostX, ga);
```

Consuming the 1st and creating the 2nd message. B only continues if the message is for B: `=B`. Event `beginB` is recorded.

```
let processB(hf:hashfunction, skB:skey) =
  OB2(hostY: host, =B, ga: G) :=
    b <-R Z;   gb <- exp(g,b);
    sig <- sign(msg2(hostY, B, ga, gb), skB);
    event beginB(hostY, B, ga, gb);
    return(hostY, B, gb, sig);
```

2nd and 3rd Message

```
let processB(hf:hashfunction, skB:skey) =
  OB2(hostY:host, =B, ga:G) :=
    (* ... *)
    return(hostY, B, gb, sig);
```

If A can verify the signature, event endA is recorded.

```
let processA(hf:hashfunction, skA:skey) =
  (* ... *)

  OA3(=A, =hostX, gb: G, s: signature) :=
    get keys(=hostX, pkX) in
    if verify(msg2(A, hostX, ga, gb), pkX, s) then
    gab <- exp(gb, a);    kA <- hash(hf, gab);
    sig <- sign(msg3(A, hostX, ga, gb), skA);
    event endA(A, hostX, ga, gb);
    return(sig);
```

3rd Message and Finish

If B can verify the signature, event `endB` is recorded.

```
OBfin(s:signature) :=
  get keys(=hostY, pkY) in
  if verify(msg3(hostY, B, ga, gb), pkY, s) then
  gab <- exp(ga, b);
  kB <- hash(hf, gab);
  event endB(hostY, B, ga, gb);
```

We want to prove secrecy only in case the two honest peers interacted. Only in this case we assign the shared secret to another variable.

```
if hostY = A then (
  keyB:key <- kB
) else
  return(kB).
```

Finish on A's Side

We could have merged that into `OA3`, but it is clearer this way.

```
OAfin() :=
  if hostX = B then (keyA:key <- kA)
  else return(kA).
```

Now we have variables `keyA` and `keyB` that are only defined for honest sessions, for which we want to prove key secrecy. Thus, we can ask CryptoVerif to prove:

```
query secret keyA.
query secret keyB.
```

Note that this way, *all* honest sessions are "test" sessions.

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... if an adversary has a negligible probability of distinguishing keys k_A from uniformly random bitstrings of same length:

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$$\mathrm{Succ}_{\mathrm{SDH}}^{\textit{key-secrecy},k_A}(t, n_A, n_B, n_K, q_H) = \max_{\mathcal{A}} \left| \Pr[\mathcal{G}_{real}(\mathcal{A}) \Rightarrow 1] - \Pr[\mathcal{G}_{random}(\mathcal{A}) \Rightarrow 1] \right|$$

- where $\mathcal{G}_{real}$ is the original game, and
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and where $\mathcal{A}$

- runs in time at most t
- starts at most n_A sessions for A, and at most n_B for B
- registers at most n_K public keys (incl. A and B)
- calls the hash oracle at most q_H times.

Correspondence Queries

Events need to be declared:

```
event endA(host, host, G, G).
event beginB(host, host, G, G).
event endB(host, host, G, G).
```

A can authenticate *B*, even if any shared secret leaks:

```
query y: G, x: G;
  inj-event(endA(A, B, x, y))
  ==> inj-event(beginB(A, B, x, y))
  public_vars keyA, keyB.
```

B can authenticate *A*, even if any shared secret leaks:

```
query y: G, x: G;
  inj-event(endB(A, B, x, y))
  ==> inj-event(endA(A, B, x, y))
  public_vars keyA, keyB.
```

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$$\max_{\mathcal{A}} \Pr\left[\begin{array}{l} \mathcal{A}^{Ostart,OA\cdot,OB\cdot,Opki,OH} : \mathcal{A} \text{ produces a sequence of events} \\ \text{such that not every } \mathrm{end}_B(A, B, g^a, g^b) \text{ is preceeded} \\ \text{by a distinct } \mathrm{end}_A(A, B, g^a, g^b) \end{array}\right]$$

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where $\mathcal{A}$

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- calls the hash oracle at most q_H times.

Proof and Result

```
(* demo *)
```

Interactive Mode

Include `interactive` in the proof environment to start the interactive mode:

```
proof {
  interactive
}
```

- `out_game "filename"` outputs the current game. Use a `.ocv` extension such that your editor highlights the syntax.
- `crypto assumption(function)` applies the assumption to the function. Example: `crypto rom(hash)`
- `success` tries to prove the queries
- `simplify` tries to simplify the current game
- `quit` leaves interactive mode and continues non-interactively.
- Ctrl+D ends the programme

What We Covered Today

- Introduction to the syntax and semantics of games
- Model simple primitives and protocols
- Use macros from the default library: symmetric encryption, MAC, signature, random oracle, basic Diffie-Hellman
- Basic interactive interaction with CryptoVerif
- Prove secrecy and correspondence properties
- Read the final result

Next Steps with CryptoVerif

- Try the exercices and reach us on VeriCrypt's Zulip during the next days
  - syntax highlighting is available for Vim and Emacs
- The reference manual is in `docs/manual.pdf`
- More examples are in the directory `examples`
  - beware, spoilers for the exercices
  - look for `.ocv` files, they use the oracle syntax presented in this tutorial. (`.pcv` and `.cv` use the *channel* frontend)
- Subscribe to the mailinglist (low activity)
  `https://sympa.inria.fr/sympa/subscribe/cryptoverif`

References

- References to the case studies are in the slides of Part I
- References for how CryptoVerif proves (titles are clickable links)
    - Secrecy:

      [1] Bruno Blanchet. A Computationally Sound Mechanized Prover for Security Protocols. IEEE Transactions on Dependable and Secure Computing, 5(4):193-207, October-December 2008. Special issue IEEE Symposium on Security and Privacy 2006.

    - Correspondence:

      [2] Bruno Blanchet. Computationally Sound Mechanized Proofs of Correspondence Assertions. Cryptology ePrint Archive, Report 2007/128.