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Chapter 7:

Probability Mass Functions

Example: Suppose in a village every couple bears six children. This gender distribution among these six children is a random phenomenon. If x is a random variable denoting the number of girl among these six children, then $x = 0, 1, 2, \ldots 6$. Here the no. of trials $n=6$ and getting a girl is a ~~small~~ success, so the probability of success $p = 1/2$.

soln:

$$p(x=x) = C(6,x)(1/2)^x (1/2)^{6-x}$$

$$x = 0, 1, 2, 3, \ldots 6$$

$$E[x] = 6(1/2) = 3$$

$$V[x] = 6(1/2)(1/2) = 1.5$$

Q) A game of ludo can be played once the die shows the face 6, so, a player can only proceed

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to next move once the die shows the face 6.

soln:,

let x be a random variable deno-ting the number of trials needed to get the third face of 6. The probability mass function of x is following.

$$P(X=x) = C(x-1, 3-1)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{x-3}$$

$$x = 3, 4, \ldots$$

$$P(X=6) = C(6-1, 3-1)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3$$
$$= 0.027 \#$$

9) If x has a poission distribution with parameters β and if $P[X=0] = 2$, evaluate $P[X>2]$.

soln:,

$$P[X=0] = \frac{e^{-\lambda}\lambda^0}{0!}$$

$$= e^{-\lambda} = 0.2$$

$$\text{or, } -\lambda = \ln(0.2) = -1.609$$

$$\Rightarrow \lambda = 1.609$$

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$$P(X>2) = 1 - P(X \leq 2)$$
$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$
$$= 1 - e^{-1.609}\left[\frac{1.609^0}{0!} + \frac{1.609^1}{1!} + \frac{1.609^2}{2!}\right]$$
$$= 1 - 0.781$$
$$= 0.219$$

Q) Suppose that a container contains 10,000 particles. The probability that such a particle escapes from the containers equals 0.0004 what is the probability that morethan 5 such escapes occur.

soln:

Denoting by X the random variable ~~cou~~ on number of escapes of particles from a container and $x = 0, 1, 2, \ldots$

Here,

$$n = 10,000, \; p = 0.0004$$
$$np = 4 \qquad np < 5$$
$$P(x=0) = e^{-\lambda} = 0.0004$$
$$\therefore \lambda = -\ln(0.0004) = 7.824 \#$$

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8) suppose that x has poission distribution If $P(x=2) = \frac{2}{3} P(x=1)$ evaluate $P(x=0)$ and $P(x=3)$.

soln:.

$$P(x=2) = \frac{2}{3} P(x=1)$$

$$\Rightarrow \frac{e^{-\lambda}\lambda^2}{2!} = \frac{2}{3} \frac{e^{-\lambda}\lambda^1}{1!}$$

$$\Rightarrow e^{-\lambda}\lambda\left[\frac{\lambda}{2} - \frac{2}{3}\right] = 0$$

so, $\lambda = 0$ or $\lambda = 4/3$

$\lambda = 0$ is not possible for the value of $\lambda = 4/3$

$$P(x=0) = e^{-\lambda} = e^{-4/3} = 0.264$$

$$P(x=3) = \frac{e^{-\lambda} \times \lambda^3}{3!} = 0.104 \#$$

9) In a certain developing country 30% children are malnaurished from a random sample of six children in this area. Find the probability that, number of undernourished are,

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(a) exactly two.
(b) more than two
(c) at most two

Also find the mean & S.D. of binomial distribution. Also use poisson distribution to find above mention probability. If there are 50 such families then how many family more than 2 children are undernourished

Binomial

$$p = \frac{30}{100} = 0.3$$

$$n = 6$$

(a) $P(x=2) = {}^6C_2 (0.3)^2 (0.7)^4$

$= C(6.2)(0.3)^2(0.7)^4$

$= 0.324135$

(b) $P(x>2) = 1 - P(x \leq 2)$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - {}^6C_0 (0.3)^0 (0.7)^6 + {}^6C_1 (0.3)^1 (0.7)^5 +$

$^6C_2 (0.3)^2 (0.7)^4$
$= 0.256$

ⓒ $P[X \le 2] = 1 - P(X > 2]$
$= 1 - 0.256 = 0.744$ #

POISSONS:

$P(X = 1) = \dfrac{e^{-\lambda} \lambda^r}{r!}$; $r = 0, 1, 2, \ldots$

$\lambda = np = 6 \times 0.3 = 1.8$

ⓐ $P[x = 2] = e^{-1.8} \dfrac{(1.8)^2}{2!} = 0.267 \cong 0.3$

ⓑ $P[X > 2] = 1 - P(X \le 2)$
$= 1 - [P(x=0) + P(x=1) + P(x=2)]$
$= 1 - e^{-1.8}\left(\dfrac{(1.8)^0}{0!} + \dfrac{(1.8)^1}{1!} + \dfrac{(1.8)^2}{2!}\right)$

$= 0.269 \cong 0.3$

ⓒ $P(X \le 2] = P(X = 0] + P(x = 1] + P(x = 2)$
$= e^{-1.8}\left[\dfrac{(1.8)^0}{0!} + \dfrac{(1.8)^1}{1!} + \dfrac{(1.8)^2}{2!}\right]$

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$= 0.73 \cong 0.7$

Q) For a binomial distribution, the mean is 6 & S.D is $\sqrt{2}$. Find the first two terms of the distribution.

soln

$mean = np = 6$

$Variance = np(1-p) = 2$

so, $1-p = 2/6 \Rightarrow p = 4/6$

$n = 6/p = \frac{6}{4/6} = 9$

$P(x=0) = c(9,0) \cdot (4/6)^0 (2/6)^9 = 0.000051$

$P(x=1) = c(9,1) \cdot (4/6)^1 (2/8)^8 = 0.00091$

Q) A consignment of 30 radius contains 4 defectives. A random sample of 5 is selected from this consignment. If X denotes the number of defectives. Find the probability function of X.

soln.

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X follows hypergeometric distribution X = 0, 1, 2, 3, 4

Here, no. of defectives are 4 and no of non-defective are 26. The probability function is;

$$P(X = x) = \frac{C(4, x) \times C(26, 5-x)}{C(30, 5)}, \quad x = 0, 1, 2, 3, 4$$

8) evaluate the probability of containing k-neutrophils out of 5 cells for k = 0, 1, 2, 3, 4, 5 where the probability that anyone cell is neutrophil is 0.6. Also find the expression for variance of no. of neutrophils.

soln,:

No. of neutrophil cells out of 5 cells is distributed as binomial with $n = 5$ and $p = 0.6$

The expectation $= np = 5 \times 0.6 = 3$

The variance $= np(1-p) = 5(0.6)(1-0.6)$

$= 1.2$ #

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Q) In a village there are 200 families with 5 children. Let X be a random variable denoting no of boys among these 5 children. Construct the probability distribution of X. Find the mathematical expectation and variance. Also find the no of families having three boys.

soln:

$P[x=0] = {}^5C_0 (1/2)^0 (1/2)^5 = 1/32$

$P(x=1] = C(5,1) \; (1/2)^1 (1/2)^4 = 5/32$

$P(x=2) = C(5,2) \; (1/2)^2 (1/2)^3 = 10/32$

$P[x=3] = C(5,3) \; (1/2)^3 (1/2)^2 = 10/32$

$P(x=4) = C(5,4) \; (1/2)^3 (1/2)^1 = 5/32$

$P(x=5) = C(5,5) \; (1/2)^5 (1/2)^0 = 1/32$

x	0	1	2	3	4	5
P(x)	1/32	5/32	10/32	10/32	5/32	1/32

$E(x) = np = 5(1/2) = 2.5$ #

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$V[x] = npq = 5(1/2)(1/2) = 1.25$

8) A pair of dice is tossed five times let X be a random variable denoting number of times a sum of seven occurs, in these five tosses. Construct probability distribution of X and find the mathematical expectation and Variance.

soln:

Sample Space = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

P(sum of seven) = $6/36$

$\because n = 5$

$p = 1/6$

$P[X=r] = C(5,r)(1/6)^r (5/6)^{5-r}$, $r = 0, 1, 2, 3, 4, 5$

$E[x] = np = 5(1/6) = 0.833$

$V[x] = npq = 5(1/6)(5/6) = 0.6944$

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Q) Suppose that it is known that in a certain area of large city, the average no of rats for quarter block is 5. Assuming that the no. of rats follows poisson. Find the probability that in randomly selected quarter block.

(a) There are exactly five rats.
(b) There are more than 5 rats.
(c) There are fewer than 5 rats.
(d) There are 5 and 7 rats inclusive.

soln:.

Given:

(a) $p(x=5) = \frac{e^{-5}5^5}{5!} = 0.175$

(b) $p(x>5) = P(x=6) + P(x=7) + P(x=8) + \dots$

$P(x>5) = 1 - P(x \leq 4)$

$= 1 - e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right]$

$= 0.394$ #

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(c) $P(X<5) = 1 - P[X \geq 5]$ or $P(X \leq 4)$

$$= e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right)$$

$$= 0.440$$

(d) $P(5 \leq x \leq 7) = e^{-5}\left[\frac{5^5}{5!} + \frac{5^6}{6!} + \frac{5^7}{7!}\right]$

$$= 0.4261$$

8) A certain surgical operation on the knee is successful 85%. of the time. Twelve people have this operation.

(a) what is the probability That the operation will be successful in all 12 cases?

(b) what is the probability that the of at least one of the operations being unsuccessful?

soln..

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$p = 95\% = 0.95$
$q = 0.05$

(a) $n = 12$
$p(\text{success}) = C(12,2)(0.95)^{12}(0.05)^{0}$
$= 0.540$

(b) $n = 11$
p(at least one unsuccesful)
$= C(12,11)(0.95)^{11} \times (0.05)^{1} +$
$C(12,10)(0.95)^{10}(0.05)^{2} + \dots$
$\dots C(12,0)(0.95)^{0}(0.05)^{12}$
$= 0.459$