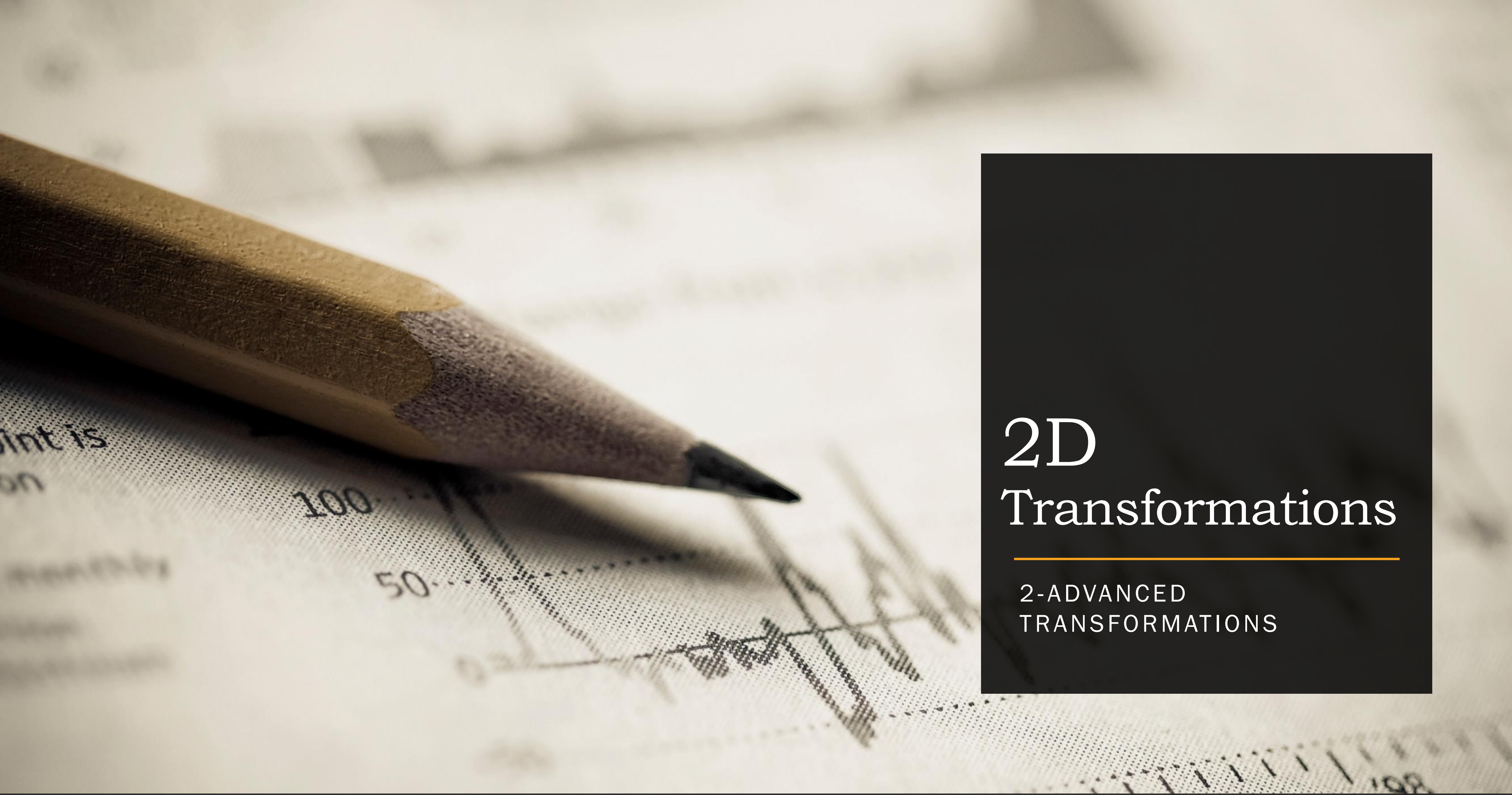


2D Transformations

2-ADVANCED TRANSFORMATIONS

Rotation About Arbitrary Point other than the Origin

If we wanted to rotate a point around something other than the origin, you need to:

First translate the whole system so that the point of rotation *is* at the origin.

Then perform the rotation.

And finally, undo the translation.

If we want to rotate point (*X*, *Y*), about point (*Xc*, *Yc*) by θ degree, then:

1) T(-Xc, -Yc)
2) R(θ) $=$ T(Xc,Yc)R(θ)T(-Xc, -Yc)
3) T(Xc, Yc)

Rotation About Arbitrary Point other than the Origin

Solving the system in matrix form

$$\begin{bmatrix} \acute{X} \\ \acute{Y} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Xc \\ 0 & 1 & Yc \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -Xc \\ 0 & 1 & -Yc \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Using matrices multiplication we get:

$$\begin{bmatrix} \acute{X} \\ \acute{Y} \\ 1 \end{bmatrix} = \begin{bmatrix} Xc + (X - Xc)cos\theta - (Y - Yc)sin\theta \\ Yc + (X - Xc)sin\theta - (Y - Yc)cos\theta \\ 1 \end{bmatrix}$$

HW: Derive this result from the above system.

Rotation About Arbitrary Point other than the Origin

Example:

The red square is the original shape with coordinates:

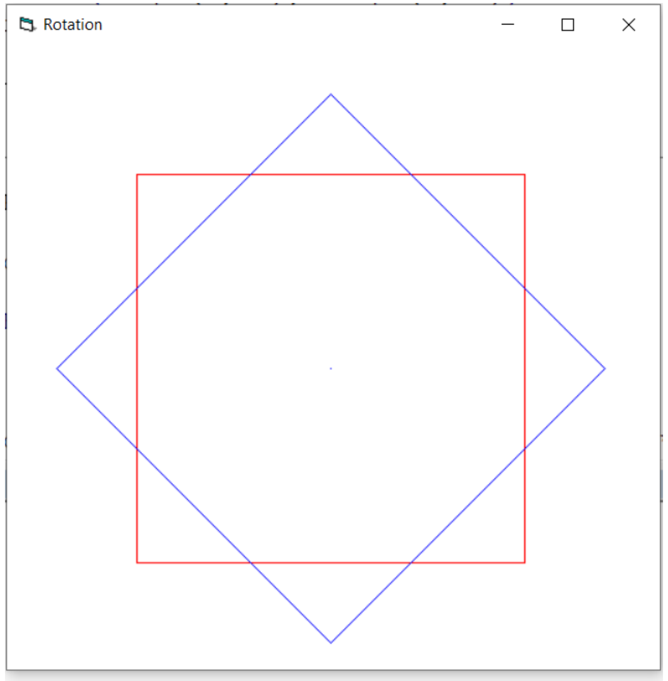
(100, 100), (400, 100). (400, 400), (100, 400)

The blue square is the same square rotated by 45 degrees

about the centre of the square (250, 250).

The rotated points are:

(250, 38), (462, 250), (250, 462), (38, 250).



Scaling About Arbitrary Point other than the Origin

If we wanted to scale a point relative something other than the origin, you need to:

First translate the whole system so that the point of scale *is* at the origin.

Then perform the scaling.

And finally, undo the translation.

If we want to scale point (*X*, *Y*), relative to point (*Xc*, *Yc*) by (Sx, Sy), then:

1) T(-Xc, -Yc)

2) S(Sx, Sy),

3) T(Xc, Yc)

Scaling About Arbitrary Point other than the Origin

Solving the system in matrix form

$$\begin{bmatrix} \acute{X} \\ \acute{Y} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Xc \\ 0 & 1 & Yc \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -Xc \\ 0 & 1 & -Yc \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Using matrices multiplication we get:

$$\begin{bmatrix} \acute{X} \\ \acute{Y} \\ 1 \end{bmatrix} = \begin{bmatrix} Tx + (X - Tx)Sx \\ Ty + (Y - Ty)Sy \\ 1 \end{bmatrix} \; OR \; \begin{bmatrix} X.Sx + Tx(1 - Sx) \\ Y.Sy + Ty(1 - Sy) \\ 1 \end{bmatrix}$$

HW: Derive this result from the above system.

Scaling About Arbitrary Point other than the Origin

Example:

The red square is the original shape with coordinates:

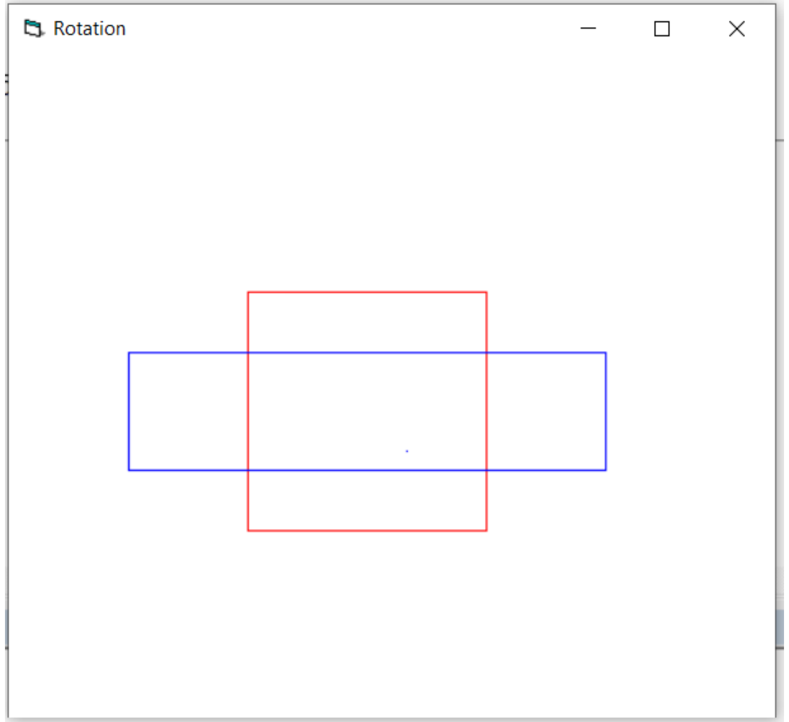
(150, 150), (300, 150). (300, 300), (150, 300)

The blue square is the same square scaled by

Sx=2, Sy=0.5, about the centre of the square (225, 225).

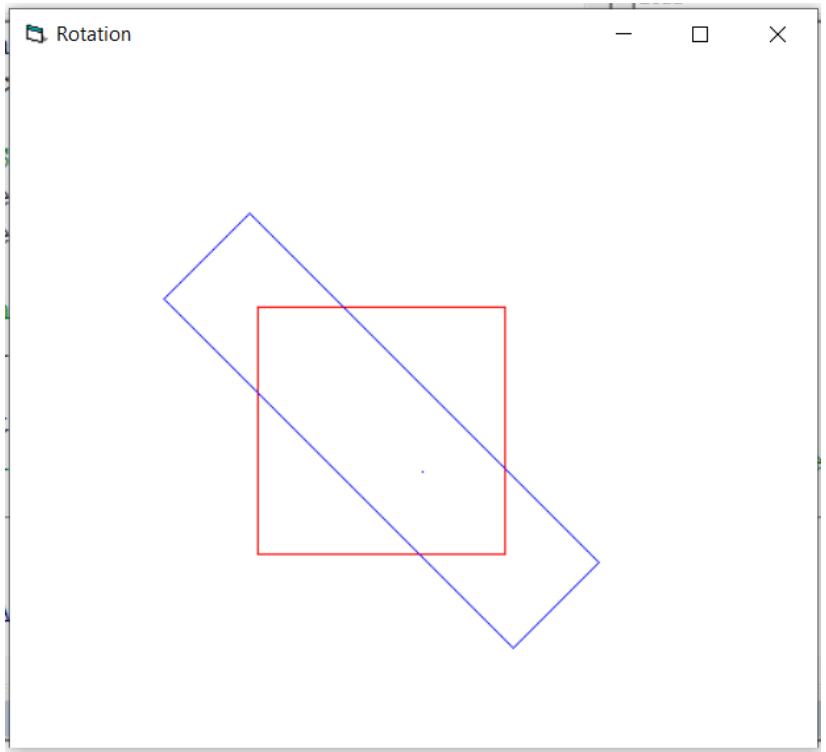
The scaled points are:

(75, 188), (375, 188), (375, 262), (75, 262).

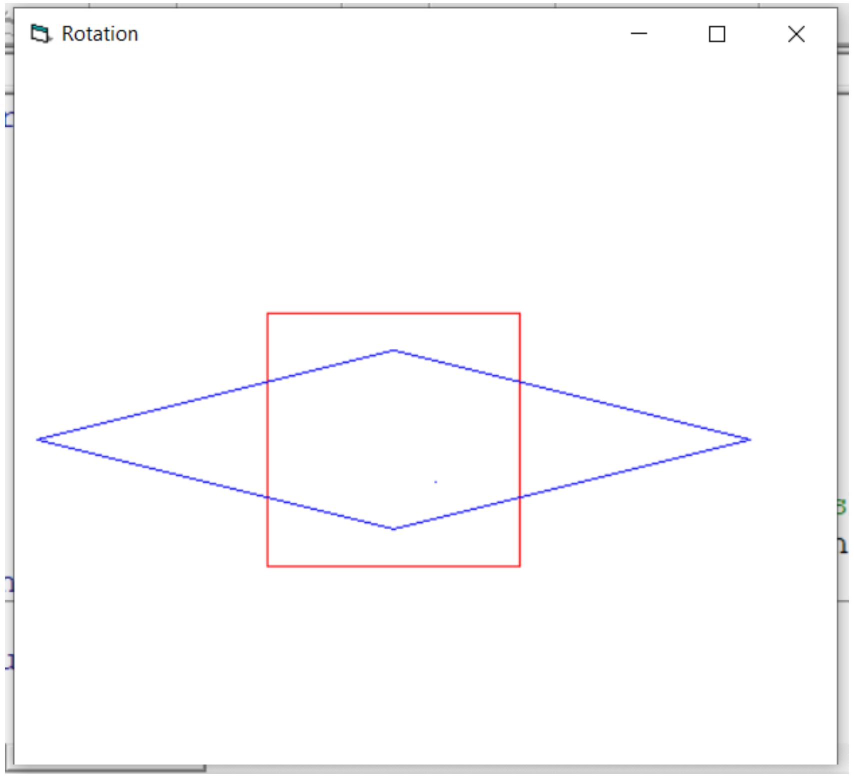


The Effect of Transformation Order

1- Scale then Rotate



2- Rotate then Scale



Other 2D Transformations

1- Reflection

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

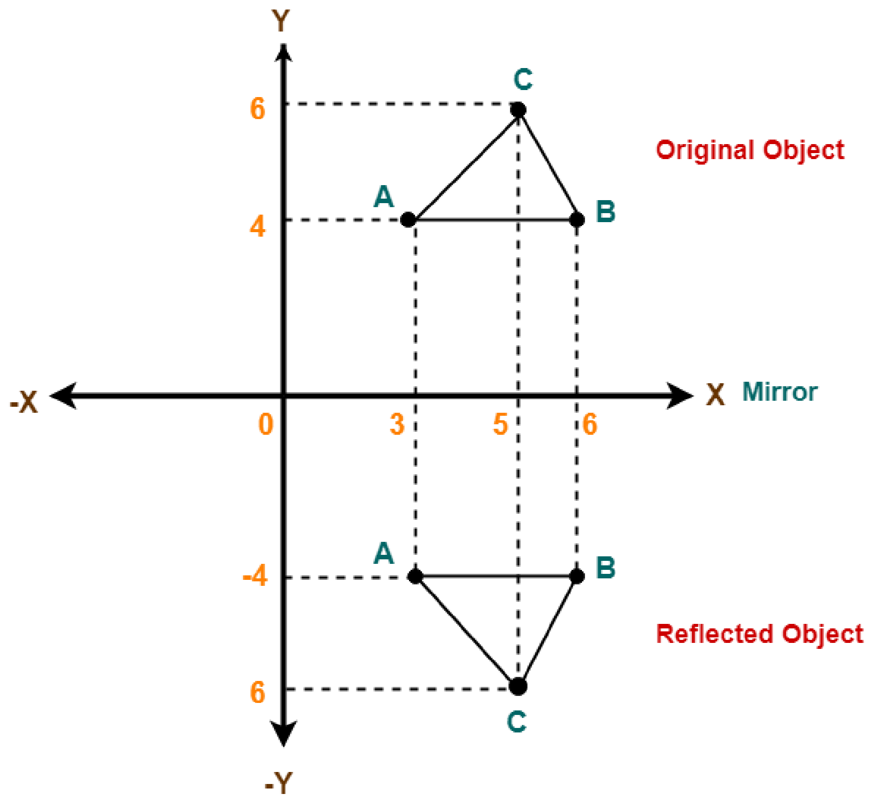
(Reflection Along X Axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

(Homogeneous Coordinates Representation)



Other 2D Transformations

1- Reflection

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

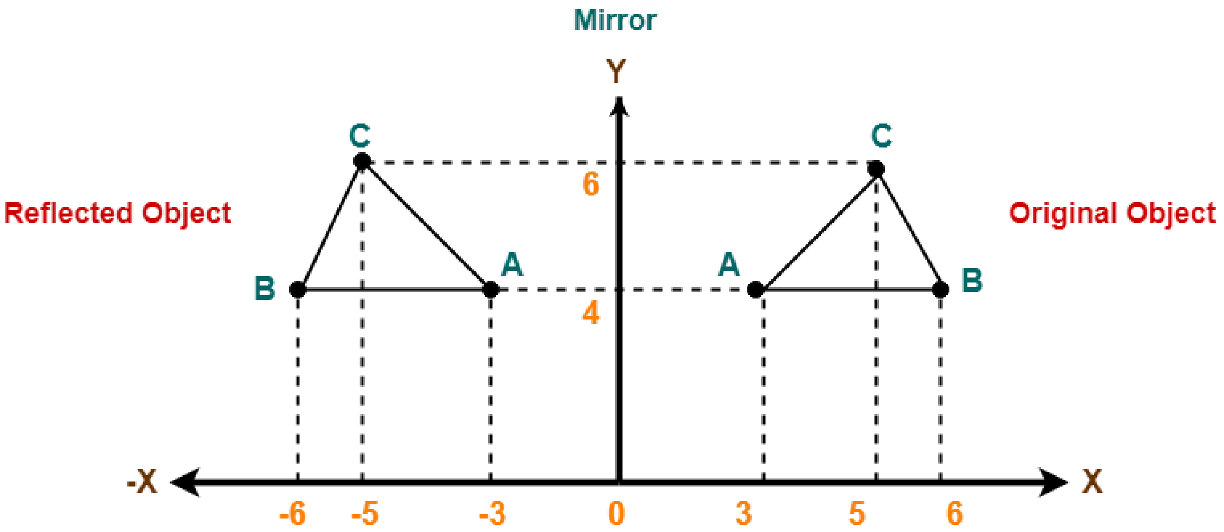
(Reflection Along Y Axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

(Homogeneous Coordinates Representation)



Other 2D Transformations

2- Sharing

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In X axis)

(Homogeneous Coordinates Representation)

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In Y axis)

(Homogeneous Coordinates Representation)

Other 2D Transformations

2- Sharing