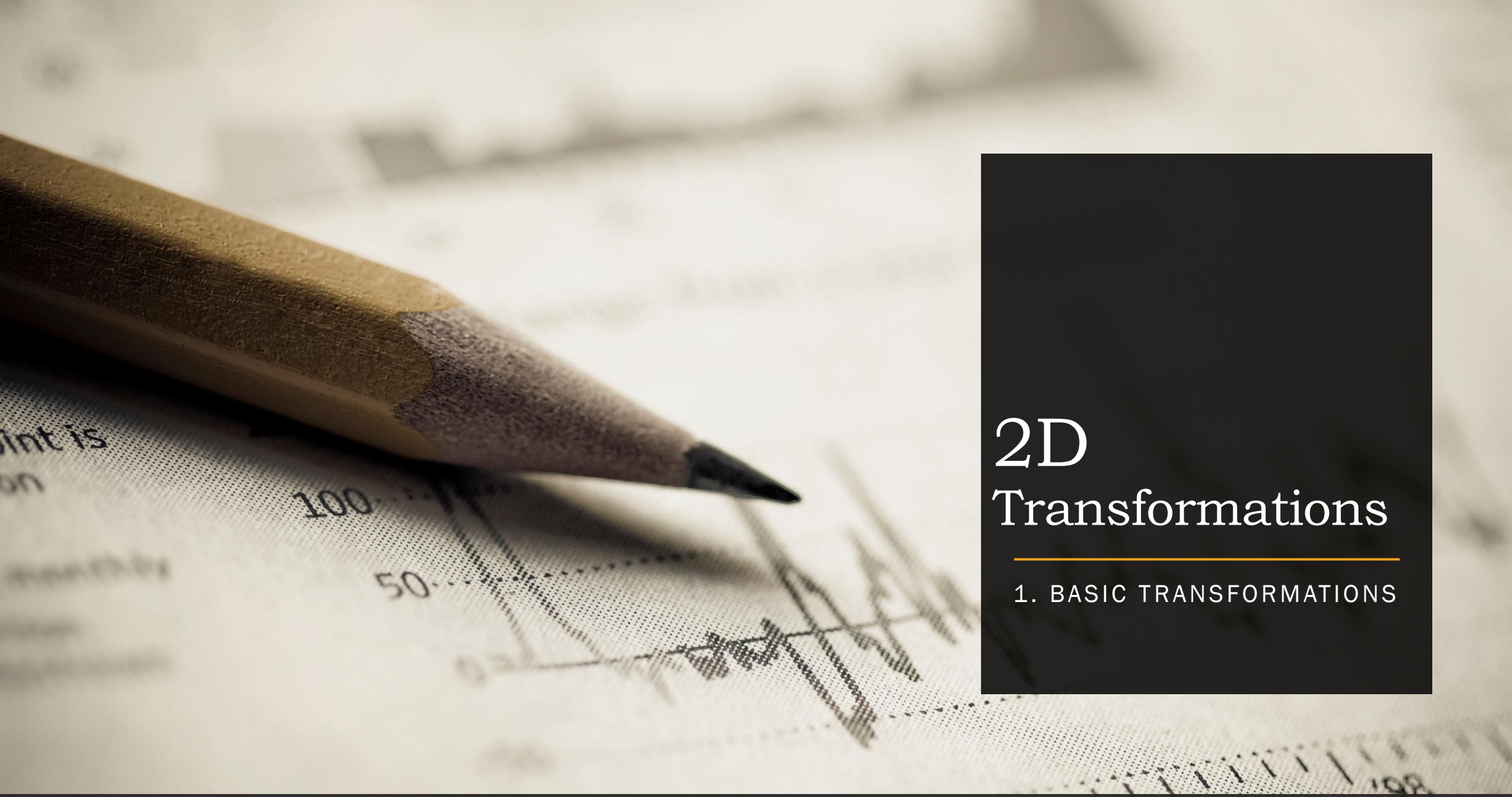


2D Transformations

1. BASIC TRANSFORMATIONS

Basic 2D Transformations

Basic 2D transformations include:

1. Translation
2. Scaling
3. Rotation

2D Translation

In Computer graphics, 2D Translation is a process of moving an object from one position to another in a two dimensional plane.

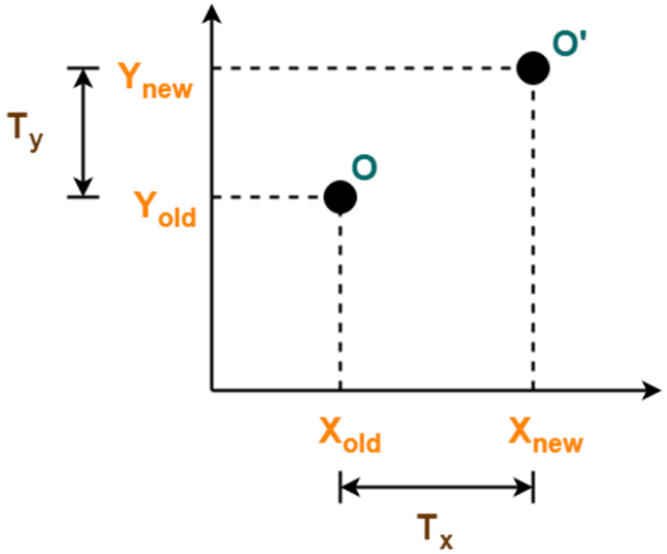
Consider a point object O has to be moved from one position to another in a 2D plane.

Let:

- Initial coordinates of the object O = (Xold, Yold)
- New coordinates of the object O after translation = (Xnew, Ynew)
- Translation vector or Shift vector = (Tx, Ty)

Given a Translation vector (Tx, Ty):

- Tx defines the distance the Xold coordinate has to be moved.
- Ty defines the distance the Yold coordinate has to be moved.



2D Translation

Translation is achieved by adding the translation coordinates to the old coordinates of the object as:

- $X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)
- $Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

- The homogeneous coordinates representation of (X, Y) is (X, Y, 1).

The above translation matrix

may be represented as a 3 x 3 matrix as:

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

2D Translation

2d translation practice problems

1. *Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.*

2. *Given a square with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the translation with distance 1 towards X axis and 1 towards Y axis. Obtain the new coordinates of the square.*

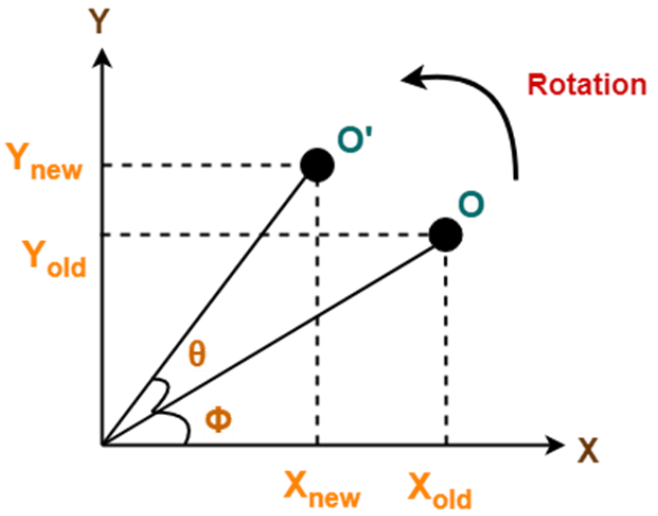
2D Rotation

In Computer graphics, 2D Rotation is a process of rotating an object with respect to an angle in a two-dimensional plane.

Consider a point object O has to be rotated from one angle to another in a 2D plane.

Let:

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = (X_{new}, Y_{new})



2D Rotation

Rotation is achieved by using the following rotation equations:

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

For homogeneous coordinates, the above rotation matrix may be represented as a 3 x 3 matrix as:

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

2D Rotation

2d translation practice problems

1. Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30-degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

2. Given a triangle with corner coordinates (0, 0), (1, 0) and (1, 1). Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

Hints: sin30 = 1/2, cos30 = √3/2. sin90=1, cos90=0

2D Scaling

In computer graphics, scaling is a process of modifying or altering the size of objects.

- Scaling subjects the coordinate points of the original object to change.
- Scaling factor determines whether the object size is to be increased or reduced.
- If scaling factor > 1, then the object size is increased.
- If scaling factor < 1, then the object size is reduced.
- Consider a point object O has to be scaled in a 2D plane.

2D Scaling

Let:

Initial coordinates of the object O = (X_{old}, Y_{old})

Scaling factor for X-axis = S_x

Scaling factor for Y-axis = S_y

New coordinates of the object O after scaling = (X_{new}, Y_{new})

2D Scaling

This scaling is achieved by using the following scaling equations:

$X_{new} = X_{old} \times S_x$

$Y_{new} = Y_{old} \times S_y$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

For homogeneous coordinates,

the above scaling matrix may

be represented as a 3 x 3 matrix as:

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

2D Scaling

2d scaling practice problems

1. Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.